THE REACTIVE SOUND POWER OF A CIRCULAR PLATE WITHIN THE LOW FREQUENCIES

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Summary

An efficient low frequency approximation for the reactive sound power of an elastically supported circular plate has been directly derived from the corresponding high frequency expressions presented earlier. The analysis of the approximation error has shown that the low frequency formulation can be applied if the error of 10% can be accepted. The result is valid for any axisymmetric boundary configuration of the plate. The influence of the boundary configuration on the reactive sound power has been examined.

Keywords: reactive sound-power, mode number, sound radiation.

MOC BIERNA DŹWIĘKU PŁYTY KOŁOWEJ W ZAKRESIE NISKICH CZĘSTOŚCI

Streszczenie

Efektywną aproksymację niskoczęstościową biernej mocy dźwięku płyty kołowej zamocowanej sprężyście wyprowadzono bezpośrednio z odpowiednich wyrażeń wysokoczęstościowych przedstawionych wcześniej. Analiza błędów aproksymacji pokazała, że wzór niskoczęstościowy może być stosowany, jeśli dopuszczalny błąd względny może wynosić do 10%. Otrzymany wynik jest słuszny dla dowolnej osiowosymetrycznej konfiguracji brzegowej płyty. Zbadano wpływ konfiguracji brzegowej na moc bierną.

Słowa kluczowe: bierna moc dźwięku, numer modu, promieniowanie dźwięku.

1. INTRODUCTION

The self power of a circular plate is a very important magnitude useful for computing numerous acoustic values like, e.g., sound pressure radiated, plate's transverse deflection or total sound power lost by an acoustic system covering the plate. The normalized complex sound power is an equivalent magnitude with the normalized impedance of the system.

So far, many investigations dealt with the magnitude. Levine, Leppington and Rdzanek presented their theoretical analysis of some efficient high frequency asymptotes for a clamped circular plate [1,2]. Czarnecki, Engel and Panuszka proposed an equivalent area as well as correlation methods to predict the total sound power radiated by a clamped circular plate [3,4]. Their theoretical results showed a good agreement with the measurements. Engel and Stryczniewicz also determined analytically the magnitude [5,6]. Stepanishen and Ebenezer used both the wavenumber as well as time domain approaches to determine the self-power of a clamped circular plate [7,8] providing some efficient theoretical expressions. Rdzanek, Rdzanek Jr. and Engel proposed a low frequency approximation for the self-power of some circular plates [9,10]. The authors also extended the theoretical results presented in Refs. [1,2] and generalized them providing some high frequency asymptotes valid for an elastically supported circular plate [11].

So far, there are no analytical approximations for the reactive sound power of an elastically supported circular plate within the low frequencies. Therefore, the authors of the paper, desiring to fill this literature gap, extend the results presented in Ref. [11] from the high to low frequencies examining carefully approximation errors arising. They show that the approximation errors can be accepted for some higher modes since the relative error does not exceed 1% within almost the whole low frequency range and for the lowest frequencies the magnitude rapidly tends to its value of zero.

2. MATHEMATICS

A thin circular plate is embedded into an infinite rigid baffle. The plate vibrates and radiates some acoustic waves into the free field. The analysis focuses on the reactive sound power for some axisymmetric time harmonic processes. This requires determining the eigenvalues of the system from the frequency equation in the form of

$$l_1 m_2 + l_2 m_1 = 0 \tag{1}$$

where

$$\left.\begin{aligned} l_1 &= J_1(\lambda_n) - qJ_0(\lambda_n) \\ m_1 &= I_1(\lambda_n) - qI_0(\lambda_n) \end{aligned}\right\} \text{ for } q < q_0 \tag{2a}$$

$$\left.\begin{aligned} l_1 &= J_0(\lambda_n) - J_1(\lambda_n)/q \\ m_1 &= I_0(\lambda_n) - I_1(\lambda_n)/q \end{aligned}\right\} \text{ for } q \ge q_0 \tag{2b}$$

$$\left.\begin{aligned} l_2 &= J_0(\lambda_n) + pJ_1(\lambda_n) \\ m_2 &= I_0(\lambda_n) + pI_1(\lambda_n) \end{aligned}\right\} \text{ for } p < p_0 \tag{2c}$$

$$\left.\begin{aligned} l_2 &= J_1(\lambda_n) + J_0(\lambda_n)/p \\ m_2 &= I_1(\lambda_n) + I_0(\lambda_n)/p \end{aligned}\right\} \text{ for } p \ge p_0 \tag{2d}$$

and q_0, p_0 are some arbitrary values set within the limits $[0;+\infty]$, e.g. as the value of unity,

$$q = \frac{K_W a^3}{\lambda_n^3 D}, \quad p = \frac{K_\psi a}{\lambda_n D} - \frac{1-\nu}{\lambda_n} \tag{3}$$

where K_W and K_ψ boundary stiffness values associated with the force resisting transverse deflection of the edge and with the boundary the bending moment at the edge, respectively, $\lambda_n = k_n a$ eigenvalue of the system, $D = Eh^3/12(1-\nu^2)$ plate stiffness, $k_n^4 = \omega_n^2 \rho h/D$ structural wavenumber, ω_n eigenfrequency, E,ν,ρ and h the plate's Young modulus, Poisson ratio, density and thickness, respectively.

The mode shape of the system is (cf., Ref. [12])

$$W_n(r) = A_n[J_0(k_n r) - C_n I_0(k_n r)] \tag{4}$$

where r radial variable, J_0 and I_0 Bessel and modified Bessel functions of the zero order, respectively, $A_n^{-2} = u_n + w_n$, $C_n = l_1/m_1 = l_2/m_2$,
$u_n = J_0^2(\lambda_n) + J_1^2(\lambda_n) - C_n S(\lambda_n)/\lambda_n$,
$w_n = C_n^2\left[I_0^2(\lambda_n) - I_1^2(\lambda_n)\right] - C_n S(\lambda_n)/\lambda_n$,
$S(\lambda_n) = J_1(\lambda_n)I_0(\lambda_n) + J_0(\lambda_n)I_1(\lambda_n)$.

The eigenfunction of the system can be formulated briefly as

$$\psi_n(w) = \frac{\lambda_n J_1(\lambda_n) J_0(w) - wJ_1(w)J_0(\lambda_n)}{w^4 - \lambda_n^4} + \frac{\lambda_n \gamma_n J_0(w) - wb_n J_1(w)}{2\lambda_n^2(w^2 + \lambda_n^2)} \tag{5}$$

where we denote J_1 and I_1 as Bessel and modified Bessel functions of the first order, respectively, $\gamma_n = J_1(\lambda_n) + C_n I_1(\lambda_n)$, $b_n = J_0(\lambda_n) - C_n I_0(\lambda_n)$, $w = \beta x$ and $w = \beta/x$ for formulating integrals representing the active and reactive sound power, respectively,

$$P_{an} = 8\lambda_n^4 A_n^2 \int_0^{\pi/2} \psi_n^2(\beta x)\beta x \, d\vartheta \tag{6a}$$

$$P_{rn} = 8\lambda_n^4 A_n^2 \int_0^{\pi/2} \psi_n^2(\beta/x)(\beta/x)^2 \, d\vartheta \tag{6b}$$

and $x = \sin\vartheta$, $\beta = ka$ dimensionless wavenumber. The complex self-power is

$$P_n = P_{an} - iP_{rn} \tag{6c}$$

for the time dependence $\exp(-i\omega t)$.

2.1. The high frequencies

The elementary expressions for the active and reactive sound power within the high frequency had been formulated for $\delta_n = k_n/k = \lambda_n/\beta = 1/\varepsilon_n < 1$ as

$$P_{an} = X_n \cos(2\beta + \pi/4) + Y_n \sin(2\beta + \pi/4) + A_n^2\left(\frac{u_n}{\sqrt{1-\delta_n^2}} + \frac{w_n}{\sqrt{1+\delta_n^2}}\right) \tag{7a}$$

$$P_{rn} = Y_n \cos(2\beta + \pi/4) - X_n \sin(2\beta + \pi/4) + \frac{2A_n^2}{\pi\beta}\left[\frac{\alpha_n^{(1)}}{1+\delta_n^2} + \frac{\alpha_n^{(2)} \arcsin\delta_n}{2\delta_n(1-\delta_n^2)^{3/2}} + \frac{\alpha_n^{(3)} \operatorname{asinh}\delta_n}{2\delta_n(1+\delta_n^2)^{3/2}}\right] \tag{7b}$$

where

$$\alpha_n^{(1)} = \frac{1}{2}\frac{1+\delta_n^2}{1-\delta_n^2}\left[J_0^2(\lambda_n) + J_1^2(\lambda_n)\right] + \frac{C_n^2}{2}\left[I_0^2(\lambda_n) - I_1^2(\lambda_n)\right], \tag{8a}$$

$$\alpha_n^{(2)} = J_0^2(\lambda_n) - J_1^2(\lambda_n)(1-2\delta_n^2) - 2C_n(1-\delta_n^2)\left[J_0(\lambda_n)I_0(\lambda_n) - J_1(\lambda_n)I_1(\lambda_n)\right], \tag{8b}$$

$$\alpha_n^{(3)} = C_n^2\left[I_0^2(\lambda_n) + I_1^2(\lambda_n)(1+2\delta_n^2)\right] - 2C_n(1-\delta_n^2)\left[J_0(\lambda_n)I_0(\lambda_n) + J_1(\lambda_n)I_1(\lambda_n)\right], \tag{8c}$$

$$X_n = 4A_n^2\sqrt{\frac{\beta}{\pi}}\left(\frac{\lambda_n^2}{\beta^2 + \lambda_n^2}\right)^2(\beta^2 d_n^2 - h_n^2), \tag{8d}$$

$$Y_n = 8A_n^2 \beta\sqrt{\frac{\beta}{\pi}}\left(\frac{\lambda_n^2}{\beta^2 + \lambda_n^2}\right)^2 d_n h_n. \tag{8e}$$

and presented earlier in the literature (cf., Ref. [11]).

2.2. The low frequencies

Given that (cf., Ref. [13])

$$\arcsin z = \begin{cases} \dfrac{\pi}{2} + i\operatorname{acosh} z & \text{for} \quad z < 1 \\ \dfrac{\pi}{2} - i\operatorname{acosh} z & \text{for} \quad z > 1 \end{cases} \tag{9}$$

and that the phase changes when moving from the high to low frequencies, namely from $\delta_n < 1$ to $\delta_n > 1$, for some of the terms in Eqs. 7, we have

$$\frac{A_n^2 u_n}{\sqrt{1-\delta_n^2}} = -i\frac{A_n^2 u_n}{\sqrt{\delta_n^2 - 1}}, \tag{10a}$$

$$-i\frac{2A_n^2\alpha_n^{(2)}\arcsin\delta_n}{2\pi\beta\delta_n(1-\delta_n^2)^{3/2}} = \frac{A_n^2\alpha_n^{(2)}}{2\beta\delta_n(\delta_n^2-1)^{3/2}} - i\frac{A_n^2\alpha_n^{(2)}\operatorname{acosh}\delta_n}{\pi\beta\delta_n(\delta_n^2-1)^{3/2}}. \tag{10b}$$

This provides the low frequency asymptotic formulations formulated as

$$P_{an} = \frac{A_n^2\alpha_n^{(2)}}{2\beta\delta_n(\delta_n^2-1)^{3/2}} + \frac{A_n^2 w_n}{\sqrt{1+\delta_n^2}} + X_n\cos(2\beta + \pi/4) + Y_n\sin(2\beta + \pi/4), \tag{11a}$$

$$P_{rn} = \frac{2A_n^2}{\pi\beta}\left[\frac{\alpha_n^{(1)}}{1+\delta_n^2} + \frac{\alpha_n^{(2)}\mathrm{acosh}\delta_n}{2\delta_n(\delta_n^2-1)^{3/2}} + \frac{\alpha_n^{(3)}\mathrm{asinh}\delta_n}{2\delta_n(1+\delta_n^2)^{3/2}}\right] + \frac{A_n^2 u_n}{\sqrt{1-\delta_n^2}} + Y_n\cos(2\beta+\pi/4) - X_n\sin(2\beta+\pi/4) \tag{11b}$$

Further, we are not interested in using Eq. (11a) since some efficient low frequency approximations are known for the active sound power [9,10]. Moreover, Eq. (11a) produces some big approximation errors. Now, we are interested in using Eq. (11b) representing the reactive sound power and in examining its approximation error if it is acceptable.

3. DISCUSSION

First, we have to examine the behaviour of the reactive sound power within the low frequencies, namely for $k/k_n < 1$. This is illustrated in Fig. 1 showing the magnitude constantly tending to its value of zero, which is plotted in the fully logarithmic coordinates. The curves plotted for some sample modes do not considerably differ each other. In consequence, we are especially interested to assure an acceptable approximation error within almost the whole low frequency range but not necessarily for the very low frequencies since they imply the very low values assumed by the magnitude.

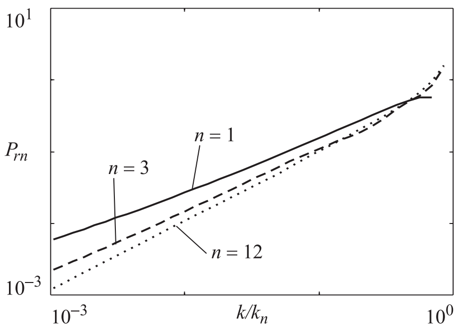

Fig. 1. The reactive sound power for some sample axisymmetric modes within the low frequencies

The absolute value of approximation error has been estimated as $\Delta P_{rn} = \left|P_{I,rn} - P_{A,rn}\right|$, where $P_{I,rn}$ and $P_{A,rn}$ have been computed from Eqs. (6b) and (11b), respectively, and represented by the curves plotted in Fig. 2. The straight lines represent the theoretical error value defined as $\Delta P_{rn} = 0.25(k/k_n)^{2/3}\lambda_n^{-2/3}$. It is easy to note that the error estimation does not considerably exceed its theoretical value within the whole low frequency range. However, we must be careful here since the magnitude assumes some comparable values as the error does for the very low frequencies, namely for $k/k_n < 0.01$ (cf., Fig. 1) and it is worth analyzing the relative error value shown in Fig. 3. The relative error does not considerably exceed its value of 10% for $k/k_n < 0.1$ for the first axisymmetric mode, $k/k_n < 0.01$ for the third mode, and for the higher modes the validity frequency range is even wider. The relative error grows up to its value of 100% for the very low frequencies where the magnitude assumes the value of zero.

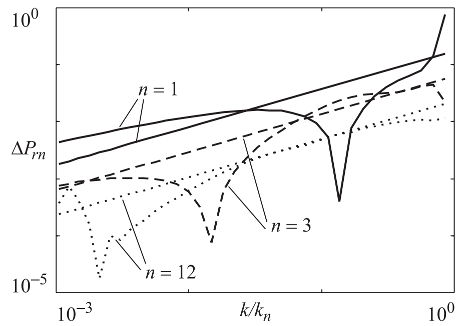

Fig. 2. The absolute approximation error

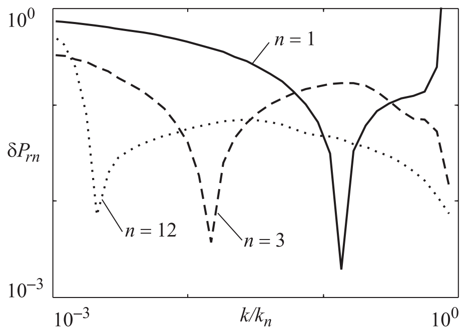

Fig. 3. The relative approximation error formulated as $\delta P_{rn} = \Delta P_{rn}/P_{rn}$

So, if we can accept the approximation error of about 10% for some engineering computations then we are equipped with an efficient low frequency asymptotic formulation for the reactive sound power of an elastically supported circular plate.

Now, let us examine the influence of an axisymmetric boundary conditions of the plate represented by the two values of boundary stiffness K_W and K_ψ (cf., with Eq. (3) and Ref. [12]). All the curves plotted in Figs. 4 and 5 have been prepared for $k/k_n = 0.5$, i.e. where the approximation error does not exceed 10% for all modes. Fig. 4 shows that the change in normalized value of K_W has the very influence on the reactive sound power within its middle range about the value of $K_W a^3/\lambda_n^3 D = 1$. The same can be observed with the change in normalized value of K_ψ which is shown in Fig. 5. Generally, the influence is bigger for the lowest modes and no influence can be observed for those higher.

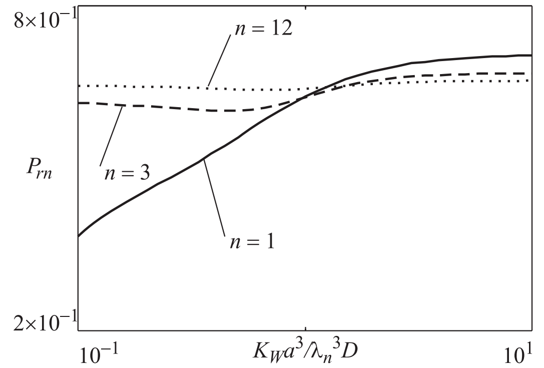

Fig. 4. The reactive sound power in terms of normalized stiffness value K_W for $K_\psi a/\lambda_n D = 1$

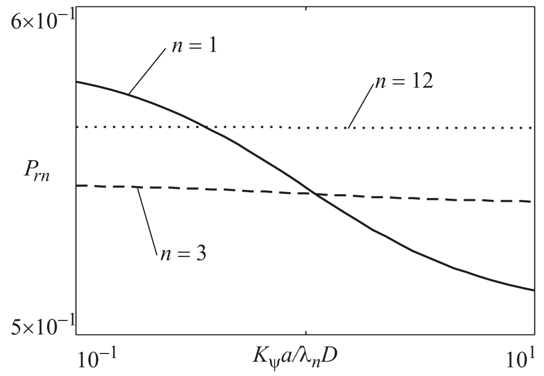

Fig. 5. The reactive sound power in terms of normalized stiffness value K_ψ for $K_W a^3/\lambda_n^3 D = 1$

4. CONCLUSIONS

Finally, we are provided with an efficient low frequency approximation for the normalized reactive sound power of a circular plate with some arbitrary but axisymmetric boundary conditions. The formulation can be used for some engineering computations if the relative error of 10% can be accepted for this purpose.

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