SUPPORTING DIAGNOSTIC INFERENCE BY MATHEMATICAL MODELLING FROM ONE-STAGE TO PLANETARY GEARBOX SYSTEMS

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Summary

The paper shows possibilities of using mathematical modelling and computer simulations for supporting fault detection in gearbox systems. The paper refers to the model given by L. Muller in which the two wheels of a gear-set are reduced to one body. The paper gives the review of the systems with one and two-stage gear-sets. The models are used to investigate influence features/factors of systems with gear-sets to vibration. The torsional and lateral vibration is considered as the signal of a gearbox condition. The factors having influence to vibration signal are divided into four groups, that is: design factors, technology factors, operational factors and change of condition factors. The paper refers to modelling of planetary gearboxes. The paper also refers to the model-based fault diagnosis. Model-based is defined as determination of faults of the system from comparison of available gearbox system measurements with a priori information represented by the system's mathematical model, through generation of residual quantities.

Keywords: Inference, modelling, gearbox, diagnostics, simulation

WSPOMAGANIE WNIOSKOWANIA DIAG NOSTYCZNEGO MODELOWANIEM
MATEMATYCZNYM W ZAKRESIE OD PRZEKŁADNI JEDNOSTOPNIOWEJ DO PLANETARNEJ

Streszczenie

Praca pokazuje możliwości wykorzystania modelowania matematycznego i symulacji komputerowej do wspomagania wykrywania uszkodzeń w układach z przekładniami zębatymi. Praca nawiązuje do modelu L. Mullera w którym dwa koła zębate redukowane są do jednego ciała. Praca jest przeglądem układów z przekładnią jednostopniową i dwustopniową. Modele są używane do badania wpływu cech układu na z przekładniami zębatymi na generację drgań. Rozpatrywane są drgania skrętne i poprzeczne jako sygnał stanu przekładni. Czynniki wpływające na sygnał drganiowy podzielone są na cztery grupy a mianowicie: czynniki konstrukcyjne, technologiczne, eksploatacyjne i czynniki związane ze zmianą stanu. Praca nawiązuje również do modelowania przekładni planetarnych. Nawiązano również do diagnostyki opartej o modelowanie matematyczne (model- based diagnostic). Ten rodzaj diagnostyki zdefiniowano jako wykrywanie uszkodzeń w systemie na podstawie porównania bieżących wyników pomiarowych z informacjami przyjętymi a priori reprezentowanymi jako model matematyczny i powstającego w wyniku tego porównania sygnału rezydualnego.

1. INTRODUCTION

Mathematical modelling and computer simulation (MMCS) [1–26] has proved to be very important tool for supporting diagnostic inference (SDI) [9-26]. We may also say that the aim of MMCS is to detect faults in a system. In the paper the term fault is used rather than failure to denote a malfunction rather than a catastrophe. The term failure suggests complete brake down of a system component of function, while the term fault may be used to indicate that malfunction may be tolerated at its present stage. A fault must be diagnosed as early as possible even it is tolerable at its early stage, to prevent any serious consequences.

Development of SDI based on MMCS has been going though many sages. For SDI of gearboxes dynamic models have been developed starting with creation one body model [1] exited by train of errors/faults moved with the speed of circumference velocity of two mating gears. Faults were separated with distance equivalent to a pitch distance of teeth. Next step is connected with development model of one stage gearbox incorporated into the system consisting of an electric engine, flexible coupling and driven machine [5-12]. Parallel model developments are given in [2 – 4]. For modelling such system and using for SDI many factors has to be taken into consideration. The factors can be divided into four groups namely: design factors

(DF), technology factors (TF), operation factors (OF), condition change factors (CCF); (collectively) DPTOCCF. All the mentioned factors have influence to dynamic behaviour of the system. Investigating influence of DPTOCCF to vibration generated by the system with a gearbox it is possible to infer relation between gearbox condition and symptoms given by vibration signal. Big progress in SDI is given by model development for system with a two-stage gearbox [13-26]. On issues of SDI of gearboxes the author in conference proceedings and journals publishes many papers. Among the conferences there are: Condition Monitoring and Diagnostic Engineering Management (COMADEM) [6,13,21] Quality Reliability & Maintenance (QRM) [9,17] Condition Monitoring (CM) [5,24] Mine Planning and Equipment Selection (MPES) [18], International Conference on Mechatronics (ICOM) [22], International Measurement Confederation (MEKO) [15], ASME Power Transmission and Gearing. Among journals: International Journal of Rotating Machinery [23], Journal of COMADEM [11], Mechanical Systems and Signal Processing [16], Transactions of the Institute of Measurement and Control [22]. All the publications are presenting DPTOCCF based way for gearbox diagnostic inference. New developments are going towards to creation a dynamic model for a system with planetary gearboxes and generated signal analysis for SDI. Till now has been obtained diagnostic vibration signal interpretation using mathematical

modelling and computer simulation. Current challenge in diagnostic method developments using mathematical modelling and computer simulation (MMCS) is to give the background for inferring process automation using neural networks [26]. The alternative way is to use MMCS for model-based fault detection for which is a need to create what is called a robust model. The robust model is used in the process of diagnostic automation as analytical redundancy [32,33].

2. MATHEMATICAL MODELLING AND COMPUTER SIMULATION FOR SUPPORTING DIAGNOSTIC INFERENCE

The discussion on MMCS for better understanding of influence of DPTOCCF to diagnostic signal we may start if we consider the scheme given in Fig.1, [12 and 25]. Primary factors are given by DF and TF, secondary factors are given by CCF, motion factors are given by OF. As it is given in Fig.1 design factors/features are divided into geometric and material factors. Geometric factors are divided into macro-geometrical and micro-geometrical. Macro-geometrical factors are described by; structural form, admissible tolerances, shape errors and others. Material factors are given by module of elasticity, damping coefficients, oil properties and so on. Seizing, pitting and so on describe change of condition factors (faults). A load and rotational speed give operation factors.
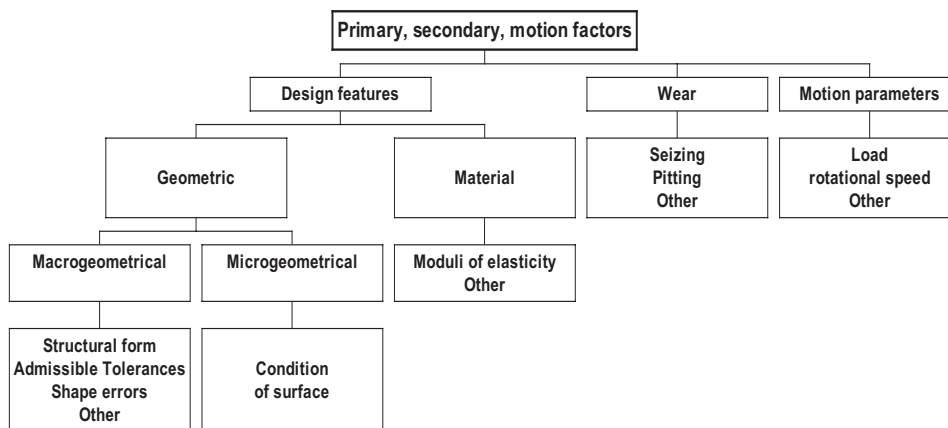

Fig.1 Factors affecting diagnostic signal [12 and 25].

Gearing co-operation errors for new gearing a), and for failed gearing by pitting b) are given in Fig.2. Fig.2a gives collective description of gearing design factors. Fig.2b gives collective description

of gearing condition after wear. Joint description of DF and one of OF as a gear load is given in Fig.3.
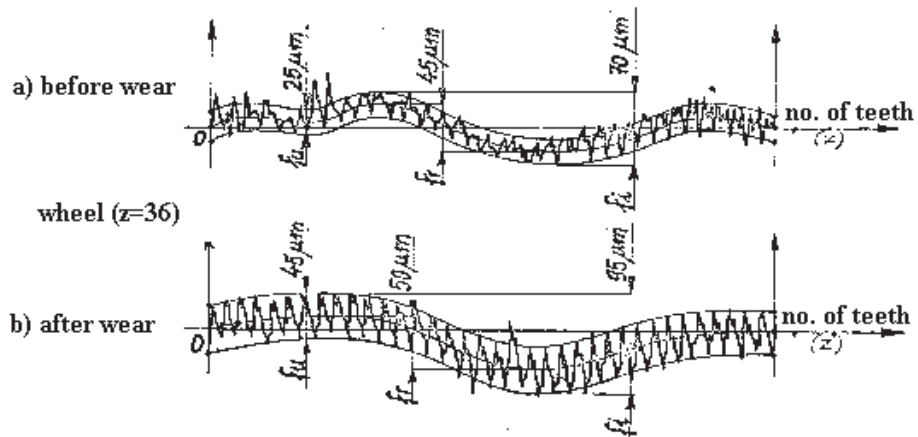
Fig. 2. Gearing co-operation errors for new gearing a), and for failed gearing by pitting b)
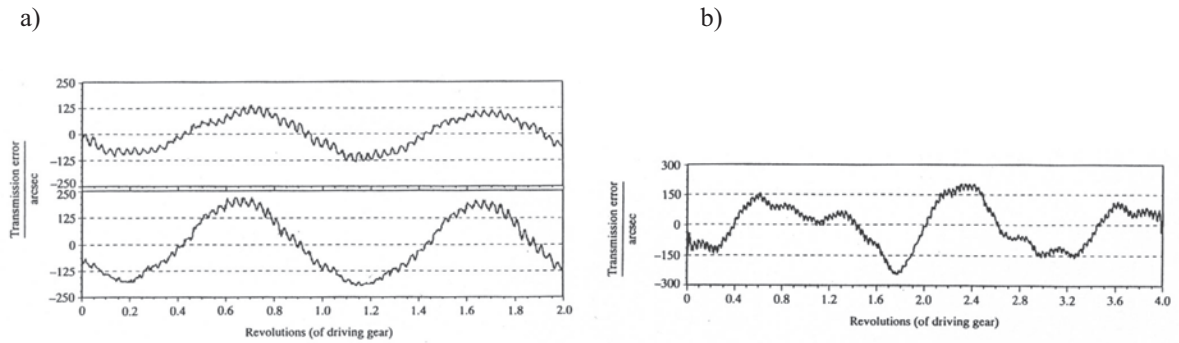
a)   b)

Fig.3. Gearing co-operation errors (transmission error) for gear-set with ratio
u = 1 - a), and for u > 1 - b), [27]

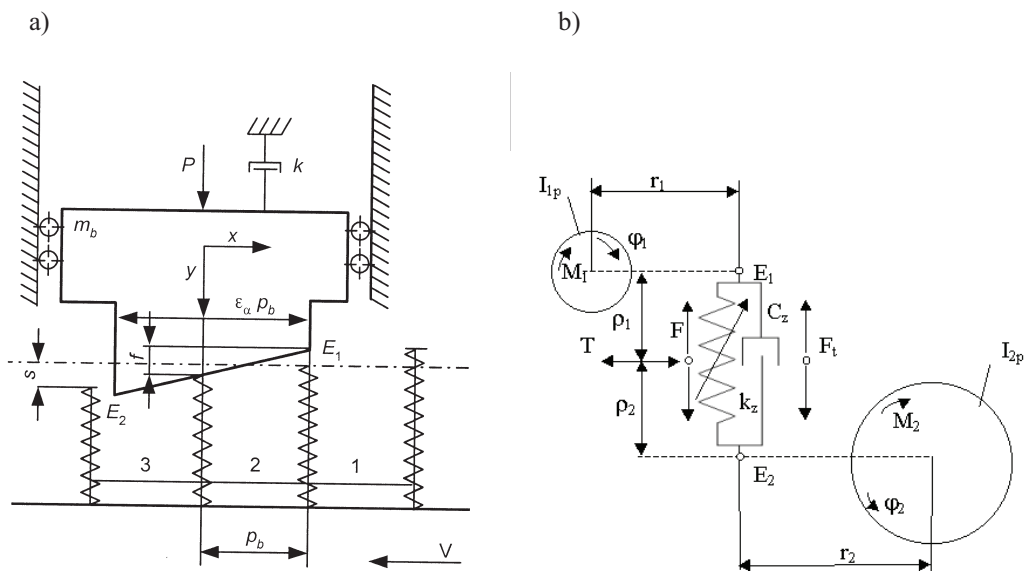a)   b)

Fig.4 One-body two-parameter model of gearbox by Muller [1] - a)
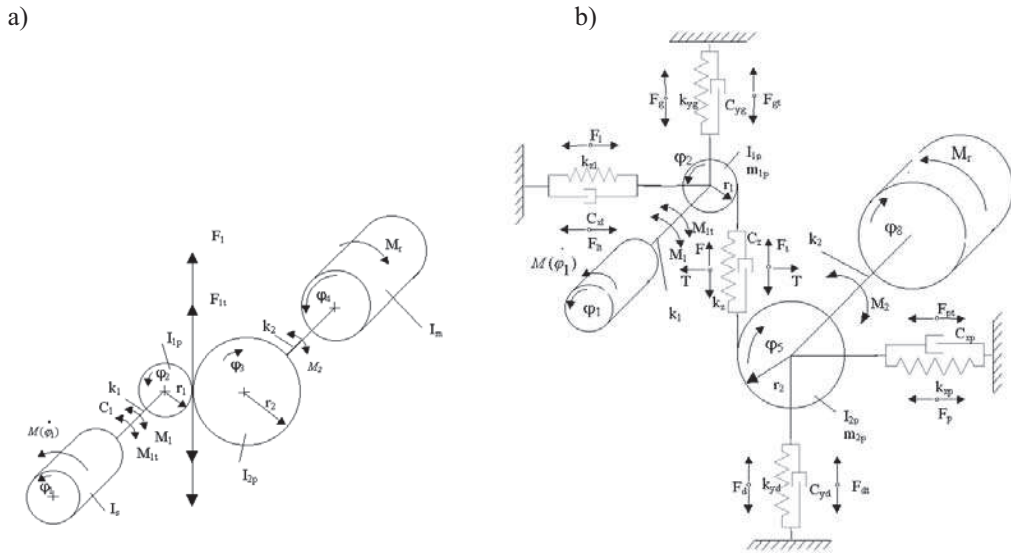two-wheel, two-parameter model of gearbox [12] - b)

a)   b)

Fig.5a) System with one-stage gearbox with: $M_s(\dot{\varphi})$ - electric motor driven moment characteristic; M_1, M_2 - moments of shaft stiffness; I_s, I_m - moments of inertia for electric motor and driven machine; M_{1t} - damping moment of clutch/coupling; C_1 – damping coefficient of coupling; F, F_t - stiffness and damping inter-tooth forces; k_1, k_2 – stiffness of shafts [5] b) Gearbox

system, with eight degrees of freedom, driven by electric motor moment $M_s(\dot{\varphi}_1)$ and loaded with external moment M_r; the system consists of: rotor inertia I_s, gear inertia I_{1p}, I_{2p}, gear masses m_{1p}; m_{2p}, driven machine inertia I_m, gearing stiffness k_z and damping C_z, gearing stiffness force F and damping force F_t, gearing friction force T, internal moments in first shaft M_1; M_{1t} (M_{1t} clutch damping moment), inner moment in second shaft M_2, internal stiffness and damping forces of upper and vertical supports (F_{v1}, F_{v1t}; F_{v2}; F_{v2t}), stiffness and damping parameters of vertical horizontal supports (k_{v1} k_{v2}, C_{v1}, C_{v2}) internal stiffness and damping forces of horizontal supports (F_{h1}, F_{h1t}, F_{h2}, F_{h2t}), stiffness and damping parameters of upper and lower support (k_{h1}, k_{h2}, C_{h1}, C_{h2}) [16, 25]
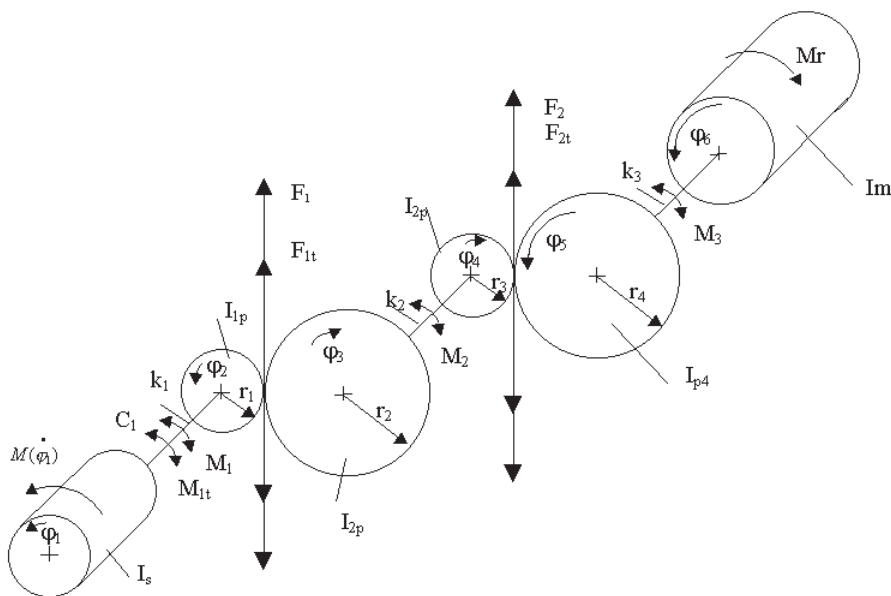

Fig.6 Two-stage gearing system with six torsional degrees of freedom, electric motor moment $M_s(\dot{\varphi}_1)$ and external load moment M_r; system consists of: rotor inertia I_s, gear inertia of first stage I_{1p}, I_{2p}, gear inertia of second stage I_{3p}, I_{4p}, driven machine inertia I_m, gearing stiffness k_z and damping C_z, gearing stiffness forces F_1, F_2, and damping force F_{1t}, F_{2t}, internal moments in first

shaft M_1; M_{1t} (M_{1t} coupling damping moment), internal moment in second and third shaft M_2 and M_3, [20, 25]

Mathematical modelling of gearbox systems is the subject of many publications [1 – 4] and many publications given by the author [5 –25] which reviewing is the subject of this paper. Muller's [1], one-stage gearbox model is shown in Fig.4a). It is a two-parameter (stiffness and damping) model. The inertia of the two gear wheels is reduced to one body. The motion of the two lumped body is equivalent to the relative motion of the two gear wheels. The motion is caused by the relative motion of springs (having different length) in contact with the body. The motion of the springs with velocity v[m/s] is equivalent to the pitch-line velocity of the wheels. As one can see in the model shown in Fig.4a) the motion of the body has no influence on the instantaneous change of v as in actual gearboxes. This weakness of the Muller 1979 model, and no possibility of building multistage gearbox models, made the author seek a new model. It is more convenient to use a model with the rotary motion of the wheels and torsional vibration, and thus overcome the weakness of the Muller's model. The torsional vibration is also given in models in [2-3]. The simplest model of this kind is shown in Fig.4b). The system with one-stage gearbox is given in Fig5a) and b). In a gearbox model given in Fig,5a) only torsional vibration is taken into consideration but in a model given in Fig.5b) both tortional and lateral vibration is taken into consideration so is given in [3, 4]. Results for computer simulations using the model given in Fig.5a) are given in papers [5 – 12, 14, 17]. Two-stage gearing system is given in Fig.6 and results of computer simulations are given in [13, 16 – 25] in the system only torsional vibration is considered. A system with the two-sage gearbox with possibility of analysing torsrional and lateral vibration is given in Fig.7.
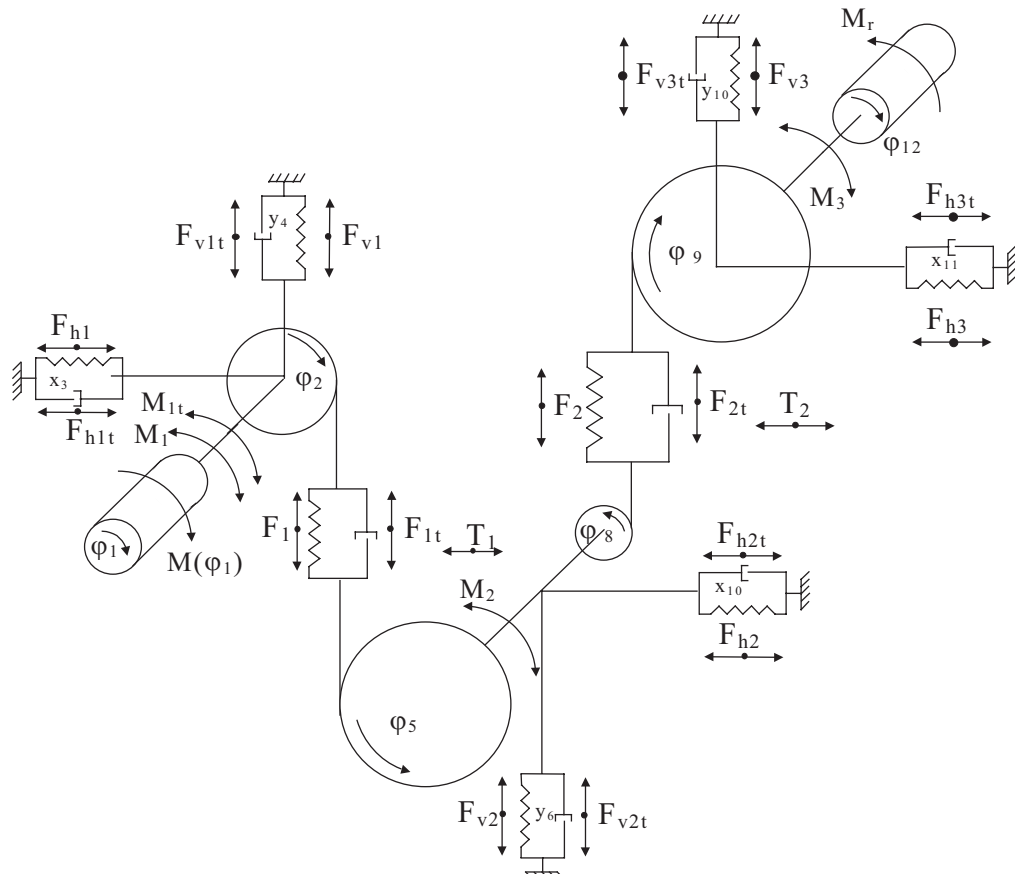

Fig.7 Two stage gearbox system, with twelve degrees of freedom, driven by electric motor

moment $M._s(\dot{\varphi}_1)$ and loaded with external moment $M._r$; system consists of: rotor inertia I_s, gear inertia I_{1p}, I_{2p}, I_{3p}, I_{4p} gear masses m_{1p}; m_{23p}, $m._{4p}$ driven machine inertia I_m, gearing stiffness k_{z1}, k_{z2} and damping C_{z1}, C_{z2}; gearing stiffness forces F_1, F_2 and damping force F_{1t}, F_{2t}; gearing friction forces T_1, T_2; internal moments in first shaft M_1; M_{1t} (M_{1t} clutch damping moment), inner moment in second shaft M_2 and third shaft M_3, internal stiffness and damping forces of horizontal and vertical supports (F_{h1}, F_{h1t}; F_{h2}, F_{h2t}; F_{h3}, F_{h3t}; F_{v1}, F_{v1t}; F_{v2}, F_{v2t}; F_{v3}, F_{v3t}), stiffness and damping parameters of horizontal and vertical supports (k_{h1}, C_{h1t}; k_{h2}, C_{h2t}; k_{h3}, C_{h3t}; k_{v1}, C_{v1t}; k_{v2}, C_{v2t}; k_{v3}, C_{v3t}).

Mathematical modelling and computer simulation can be applied to gearbox dynamic examinations to support diagnostic signal evaluation for diagnostic inference. This is the main aim of the presented research. General information, on gearing, needed for the computer simulation of gearbox behaviour is given in [12] and [25]. The papers show that MMCS enable the detailed investigation of the dynamic properties of a gearing system. All the basic factors such as: design, production technology, operation and change of the gearing system condition, which have a bearing on the vibration generated by a gear-set, can be investigated. Using computer simulation and taking these factors into account: Design, Production Technology, Operation and Condition Change factors leads to DPTOCC inferring diagnostic information of the gearing system condition. The causes of vibration in gearboxes are mainly tooth errors Fig2 and 3, which together with a gearing deflection, show the gearing condition and the vibration is an indication of them. The computer simulation results are referred to the laboratory rig investigation results presented in [28] and to the field measurements reported in [29 - 31]. As mentioned above, the vibration of a gearbox indicates whether there are tooth errors in it. The errors appear at the production stage and during change of condition. The nature of the gear wheel interaction is such that non-linear phenomena occur caused by friction, inter-tooth backlash, impact-like inter-tooth forces and periodic changes in tooth stiffness. As a result, inter-tooth forces may exceed the force values, which follow from the gearbox system's rated moment. Mathematical description allowing one to include these phenomena in the equation of motion is given in [12], [25]. The inter-tooth forces increase dramatically in unstable conditions. A one-stage gear system operates in resonance conditions and is unstable when the gearbox system's mesh frequency is equal to its natural frequency . In such conditions the inter-tooth forces are two times or more grater than the rated forces. The phenomenon of resonance has not been investigated fully for gearbox systems but some considerations are given in [6], [11] and [25].

Computer simulations reveal that conditions similar to those occurring at resonance may result as errors (pitting, scuffing of teeth flanks and failure of bearings) increase during the service of a gearbox system. In the present paper refers to current developments in gearbox modelling are presented. In papers shown that a flexible coupling and an error mode random parameter have an influence on gearbox stability (tooth separation). An error mode is described by several parameters, i.e. maximum error value, shape of error plot and random error fluctuation depth [11, 12, 25].

3. MODELLING PLANETARY GEARBOXES

Modelling planetary gearboxes we have quite new situation. The case of planetary gearboxes gives new problems first is a choice of a planetary gearbox model with suitable simplifications. The most important difference in comparing to presented above models is planetary movement of some wheels called planets. The main advantage of planetary gearbox is transmitted power concentration per unite volume of space taken by a planetary gearbox. The simplest planetary gearbox consists of gears called a sun, planet and ring, and an arm. One of the cases is when the sun is standstill and the planet makes a planetary movement, rotation about its axe with rotation about the sun axe. In this case a number of ring rotation is an input rotation – n_{ri} [RPM] and rotation of an arm is the output n_o [RPM]. In this case the ring also rotates about its axe, which is also the sun axe. The most frequent case is when the sun rotates with its input rotation – n_{si} [RPM] and the planet makes the planetary movement, the ring is standstill, and rotation of an arm is the output n_o [RPM]. The third case is when all three elements of a planetary gearbox are in rotation. In the third case we have two inputs rotation of the sun n_{si1} [RPM] and the ring rotation n_{ri2} [RPM], the output rotation n_o [RPM]. In this third case in use of this type of a planetary gearbox for driving systems for bucket wheels, slewing gearing in bucket wheel excavators where a gearbox has two outputs with equal rotation n_o [RPM]. Taking into consideration number of teeth: for the sun z_1 for the planet z_2 and for the ring z_3. The ratio for the three cases is given: for the first case $u_1 = 1 + z_1/z_3$, for the second case $u_2 = 1 + z_3/z_1$, for the third case $u_3 = (1 + z_3/z_1)/(1 - n_{ri2}/n_{si1} z_3/z_1)$. If we take for further consideration the second case the meshing frequency generated between a sun and planet $f_{12} = n_{si}z_1z_3/[60(z_1 + z_3)]$, and frequency generated between planet and ring $f_{23} = n_{ri}z_1/120$.

4. MODEL-BASED FAULT DIAGNOSIS

Model-based fault detection and isolation (FDI) makes use of mathematical models of the gearbox system. Model-based fault diagnosis [32] can be defined as the determination of faults of a system from comparison of available system measurements with a priori information represented by the system's mathematical model, through generation of residual quantities and their analysis. A residual is a fault indicator or an accentuating signal, which reflects the faulty situation of the monitored system. A traditional approach to fault diagnosis in the wider application context is based on "hardware (or physical /parallel redundancy" methods which use multiple
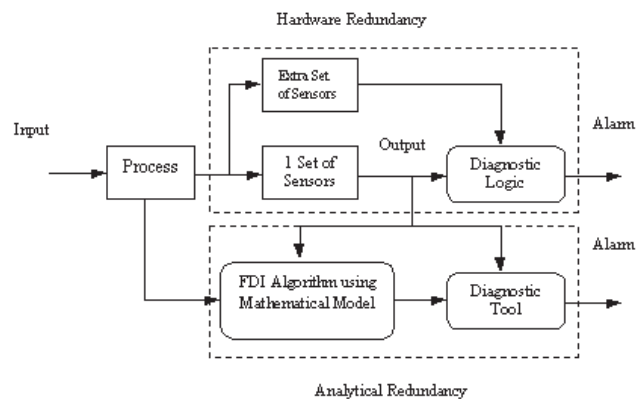
Fig.8 Hardware vs. analytical redundancy [32]

lanes of sensors, actuators, computers and software to measure and/or control a particular variable. The major problems encountered with hardware redundancy are extra equipment and maintenance and cost and, further more the additional space required to accommodate the equipment. Fig.8 illustrated the hardware vs. analytical redundancy concepts. No additional hardware faults are introduced into an analytical redundant scheme, because no extra hardware is required, hence analytical redundancy is potentially more reliable than hardware redundancy.

5. CONCLUSIONS

The paper gives review of achievements in mathematical modelling and computer simulations for supporting diagnostic inference for fault detection. Presenting models give possibility of investigation all factors, which have influence to vibration signal generation. The factors are divided into four groups: design factors, technology factors, operation factors and change of condition factors. It is given an introduction to planetary gearbox modelling and what is called model - based diagnostic for fault detection which can be used in mechatronic systems.

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