Paper

On the performance analysis of a heuristic approach dedicated to a multiobjective dynamic routing model

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Abstract—In previous works the features and a complete formulation for circuit-switched networks of a multiple objective dynamic routing method (MODR) of periodic state dependent routing type were presented. The aim of the model is to resolve a very complex network bi-objective dynamic routing problem, by recurring to a heuristic for synchronous path selection enabling to obtain a good compromise solution in terms of two network performance measures. In this paper we present a study on the performance of variants of the MODR heuristic of synchronous path selection by using relaxations of the values previously calculated for the two network objective functions. This study permitted the development of an improved version of the initial heuristic. Also a comparison of the analytical values of the network objective functions obtained with selected variants of the initial heuristic with the corresponding results from a known reference method, the real time network routing (RTNR) method, given by a discrete-event simulator for single-service networks, is presented.

Keywords— *multiple criteria analysis, routing, heuristics, telecommunications.*

1. Introduction

Routing is an essential component of the functional structure of any type of telecommunication network. It has a decisive impact on the quality of service (QoS) performance of the various services provided by the network as well as on its cost and return structure. A routing method is focused on the calculation and selection of a path or set of paths between every pair of nodes, for each service request. The choice of path(s) seeks to optimise certain objective(s) and satisfy certain constraints of a technical or economical nature. The evolution of present multiservice telecommunication network functionalities leads to the necessity of dealing with multiple and heterogeneous QoS requirements. Hence the formulation of the routing problems involves the consideration (as objective functions and/or constraints) of various metrics such as delay, blocking probability, number of arcs (or "hops") or cost.

A new routing concept designated as QoS routing has emerged [22, 23] which involves the selection of a chain of network resources satisfying certain QoS requirements while seeking simultaneously to optimise the route associated metric(s). In these type of models the path calculation problem has been usually formulated as a shortest path problem where QoS requirements are often incorporated through specific constraints. Such problems are typically solved through heuristics often based on Dijkstra or Bellman-Ford shortest path algorithms. A review on QoS routing algorithms with applications can be seen in [11]. Note that these "classical" types of QoS routing models are single objective thence do not enable an explicit mathematical representation (in the form of objective functions) of potentially conflicting routing objectives.

We think there are potential advantages in considering the routing problem in integrated communication networks as a multiobjective problem, having in mind to grasp eventual conflicts and trade-offs among distinct objectives and QoS constraints. In fact multiple objective routing models enable the trade-offs among distinct QoS metrics to be treated in a mathematically consistent manner. In this context paths are normally selected in the set of non-dominated paths, i.e., paths for which (in minimisation problems) it is not possible to decrease the value of an objective function without increasing on at least the value of one of the other objective functions. Examples of multiple objective routing models in specific types of telecommunication networks can be seen in [20] and [5] (focusing on applications to asynchronous transfer mode (ATM) – networks) and [8] (dealing with a routing problem in multiprotocol label switched (MPLS) – networks). A review on multicriteria models and algorithms for telecommunication network routing problems can be seen in [4].

On the other hand the utilisation of dynamic routing methods in various types of networks is well known to have significant impact on network performance and cost, namely in overload and failure conditions [1]. This is due to the adaptive nature of dynamic routing characterised by the fact that selected routes vary dynamically as a function of varying network conditions. The routing changes are made, for example, in response to fluctuations in traffic intensities or to the state of occupation of the transmission links, corresponding to the arcs of the network representation.

In previous papers [6, 16] the essential features of a multiobjective dynamic routing method (MODR) of periodic state-dependent routing type, based on a bi-objective shortest path model, were presented. A major aspect of the MODR method (in the version for single-service traffic), beyond its specific multiobjective nature, is the explicit consideration of a "fairness" objective to be optimised together with a "classical" objective function in this type of models (network mean blocking probability). Also the consideration of a dynamic alternative routing optimisation problem (reviewed in this paper) formulated at network level is an added value with respect to classical flow-oriented QoS routing models where the paths for each node-to-node traffic flow are calculated separately, each at a time, hence giving no guarantee of obtaining "good" approximately "optimal" solutions in terms of the routes selected for all the network traffic flows. In its initial formulation, for circuit-switched networks, the model uses implied costs and blocking probabilities as metrics for the path (or route) calculation problem. Also an analytical model and a heuristic were developed [15, 17] for synchronous selection of a first choice path and an alternative path between every pair of nodes in single-service networks, seeking to obtain a set of routes which is a satisfactory compromise solution from the point of view of two global network performance objectives, namely the network mean blocking probability and the maximal end-to-end blocking probability (for all traffic flows). In [15, 17] the performance of the routing method (MODR-1) using that heuristic was compared in terms of the two network global performance metrics with the corresponding results given by a discrete event simulation model for a reference dynamic routing method, real-time network routing (RTNR) developed by AT&T, known for its efficiency and sophistication in terms of service protection mechanisms. This comparative study revealed that the method globally performed well in most situations. The extension of the MODR method to multiservice networks was outlined in [13] using a hierarchical multiobjective formulation of the dynamic alternative routing optimisation problem with $2(1+|S|)$ objectives where S represents the set of service types.

In the present work we present a study on the performance of variants of the previous MODR heuristic of synchronous path selection by using relaxations of the values previously calculated for the two network objective functions (g.o.f. in short). The consideration of these adaptations of the heuristic has in mind to enable the obtainment of approximate non-dominated solutions by travelling on the g.o.f. space in order to improve either one or the other g.o.f. with respect to the values corresponding to the solution obtained by the initial version of the heuristic. This study permitted the development of an improved version of the initial heuristic. Also a comparison of the analytical values of the g.o.f. obtained through an analytical model, with this variant of the initial heuristic and the corresponding results from the RTNR method given by a discrete-event simulator, for single-service networks, will be presented.

The major contributions of this paper with respect to previous works of the authors on the MODR model are:

- the exploration of possible variants to the heuristic described in [17] for the MODR version for single service networks;
- to analyse the relative performance of these variants in terms of the two network routing metrics for different overload factors by using an analytical model;
- to show that one of these variants, based on a simple relaxation of the value of one of global objective functions (with respect to the current minimum) may be advantageous in some practical network engineering conditions, by enabling a slight improvement in total average revenue at the cost of a small degradation of the maximal node-to-node blocking probability.

Finally note that in the context of MODR the selection of routes for every node-to-node traffic flow has to be performed in a fully automatic manner. This raises specific difficulties concerning the representation of the system of preferences, which, in a certain manner, is imbedded in key points of the considered variants of the heuristic of synchronous path selection.

The paper is organised as follows. Section 2 reviews the essential features of the MODR model and the bi-objective shortest path algorithm used as a basis for its resolution. Also the main features of the heuristic previously developed for synchronous path selection are outlined. Section 3 describes the considered new versions of the heuristic obtained by using certain relaxations of the values previously calculated for the two global objective functions. Also the behaviour of theses variants in the g.o.f. space, are analysed in this section. Section 4 presents a comparison of the network performance results obtained with a specific new variant of the heuristic with a reference dynamic routing method (RTNR) for some test networks by recurring to a discrete-event simulator. This will enable some conclusions to be drawn concerning the potential advantages and difficulties of the model and an outline of developments of this work.

2. Review of the multiobjective dynamic routing model

2.1. The MODR model

The MODR method for single-service networks, the model of which was presented in [6, 16] is a periodic state-dependent routing method, where the (loopless) paths $\{r^1(f),\dots,r^M(f)\}$ that may be attempted by a call of each node to node traffic flow f (from node v_s to v_t) change periodically as a function of a measure of the network working conditions. The calculation of paths is based on a bi-objective shortest path model that is resolved by a very efficient algorithmic approach designated as modified multiobjective routing algorithm (MMRA). This procedure uses an extremely efficient k-shortest path algorithm [12] to search for non-dominated, including unsupported non-dominated solutions located in the interior of the convex-hull of the feasible solutions set. In the formulation of MODR for networks equivalent to circuit-switched loss networks this underlying bi-objective shortest path static routing model uses blocking probabilities and implied costs (in the sense defined by Kelly [10]) as path metrics. This model uses soft-constraints (that is constraints not directly incorporated into the bi-objective shortest path mathematical model) in the form of required and/or accepted values for each metric which define preference regions in the objective function space.

In terms of global network performance the MODR method seeks good compromise solutions to a network bi-objective alternative dynamic routing problem. In the formulation of this problem in the case of single-service networks (see [16]) the first objective is the minimisation of the network mean blocking probability B_m (this is the objective function in classical single-objective routing models). The second objective is the minimisation of the maximal marginal blocking probability B_M (maximal value of the marginal blocking probabilities $B(f)$ experienced by all traffic flows f). In the present formulation of the method a call of each traffic flow may attempt two paths (or routes) according to the alternative routing principle ($M=2$): the first choice path $r^1(f)$ (which is the direct arc (v_s,v_t) whenever it exists) and (when $r^1(f)$ is blocked) the alternative path, $r^2(f)$. Therefore the network bi-objective alternative dynamic routing problem in the decision variables $\overline{R}_t$ is formulated as:

(Problem $\mathscr{P}_G^{(2)}$)

$$\min_{\overline{R}_t} B_m = \sum_{f\in\mathscr{F}} \frac{A_t(f)B(f)}{A_t^0} \qquad (1)$$

$$\min_{\overline{R}_t} B_M = \max_{f\in\mathscr{F}} \{B(f)\} \qquad (2)$$

s. t.

$$B(f) = L_{r^1(f)} L_{r^2(f)} \qquad (3)$$

and equations of the teletraffic model enabling to calculate $\{B(f),\ \text{all } f\in F\}$ in terms of $A_t(f)$, for given route set and arc capacities C_j (for all arcs l_j,)

where $A_t^o = \sum_{f\in F} A_t(f)$ is the total traffic offered to the network, $A_t(f)$ is the traffic offered by flow f (in Erlangs) at time period t, $L_{r^i(f)}$ is the blocking probability of a call of f on route $r^i(f)$ $(i=1,2)$ and $\overline{R}_t$ is the set of the route sets of all traffic flows $f\in F$ at time period $t=nT$ $(n=1,2,\ldots)$:

$$\overline{R}_t = \{R_t(f_1),\ldots,R_t(f_{|F|})\}, \qquad (4)$$

$$R_t(f) = \{r^1(f), r^2(f)\}, \qquad (5)$$

The complete analytical model is described in [15, 17]. In [7] it is proved that, assuming quasi-stationary conditions in successive route updating periods (i.e., the offered traffic stochastic features remain stationary during periods which are relatively long compared to the solution time) the single-objective adaptive alternative routing problem (corresponding to the g.o.f. Eq. (1)) is NP-complete in the strong-sense, even in the "degenerated" simpler case where $M=1$ (no alternative route provided). It should be noted that our model is a bi-objective formulation of this type of problem.

The basis of the problem resolution procedure is an algorithmic approach (designated as MMRA) which seeks good compromise non-dominated and possibly dominated solution(s) (when there is a dominated solution located in the first priority region(s) of the objective function space of the Problem $\mathscr{P}^{(2)}$ (Eq. (6)) which may be selected corresponding to some second choice route, see [6]) to the following bi-objective shortest path problem (for each flow f from node v_s to node v_t) defined in the network (V,N), where V is the node set and L the arc set:

(Problem $\mathscr{P}^{(2)}$)

$$\min\ z^n = \sum_{l_k=(v_i,v_j)\in L} \mathscr{C}_k^n x_{ij} \quad (n=1,2) \qquad (6)$$

s.t.

$$\begin{aligned}
&\sum_{v_j\in V} x_{sj} = 1\\
&\sum_{v_i\in V} x_{ij} - \sum_{v_q\in V} x_{jq} = 0 \quad \forall v_j\in V, (v_j\neq s,t)\\
&\sum_{v_i\in V} x_{it} = 1 \qquad (7)\\
&x_{ij}\in\{0,1\}, \quad \forall l_k=(v_i,v_j)\in L\\
&(x_{ij}=1 \text{ iff } l_k=(v_i,v_j)\ \in\ r^i(f)),
\end{aligned}$$

where

$\mathscr{C}_k^1 = c_k(\text{implied cost on link } l_k)$ and $\mathscr{C}_k^2 = -\log(1-B_k)$.

The call blocking probability B_k (or call congestion) on arc l_k and the application of log is necessary for obtaining an additive metric. The implied cost c_k associated with link l_k is an important concept in teletraffic routing theory due to Kelly [10]. It represents the expected value of the increase in lost calls (on all routes of all traffic flows which use l_k) which results from accepting a call of a given traffic flow on link l_k. Note that each c_k depends on $\{c_j\}$, $\{C_j\}$, $\{B_j\}$, $\{A_t(f)\}$ and $\overline{R}_t$. The equations of the teletraffic model (in [15]) also imply that each B_k depends on $\{B_j\}$, $\{C_j\}$, $\{A_t(f)\}$ and $\{R_k\}$ (set of routes which at a given period may use link l_k). The arcs are supposed undirected and the paths for each flow f are node disjoint, loopless, and have a predefined maximal number of arcs.

From the analytical model (see [17]) it can be easily shown that there are interdependencies between the objective functions coefficients $\{c_k\}$ and $\{B_k\}$ in $\mathscr{P}^{(2)}$ and between these two sets and the current total route set $\overline{R}_t$, via the set of routes R_k, which, at a given time t, may use link l_k. MMRA enables solutions to $\mathscr{P}^{(2)}$ to be computed assuming fixed values of $\{c_k\}$ and $\{B_k\}$ and given $\{A_t(f), \text{all } f \in F\}$ and the capacities C_k of all links l_k.

Taking into account the NP completeness (in the strong sense) nature of the network problem $\mathscr{P}_G^{(2)}$ and the aforementioned interdependencies between the mathematical entities $\{c_k\}$, $\{B_k\}$ and $\overline{R}_t$, it can be concluded of the extreme intractability of the network problem $\mathscr{P}_G^{(2)}$.

Concerning the possible conflict between the objective functions in $\mathscr{P}_G^{(2)}$ it can be said that in many situations (in networks using alternative routing) the minimisation of B_m is associated with a deterioration on $B(f)$ for "small" traffic flows $A_t(f)$, leading to an increase in B_M. In conventional single-objective routing models this effect is usually limited by imposing upper bounds on $B(f)$.

The use of MMRA as a basis for seeking approximate solutions to $\mathscr{P}_G^{(2)}$ relies on the property that minimising z^1 in $\mathscr{P}^{(2)}$ corresponds to minimising B_m, when searching for a path for flow f assuming all the remaining conditions in the network (namely the routes assigned to all other flows and all the link implied costs) were maintained constant while the minimisation of z^2 in $\mathscr{P}^{(2)}$ tends to achieve the minimisation of B_M, under similar assumptions. Of course from the analysis on the problem overall complexity it is clear that these assumptions (all remaining conditions in the network are maintained constant) do not hold, which leads to an unstable behaviour of MMRA solutions as reviewed in the next section.

Concerning the traffic modelling aspects, underlying the calculation of B_k and c_k, we must clarify that we used a one-parameter simplification, based, for the multiservice networks case, on the Kaufman [9] or Roberts [21] algorithms [14]. It is well known in teletraffic theory that these models represent an oversimplification (from a stochastic point of view) which leads to significant errors, specially for low blocking probabilities. The reason for this choice was purely instrumental taking into account the great numerical efficiency of the used procedures which is absolutely critical in a model of this nature. In fact the traffic calculation subroutines used for resolving the system of equations (involving implied costs and blocking probabilities for each traffic type in every link) enabling to estimate $(\overline{c}, \overline{B})$ have to be executed a very large number of times in each run of the heuristic for final route selection. Note that the importance of the accuracy of the results given by the traffic calculation model, in the context of MODR, is in terms of relative values of the associated route metrics (since the aim is just to compare routing solutions with respect to those metrics) rather than in terms of absolute errors. Also note that these simplified models were used/recommended in single-objective global routing optimisation models, for off-line application, such as in Mitra *et al.* [18]. In a dynamic routing environment, specially when a very complex and lengthy calculation procedure is at stake, the need for a very efficient approximation (albeit simplistic) is unavoidable for tractability reasons.

2.2. First version of a heuristic of path selection

The interdependencies between key mathematical entities of the model $\mathscr{P}^{(2)}$ and the great complexity of the global problem $\mathscr{P}_G^{(2)}$ make the direct application of the bi-objective algorithm MMRA (to every pair of nodes) to generate unstable solutions, possibly leading to poor network performance (under the bi-objective model (B_m, B_M)) as shown in [16]. In fact direct application of MMRA to obtain the "best" compromise alternative paths for every node to node traffic flow as a function of the network state leads typically to situations where the chosen path sets $\overline{R}_t$ may oscillate between a few sets of solutions. This is associated with the fact that in a certain iteration certain links will be very loaded (as a result of contributing to many paths) while others are lightly loaded; in the following iteration the more loaded and the less loaded links will tend to reverse their condition. This is a new and specific "bi-objective" case of a known instability problem in single objective adaptive shortest path routing models which was extensively studied in packet switched networks (see for example [3, Chapter 5]) and also analysed in some single-objective dynamic alternative routing models.

This path instability phenomena in the context of MODR was extensively analysed in [16].

A heuristic was developed in [17] for selecting path sets $\overline{R}_t$ $(t = nT; \quad n = 1, 2, \dots)$ capable of guaranteeing a good compromise solution in terms of the two network performance criteria (B_m, B_M), at every updating period. The basis of that procedure is to search for the subset of the alternative path set

$$\overline{R}^a_{t-T} = \{r^2(f), \quad f \in \mathscr{F}\} \tag{8}$$

the elements of which should possibly be changed in the next updating period, seeking to minimise B_m while not letting an excessive increase in max $\{B(f)\}$. The authors proposed in [16] the following criterion for choosing candidate paths for possible improvement which depends explicitly both on the first choice path $r^1(f)$ and on the alternative path $r^2(f)$:

$$\xi(f) = F_1 F_2 = \left(2C^1_{r^1(f)} - C^1_{r^2(f)}\right)\left(1 - L_{r^1(f)} L_{r^2(f)}\right), \tag{9}$$

$$C^1_{r^i(f)} = \sum_{l_k \in r^i(f)} c_k \,. \tag{10}$$

The objective of the factor F_1 is to favour (concerning the need to change the 2nd route) the flows for which the 2nd route has a high implied cost and the 1st route a low implied cost. The factor 2 of $C^1_{r^1(f)}$ was introduced for normalising reasons taking into account that $r^1(f)$ has one arc and $r^2(f)$ two arcs, in fully meshed networks. In a more general case, where $r^1(f)$ has n_1 arcs and $r^2(f)$ n_2 arcs $(n_1 \le n_2)$:

$$F_1 = (n_2 - n_1)c'_1 + C^1_{r^1(f)} - C^1_{r^2(f)}, \tag{11}$$

c'_1 being the average implied cost of the arcs in $r^1(f)$. The second factor F_2 intends to favour the flows with worse end-to-end blocking probability. An important issue tackled in the procedure is the specification of how many and which of the second choice routes $r^2(f)$ with smaller value of $\xi(f)$ should possibly be changed by applying MMRA once again. Among the recalculated routes only those which lead to solutions which dominate previous ones (in terms of B_m and B_M) are finally selected as routes to be changed in each path updating period. This implies that the effect of each candidate route (in terms of B_m and B_M) is previously anticipated by solving the corresponding analytical model. This heuristic procedure uses two variables that control the current number of candidate paths for improvement in the two main cycles of the heuristic. The first variable is initialised to the total number of node pairs and controls an external cycle where the second variable is initialised; the second variable is used in an internal cycle that seeks to obtain new alternative paths $r^2(f)$ able of improving B_m and/or B_M.

The MODR heuristic uses a specific "service protection scheme", aimed at preventing excessive network blocking degradation in overload situations, associated with the utilisation of alternative routes for all node-to-node traffic flows. This mechanism designated as alternative path removal (APR) is based on the elimination of the alternative paths of all traffic flows for which the value of the scalar function (convex combination of the two objective functions) of the bi-objective shortest path model $\mathscr{P}^{(2)}$ is greater than or equal to a certain parameter z_{APR} that is adapted dynamically to overload conditions. Details and a formal description of this heuristic are in [17].

In [15] the performance of the global routing method using that heuristic (MODR-1) was compared in terms of the two global performance network metrics with the corresponding results given by a discrete event simulation model for a reference dynamic routing method, real-time network routing developed by ATT&T, known for its efficiency and sophistication in terms of service protection mechanisms. This comparative study revealed that the method globally performed well in most situations.

3. New versions of the heuristic

Having in mind the very complex nature of the network bi-objective dynamic alternative routing problem $\mathscr{P}_G^{(2)}$ we have considered the analysis of variants of the previously described version of the heuristic, namely by using relaxations of the values calculated for the two network objective functions in $\mathscr{P}_G^{(2)}$, B_m and B_M. This had in mind to enable the calculation of approximate non-dominated solutions by travelling in the network objective function's space in order to improve one of the objective functions, relaxing the other with respect to the values corresponding to the solution obtained by the initial version of the heuristic (designated hereafter as MODR-1). This also enabled the analysis of the behaviour of the variants of the heuristic with respect to the objective function values and test possible improvements of MODR-1. The test networks are the same which were used in previous studies: the network in [19] (fully meshed, with six nodes) widely used in studies on dynamic routing methods (network M in short) and two networks with the same topology designated as networks B and A. Network M has strong asymmetries in many arc capacities, with respect to the direct traffic offered to them. Network B was engineered by recalculating the link capacities of network M for the same values of traffic offered $A_{t_0}(f)$ with a standard dimensioning method for dynamic routing circuit-switched networks [2]. Network A has a different matrix of nominal traffic offered with a smaller variation in traffic intensities than in network B and M; its link capacities were obtained by the same method as network B. The specification of each of these networks, including the initial route set $\overline{R}_{t_0}$ computed by the mentioned method [2], is given in Table 1.

3.1. Versions of the heuristic

Firstly the path selection procedure (heuristic) was changed so that the routes are chosen by seeking to minimise separately one of the network metrics: B_m and B_M. The solutions obtained are denoted by (B_m^*, B_M^+) and (B_m^+, B_M^*) and correspond to the approximations to the minimum of B_m (B_m^*) and B_M (B_M^*) which the heuristic was capable of obtaining. These solutions are designated as **extremes-H**.

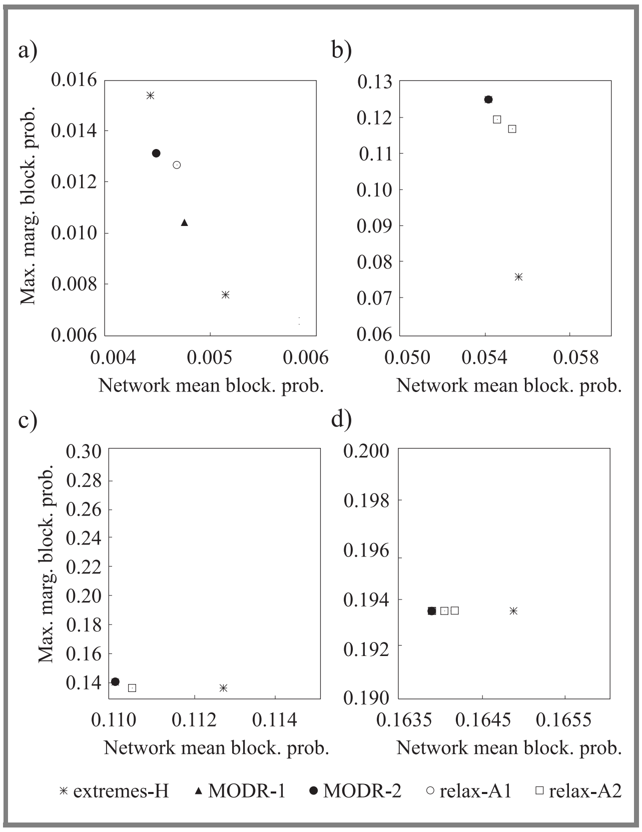

Fig. 1. Network B: overload factor (a) 0%; (b) 10% (c) 20%; (d) 30% .

Table 1
Test networks A, B and M

O-D pair	Network A link capac.	Network A offer. traf.	Network A intermed. node	Network B link capac.	Network B offer. traf.	Network B intermed. node	Network M link capac.	Network M offer. traf.	Network M intermed. node
1-2	36	27	3	41	27.47	3	36	27.47	3
1-3	13	6	4	13	6.97	4	24	6.97	5
1-4	33	25	5	276	257.81	5	324	257.81	–
1-5	27	20	6	33	20.47	6	48	20.47	3
1-6	31	20	2	45	29.11	2	48	29.11	5
2-3	29	25	4	29	25.11	4	96	25.11	–
2-4	17	10	5	112	101.61	5	96	101.61	3
2-5	37	30	6	88	76.78	6	108	76.78	3
2-6	25	20	1	94	82.56	1	96	82.56	3
3-4	17	11	5	18	11.92	5	12	11.92	1
3-5	14	8	6	11	6.86	6	48	6.86	6
3-6	19	13	1	21	13.25	1	24	13.25	2
4-5	13	9	6	87	79.42	6	192	79.42	1
4-6	27	20	1	94	83.0	1	84	83.0	5
6-6	18	12	1	137	127.11	1	336	127.11	–

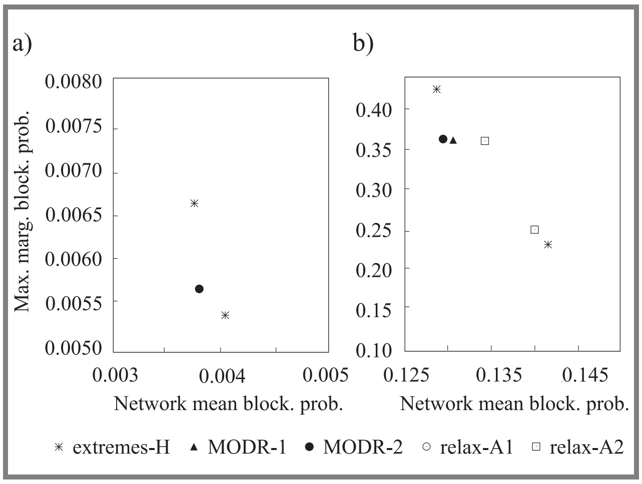

Fig. 2. (a) Network A: overload factor 0%; (b) network M: overload factor 60%.

Next two new versions of the heuristic were implemented which seek solutions $\overline{R}_t$ satisfying:

$$(\mathrm{A1})\quad \min B_m : B_M < B_M^+ - n\Delta^1,\ \ n = 9, \ldots, 1 \qquad (12)$$

$$(\mathrm{A2})\quad \min B_M : B_m < B_m^+ - n\Delta,\quad n = 9, \ldots, 1 \qquad (13)$$

where

$$\Delta = \frac{B_m^+ - B_{m_{MODR}}}{10},$$

$$\Delta^1 = \frac{B_M^+ - B_{M_{MODR}}}{10}$$

and $(B_{m_{MODR}}, B_{M_{MODR}})$ are the objective function values corresponding to the solution obtained by the initial version of the MODR heuristic. A1 (A2) corresponds to the relaxation of B_M (B_m) by successive increments equal to Δ^1 (Δ) in the interval $]B_{M_{MODR}}, B_M^+[$ ($]B_{m_{MODR}}, B_m^+[$). In Figs. 1 and 2 the solutions obtained with A1 and A2 correspond to the points signalled as **relax-A1** and **relax-A2**.

3.2. Insight on the heuristic

Finally to give some insight on the behaviour of the solutions generated by the major cycles of this type of heuristic a fourth version of the heuristic was implemented.

This is a variant of the heuristic where, in the search for solutions which minimise B_M and B_m, the currently selected solutions have to satisfy the condition:

$$(\mathrm{B})\qquad B_m^* \le B_m \le B_m^+ \quad \text{and} \quad B_M^* \le B_M \le B_M^+ . \qquad (14)$$

In Figs. 3 and 4 the solutions from this version correspond to the points signalled as **val. interv**. The consideration of this version has to do with the fact that in the initial version of the heuristic from one iteration to the next it is not accepted a generated solution which worsens any of the two objective functions values. It was observed that this condition was too strict regarding the prosecution of the main search for solutions. In the new version the controlled relaxation of this condition with respect to the two

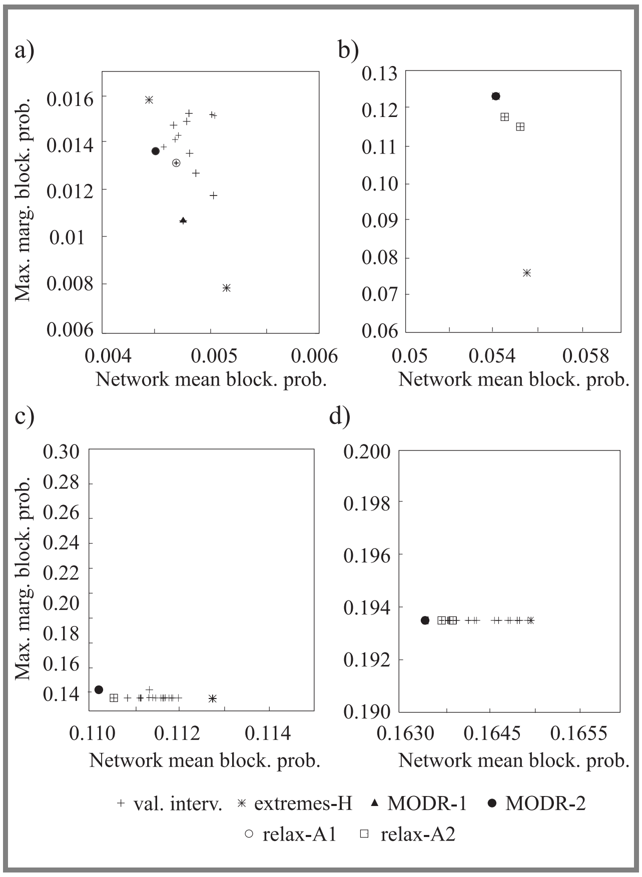

Fig. 3. Network B: overload factor (a) 0%; (b) 10%; (c) 20%; (d) 30%.

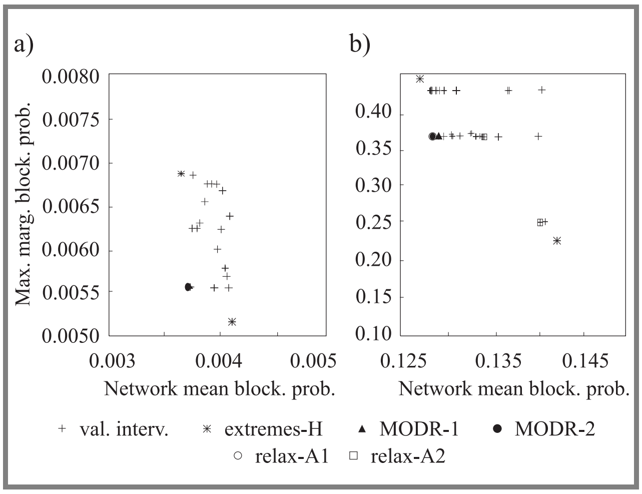

Fig. 4. (a) Network A: overload factor 0%; (b) network M: overload factor 60%.

metrics might enable that the solutions calculated in this manner could lead at a later iteration to solution(s) worthwhile considering.

3.3. Analysis of results

The most significant results obtained with the described versions of the heuristic are depicted in Figs. 1 and 2, for the three test networks.

The first conclusion is that the solutions obtained by MODR-1 are in almost all the cases non-dominated with respect to the solutions from all the other versions of the heuristic and are good compromise solutions in terms of B_m and B_M. The only exception was in case Fig. 2b, where the MODR-1 solution was slightly dominated by the solution from **relax-A1** with respect to the metric B_m. This situation can be explained by the very complex nature of the problem $\mathscr{P}_G^{(2)}$, previously reviewed, namely the strong interdependencies between the objective functions and between the parameters of the functions and the calculated path sets. Related to the situation in Fig. 2b, we can say that by considering some relaxation of B_M (version **relax-A1** of the heuristic) we might also obtain solutions which are non-dominated with respect to those of MODR-1 but for which B_m is better than for MODR-1 while B_M is just slightly worse. It may also happen that some of these slight differences in the values of B_M or B_m result from numerical imprecision associated with the lengthy and complicated numerical procedures involved in the resolution of the network teletraffic model. Having all this in mind (as well as other experiments) and to enable that such solutions may be selected, a new version of the MODR heuristic, designated as MODR-2 was implemented which seeks solutions which tend to minimise B_m and B_M while accepting those for which B_M is 3% worse than the current minimum. This new version of the heuristic enabled slight improvements in the network performance results in terms of B_m in some situations (as illustrated in the next section). Note that from an engineering point of view it is correct to accept solutions with somehow better B_m at the cost of a slight worsening in B_M, since the former metric is directly related to the average revenue associated with the total traffic carried in the network.

Other interesting aspect to be assessed in these results concerns the sets of solutions with the same value of B_M, which occur for higher overloads (Figs. 1c, 1d, 2b, 3c, 3d, 4b). This phenomenon can be explained as a result of the elimination of the alternative route for some traffic flows which are then the only flows in the corresponding direct arcs. One of the flows in these conditions (the one which suffers the highest congestion) determines the value of B_M. Hence the value B_M does not change while (in the solutions obtained from the different versions of the heuristic) there is no alternative route of other flow(s) which uses the direct arc associated with that flow or while that flow does not have an alternative route.

4. Network performance

In order to evaluate the network performance in terms of the two metrics B_m, B_M obtained with the initial version of the heuristic (MODR-1) and the new version, MODR-2, described in the previous section, Tables 2 to 4 show the corresponding analytical results, for the three test networks

Table 2
Global network performance for network M

Overl. factor [%]	MODR–1 Analytical model		MODR–2 Analytical model		RTNR Simulation model	
	B_m	B_M	B_m	B_M	$B_m \pm \Delta$	$B_M \pm \Delta$
0	$< 10^{-3}$	0.001	$< 10^{-3}$	0.001	$\mathbf{< 10^{-3}}$	$\mathbf{< 10^{-3}}$
10	0.001	0.009	0.001	0.009	$\mathbf{0.001 \pm 1.1 \cdot 10^{-4}}$	$\mathbf{0.005 \pm 1.1 \cdot 10^{-3}}$
20	0.005	0.035	0.005	0.035	$\mathbf{0.004 \pm 3.0 \cdot 10^{-4}}$	$\mathbf{0.025 \pm 2.4 \cdot 10^{-3}}$
30	0.019	0.076	**0.019**	**0.076**	$0.027 \pm 1.5 \cdot 10^{-3}$	$0.144 \pm 1.3 \cdot 10^{-2}$
40	0.063	0.141	**0.063**	**0.141**	$0.063 \pm 1.6 \cdot 10^{-3}$	$0.257 \pm 5.5 \cdot 10^{-3}$
50	0.103	0.192	0.103	**0.192**	$\mathbf{0.101 \pm 1.8 \cdot 10^{-3}}$	$0.335 \pm 3.3 \cdot 10^{-3}$
60	0.130	0.361	**0.130**	**0.362**	$0.138 \pm 1.5 \cdot 10^{-3}$	$0.397 \pm 3.7 \cdot 10^{-3}$
70	0.169	0.397	**0.166***	**0.398**	$0.173 \pm 1.7 \cdot 10^{-3}$	$0.446 \pm 2.9 \cdot 10^{-3}$
80	0.203	0.429	**0.196***	0.484	$0.204 \pm 1.6 \cdot 10^{-3}$	$\mathbf{0.479 \pm 1.4 \cdot 10^{-3}}$

Table 3
Global network performance for network B

Overl. factor [%]	MODR–1 Analytical model		MODR–2 Analytical model		RTNR Simulation model	
	B_m	B_M	B_m	B_M	$B_m \pm \Delta$	$B_M \pm \Delta$
0	0.005	0.011	**0.005**	**0.011**	$0.007 \pm 6.7 \cdot 10^{-4}$	$0.029 \pm 6.4 \cdot 10^{-3}$
10	0.054	0.124	**0.054**	**0.124**	$0.058 \pm 1.1 \cdot 10^{-3}$	$0.180 \pm 9.7 \cdot 10^{-3}$
20	0.110	0.140	**0.110**	**0.140**	$0.111 \pm 1.3 \cdot 10^{-3}$	$0.257 \pm 1.2 \cdot 10^{-2}$
30	0.164	0.194	**0.164**	**0.194**	$0.193 \pm 2.1 \cdot 10^{-3}$	$0.296 \pm 3.8 \cdot 10^{-3}$
40	0.214	0.246	**0.214**	**0.246**	$0.216 \pm 1.2 \cdot 10^{-3}$	$0.315 \pm 7.7 \cdot 10^{-3}$

Table 4
Global network performance for network A

Overl. factor [%]	MODR–1 Analytical model		MODR–2 Analytical model		RTNR Simulation model	
	B_m	B_M	B_m	B_M	$B_m \pm \Delta$	$B_M \pm \Delta$
0	0.004	0.006	0.004	0.006	$\mathbf{0.003 \pm 5.3 \cdot 10^{-4}}$	$\mathbf{0.006 \pm 1.5 \cdot 10^{-3}}$
10	0.031	0.038	**0.031**	**0.038**	$0.041 \pm 2.9 \cdot 10^{-3}$	$0.061 \pm 4.4 \cdot 10^{-3}$
20	0.078	0.153	**0.077**	0.153	$0.090 \pm 2.7 \cdot 10^{-3}$	$\mathbf{0.133 \pm 8.9 \cdot 10^{-3}}$
30	0.119	0.198	**0.118**	0.198	$0.129 \pm 2.2 \cdot 10^{-3}$	$\mathbf{0.186 \pm 8.7 \cdot 10^{-3}}$
40	0.157	0.242	**0.156**	0.242	$0.167 \pm 1.8 \cdot 10^{-3}$	$\mathbf{0.226 \pm 1.1 \cdot 10^{-2}}$

and different overload factors. Since the major objective of this study was to perform a comparison between the relevant variants of the heuristic only analytical results are given in these tables. A simulation study using a discrete-event platform (in report [15]) confirmed the relations between the results obtained from the two variants of MODR for the test networks and the different overload factors. Also the results from a reference dynamic routing method (RTNR), obtained from a discrete event simulator, are displayed with 95% confidence intervals. The results presented for RTNR are intended as reference values for each case.

The major conclusion is that MODR-2 enables slight improvements in B_m at the cost of slight increases in B_M, specially in high overload conditions. These results also confirm that both versions of the heuristic globally perform well when compared to RTNR, specially in overload conditions, as already concluded in [15] and in [17] for the MODR-1 case. In fact, excepting for the case of the poorly engineered network M for low and moderate overload (where the values B_m and B_M obtained by the heuristic were even so very low and always below standardised requested values) and for very low blocking in network A and B the solutions of the heuristics either dominate the RTNR solutions or are non-dominated with respect to the latter. Only for low or very low overload where even so MODR-1 and MODR-2 values for B_m are normally below typical required values (e.g., $\leq 0.5\%$ at 0% overload), RTNR tends to give better results than MODR-1 in terms of B_m. A detailed comparison of the network performance with the solutions from the MODR-1 heuristic, with the corresponding results for the RTNR solutions, using discrete event simulation models for both dynamic routing methods is described in [15, 17]. Those simulation studies have shown that MODR-1 globally tends to have better performance than RTNR, specially in overload conditions. Note that MODR-2 performs at least as well as MODR-1 with respect to the total average network revenue (or network mean blocking probability).

5. Conclusions and further work

A study was presented on the performance of variants of a heuristic for synchronous path selection in a bi-objective dynamic alternative routing model, by using relaxations of the values previously calculated for the two network objective functions.

This work permitted the specification of a new version of the heuristic which enables slight improvements in the network mean blocking probability possibly at the cost of a slight increase in the maximal node to node blocking probability which is advantageous in some practical network engineering situations. Also a comparison of the network performance (as measured by the two metrics) obtained with two versions of the heuristic and the dynamic routing method RTNR enabled the confirmation of the globally good performance of the MODR method, namely in overload conditions.

Further work concerns the extension and complete formulation of the MODR model for multiclass traffic loss networks as outlined in [13]. This includes the development of a heuristic capable of finding good compromise solutions for a bi-hierarchical multiple objective dynamic alternative routing problem where the first priority global objective functions concern the global network level metrics and the second priority network objective functions are concerned with the quality of service metrics associated with the different services. Also extensive simulational comparative studies have to be carried out in this context, in order to evaluate with more precision the results of the heuristics for various test networks.

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