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Positioning with Interactive Navigational Structures Implementation

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INTRODUCTION

A rapid growth of merchant fleet tonnage after the II World War entailed a considerable rise in a number of ships collisions. The above circumstances became then a reason for the seaside countries governments to take their interest in the problem of navigation safety. Therefore the search for new types of navigational signs and new positioning methods became focused on increase of the navigation safety level. Till now the basic assignment for the maritime administration authorities has been a continuous rising and upholding the adequate navigation safety at the subordinated water areas. The most important and valuable factor for solving the above issue appeared to be the prompt development of technique. In consequence, for the last fifty years a sharp progress in the existing navigational signs system reliability has taken place. Designing new radio-navigational systems, as for example DECCA, TRANSIT, LORAN (A and C versions), OMEGA, BRAS, MARS enabled heightening accuracy in determination of vessels proper positions.

However, the most significant improvement of maritime navigation quality happened at the time of putting into service the satellite positioning system (GPS) and also its differential version (DGPS). High accuracy and frequency in determining positions, also the system reliability, are only some of a long list of the system advantages. Common usage of the system would not be possible, but for the fact that a sudden electronic computing development has turned out. Extensive implementation of computers, miniaturization thereof as well as low prices, have been the main reasons for using satellite survey techniques. Maximal shortening of a time, required for determining positions of vessels at sea with adequate accuracy level, is for every navigator one of priorities on watch.
In fact, the techniques development has contributed to a procedure of providing navigating bridges with the "one – man – bridge" type aids. At present, automation in positioning a vessel at sea is based on, first of all, radio-navigational systems, mainly of the satellite type.

Satellite systems for objects positioning appeared indispensable for performing basic tasks of maritime navigation. Navigation, understood as safe and effective conducting a vehicle from one point to another, within a specific physical–geographical environment [Kopacz, Urbański, 1998]. However, the systems have not solved the problem of accessibility to reliable and highly accurate information about a position of an object, especially if surveyed toward on-shore navigational signs or in sea depth. And it's of considerable significance for many navigational tasks, carried out within the frameworks of special works performance and submarine navigation.

In addition, positioning precisely the objects other than vessels, while executing hydrographical works, is not always possible with a use of any satellite system. The problem is, for example, to locate precisely floating signs along the state borders at sea. Difficulties with GPS application show up also while positioning such off–lying dangers as wrecks, underwater and aquatic rocks also other natural and artificial obstacles. It is caused by impossibility of surveyors approaching directly any such object while its positioning. Moreover, determination of vessels positions mutually (mutual geometrical relations) by teams carrying out one common tasks at sea, demands applying the navigational techniques other than the satellite ones. Vessels' staying precisely on specified positions is of special importance in, among the others, the cases as follows:

- surveying vessels while carrying out bathymetric works, wire dragging;
- war ships while searching for submarines, minesweeping, performing common artillery and rocket tasks;
- special tasks watercraft in course of carrying out scientific research, sea bottom exploration etc.

The problems are essential for maritime economy and the Country defence readiness. Resolving them requires applying not only the satellite navigation methods, but also the terrestrial ones.

The condition for implementation of the geo-navigation methods is at present the methods development – both: in aspects of their techniques and technologies as well as survey data evaluation. Now, the classical geo-navigation comprises procedures, which meet out-of-date accuracy standards. To enable meeting the present-day requirements, the methods should refer to well-recognised and still developed methods of contemporary geodesy. Moreover, in a time of computerization and automation of calculating, it is feasible to create also such software, which could be applied in the integrated navigational systems, allowing carrying out navigation, provided with combinatory systems as well as with the new positioning methods. Whereas, as regards data evaluation, there should be applied the most advanced achievements in that subject; first of all the newest, although theoretically well-recognised estimation methods, including *M*-estimation (currently being under development in many research centres, which carry on studies on the observational data evaluation subject). Such approach to the problem consisting in positioning a vehicle in motion and solid objects under observation enables an opportunity of creating dynamic and interactive navigational structures.

1. AIM AND SCOPE OF THE WORK

The main subject of the propositions suggested in this work and the detailed theoretical and empirical analyses, is the Interactive Navigational Structure (here in after called *IANS*). In this paper, the Structure will stand for the existing navigational signs systems, any observed solid objects and also vehicles, carrying out navigation (submarines inclusive), which, owing to mutual dependencies, (geometrical and physical) allow to determine coordinates of this new Structure's elements and to correct the already known coordinates of other elements.

Interactivity, or mutual influence in the presented Structure, consists in a possibility of continuous intervention of a navigator (observer) into its formation. Thus, it has been assumed that, depending on necessities, one remains capable to develop optionally the Structure and to change its configuration as well. Apart of the above, there is also an opportunity of selecting the positioning methods, limited only with the activity sphere and the Structure elements type.

If it concerns *IANS,* the most essential is to define the transmitting and receiving elements, and also quantities joining them together. Therefore the transmitting elements include any sorts of navigational signs, which can be used in a process of determining or correcting positions of any other Structure elements. On the basis of their proper definitions, the navigational signs have their specific place in space (two or three-dimensional space). Navigational signs include also the elements of optical and radar navigational signing and on-shore stations of radio-navigational and satellite systems (i.e.: reference stations). A place of their foundation is defined with coordinates, generally defined in the two-dimensional system. On the other hand, the receiving elements are these of the navigational structure elements, positions of which were just determined or previous determinations were updated (corrected). They may also be the observed solid objects, additional navigational signs, which complement the existing navigational structure, the signs having positions already corrected and vehicles, the proper positions of which were just determined. The transmitting and receiving elements are connected either with geometrical quantities (bearings, distances) subject to survey, or sometimes with physical quantities (velocity and frequency of electromagnetic waves, the parameters changes etc.).

Determination of solid objects foundation coordinates, carried out at other observational stations is a geodesy domain, whereas maritime navigation deals with, first of all, surveying location of proper positions of vehicles, with an observer on-board. Anyhow, there are many special navigational tasks, which require determining a position of an object, remaining under a navigator observation.

The tasks are, among the others, as follows:

- positioning maritime obstructions, danger for navigation, having no chance to approach them;
- positioning objects sunk in sea depth;
- verification of navigational signs positions, carried out from the ship with no need of any navigational service assistance;
- establishing additional inaccessible navigational signs.

One of *IANS* tasks is precise positioning of submarines. Travelling deep in the sea eliminates a possibility of continuous using the GPS system for determination of the vessel's proper positions. Also a lack of any navigational signs systems located at the sea bottom, indicates a necessity of designing any "substitutive navigational systems", internally coherent and assigned for positioning objects in the sea depths. Literature on the above mentioned subject matter is rather insufficient [Sea technology 2003, Saclantcen 2002, Kaniewski 2003a,b]. In that paper, [Sea technology 2003], the joint American and British teams have presented the main results of the practical tests, discussed on the subject on issue. They informed that an autonomous submarine vehicle had travelled a fairly short distance, provided with aids offering an opportunity of determining its proper position. However, neither the navigation method, nor the achieved accuracies have been revealed. The work [Saclantcen 2002] presents also results of theoretical studies, carried out in scientific research centres of Italian Navy. On the other hand, contents of the work performed in France [Kaniewski 2003a,b] are results comprising the efforts, connected with construction of a self-propelled submarine, equipped with a navigational block, operated on the basis of Kalman's filter and its mutations.

In the available Russian literature no such research results are shown, however, according to the author's opinion, it gives no evidence that the problem is out of the Eastern scientific centres interest. It is also confirmed by the fact that, in the whole world, the problems referring to positioning vehicles and any external objects within the sea depth are included in the Navies' activities scope. In Poland the preparatory research aimed at undertaking the subject, were carried out in the Naval University of Gdynia. The research was focused on integration and joint processing navigational data, obtained using acoustic techniques and classical sensors, such as gyroscopes, magnetic gauges or accelerometers, having made the assumption of knowing an initial and final positions (draught and emergence), measured applying satellite techniques. An advantage of acoustic measurements may be taken for comparison with a sea bottom map or, through correlation of pictures, obtained at very specific moments, for determination of the object movement components in reference to the sea bottom. The research results were published in the works [Mięsikowski 2002; Meller, Wąż 2003; Praczyk, Wąż 2003].

The basic rules of constructing the Interactive Navigational Structure have already been a subject of previous works of the author. Mainly they were of recognizable character and comprised usually some problems connected with such structure, but standing apart.

Among the other studies, there were carried out investigations concerning the ways of positioning new navigational signs while coastal navigation proceeding. On searching the optimal values of those signs coordinates in coastal navigation, there were applied the sequential estimation principles.

Such a task, carried out with a use of gyro bearings and distances (on the basis of the vertical angle measurement and a given height of an object), has been presented and resolved in the works [Czaplewski 2002b, Czaplewski, Wiśniewski 2002]. The existing navigational signs K_i and a characteristic on-shore field object *M*, which played a part of an additional signing, were the points, subject to observations. Its optimal coordinates were determined sequentially at positions P_1, P_2, P_3 of the sailing vessel (Fig.1.1, basing on the work [Czaplewski 2002b]).

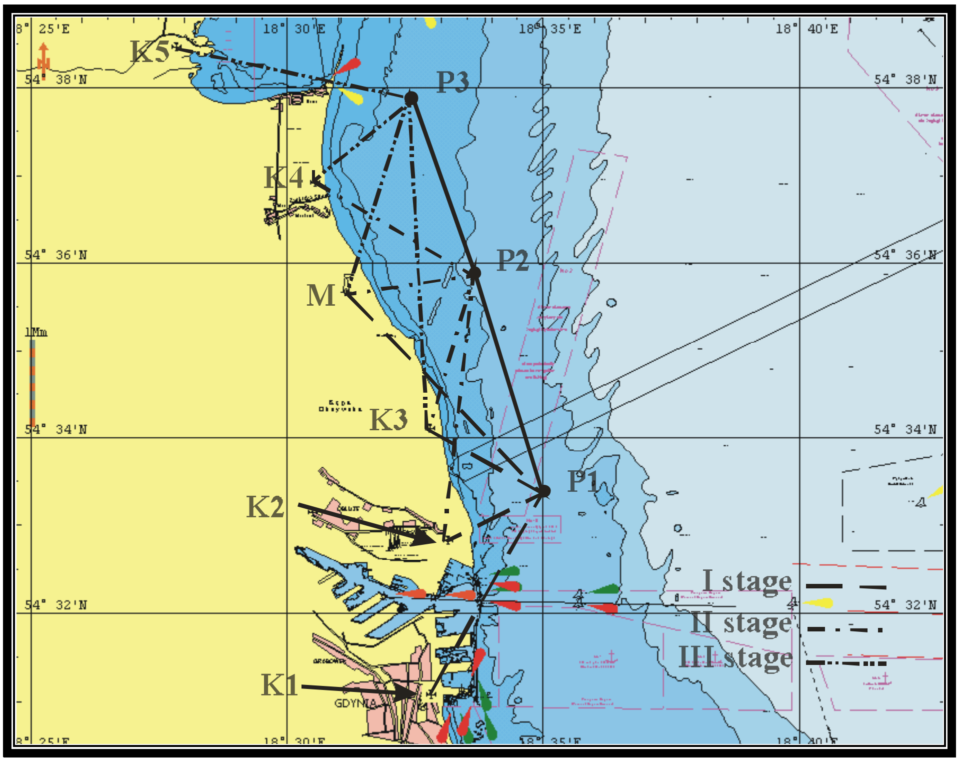

Fig. 1.1. Navigational task considered in [Czaplewski 2002b]

In the work [Czaplewski 2002a], applying the sequential method, on the basis of optical bearings, there has been determined the lattice mast K (here in after called the station K). Positions $P_1,...,P_4$ of the vessel, from which the observations were carried out, were determined basing on optical bearings taken toward the navigational signs $L_1,...,L_4$ (Fig.1.2., on the basis of the work [Czaplewski 2002a]).

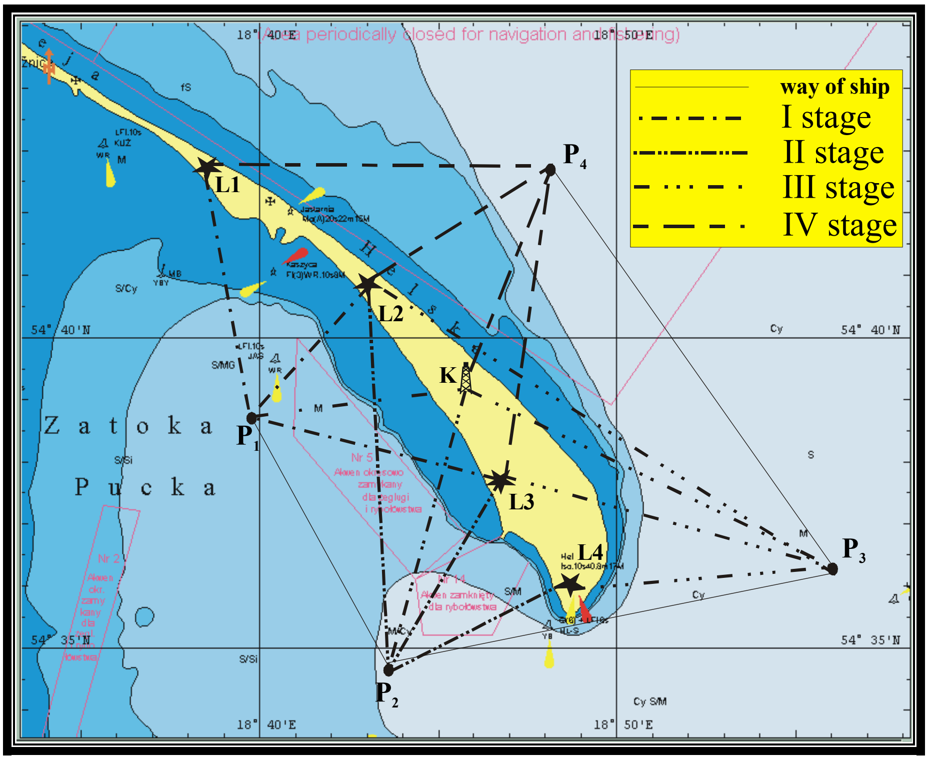

Fig. 1.2. Navigational task considered in [Czaplewski 2002a]

In the Navigational Structure being under development, there is also an opportunity (under some certain conditions, resulting from accuracy nature) of taking advantage of a vessel's route vector elements. The elements, by joining positions P_i, P_{i+1} may "consolidate" (strengthen) the obtained structure, mainly in respect of its reliability (sensitivity to inadmissible survey errors). Such consolidation of the navigational structure was applied in the works [Czaplewski, Wiśniewski 2003b; 2003c], also in [Czaplewski 2004a; 2004d]. The navigational structure, evaluated in the work [Czaplewski 2004b] is displayed in Fig. 1.3. (Z-navigational signs, R-the point complementary to the navigational system, P-proper position, determined basing on bearings toward signs Z).

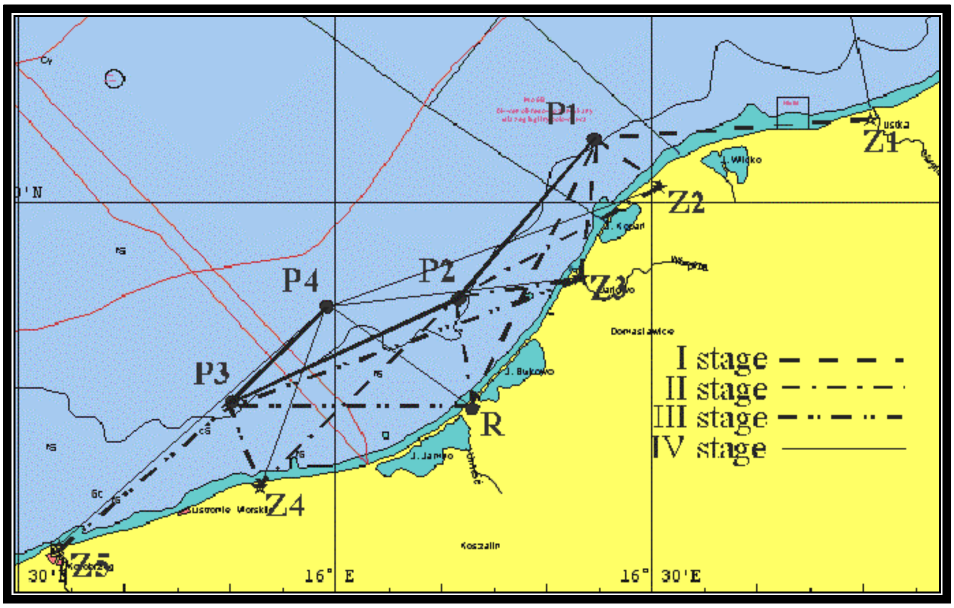

Fig. 1.3. Navigational task considered in [Czaplewski 2004d]

In the works [Czaplewski Wiśniewski 2003b, 2003c] there was undertaken a trial of supplementing the set of observations with the DGPS observations. The position P_i coordinates of a vessel, determined in this way, have been treated in the sequential process of the navigational structure adjustment as pseudo-observations characterized with the established covariance matrix [Czaplewski Wiśniewski 2003b] or only as the initial data in the structure development [Czaplewski Wiśniewski 2003c]. The navigational structures analyzed in the mentioned papers are presented in Fig. 1.4 (S_i-the navigational systems stations, R,T -the signs supplementary for the systems, P_i-the proper positions of the vessel determined on the basis of bearings toward the stations and on the basis of DGPS measurements).

The essential issue, even so not evaluated in this paper, is an optimization of navigational signs number and location, carried out from the viewpoint of the this way developed navigational system accessibility and the required navigation accuracies. The problem can be resolved applying the methods presented in the papers [Czaplewski 1999; Czaplewski, Wiśniewski 1999a,b]. The complex methodology, referring to optimal installation of navigational systems' stations, (following the example of the Quotient Navigational System), advised in the above mentioned publications, enables a possibility of finding out a required number and location of such stations, necessary for navigation of the vessels, positions of which have to be determined in conformity with the defined positioning accuracy requirements.

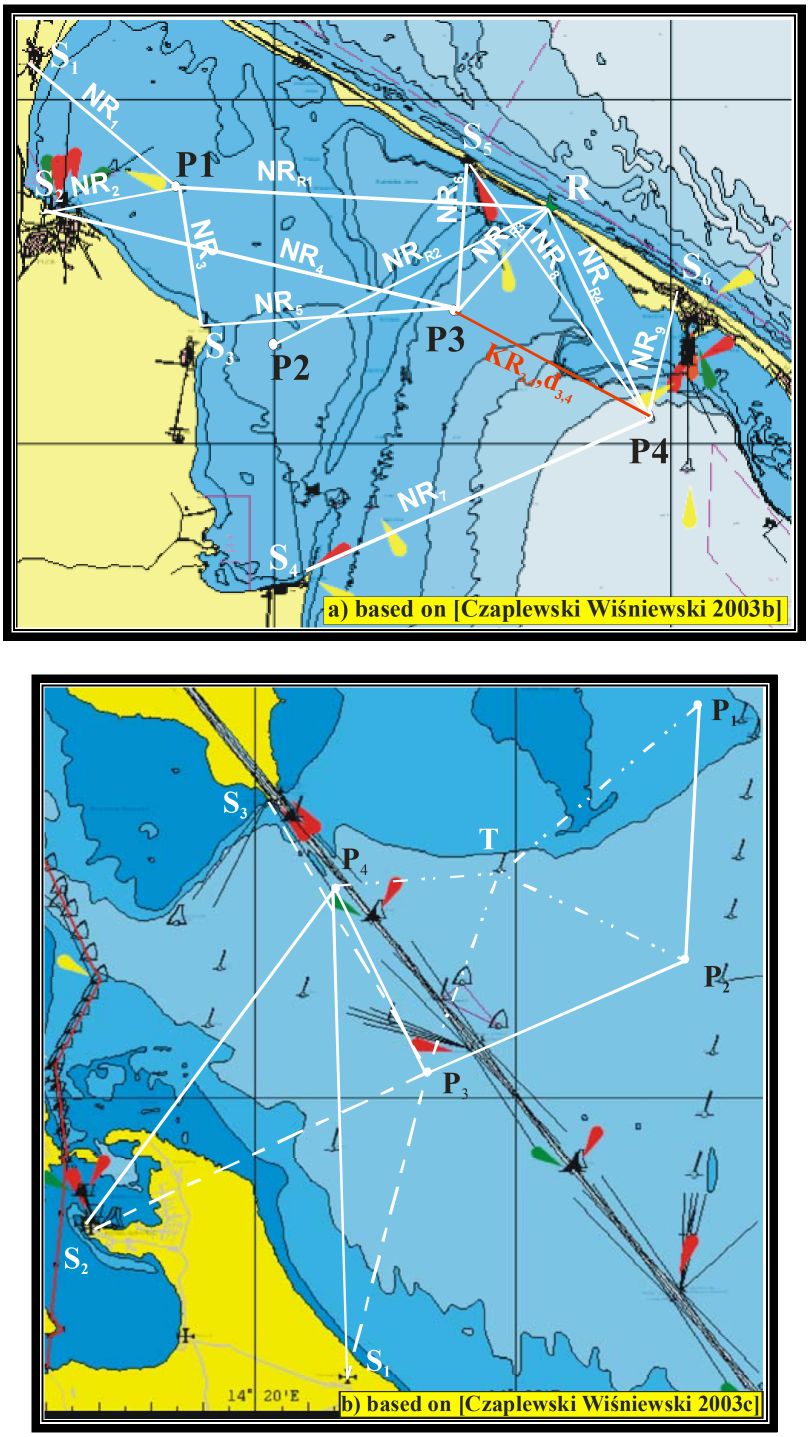

Fig. 1.4. Considered navigational tasks

Introduction of integral calculus and the simplexes method [Czaplewski 2000a, 2002c] in the designing works connected with arrangement of navigational marks indicates a way how to make optimal selection of new signs in the suggested navigational structure.

Another, extremely important problem of present-day navigation at sea (highly automated), is an eventuality of occurring gross errors biased observations in sets of observations. The errors, not containing in the intervals, admissible for random errors, are in literature of the subject called usually gross errors, and the observations biased with such the errors – outliers. [Kadaj 1984]. Sources of such errors are generally the errors, which occur upon identification of navigational signings, faults in data transmission, momentary disturbances in survey aids operation etc. In the instructions of international organizations, first of all in the resolutions of the International Maritime Organization (IMO) it is stated, that gross errors biased positions should be rejected and the concerned survey - repeated. However, due to the vessel continuous displacing, it is not possible to take the measurements again at the same, for some reasons significant point.

In such situations, aimed also at improvement of reliability of the IASN under development, in working it out, there should be applied the methods of robust estimation, coping with gross errors. A class of the coping- with- errors methods of robust adjustment is "generated" by, among the others, M -estimation, formulated applying specifically selected attenuation functions. Application of the so-called the function of attenuation (ex-potential) in radar navigation, was suggested in the works [Czaplewski 2003, 2004b,c,d; Czaplewski, Wąż 2004; Czaplewski, Wiśniewski 2003a]

Literature referring to the subject, also the briefly reported above results of the author's studies, prove raising the following possibilities in the present-day navigation:

- determination of proper positions by taking advantage of non homogeneous observations obtained on using various navigational systems jointly (i.e. DGPS, optical, radar observations etc.) and with consideration to the route vector elements as well;
- optimization of such positions (with accuracy evaluation) applying the contemporary estimation methods, the robust estimation (M -estimation) inclusive;
- development of the navigational structure in the fundamental navigation process;
- optimization of new points in this structure with applying sequential estimation (including the one, coping with gross errors).

The advanced navigation enables to determine the proper position of a vessel with a use of non homogeneous observations, obtained with different navigational systems. Thus, in the paper [Czaplewski 2004d] there was presented the opportunity of using jointly the observations obtained with DGPS and optical bearings, considering at the same time also the sailing vessel movement elements.

Besides, a try of exercising jointly the various navigational data (DGPS, optical and radar observations) in multi-variant navigational structure being subject to development, is contained in [Czaplewski, Wiśniewski 2003b,c].

The above indications have been the basis for the further propositions and generalizations, which stand for the main contents of this work and refer to, first of all, rules governing structuring and working out the geometrical survey structures of geo-navigation, supported with DGPS surveys. The suggested generalization of the geometrical measurement structure, developed on navigation process, is (as said before) the Interactive Navigational Structure. The descriptive and functional definition of this structure, mutual relationship between its main elements and basic principles of *IANS* development are presented in Chapter 2. As for reasons of the general assumptions made as concerns survey conditions, the *IANS* is a complete structure, a choice of its real variant, adjusted to a certain navigational situation, is made with a use of the decisive function. General assumptions connected with the function, treated as a specific case of the attenuation function applied in robust estimation, is this Chapter contents as well.

In Chapter 3 there has been formulated and resolved the adjustment task, concerning the Interactive Navigational Structure element. The element is constructed with stations (signs) of the navigational system, proper position of the vessel, newly determined points, complementary for the navigational system (or other points of navigational importance), as well as sets of terrestrial observations (e.g. of radar system) supplemented with DGPS measurements. The adjustment task has been formulated basing on the functional-decisive model of *IANS* element, the survey results covariance matrix model, in robust resolution substituted with an equivalent model (applying with decisive-equivalent weights matrix) and the target function of the least squares method. The adjustment task and its solutions are referred to the basic principles of robust *M*-estimation, capable to cope with survey gross errors. Due to a decisive character of the functional model, the equivalent weights matrix, applied in this estimation, was substituted with the decisive-equivalent matrix. Decisions on selection of the functional model's variant have been at this point realized by the decisive matrix, whereas robustness, it means capability of coping with gross errors has been obtained by applying the attenuation matrix. The product of the mentioned matrixes is the decisive-attenuation matrix, which makes a basis for, the advised in Chapter 3 function of the robust-decisive adjustment task target of the *IANS* chain element. The described Chapter has been finished with evaluation of special cases of the obtained general solution. In this evaluation there has been indicated also, which of them and in what cancellation, were a subject of the previous works of the author.

Two elements of *IANS,* connected with mutual observations (in the simplest, specific case they may be the route vector elements), create the *IANS* module. Formulating the decisive-functional model of such module, the decisive-equivalent statistic model and the resulting therefrom adjustment task (with its solution) is the basic contents of Chapter 4 in this work.

The solution presented in this Chapter is not only a theoretical basis for development of the *IANS* chain, is also determines technological conditions of such navigational task. Two elementary situations have been distinguished within this subject matter. In the first of them the *IANS* is developed by a singular watercraft, whereas insufficiently accurate elements of the route vector are neglected. instead, in the second case, the task is executed by two vessels which, apart from the principal surveys (supported with DGPS measurements), carry out the mutual observations as well. Then, for the final evaluation of the *IANS* parameters, the watercraft should exchange information about the values of the partial evaluations (carried out by stages). A range of such evaluations and a way of obtaining thereof are contained in Chapter 4 as well. Besides, there have been specified also the special cases of the achieved solution; the subject had been of the author's interest previously.

A result of developing *IANS* is the new signs which, in some certain situations, can become an intrinsic basis for objects navigation. Such signs positions are determined with no physical contact therewith, what may lead to mistakes in the signs descriptions. A user may get provided with such mistaken descriptions of other, similar and situated nearby objects, instead of navigational signs. In difficult conditions of navigation, as, for example, of submarines navigation, one may also expect mistaken identification of the products, even if no mistakes occur in their location. Referring to such extraordinary situations, however possible in practice, in Chapter 5 of this work there has been advised the object positioning method, in which not only the outlying observations are considered (gross errors biased survey), but also "outlying" (misidentified) adjustment points. The theoretical bases for this method are the principles of free, robust adjustment, supplemented with the decisive-equivalent observations weights matrix. In such a sense, the suggested solution may conventionally be treated as hybrid *M*-estimation.

The work is finished with Chapter 6, in which some numerical tests have been presented. Mainly they concern (not exhausting the entire set of eventual practical situations) those variants, which have not been a subject of evaluations carried out by the author before. The tests should be treated, first of all, as illustrations for the theoretical solutions described in the work. It seams that practical realization of *IANS* requires further analyses, especially of empirical character, anyhow including also the practical experiments. Nevertheless such sort of tests has not been a subject of this work, as its character remains mainly theoretical (still with consideration of the practical implementations).

2. INTERACTIVE NAVIGATIONAL STRUCTURE. THE BASIC ASSUMPTIONS.

The suggested Interactive Navigational Structure - if considered from the geometrical point of view - is created by a set of points $\mathfrak{Z} = \left\{ Z_j : j = 1,...,n_z \right\}$ with coordinates given in a certain configuration (e.g. (X, Y)), also subsets of determined points $\mathfrak{P} = \left\{ P_i : i = 1,...,n_p \right\}$ and $\mathfrak{R} = \left\{ R_1 : 1 = 1,...,n_R \right\}$. The $\mathfrak{Z}$ set can be created by optical navigation systems' signs, radionavigation systems' stations, elements of radar navigation systems, reference stations of DGPS system or stations of other navigation systems, known in the navigation theory (e.g. of the quotient navigational system). The determined points $\mathfrak{P}$ are specific positions P_i of a watercraft in motion or positions of a group of crafts, which carry on a common navigation task (e.g. hydrographic survey sweeping, fighting vessels task force formation etc.). The $\mathfrak{R}$ subset is created by the points, determined throughout P points, which, after fulfilling the settled requirements (especially within the accuracy scope) are to complement the set of adjustment $\mathfrak{Z}$.

Let's assume that within a certain navigational area and within a conventional stage (*k*) a set of points $\mathfrak{Z}^{(k)}$ is available. For some reasons, (for example other navigation tasks or widening the water area where navigation is carried out etc.) the set is insufficient to carry on navigation within the next following stage (*k+1*). Basing on the $\mathfrak{Z}^{(k)}$ set, there are determined the specific $\mathfrak{P}^{(k)}$ positions, and throughout them, the coordinates of new $\mathfrak{R}^{(k)}$ points. Thus, within the (k+1) stage there is available a set of adjustment points $\mathfrak{Z}^{(k+1)} = \left\{ \mathfrak{Z}^{(k)}, \mathfrak{R}^{(k)} \right\}$ (or possibly fill up additional systems stations).

A chain of Interactive Navigational Structure may have many links, unless at each of them the settled accuracy criteria are fulfilled). Generally, the relations between the elements of such chain are presented in Fig. 2.1.

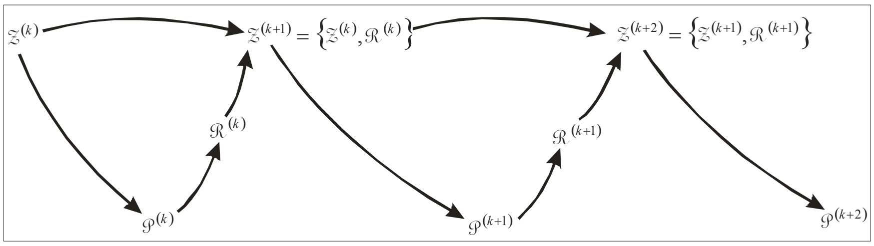

Fig. 2.1. Conception of the Interactive Navigational Structure.

The sets and subsets discriminated above, are joined into a common observational arrangement by the geometrical quantities, which are subject to survey and, for some navigational systems, also by the physical quantities (for example – for the quotient navigational system – a run time of carrier wave in water environment).

Let us assume that determination of a specific position P_i (element of $\mathcal{P}$ set) is carried out through bearings *NR* and distances *d* from the point P_i to some points of $\mathcal{Z}$ set (for example with a use of radar and gyroscope systems). The position P_i may also be determined (intrinsically or parallel to the survey described above), basing on the satellite navigation systems, as, for example, DGPS. Moreover, with expectation that from the position P_i there is to be carried out a bearing of point R (or several such points covered by $\mathcal{R}$ point) and a distance thereto is measured, there is achieved an observational arrangement (Fig.2.2) which stands for intrinsic, basic *IANS* element.

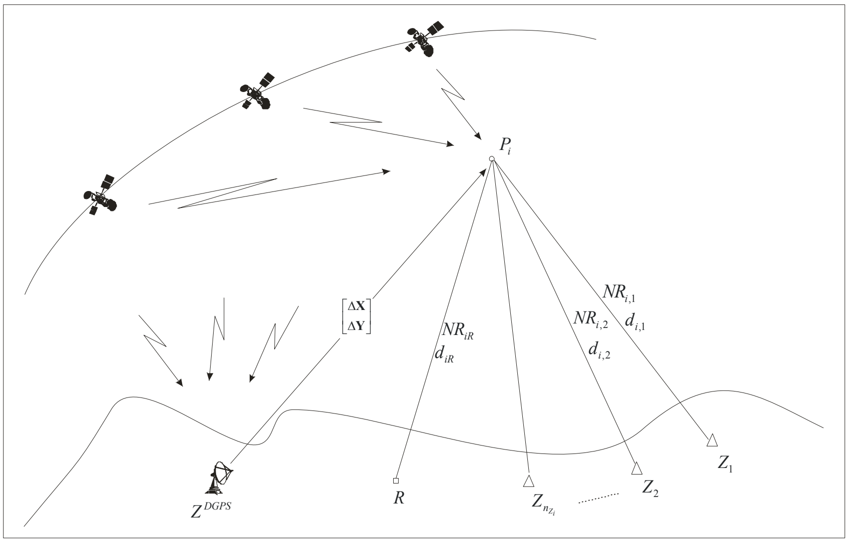

Fig. 2.2 Basic *IANS* element.

The interactive character of a navigational structure including, first of all, an assumed possibility of „transferring" points $\Re$ to $\mathfrak{Z}$ set, is demanding creation of such observational arrangement, which, on one hand, meets the stipulation concerning *IANS* area development and on the other hand, enables carrying out the obtained determinations control. A singular *IANS* element fails to meet those requirements. In such an arrangement the points $\Re$ are, at the utmost, uniquely determinable, thus unable to carry out reliable estimation of its positioning accuracy. The *IANS* element, as an intrinsic navigational structure, is also slightly robust to significant survey errors (rejection of one observation because of that reason, may result, in some cases, in non determinability of point R or even P_i position). A solution which considerably eliminates such sorts of inconvenience is connecting basic elements into the *IANS* chain. The observations to connect are in such case the observations, which refer to the $\Re$ points'coordinates and the elements in two vessels arrangement, also mutual bearings and mutual distances. A fragment of the Interactive Navigational Structure chain is presented in Fig. 2.3.

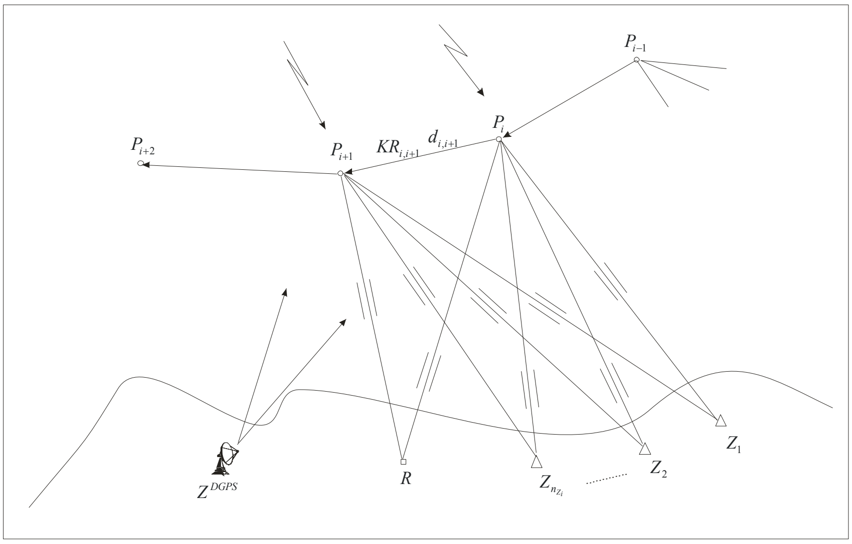

Fig. 2.3 Interactive Navigational Structure Modulus

The Navigational Structure, presented in this study, is of interactive character and apart of the above, integrates different types of available information (bearings, distances, path vector elements, DGPS measurements). One should expect that only some variants of mutual connections of $\mathfrak{Z}, \wp, \Re$ sets' elements will be used in practical application. It is also predicted, (as described in the part of the study below) that even if any of the observations are practically executed, due to biasing thereof with major errors, they should also be rejected or attenuated applying any justified way. Thus, in general, let's accept that t functions, assuming values of $\langle 0;1 \rangle$ interval, are subordinated to the observations and coordinates of the points.

The functions assume the extreme values when the observations (coordinates) are not taken into the commonly worked out observational arrangement $(t = 0)$ or a part thereof is full $(t = 1)$. However, if from some reason the observations are only damped, then $(0 < t < 1)$. The t functions, of such the general properties, in reference to the robust estimation principles, are called in the subject' literature the attenuation functions (e.g. [Hampel, Ronchetti, Rousseeuw, Stahel 1986; Yang, Cheng, Chum, Tampley 1999]). A particular case of the attenuation function t may be a such double value decisive function ι, that (more detailed relations between the functions t and ι are presented in chapter 3):

$$\iota(s) = \begin{cases} 1 & \text{if } s \text{ is acceptable} \\ 0 & \text{if } s \text{ is rejected} \end{cases}$$

where: s – is an optional element of the points or observations set.

Description of the suggested Interactive Navigational Structure will simplify generalization of the decisive function ι. The subsets of $\mathcal{Z}, \mathcal{Z}^{DGPS}, \mathcal{P}$ points and subsets of the observations discriminated above are the arguments for this generalization. Let's assume that $\mathcal{O}^Z$ is a subset of the observations carried out from the points $\mathcal{P}$ towards the adjustment points $\mathcal{Z}$ (aimed at determining positions $\mathcal{P}^Z$ of the points $\mathcal{P}$), $\mathcal{O}^P$ is a subset of mutual observations between the points $\mathcal{P}$ (for example: the path vector elements), $\mathcal{O}^{DGPS}$ is the set of the observations DGPS (on the basis thereof there are determined alternative or intrinsic positions $\mathcal{P}^{DGPS}$ of $\mathcal{P}$ points), whereas $\mathcal{O}^R$ is a set of the observations carried out at the points $\mathcal{P}$ towards newly determined points $\mathcal{R}$. It is assumed that generalization of the function ι having s argument proceeding through the elements (subsets) of the following set:

$$\mathcal{S} = \left\{ \mathcal{O}^Z, \mathcal{O}^P, \mathcal{O}^{DGPS}, \mathcal{O}^R, \mathcal{Z}, \mathcal{Z}^{DGPS}, \mathcal{P} \right\}$$

with, in general, $\mathcal{P} = \left\{ \mathcal{P}^Z, \mathcal{P}^{DGPS} \right\}$, is a function

$$\mathcal{I}_{\mathcal{S}}(s) = \begin{cases} 1 & \text{if in the subset } s \text{ the acceptable elements does exist} \\ 0 & \text{if } s \text{ is an empty set or all its elements are unacceptable} \end{cases}$$

It is worth to add, that $\mathcal{S}$ is a set which corresponds to the complete (in accordance with the assumptions made as yet) structure; however it enables to select one of its variants, adjusted to a specific navigational task.

An example thereof based on the adjustment points $\mathcal{Z}$ and observations $\mathcal{O}^Z$, positions $\mathcal{P}^Z$ of the points $\mathcal{P}$, is shown in the logical diagram presented in Fig. 2.4.

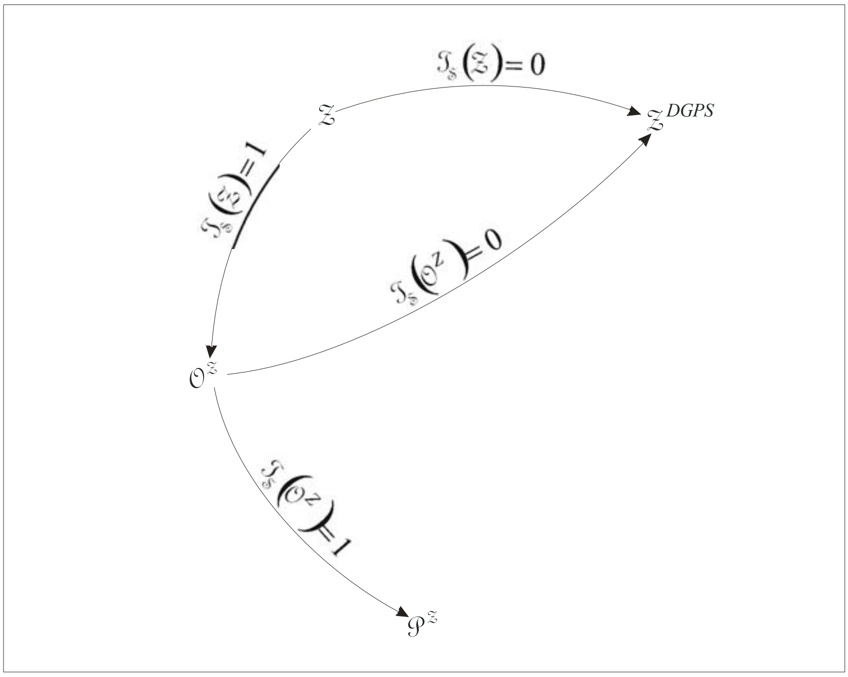

Fig. 2.4. The selected variant of the decisive function application in *IANS*

In the example, a condition of determination of the position $\mathcal{P}^Z$ of the points $\mathcal{P}$ is acceptance for the sets $\mathcal{Z}$ and $\mathcal{O}^Z$ (or at least some elements of these sets, sufficient for determination $\mathcal{P}^Z$, elements of these sets). The situation when $\mathcal{F}_{\mathcal{S}}(\mathcal{Z}) = 0$ or $\mathcal{F}_{\mathcal{S}}(\mathcal{O}^Z) = 0$, is forcing to choose an alternative way. According to the assumptions made in this work, the starting point for this path is a set of reference stations $\mathcal{Z}^{DGPS}$. Its accessibility, also technical ability for execution of the set of observations $\mathcal{O}^{DGPS}$, allows to develop *IANS*, to achieve the form presented in Fig. 2.5.

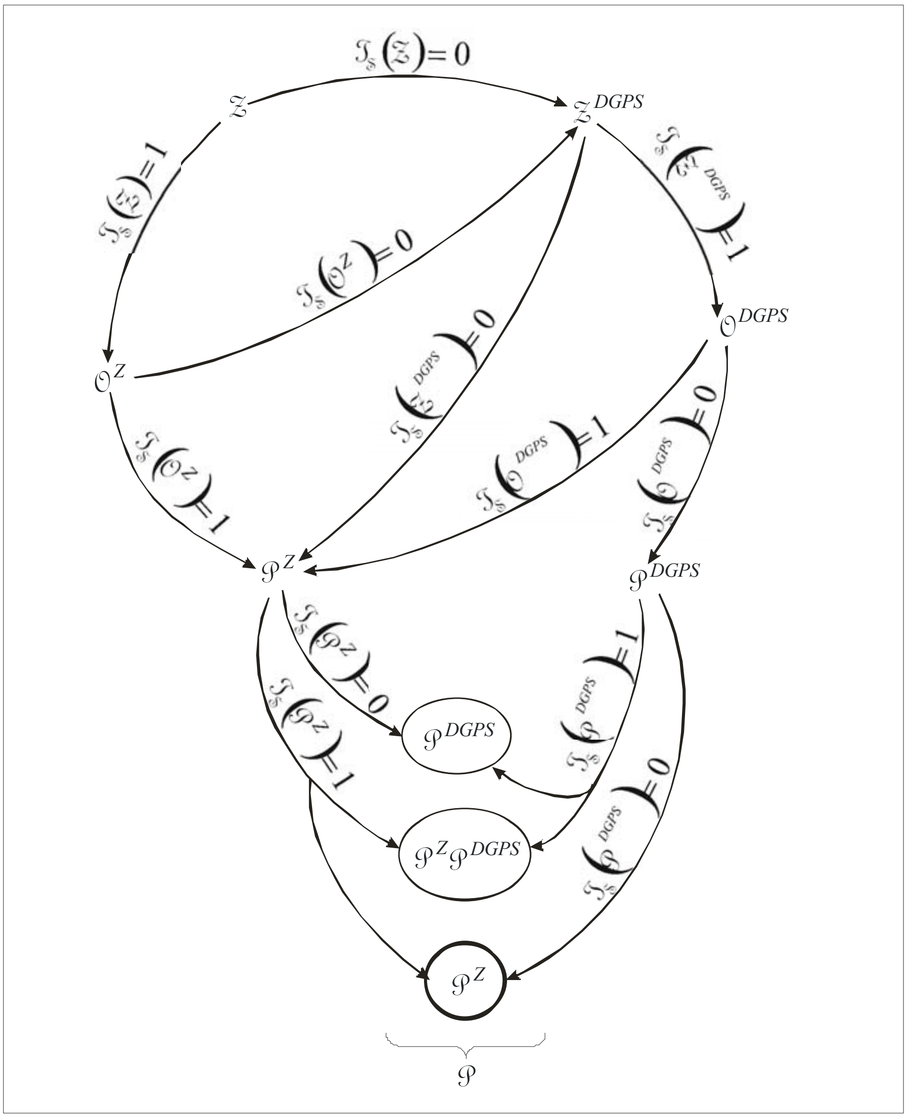

Fig. 2.5. The possible *IANS* development in (k) stage

Let's presume at this moment, that according to the assumptions made before, *IANS* is still developed through newly determined points $\Re$. After fulfilment of the set up criteria, what in convention of the function $\Im_1$ stands for meeting the condition $\Im_1(\Re)=1$, the points, in the successive stage $(k+1)$, will become complementary for the previous (for *k* stage) set of the adjustment points. A diagram of such development, which at the same time is a supplement for the logical diagram presented in Fig. 2.5., is shown in Fig. 2.6.

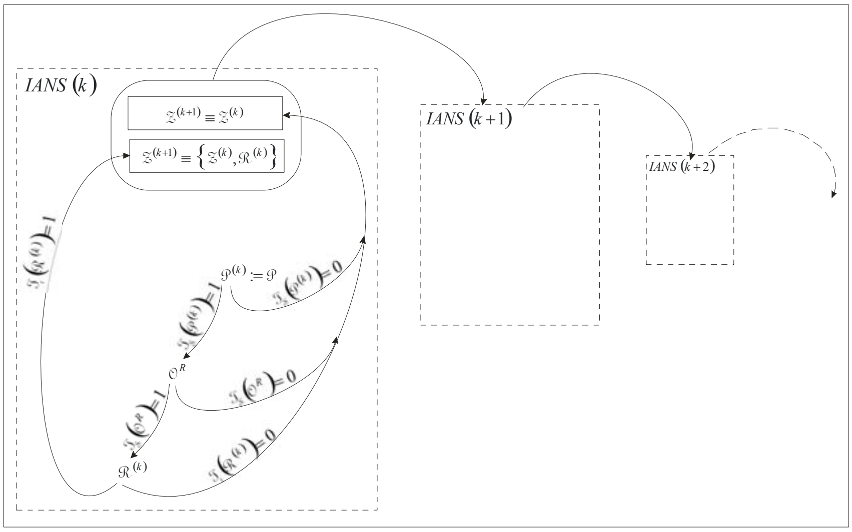

Fig. 2.6. Possible *IANS* development in $(k+1)$ stage

3. BASIC ELEMENT OF *IANS*

3.1. Basic Assumptions and Models

Let us assume, that at the $P_i \in \mathcal{P}$ position of the vessel, starting to create *IANS*, the sets of points $\mathcal{Z}_i \subset \mathcal{Z}$ and reference stations of DGPS system $\mathcal{Z}_i^{DGPS} \subset \mathcal{Z}^{DGPS}$ are available. The bearings and distances to the points Z_i, and additionally to the points $R_1,\ldots,R_{n_{R_i}}$, which form the set $\mathfrak{R}_i \subset \mathfrak{R}$, are carried out at the point P_i. After the settled criteria are fulfilled, the above mentioned points shall become complementary for the previously formed set $\mathcal{Z}$, supporting the further process of navigation. Let us also assume, that on the basis of the set $\mathcal{Z}_i^{DGPS}$ and the results of $\mathcal{O}^{DGPS}$ DGPS survey (however with the set's structure details omitted), the coordinates $X_{P_i}^{DGPS}, Y_{P_i}^{DGPS}$ of the point P_i are known as well. The undertaken, elementary navigational situation is displayed in Fig. 3.1.

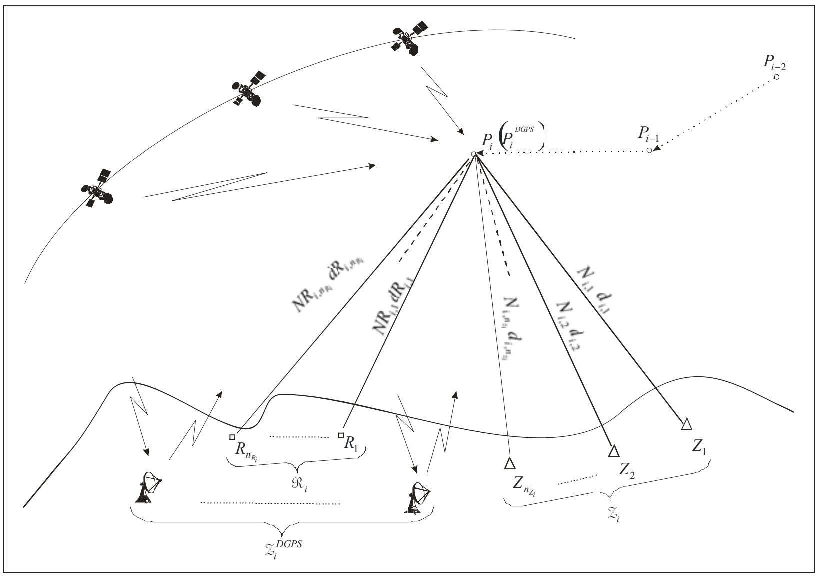

Fig.3.1. The first stage of creating *IANS* within the *i* moment

And now let's assume that the navigational structure, described above, corresponds with the following sets of observations:

$$\mathcal{O}_i^{\mathcal{Z}} = \Big\{ \underbrace{N_{i,1}, N_{i,2}, ..., N_{i,n_{Z_i}}}_{\text{bearings to } Z_i \text{ - points}}, \underbrace{d_{i,1}, d_{i,2}, ..., d_{i,n_{Z_i}}}_{\text{distances to } Z_i \text{ - points}} \Big\}$$

$$\mathcal{O}_i^{\mathcal{R}(i)} = \Big\{ \underbrace{NR_{i,1}^{(i)}, NR_{i,2}^{(i)}, ..., NR_{i,n_R}^{(i)}}_{\substack{\text{bearings to } R_i \text{ - points} \\ \text{from } P_i \text{ - position}}}, \underbrace{dR_{i,1}^{(i)}, dR_{i,2}^{(i)}, ..., dR_{i,n_R}^{(i)}}_{\substack{\text{distances to } R_i \text{ - points} \\ \text{from } P_i \text{ - position}}} \Big\}$$

Basing on the above sets, the following survey results vectors can be created:

$$\boldsymbol{x}_{Z_i} = \left[N_{i,1}, N_{i,2}, ..., N_{i,n_{Z_i}}, d_{i,1}, d_{i,2}, ..., d_{i,n_{Z_i}} \right]^T$$

$$\boldsymbol{x}_{R_i}^{(i)} = \left[NR_{i,1}^{(i)}, NR_{i,2}^{(i)}, ..., NR_{i,n_R}^{(i)}, dR_{i,1}^{(i)}, dR_{i,2}^{(i)}, ..., dR_{i,n_R}^{(i)} \right]^T$$

(in some navigational systems the vectors $\boldsymbol{x}_{Z_i}, \boldsymbol{x}_{R_i}^{(i)}$ stand for the basis for creation of pseudo-observation vectors in the quotient system e.g. [Czaplewski 1998, Kołaczyński 1995]).

According to the assumptions made previously, each of the observations set elements $\mathcal{O}_i = \left\{ \mathcal{O}_i^{\mathcal{Z}}, \mathcal{O}_i^{\mathcal{R}(i)} \right\}$ is an argument of the two-value decisive function $\iota(s), s \in \mathcal{O}_i$. A set of this function values can be presented in a form of the following diagonal decisive matrix:

$$\mathfrak{I}(\boldsymbol{x}_i) = \mathrm{Diag}\left\{ \mathfrak{I}(\boldsymbol{x}_{Z_i}), \mathfrak{I}(\boldsymbol{x}_{R_i}^{(i)}) \right\}$$

where:

$$\mathfrak{I}(\boldsymbol{x}_{Z_i}) = Diag\left\{ \iota\left(N_{i,1}\right), ..., \iota\left(N_{i,n_{Z_i}}\right), \iota\left(d_{i,1}\right), ..., \iota\left(d_{i,n_{Z_i}}\right) \right\}$$

$$\mathfrak{I}(\boldsymbol{x}_{Z_i}) = Diag\left\{ \iota\left(N_{i,1}\right), ..., \iota\left(N_{i,n_{Z_i}}\right), \iota\left(d_{i,1}\right), ..., \iota\left(d_{i,n_{Z_i}}\right) \right\}$$

$$\boldsymbol{x}_i = \left[\boldsymbol{x}_{Z_i}^T, \boldsymbol{x}_{R_i}^{(i)T} \right]^T$$

In the discussed navigational task, the elements sought are coordinates $\mathbf{X}_{P_i} = \left[X_{P_i}, Y_{P_i} \right]^T$ of the point P_i and coordinates $\mathbf{X}_{R_i} = \left[X_{R_1}, Y_{R_1}, ..., X_{R_{nR_i}}, Y_{R_{nR_i}} \right]^T$ (the coordinate system (X, Y) is adopted to simplify further considerations, accepting thereby transformations of systems, required in such situations). Let's assume, that $\hat{\mathbf{X}}_{P_i} = \left[\hat{X}_{P_i}, \hat{Y}_{P_i} \right]^T$ is an estimator of the coordinates $\mathbf{X}_{P_i}$. Whereas the vector $\hat{\mathbf{X}}^{(i)}_{R_i} = \left[\hat{X}^{(i)}_{R_1}, \hat{Y}^{(i)}_{R_1}, ..., \hat{X}^{(i)}_{R_{nR_i}}, \hat{Y}^{(i)}_{R_{nR_i}} \right]^T$ is an estimator of the coordinates $\mathbf{X}_{R_i}$ of the points $\Re = \left\{ R_1, ..., R_{nRi} \right\}$, obtained basing on the observations, carried out at the position P_i.

Moreover (e.g. [Baran 1999], [Wiśniewski 2000, 2002]:

$$\left.\begin{aligned} \boldsymbol{x}_{\boldsymbol{Z_i}} &= \bar{\boldsymbol{x}}_{\boldsymbol{Z_i}} + \varepsilon_{\boldsymbol{Z_i}} \\ \boldsymbol{x}^{(i)}_{\boldsymbol{R_i}} &= \bar{\boldsymbol{x}}^{(i)}_{\boldsymbol{R_i}} + \varepsilon^{(i)}_{\boldsymbol{R_i}} \end{aligned}\right\} \rightarrow \left\{\begin{aligned} \mathbf{V}_{\boldsymbol{Z_i}} &= \hat{\bar{\boldsymbol{x}}}_{\boldsymbol{Z_i}} - \boldsymbol{x}_{\boldsymbol{Z_i}} \\ \mathbf{V}^{(i)}_{\boldsymbol{R_i}} &= \hat{\bar{\boldsymbol{x}}}^{(i)}_{\boldsymbol{R_i}} - \boldsymbol{x}^{(i)}_{\boldsymbol{R_i}} \end{aligned}\right.$$

where: $\mathbf{V}_{Z_i} = -\hat{\varepsilon}_{Z_i}, \mathbf{V}^{(k)}_{R_i} = -\hat{\varepsilon}^{(k)}_{R_i}$ are estimations of survey errors vectors,

$\varepsilon_{Z_i}, \varepsilon^{(k)}_{R_i}$ - corrections vectors, whereas $\bar{\boldsymbol{x}}$ - a vector of true measured quantities values,

$\hat{\bar{\boldsymbol{x}}}$ - an estimator of $\bar{\boldsymbol{x}}$ vector.

Then if

$$\begin{aligned} \bar{\boldsymbol{x}}_{\boldsymbol{Z_i}} &= \mathbf{F}_{\boldsymbol{Z_i}}\left(\mathbf{X}_{P_i}, \mathbf{X}_{R_i}\right) \\ \bar{\boldsymbol{x}}^{(i)}_{\boldsymbol{R_i}} &= \mathbf{F}^{(i)}_{\boldsymbol{R_i}}\left(\mathbf{X}_{P_i}, \mathbf{X}_{R_i}\right) \end{aligned}$$

so

$$\left.\begin{aligned} \mathbf{V}_{Z_i} &= \mathbf{F}_{Z_i}(\hat{\mathbf{X}}^{(i)}_{P_i}, \hat{\mathbf{X}}^{(i)}_{R_i}) \qquad - \boldsymbol{x}_{\boldsymbol{Z_i}} \\ \mathbf{V}^{(i)}_{R_i} &= \mathbf{F}^{(i)}_{R_i}(\hat{\mathbf{X}}^{(i)}_{P_i}, \hat{\mathbf{X}}^{(i)}_{R_i}) \qquad - \boldsymbol{x}^{(i)}_{R_i} \end{aligned}\right\} \tag{3.1}$$

which is an a posteriori model*) o fan adjustment task in an elementary navigational problem.

---

*) Due to the solutions applied below (sequential estimation) in functional models notation and related adjustment tasks, instead of **X** parameters there will be used their estimators $\hat{\mathbf{X}}$ (a posteriori models).

Due to the non-linear character of the vector functions' elements $\mathbf{F}_Z$ and $\mathbf{F}_R$, (bearings and distances are the quantities measured) we may expand them into Taylor's series, limited to the first (linear) terms. Upon carrying on these resolving within the neighbourhoods $\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}}, \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}$ of points $\mathbf{X}^0_{P_i}, \mathbf{X}^0_{R_i}$, we obtain as follows:

$$\left.\begin{aligned}\mathbf{F}_{Z_i}(\hat{\mathbf{X}}^{(i)}_{P_i},\hat{\mathbf{X}}^{(i)}_{R_i}) &= \mathbf{F}_{Z_i}(\mathbf{X}^0_{P_i},\mathbf{X}^0_{R_i}) + \partial_{\mathbf{X}_{P_i}}\mathbf{F}_{Z_i}(\mathbf{X}^0_{P_i},\mathbf{X}^0_{R_i})\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} + \partial_{\mathbf{X}_{R_i}}\mathbf{F}_{Z_i}(\mathbf{X}^0_{P_i},\mathbf{X}^0_{R_i})\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}} \\ \mathbf{F}^{(i)}_{R_i}(\hat{\mathbf{X}}^{(i)}_{P_i},\hat{\mathbf{X}}^{(i)}_{R_i}) &= \mathbf{F}^{(i)}_{R_i}(\mathbf{X}^0_{P_i},\mathbf{X}^0_{R_i}) + \partial_{\mathbf{X}_{P_i}}\mathbf{F}^{(i)}_{R_i}(\mathbf{X}^0_{P_i},\mathbf{X}^0_{R_i})\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} + \partial_{\mathbf{X}_{R_i}}\mathbf{F}^{(i)}_{R_i}(\mathbf{X}^0_{P_i},\mathbf{X}^0_{R_i})\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}\end{aligned}\right\} \quad (3.2)$$

(elements of the vectors $\mathbf{X}^0_{P_i}, \mathbf{X}^0_{R_i}$ are approximated coordinates of the points P_i and $\Re_i$). Through implementation of the following designations:

$$\partial_{\mathbf{X}_{P_i}}\mathbf{F}_{Z_i}(\mathbf{X}^0_{P_i},\mathbf{X}^0_{R_i}) = \mathbf{A}_{Z_iP_i} \in \mathfrak{M}\left(2n_{Z_i},2\right) \quad \partial_{\mathbf{X}_{R_i}}\mathbf{F}_{Z_i}(\mathbf{X}^0_{P_i},\mathbf{X}^0_{R_i}) = \mathbf{A}_{Z_iR_i} \in \mathfrak{M}\left(2n_{Z_i},2n_{R_i}\right)$$

$$\partial_{\mathbf{X}_{P_i}}\mathbf{F}^{(i)}_{R_i}(\mathbf{X}^0_{P_i},\mathbf{X}^0_{R_i}) = \mathbf{A}_{R_iP_i} \in \mathfrak{M}\left(2n_{R_i},2\right) \quad \partial_{\mathbf{X}_{R_i}}\mathbf{F}^{(i)}_{R_i}(\mathbf{X}^0_{P_i},\mathbf{X}^0_{R_i}) = \mathbf{A}_{R_iR_i} \in \mathfrak{M}\left(2n_{R_i},2n_{R_i}\right)$$

the model (3.1) can be presented in the following form:

$$\left.\begin{aligned}\mathbf{V}_{Z_i} &= \mathbf{A}_{Z_iP_i}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} + \mathbf{A}_{Z_iR_i}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}} + \mathbf{L}_{Z_i} \\ \mathbf{V}^{(i)}_{R_i} &= \mathbf{A}_{R_iP_i}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} + \mathbf{A}_{R_iR_i}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}} + \mathbf{L}^{(i)}_{R_i}\end{aligned}\right\} \quad (3.3)$$

where:

$$\mathbf{L}_{Z_i} = \mathbf{F}_{Z_i}(\mathbf{X}^0_{P_i},\mathbf{X}^0_{R_i}) - \boldsymbol{x}_{Z_i}$$

$$\mathbf{L}^{(i)}_{R_i} = \mathbf{F}^{(i)}_{R_i}(\mathbf{X}^0_{P_i},\mathbf{X}^0_{R_i}) - \boldsymbol{x}^{(i)}_{R_i}$$

are the vectors of free terms ($\mathfrak{M}_{(a,b)}$ - the set of real matrixes of $a \times b$ dimensions).

The equations system (3.3) is a functional model of an adjustment task in elementary navigational problem, in the paper considered to be as an autonomous *IANS* element. However, it is not a complete model in the situation when the coordinates $\left(X_i^{DGPS}, Y_i^{DGPS}\right)$ of the point P_i are available. Let us make the most general assumption, that the result of handling data $\mathcal{O}^{DGPS}$ referred to the set $\mathcal{Z}_i^{DGPS}$ is a vector of coordinates $\mathbf{X}_{P_i}^{DGPS} = \begin{bmatrix} X_{P_i}^{DGPS} & , Y_{P_i}^{DGPS}\end{bmatrix}^T$ of the point P_i of the covariance matrix $\mathbf{C}_{X_i^{DGPS}}$. As it has been previously assumed that $\mathcal{P}^{DGPS}$ is an argument of the decisive function $\Im$, so the vector $\mathbf{X}_{P_i}^{DGPS}$ is also an argument of the decisive matrix $\Im\left(\mathbf{X}_{P_i}^{DGPS}\right)$. The coordinates $\mathbf{X}_{P_i}^{DGPS}$, as in the study [Czaplewski, Wiśniewski 2003b] and in conformity with the basic principles of sequence adjustment (e.g. [Sikorski 1979, 1991]), shall still be treated as a pseudo-observation of the weights matrix:

$$\mathbf{P}_{X_i^{DGPS}} = \mathbf{Q}^{-1}_{X_i^{DGPS}} \tag{3.4}$$

where $\mathbf{Q}_{X_i^{DGPS}}$ - is the cofactors matrix, represented in the model:

$$\mathbf{C}_{X_i^{DGPS}} = \sigma_0^2 \mathbf{Q}_{X_i^{DGPS}} \tag{3.5}$$

of the coordinates covariance matrix $\mathbf{X}_{P_i}^{DGPS}$. Thus, in case $\hat{\mathbf{X}}_{P_i}$ is an ultimate estimator of the point P_i (in *IANS* element) coordinates vector, which is a function of both – the direct observations $\boldsymbol{x}_i$ and coordinates $\mathbf{X}_{P_i}^{DGPS}$, then the non zero residuum is to be expected:

$$\left(\hat{\mathbf{X}}_{P_i}^{(i)} = \mathbf{X}_{P_i}^{DGPS} + \mathbf{V}_{X_i^{DGPS}}\right) \Rightarrow \mathbf{V}_{X_i^{DGPS}} = \hat{\mathbf{X}}_{P_i}^{(i)} - \mathbf{X}_{P_i}^{DGPS} \tag{3.6}$$

but because $\hat{\mathbf{X}}_{P_i}^{(i)} = \mathbf{X}_{P_i}^0 + \hat{\mathbf{d}}_{X_{P_i}}^{(i)}$, so

$$\mathbf{V}_{X_i^{DGPS}} = \hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)} + \mathbf{X}_{P_i}^0 - \mathbf{X}_{P_i}^{DGPS} \tag{3.7}$$

By complementing the system (3.3) with the equation (3.7), the complete (in relation to the general assumptions adopted in the chapter 2) functional model of the adjustment task in *IANS* element is obtained in the form as follows:

$$\left.\begin{aligned}
\mathbf{V}_{Z_i} &= \mathbf{A}_{Z_iP_i}\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)} + \mathbf{A}_{Z_iR_i}\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)} + \mathbf{L}_{Z_i} \\
\mathbf{V}_{R_i}^{(i)} &= \mathbf{A}_{R_iP_i}\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)} + \mathbf{A}_{R_iR_i}\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)} + \mathbf{L}_{R_i}^{(i)} \\
\mathbf{V}_{X_i^{DGPS}} &= \hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)} + \mathbf{X}_{P_i}^0 - \mathbf{X}_{P_i}^{DGPS}
\end{aligned}\right\} \tag{3.8}$$

To each of three system equations (3.8) there is subordinated the respective decisive matrix. Process of acceptance or rejection of one of them (as a whole or only of some of its equations), from the functional model viewpoint, is to be treated as multiplying the equations by the matrix $\Im$, it means

$$\left.\begin{array}{ll}\mathfrak{I}(\boldsymbol{x}_{Z_i})\cdot & \left(\mathbf{V}_{Z_i} = \mathbf{A}_{Z_iP_i}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} + \mathbf{A}_{Z_iR_i}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}} + \mathbf{L}_{Z_i}\right)\\ \mathfrak{I}(\boldsymbol{x}^{(i)}_{R_i})\cdot & \left(\mathbf{V}^{(i)}_{R_i} = \mathbf{A}_{R_iP_i}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} + \mathbf{A}^{(i)}_{R_iR_i}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}} + \mathbf{L}^{(i)}_{R_i}\right)\\ \mathfrak{I}(\mathbf{X}^{DGPS}_i)\cdot & \left(\mathbf{V}_{\mathbf{X}^{DGPS}_i} = \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} + \mathbf{X}^0_{P_i} - \mathbf{X}^{DGPS}_{P_i}\right)\end{array}\right\} \tag{3.9}$$

or

$$\left.\begin{array}{ll}\mathfrak{I}(\boldsymbol{x}_i)\cdot & \left(\mathbf{V}_{\boldsymbol{x}_i} = \mathbf{A}_{P_i}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} + \mathbf{A}^{(i)}_R\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}} + \mathbf{L}_{\mathbf{x}_i}\right)\\ \mathfrak{I}(\mathbf{X}^{DGPS}_i)\cdot & \left(\mathbf{V}_{\mathbf{X}^{DGPS}_i} = \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} + \mathbf{X}^0_{P_i} - \mathbf{X}^{DGPS}_{P_i}\right)\end{array}\right\} \tag{3.10}$$

where:

$$\mathbf{V}_{\boldsymbol{x}_i} = \begin{bmatrix}\mathbf{V}_{Z_i}\\ \mathbf{V}^{(i)}_{R_i}\end{bmatrix} \in \mathfrak{M}(n_i,1) \qquad \mathbf{A}_{P_i} = \begin{bmatrix}\mathbf{A}_{Z_iP_i}\\ \mathbf{A}_{R_iP_i}\end{bmatrix} \in \mathfrak{M}(n_i,2)$$

$$\mathbf{A}^{(i)}_R = \begin{bmatrix}\mathbf{A}_{Z_iR_i}\\ \mathbf{A}^{(i)}_{R_iR_i}\end{bmatrix} \in \mathfrak{M}(n_i,2n_{R_i}) \qquad \mathbf{L}_{\boldsymbol{x}_i} = \begin{bmatrix}\mathbf{L}^T_{Z_i} & \mathbf{L}^T_{R_i}\end{bmatrix}^T$$

$n_i = 2(n_{Z_i} + n_{R_i})$ - a number of observations

$r_i = 2 + 2n_{R_i}$ - a number of unknowns

The following statistical model (covariance matrix model) is being subordinated to the functional – decisive model (3.10):

$$\left.\begin{array}{l}\mathbf{C}_{\boldsymbol{x}_{Z_i}} = \sigma_0^2\mathbf{Q}_{\boldsymbol{x}_{Z_i}} = \sigma_0^2\mathbf{P}^{-1}_{\boldsymbol{x}_{Z_i}}\\ \mathbf{C}_{\boldsymbol{x}^{(i)}_{R_i}} = \sigma_0^2\mathbf{Q}_{\boldsymbol{x}^{(i)}_{R_i}} = \sigma_0^2\mathbf{P}^{-1}_{\boldsymbol{x}^{(i)}_{R_i}}\\ \mathbf{C}_{\mathbf{X}^{DGPS}_i} = \sigma_0^2\mathbf{Q}_{\mathbf{X}^{DGPS}_i} = \sigma_0^2\mathbf{P}^{-1}_{\mathbf{X}^{DGPS}_i}\end{array}\right\} \Leftrightarrow \begin{cases}\mathbf{C}_{\boldsymbol{x}_i} = \sigma_0^2\mathbf{Q}_{\boldsymbol{x}_i} = \sigma_0^2\mathbf{P}^{-1}_{\boldsymbol{x}_i}\\ \mathbf{C}_{\mathbf{X}^{DGPS}_i} = \sigma_0^2\mathbf{Q}_{\mathbf{X}^{DGPS}_i} = \sigma_0^2\mathbf{P}^{-1}_{\mathbf{X}^{DGPS}_i}\end{cases} \tag{3.11}$$

where:

$$\mathbf{Q}_{\boldsymbol{x}_i} = Diag\left(\mathbf{Q}_{\boldsymbol{x}_{Z_i}}, \mathbf{Q}_{\boldsymbol{x}^{(i)}_{R_i}}\right) = Diag\left(\mathbf{P}^{-1}_{\boldsymbol{x}_{Z_i}}, \mathbf{P}^{-1}_{\boldsymbol{x}^{(i)}_{R_i}}\right) = \mathbf{P}^{-1}_{\boldsymbol{x}_i}$$

Quasi-diagonal character of $\mathbf{Q}_{\boldsymbol{x}_i}$ and $\mathbf{P}^{-1}_{\boldsymbol{x}_i}$ matrixes results from the assumed mutual independency of the observations, which belong to different sets $\mathcal{O}$.

It's worth to consider that in the statistical model (3.11) there has been introduced a coefficient σ_0^2, common for all the covariance matrixes. According to the idea presented, among the others, in the papers [Wiśniewski 1989, 1999] it is possible also at this point to apply local variances coefficients, subordinated to specific, discriminated covariance matrixes. Then

$$\left.\begin{array}{l}\mathbf{C}_{\boldsymbol{x}_{Z_i}} = \sigma_0^2(\boldsymbol{x}_{Z_i})\mathbf{Q}_{\boldsymbol{x}_{Z_i}} = \sigma_0^2(\boldsymbol{x}_{Z_i})\mathbf{P}_{\boldsymbol{x}_{Z_i}}^{-1} \\ \mathbf{C}_{\boldsymbol{x}_{R_i}^{(i)}} = \sigma_{0\left(\boldsymbol{x}_{R_i}^{(i)}\right)}^2\mathbf{Q}_{\boldsymbol{x}_{R_i}^{(i)}} = \sigma_{0\left(\boldsymbol{x}_{R_i}^{(i)}\right)}^2\mathbf{P}_{\boldsymbol{x}_{R_i}^{(i)}}^{-1} \\ \mathbf{C}_{\mathbf{X}_i^{DGPS}} = \sigma_0^2(\mathbf{X}_i^{DGPS})\mathbf{Q}_{\mathbf{X}_i^{DGPS}} = \sigma_0^2(\mathbf{X}_i^{DGPS})\mathbf{P}_{\mathbf{X}_i^{DGPS}}^{-1}\end{array}\right\} \Leftrightarrow \begin{cases}\mathbf{C}_{\boldsymbol{x}_i} = \mathbf{G}\mathbf{Q}_{\boldsymbol{x}_i} = \mathbf{G}\mathbf{P}_{\boldsymbol{x}_i}^{-1} \\ \mathbf{C}_{\mathbf{X}_i^{DGPS}} = \sigma_0^2(\mathbf{X}_i^{DGPS})\mathbf{Q}_{\mathbf{X}_i^{DGPS}} = \sigma_0^2(\mathbf{X}_i^{DGPS})\mathbf{P}_{\mathbf{X}_i^{DGPS}}^{-1}\end{cases} \quad 3.12$$

where:

$$\mathbf{G} = Diag\left\{\sigma_0^2(\boldsymbol{x}_{Z_i})\mathbf{I}(2n_{Z_i}), \sigma_{0\left(\boldsymbol{x}_{R_i}^{(i)}\right)}^2\mathbf{I}(2n_R)\right\}$$

$\mathbf{I}_{(a)} \in \mathfrak{M}_{(a,1)}$ - unit matrix.

3.2. The Function of Target

On forming the target function of the adjustment task (adjustment criterion), let's refer to the above assumed functional –decisive models, statistical models (models of covariance matrix of (3.11) form and the basic principles of estimation carried out with the least squares method. The optimization criterion, resulting from the assumptions and the principles made, can be presented in the following form:

$$\begin{aligned}&\min_{\Omega}\left\{\Phi\left(\mathrm{d}_{\mathrm{X}_{\mathrm{P_i}}}^{(\mathrm{i})}, \mathrm{d}_{\mathrm{X}_{\mathrm{R_i}}}^{(\mathrm{i})}\right) = \Phi_x\left(\mathrm{d}_{\mathrm{X}_{\mathrm{P_i}}}^{(\mathrm{i})}, \mathrm{d}_{\mathrm{X}_{\mathrm{R_i}}}^{(\mathrm{i})}\right) + \Phi_{\mathrm{DGPS}}\left(\mathrm{d}_{\mathrm{X}_{\mathrm{P_i}}}^{(\mathrm{i})}, \mathrm{d}_{\mathrm{X}_{\mathrm{R_i}}}^{(\mathrm{i})}\right)\right\} = \\ &= \mathrm{V}_{x_i}^T \mathrm{P}_{x_i} \mathrm{V}_{x_i} + \mathrm{V}_{\mathrm{X_i^{DGPS}}} \mathrm{P}_{\mathrm{X_i^{DGPS}}} \mathrm{V}_{\mathrm{X_i^{DGPS}}}, \\ &\Omega = \left(\mathrm{d}_{\mathrm{X}_{\mathrm{P_i}}}^{(\mathrm{i})}, \mathrm{d}_{\mathrm{X}_{\mathrm{R_i}}}^{(\mathrm{i})}\right)\end{aligned} \quad (3.13)$$

where:

$$\mathrm{V}_{x_i}^{\mathrm{T}} \mathrm{P}_{x_i} \mathrm{V}_{x_i} = \Phi_x\left(\hat{\mathrm{d}}_{\mathrm{X}_{\mathrm{P_i}}}^{(\mathrm{i})}, \hat{\mathrm{d}}_{\mathrm{X}_{\mathrm{R_i}}}^{(\mathrm{i})}\right) \quad (3.14)$$

$$\mathbf{V}_{\mathbf{X}_i^{DGPS}}^T \mathbf{P}_{\mathbf{X}_i^{DGPS}} \mathbf{V}_{\mathbf{X}_i^{DGPS}} = \Phi_{DGPS}\left(\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)}, \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}\right) \quad (3.15)$$

Taking into consideration the decisive character of the functional models (3.10) in components (3.14) and (3.15) of the function of target $\Phi\left(\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}, \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}\right)$ we obtain as follows:

$$\begin{aligned} &V_{x_i} := \Im(\boldsymbol{x}_i)\cdot V_{x_i} \rightarrow \Phi_x\left(\hat{d}^{(i)}_{X_{P_i}}, \hat{d}^{(i)}_{X_{R_i}}\right) = \left[\Im(\boldsymbol{x}_i)\cdot V_{x_i}\right]^T P_{x_i}\Im(\boldsymbol{x}_i)\cdot V_{x_i} = \\ &= V^T_{x_i}\Im(\boldsymbol{x}_i)P_{x_i}\Im(\boldsymbol{x}_i)V_{x_i} = V^T_{x_i}\widetilde{P}_{x_i}V_{x_i} \end{aligned} \tag{3.16}$$

$$\begin{aligned} &V_{X_i^{DGPS}} := \Im(X_i^{DGPS})\cdot V_{X_i^{DGPS}} \rightarrow \Phi_{DGPS}\left(\hat{d}^{(i)}_{X_{P_i}}, \hat{d}^{(i)}_{X_{R_i}}\right) = \\ &= \left[\Im(X_i^{DGPS})V_{X_i^{DGPS}}\right]^T P_{X_i^{DGPS}}\Im(X_i^{DGPS})V_{X_i^{DGPS}} = \\ &= V^T_{X_i^{DGPS}}\Im(X_i^{DGPS})P_{X_i^{DGPS}}\Im(X_i^{DGPS})V_{X_i^{DGPS}} = \\ &= V^T_{X_i^{DGPS}}\widetilde{P}_{X_i^{DGPS}}V_{X_i^{DGPS}} \end{aligned} \tag{3.17}$$

where:

$$\widetilde{\mathbf{P}}_{\boldsymbol{x}_i} = \Im(\boldsymbol{x}_i)\mathbf{P}_{\boldsymbol{x}_i}\Im(\boldsymbol{x}_i) \tag{3.18}$$

$$\widetilde{\mathbf{P}}_{\mathbf{X}_i^{DGPS}} = \Im(\mathbf{X}_i^{DGPS})\mathbf{P}_{\mathbf{X}_i^{DGPS}}\Im(\mathbf{X}_i^{DGPS}) \tag{3.19}$$

Let's assume that in the set of observations $\mathcal{O}_i$ one of them is not accepted. Let the observation be corresponded to by *l-th* element of the observation vector $\boldsymbol{x}_i$ of the following weights matrix:

$$\mathbf{P}_{\boldsymbol{x}_i} = \begin{bmatrix} P_{11} & \cdots & P_{1,l-1} & P_{1,l} & P_{1,l+1} & \cdots & P_{1,n} \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ P_{l-1,1} & \cdots & P_{l-1,l-1} & P_{l-1,l} & P_{l-1,l+1} & \cdots & P_{l-1,n} \\ P_{l,1} & \cdots & P_{l,l-1} & P_{l,l} & P_{l,l+1} & \cdots & P_{l,n} \\ P_{l+1,1} & \cdots & P_{l+1,l-1} & P_{l+1,l} & P_{l+1,l+1} & \cdots & P_{l+1,n} \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ P_{n1} & \cdots & P_{n,l-1} & P_{n,l} & P_{n,l+1} & \cdots & P_{n,n} \end{bmatrix}$$

As in such situation, a form of the decisive matrix $\Im(\boldsymbol{x}_i)$ is as follows:

$$\Im(\boldsymbol{x}_i) = Diag\left(1,...,1,0_l,1,...,1\right)$$

(0_i - zero at *l-th* position), so

$$\widetilde{\mathbf{P}}_{\boldsymbol{x}_i} = \Im(\boldsymbol{x}_i)\mathbf{P}_{\boldsymbol{x}_i}\Im(\boldsymbol{x}_i) = \begin{bmatrix} P_{11} & \cdots & P_{1,l-1} & 0 & P_{1,l+1} & \cdots & P_{1,n} \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ P_{l-1,1} & \cdots & P_{l-1,l-1} & 0 & P_{l-1,l+1} & \cdots & P_{l-1,n} \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ P_{l+1,1} & \cdots & P_{l+1,l-1} & 0 & P_{l+1,l+1} & \cdots & P_{l+1,n} \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ P_{n1} & \cdots & P_{n,l-1} & 0 & P_{n,l+1} & \cdots & P_{n,n} \end{bmatrix}$$

In case the observations are the mutually independent elements quantities, then:

$$\mathbf{P}_{x_i} = Diag\left(P_{11},...,P_{l-1,l-1},P_{l,l},P_{l+1,l+1},P_{n,n}\right)$$

Thus

$$\Im(\boldsymbol{x}_i)\mathbf{P}_{\boldsymbol{x}_i} = Diag\left(P_{11},...,P_{l-1,l-1},0,P_{l+1,l+1},P_{n,n}\right) = \widetilde{\mathbf{P}}_{\boldsymbol{x}_i}$$

$$\mathbf{P}_{\boldsymbol{x}_i}\Im(\boldsymbol{x}_i) = Diag\left(P_{11},...,P_{l-1,l-1},0,P_{l+1,l+1},P_{n,n}\right) = \widetilde{\mathbf{P}}_{\boldsymbol{x}_i}$$

The relation existing for the independent variables

$$\widetilde{\mathbf{P}}_{\boldsymbol{x}_i} = \Im(\boldsymbol{x}_i)\mathbf{P}_{\boldsymbol{x}_i}\Im(\boldsymbol{x}_i) = \widetilde{\mathbf{P}}_{\boldsymbol{x}_i}\Im(\boldsymbol{x}_i) = \Im(\boldsymbol{x}_i)\widetilde{\mathbf{P}}_{\boldsymbol{x}_i} = \Im(\boldsymbol{x}_i)\mathbf{P}_{\boldsymbol{x}_i} \quad (4.20)$$

has direct reference to the robust *M*-estimation principles (for independent variables). According to the above principles, the original weights matrix $\mathbf{P}_{x_i}$ should be substituted with the following equivalent matrix (e.g. [Yang 1994, Wiśniewski 2002]

$$\widehat{\mathbf{P}}_{\boldsymbol{x}_i} = \mathbf{T}(\mathbf{V}_{\boldsymbol{x}_i})\mathbf{P}_{\boldsymbol{x}_i} \quad (3.21)$$

The matrix:

$$\mathbf{T}(\mathbf{V}_{x_i}) = Diag\left\{t(\upsilon_1),t(\upsilon_2),...,t(\upsilon_n)\right\}$$

is an attenuation matrix, whereas $t(\upsilon_l) \in \langle 0;1 \rangle$ is the attenuation function, mentioned already in chapter 2, (υ_l - *l-th* element of the correction vector $\mathbf{V}_{\boldsymbol{x}_i}$).

Let's assume that the attenuation function $t(\upsilon)$ is characterized with the following general properties:

$$t(\upsilon)=1 \qquad \text{for} \qquad \upsilon \in \Delta_\upsilon^+$$

$$e < t(\upsilon) < 1 \qquad \text{for} \qquad |\upsilon| \in \Delta_\upsilon^\pm$$

$$t(\upsilon) < e \qquad \text{for} \qquad |\upsilon| \in \Delta_\upsilon^-$$

where e is a numerical boundary of the attenuation function "zero adjustment". With Δ_υ there have been marked the following intervals (Fig.3.2):

Δ_υ^+ - interval admissible for corrections, which represent the random survey errors

$\Delta_\upsilon^\pm$ - interval permissible

Δ_υ^- - interval of corrections which represent the gross survey errors

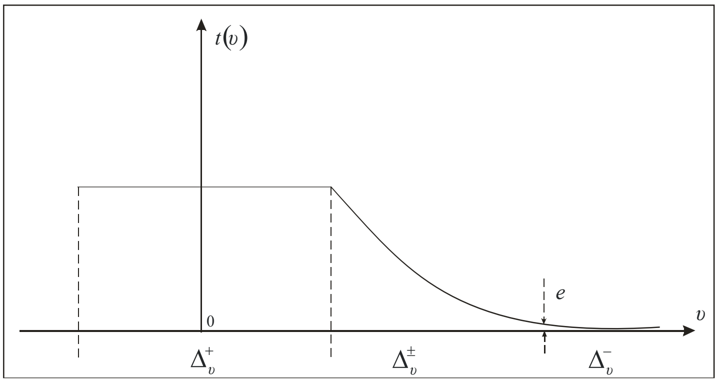

Fig. 3.2. Graphic interpretation of intervals for corrections υ

Moreover, in case it is assumed that:

if $\left.\begin{matrix} \upsilon \in \Delta_\upsilon^+ \\ |\upsilon| \in \Delta_\upsilon^\pm \end{matrix}\right\}$ then the observation x, corresponding to the correction υ, is accepted

if $|\upsilon| \in \Delta_\upsilon^-$ then the observation x, corresponding to the correction υ, is rejected

thus the function of attenuation

$$\left\{\iota(x)=0\right\} \le t(\upsilon) \le \left\{\iota(x)=1\right\}$$

thereby the attenuation matrix

$$\left\{\mathfrak{I}(\boldsymbol{x}_i)=\mathbf{0}\right\} \le \mathbf{T}(\mathbf{V}_{\boldsymbol{x}_i}) \le \left\{\mathfrak{I}(\boldsymbol{x}_i)=\mathbf{I}_{(n)}\right\}$$

are to play, at the extreme cases, which are of our interest, a similar parts as the decisive function and the decisive matrix.

Carrying on the same generalization is possible for the independent variables as well; as in the paper [Yang, Song, Xu 2002] it is displayed that in such situation, in relation to the matrix $\mathbf{P}_{x_i}$, the equivalent weights matrix form is as follows:

$$\widehat{\mathbf{P}}_{x_i} = \begin{bmatrix} \gamma_{1,1}p_{1,1} & \gamma_{1,2}p_{1,2} & \vdots & \gamma_{1,n}p_{1,n} \\ \gamma_{21}p_{21} & \gamma_{22}p_{22} & \vdots & \gamma_{2,n}p_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ \gamma_{n,1}p_{n,1} & \gamma_{n,2}p_{n,2} & \vdots & \gamma_{n,n}p_{n,n} \end{bmatrix} \tag{3.22}$$

where: $\gamma_{l,k} = \sqrt{\gamma_{l,l}\gamma_{k,k}}$

$(l, k = 1,2,...,n)$.

Bifactors $\gamma_{l,l}, \gamma_{k,k}$ are the attenuation function values:

$$\gamma_{l,l} = t(\upsilon_l), \quad \gamma_{k,k} = t(\upsilon_k)$$

Therefore, assuming that

$$\mathbf{T}_{sqr}(\mathbf{V}_{x_i}) = Diag\left\{\sqrt{t(\upsilon_1)}, \sqrt{t(\upsilon_2)}, ..., \sqrt{t(\upsilon_n)}\right\} \tag{3.23}$$

for the dependent variables it may be written that

$$\widehat{\mathbf{P}}_{x_i} = \mathbf{T}_{sqr}(\mathbf{V}_{x_i}) \cdot \mathbf{P}_{x_i} \cdot \mathbf{T}_{sqr}(\mathbf{V}_{x_i}) \tag{3.24}$$

As $\mathfrak{I}(\boldsymbol{x}_i) = \mathfrak{I}_{sqr}(\boldsymbol{x}_i) = Diag\left\{\sqrt{\iota(x_1)}, \sqrt{\iota(x_2)}, ..., \sqrt{\iota(x_n)}\right\}$

(the decisive function assumes the values 0 or 1 only), so

$$\widetilde{\mathrm{P}}_{x_i} = \mathfrak{I}(\boldsymbol{x}_i) \cdot \mathrm{P}_{x_i} \cdot \mathfrak{I}(\boldsymbol{x}_i) = \mathfrak{I}_{sqr}(\boldsymbol{x}_i) \cdot \mathrm{P}_{x_i} \cdot \mathfrak{I}_{sqr}(\boldsymbol{x}_i) \tag{3.25}$$

The carried out analysis proves, that in a general case, it means in such a case when not only acceptance or rejection are assumed, but also justified attenuation of their influence on the final results of the work, the function of target

$$\Phi_{\boldsymbol{x}}\left(\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}}, \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}\right) = \mathbf{V}^T_{x_i}\mathfrak{I}(\boldsymbol{x}_i) \cdot \mathbf{P}_{x_i} \cdot \mathfrak{I}(\boldsymbol{x}_i)\mathbf{V}_{x_i} = \mathbf{V}^T_{x_i} \cdot \widetilde{\mathbf{P}}_{x_i} \cdot \mathbf{V}_{x_i}$$

can be replaced by the function

$$\Phi^R_{\boldsymbol{x}}\left(\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}}, \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}\right) = \mathbf{V}^T_{x_i}\mathbf{T}_{sqr}(\mathbf{V}_{x_i}) \cdot \mathbf{P}_{x_i} \cdot \mathbf{T}_{sqr}(\mathbf{V}_{x_i})\mathbf{V}_{x_i} = \mathbf{V}^T_{x_i} \cdot \widehat{\mathbf{P}}_{x_i} \cdot \mathbf{V}_{x_i} \tag{3.26}$$

There is no objection in carrying out the same substitution also for the function

$$\Phi_{DGPS}\left(\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}},\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}\right).$$

Then

$$\begin{aligned}\Phi^{R}_{DGPS}\left(\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}},\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}\right)&=\mathbf{V}^{T}_{\mathbf{X}_i^{DGPS}}\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\cdot\mathbf{P}_{\mathbf{X}_i^{DGPS}}\cdot\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{V}_{\mathbf{X}_i^{DGPS}}=\\&=\mathbf{V}^{T}_{\mathbf{X}_i^{DGPS}}\widehat{\mathbf{P}}_{\mathbf{X}_i^{DGPS}}\mathbf{V}_{\mathbf{X}_i^{DGPS}}\end{aligned}\tag{3.27}$$

where

$$\widehat{\mathbf{P}}_{\mathbf{X}_i^{DGPS}}=\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\tag{3.28}$$

We assume at that point, that the function of attenuation $t\left(\overset{DGPS}{\upsilon}\right)$ satisfies similar properties as the function $t(\upsilon)$, and that

$$\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})=Diag\left\{\sqrt{t\left(\overset{DGPS}{\upsilon_{\mathbf{X}_i}}\right)},\sqrt{t\left(\overset{DGPS}{\upsilon_{\mathbf{Y}_i}}\right)}\right\}$$

where

$$\left[\overset{DGPS}{\upsilon_{\mathbf{X}_i}},\overset{DGPS}{\upsilon_{\mathbf{Y}_i}}\right]^{T}=\mathbf{V}_{\mathbf{X}_i^{DGPS}}$$

Making an assumption concerning independence of DGPS coordinates; the weights matrix (3.28) is taking the following form:

$$\begin{aligned}\widehat{\mathbf{P}}_{\mathbf{X}_i^{DGPS}}&=\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})=\\&=\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}=\mathbf{T}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}\end{aligned}\tag{3.29}$$

where:

$$\mathbf{T}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})=\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})=Diag\left\{t\left(\overset{DGPS}{\upsilon_{\mathbf{X}_i}}\right),t\left(\overset{DGPS}{\upsilon_{\mathbf{Y}_i}}\right)\right\}$$

Due to the form of the equivalent weights matrix $\widehat{\mathbf{P}}_{\boldsymbol{x}_i}$ and $\widehat{\mathbf{P}}_{\mathbf{X}_i^{DGPS}}$ (their dependence on corrections vectors) a process of searching $\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}},\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}$:

$$\min_{\Omega}\Phi^{R}\left(\mathbf{d}^{(i)}_{\mathbf{X}_{P_i}},\mathbf{d}^{(i)}_{\mathbf{X}_{R_i}}\right)=\Phi^{R}\left(\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}},\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}\right)=\Phi^{R}_{\boldsymbol{x}}\left(\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}},\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}\right)+\Phi^{R}_{DGPS}\left(\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}},\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}\right)\tag{3.30}$$

$\left(\Omega = \left(\mathbf{d}^{(i)}_{\mathbf{X}_{P_i}}, \mathbf{d}^{(i)}_{\mathbf{X}_{R_i}}\right)\right)$ is of iterative character. It means that especially in such, basically multi-step process, the corrections which correspond to unaccepted observations, tend to the intervals Δ^{-}_{υ}, subordinated thereto. Thus, finding out which of the observations is unaccepted is an a posteriori process (after adjustment).

However, in practical problems of navigation there is a possibility to indicate, yet before carrying out an adjustment, (a priori) such elements of the set $\mathcal{O}$, which should not affect the ultimate determinations (e.g. the identified false radar echoes, toward which automatically the bearings were carried out; therefrom their presence in the observations set). According to the assumptions made in this work, such the observations are corresponded by zero values of the decisive function. Substitution of the decisive function with the more general attenuation function in the robust adjustment process leads finally to the required results. Anyhow, in circumstances when information about unaccepted observations is available a priori, there may be suggested another, from the practical viewpoint more rational solution. A basis for such the suggestion is composing the decisive function (a priori object) and the attenuation function (a posteriori object). Thus we can obtain the following decisive-attenuation function:

$$\widetilde{t}\left(x,\upsilon\right) = \iota\left(x\right)\cdot t\left(\upsilon\right) \tag{3.32}$$

of the general properties as follows:

$$\widetilde{t}\left(x,\upsilon\right) = \begin{cases} t\left(\upsilon\right) & \text{for } x \text{ participating in the adjustment} \\ 0 & \text{for } x \text{ unaccepted a priori} \end{cases}$$

The decisive–attenuation function can be a basis for formulation the following decisive–attenuation matrixes:

$$\widetilde{\mathbf{T}}_{sqr}(\boldsymbol{x}_i, \mathbf{V}_{\boldsymbol{x}_i}) = \mathfrak{I}_{sqr}(\boldsymbol{x}_i)\mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_i}) = \mathfrak{I}(\boldsymbol{x}_i)\mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_i}) \tag{3.33}$$

and

$$\widetilde{\mathbf{T}}_{sqr}(\mathbf{X}^{DGPS}_{P_i}, \mathbf{V}_{\mathbf{X}^{DGPS}_i}) = \mathfrak{I}_{sqr}(\mathbf{X}^{DGPS}_{P_i})\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}^{DGPS}_i}) = \mathfrak{I}(\mathbf{X}^{DGPS}_{P_i})\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}^{DGPS}_i}) \tag{3.34}$$

with

$$\left.\begin{aligned} \widetilde{T}_{sqr}(\boldsymbol{x}_i, V_{\boldsymbol{x}_i})\widetilde{T}_{sqr}(\boldsymbol{x}_i, V_{\boldsymbol{x}_i}) &= \mathfrak{I}(\boldsymbol{x}_i)\mathfrak{I}(\boldsymbol{x}_i)T_{sqr}(V_{\boldsymbol{x}_i})T_{sqr}(V_{\boldsymbol{x}_i}) = \\ &= \mathfrak{I}(\boldsymbol{x}_i)T(V_{\boldsymbol{x}_i}) = \widetilde{T}(\boldsymbol{x}_i, V_{\boldsymbol{x}_i}) \\ \widetilde{T}_{sqr}(X^{DGPS}_{P_i}, V_{X^{DGPS}_i})\widetilde{T}_{sqr}(X^{DGPS}_{P_i}, V_{X^{DGPS}_i}) &= \mathfrak{I}(X^{DGPS}_{P_i})\mathfrak{I}(X^{DGPS}_{P_i})T_{sqr}(V_{X^{DGPS}_i})T_{sqr}(V_{X^{DGPS}_i}) = \\ &= \mathfrak{I}(X^{DGPS}_{P_i})T(V_{X^{DGPS}_i}) = \widetilde{T}(X^{DGPS}_{P_i}, V_{X^{DGPS}_i}) \end{aligned}\right\} \tag{3.35}$$

Whereas basing on the decisive – attenuation matrixes there are obtained the following, equivalent weights matrixes:

$$\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} = \widetilde{\mathbf{T}}_{sqr}(\boldsymbol{x}_i, \mathbf{V}_{\boldsymbol{x}_i})\mathbf{P}_{\boldsymbol{x}_i}\widetilde{\mathbf{T}}_{sqr}(\boldsymbol{x}_i, \mathbf{V}_{\boldsymbol{x}_i})$$

$$\widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} = \widetilde{\mathbf{T}}_{sqr}(\mathbf{X}_i^{DGPS}, \mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}\widetilde{\mathbf{T}}_{sqr}(\mathbf{X}_i^{DGPS}, \mathbf{V}_{\mathbf{X}_i^{DGPS}}) \tag{3.36}$$

and at the same time

$$\begin{aligned}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} &= \Im(\boldsymbol{x}_i)\mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_i})\mathbf{P}_{\boldsymbol{x}_i}\Im(\boldsymbol{x}_i)\mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_i}) = \Im(\boldsymbol{x}_i)\underbrace{\mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_i})\mathbf{P}_{\boldsymbol{x}_i}\mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_i})}_{\widehat{\mathbf{P}}_{\mathbf{x}_i}}\Im(\boldsymbol{x}_i) = \\ &= \Im(\boldsymbol{x}_i)\widehat{\mathbf{P}}_{\boldsymbol{x}_i}\Im(\boldsymbol{x}_i)\end{aligned} \tag{3.36a}$$

and

$$\begin{aligned}\widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} &= \Im(\mathbf{X}_i^{DGPS})\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}\Im(\mathbf{X}_i^{DGPS})\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}}) = \\ &= \Im(\mathbf{X}_i^{DGPS})\widehat{\mathbf{P}}_{\mathbf{X}_i^{DGPS}}\Im(\mathbf{X}_i^{DGPS})\end{aligned} \tag{3.37}$$

In case the observations are mutually independent $\left(\mathbf{P}_{\boldsymbol{x}_i} = Diag\left(\mathbf{P}_{\boldsymbol{x}_i}\right)\right)$, then

$$\begin{aligned}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} &= \Im(\boldsymbol{x}_i)\widehat{\mathbf{P}}_{\boldsymbol{x}_i}\Im(\boldsymbol{x}_i) = \Im(\boldsymbol{x}_i)\Im(\boldsymbol{x}_i)\widehat{\mathbf{P}}_{\boldsymbol{x}_i} = \Im(\boldsymbol{x}_i)\widehat{\mathbf{P}}_{\boldsymbol{x}_i} = \\ &= \Im(\boldsymbol{x}_i)\mathbf{T}(\mathbf{V}_{\boldsymbol{x}_i})\mathbf{P}_{\boldsymbol{x}_i} = \widetilde{\mathbf{T}}(\boldsymbol{x}_i, \mathbf{V}_{\boldsymbol{x}_i})\mathbf{P}_{\boldsymbol{x}_i}\end{aligned} \tag{3.38}$$

Similarly, with neglecting mutual dependence between the coordinates $X_{P_i}^{DGPS}, Y_{P_i}^{DGPS}$, the following is achieved:

$$\begin{aligned}\widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} &= \Im(\mathbf{X}_i^{DGPS})\widehat{\mathbf{P}}_{\mathbf{X}_i^{DGPS}}\Im(\mathbf{X}_i^{DGPS}) = \Im(\mathbf{X}_i^{DGPS})\widehat{\mathbf{P}}_{\mathbf{X}_i^{DGPS}} = \\ &= \Im(\mathbf{X}_i^{DGPS})\mathbf{T}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}} = \widetilde{\mathbf{T}}(\mathbf{X}_i^{DGPS}, \mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}\end{aligned} \tag{3.39}$$

By substitutions of the equivalent weights matrixes $\widehat{\mathbf{P}}_{\boldsymbol{x}_i}, \widehat{\mathbf{P}}_{\mathbf{X}_i^{DGPS}}$ in components of the function $\Phi^R\left(\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)}, \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}\right)$, with matrixes $\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i}, \widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}}$, the following function is obtained:

$$\Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)}, \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}\right) = \Phi_{\boldsymbol{x}}^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)}, \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}\right) + \Phi_{DGPS}^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)}, \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}\right) \tag{3.40}$$

where:

$$\begin{aligned}\Phi_{x}^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)},\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}\right)&=\mathbf{V}_{x_i}^{T}\widetilde{\widehat{\mathbf{P}}}_{x_i}\mathbf{V}_{x_i}=\mathbf{V}_{x_i}^{T}\Im(x_i)\widehat{\mathbf{P}}_{x_i}\Im(x_i)\mathbf{V}_{x_i}=\\&=\mathbf{V}_{x_i}^{T}\Im(x_i)\mathbf{T}_{sqr}(\mathbf{V}_{x_i})\mathbf{P}_{x_i}\mathbf{T}_{sqr}(\mathbf{V}_{x_i})\Im(x_i)\mathbf{V}_{x_i}=\\&=\mathbf{V}_{x_i}^{T}\widetilde{\mathbf{T}}_{sqr}(x_i,\mathbf{V}_{x_i})\mathbf{P}_{x_i}\widetilde{\mathbf{T}}_{sqr}(x_i,\mathbf{V}_{x_i})\mathbf{V}_{x_i}\end{aligned}\tag{3.41}$$

and

$$\begin{aligned}&\Phi_{DGPS}^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)},\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}\right)=\mathbf{V}_{\mathbf{X}_i^{DGPS}}^{T}\widetilde{\widehat{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}}\mathbf{V}_{\mathbf{X}_i^{DGPS}}=\\&=\mathbf{V}_{\mathbf{X}_i^{DGPS}}^{T}\Im(\mathbf{X}_{P_i}^{DGPS})\widehat{\mathbf{P}}_{\mathbf{X}_i^{DGPS}}\Im(\mathbf{X}_{P_i}^{DGPS})\mathbf{V}_{\mathbf{X}_i^{DGPS}}=\\&=\mathbf{V}_{\mathbf{X}_i^{DGPS}}^{T}\Im(\mathbf{X}_{P_i}^{DGPS})\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\Im(\mathbf{X}_{P_i}^{DGPS})\mathbf{V}_{\mathbf{X}_i^{DGPS}}=\\&=\mathbf{V}_{\mathbf{X}_i^{DGPS}}^{T}\widetilde{\mathbf{T}}_{sqr}(\mathbf{X}_{P_i}^{DGPS},\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}\widetilde{\mathbf{T}}_{sqr}(\mathbf{X}_{P_i}^{DGPS},\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{V}_{\mathbf{X}_i^{DGPS}}\end{aligned}\tag{3.42}$$

For the independent variables we may write:

$$\begin{aligned}\Phi_{x}^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)},\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}\right)&=\mathbf{V}_{x_i}^{T}\widetilde{\widehat{\mathbf{P}}}_{x_i}\mathbf{V}_{x_i}=\mathbf{V}_{x_i}^{T}\Im(x_i)\widehat{\mathbf{P}}_{x_i}\mathbf{V}_{x_i}=\\&=\mathbf{V}_{x_i}^{T}\Im(x_i)\mathbf{T}(\mathbf{V}_{x_i})\mathbf{P}_{x_i}\mathbf{V}_{x_i}=\mathbf{V}_{x_i}^{T}\widetilde{\mathbf{T}}(x_i,\mathbf{V}_{x_i})\mathbf{P}_{x_i}\mathbf{V}_{x_i}\end{aligned}\tag{3.43}$$

and

$$\begin{aligned}\Phi_{DGPS}^{D-R}\left(\hat{\mathbf{d}}_{X_{P_i}},\hat{\mathbf{d}}_{X_{R_i}}^{(i)}\right)&=\mathbf{V}_{\mathbf{X}_i^{DGPS}}^{T}\widetilde{\widehat{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}}\mathbf{V}_{\mathbf{X}_i^{DGPS}}=\mathbf{V}_{\mathbf{X}_i^{DGPS}}^{T}\Im(\mathbf{X}_{P_i}^{DGPS})\widehat{\mathbf{P}}_{\mathbf{X}_i^{DGPS}}\mathbf{V}_{\mathbf{X}_i^{DGPS}}=\\&=\mathbf{V}_{\mathbf{X}_i^{DGPS}}^{T}\Im(\mathbf{X}_{P_i}^{DGPS})\mathbf{T}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}\mathbf{V}_{\mathbf{X}_i^{DGPS}}=\\&=\mathbf{V}_{\mathbf{X}_i^{DGPS}}^{T}\widetilde{\mathbf{T}}(\mathbf{X}_{P_i}^{DGPS},\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}\mathbf{V}_{\mathbf{X}_i^{DGPS}}\end{aligned}\tag{3.44}$$

The function (3.40) of the components (3.41), (3.42) (or their version for the independent variables) is the target function of the decisive-robust Interactive Navigational Structure chain's element adjustment task.

3.3. The Adjustment Task and its Solution

By joining the functional models in the form (3.10), the statistical model in the form (3.11) and the function of target $\Phi^{D-R}\left(\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}},\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}\right)$, the following adjustment assignment can be obtained:

$$\left.\begin{array}{l}
\left.\begin{array}{l}
\mathbf{V}_{x_i}=\mathbf{A}_{P_i}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}}+\mathbf{A}^{(i)}_{R}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}+\mathbf{L}_{x_i}\\
\mathbf{V}_{\mathbf{X}_i^{DGPS}}=\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}}+\mathbf{X}^0_{P_i}-\mathbf{X}^{DGPS}_{P_i}
\end{array}\right\}\textit{functional models}\\
\left.\begin{array}{l}
\mathbf{C}_{x_i}=\sigma_0^2\mathbf{Q}_{x_i}=\sigma_0^2\mathbf{P}^{-1}_{x_i}\\
\mathbf{C}_{\mathbf{X}^{DGPS}_{P_i}}=\sigma_0^2\mathbf{Q}_{\mathbf{X}^{DGPS}_{P_i}}=\sigma_0^2\mathbf{P}^{-1}_{\mathbf{X}^{DGPS}_{P_i}}
\end{array}\right\}\rightarrow
\left.\begin{array}{l}
\tilde{\tilde{\mathbf{C}}}_{x_i}=\sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{x_i}\\
\tilde{\tilde{\mathbf{C}}}_{\mathbf{X}^{DGPS}_{P_i}}=\sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{\mathbf{X}^{DGPS}_{P_i}}
\end{array}\right\}\textit{statistic models}\\
\min\limits_{\Omega}\Phi^{D-R}\left(\mathbf{d}^{(i)}_{\mathbf{X}_{P_i}},\mathbf{d}^{(i)}_{\mathbf{X}_{R_i}}\right)=\Phi^{D-R}\left(\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}},\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}\right)=\mathbf{V}^T_{x_i}\tilde{\tilde{\mathbf{P}}}_{x_i}\mathbf{V}_{x_i}+\mathbf{V}^T_{\mathbf{X}^{DGPS}_{P_i}}\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}^{DGPS}_{P_i}}\mathbf{V}_{\mathbf{X}^{DGPS}_{P_i}}\\
\Omega=\left(\mathbf{d}^{(i)}_{\mathbf{X}_{P_i}},\mathbf{d}^{(i)}_{\mathbf{X}_{R_i}}\right)
\end{array}\right\}\quad(3.45)$$

With the equivalent covariance matrixes $\tilde{\tilde{\mathbf{C}}}_{x_i}=\sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{x_i}$, $\tilde{\tilde{\mathbf{C}}}_{\mathbf{X}^{DGPS}_{P_i}}=\sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{\mathbf{X}^{DGPS}_{P_i}}$.

With the following designations:

$$\mathbf{V}_i=\begin{bmatrix}\mathbf{V}_{x_i}\\ \mathbf{V}_{\mathbf{X}_i^{DGPS}}\end{bmatrix},\ \mathbf{A}^{(i)}_i=\begin{bmatrix}\mathbf{A}_{P_i} & \mathbf{A}^{(i)}_R\\ \mathbf{I}_{(2)} & \mathbf{0}\end{bmatrix},\ \mathbf{L}_i=\begin{bmatrix}\mathbf{L}_{x_i}\\ \mathbf{X}^0_{P_i}-\mathbf{X}^{DGPS}_{P_i}\end{bmatrix},\ \hat{\mathbf{d}}_{\mathbf{X}_i}=\begin{bmatrix}\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}}\\ \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}}\end{bmatrix},$$

$$\tilde{\tilde{\mathbf{C}}}_i=Diag\left(\tilde{\tilde{\mathbf{C}}}_{x_i},\tilde{\tilde{\mathbf{C}}}_{\mathbf{X}^{DGPS}_{P_i}}\right),\ \tilde{\tilde{\mathbf{P}}}_i=Diag\left(\tilde{\tilde{\mathbf{P}}}_{x_i},\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}^{DGPS}_{P_i}}\right)$$

the assignment (3.45) can also be presented in the form as follows:

$$\left.\begin{array}{l}
\mathbf{V}_i=\mathbf{A}_i\hat{\mathbf{d}}_{\mathbf{X}_i}+\mathbf{L}_i\\
\mathbf{C}_i=\sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_i\\
\min\limits_{\mathbf{d}_{\mathbf{X}_i}}\Phi^{D-R}\left(\mathbf{d}_{\mathbf{X}_i}\right)=\Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_i}\right)=\mathbf{V}^T_i\tilde{\tilde{\mathbf{P}}}_i\mathbf{V}_i
\end{array}\right\}\quad(3.46)$$

The classic form of the above allows for presenting (without unnecessary derivations) the following solution (e.g.[Baran1999,Wiśniewski 2000, 2004]:

$$\hat{\mathbf{d}}_{\mathbf{X}_i} = -\left(\mathbf{A}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{A}_i\right)^{-} \mathbf{A}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{L}_i \tag{3.47}$$

Moreover, in case a row $\left(\mathbf{A}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{A}_i\right) = r_i$ (as in classic solutions), then $\left(\mathbf{A}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{A}_i\right)^{-} = \left(\mathbf{A}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{A}_i\right)^{-1}$. Instead, the variance coefficient estimator can be determined applying the formula [Wiśniewski 1999, Yang 1997]:

$$\hat{\sigma}_0^2 = \frac{1}{f_i} \mathbf{V}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{V}_i \tag{3.48}$$

where:

$$f_i = n_i + 2 - r_i = 2\left(n_{Z_i} + n_{R_i}\right) + 2 - 2\left(1 + n_{R_i}\right) = 2n_{Z_i}$$

number of direct observations (n_i); number of pseudo-observations (2)

The estimator of the vector covariance matrix $\hat{\mathbf{d}}_{\mathbf{X}_i} = \left[\left(\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)}\right)^T, \left(\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}\right)^T\right]^T$ and thereby the estimator of the adjusted coordinates covariance matrix

$$\hat{\mathbf{X}}_i = \mathbf{X}_i^0 + \hat{\mathbf{d}}_{\mathbf{X}_i} = \begin{bmatrix} \mathbf{X}_{P_i}^0 \\ \mathbf{X}_{R_i}^0 \end{bmatrix} + \begin{bmatrix} \hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)} \\ \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{X}}_{P_i}^{(i)} \\ \hat{\mathbf{X}}_{R_i}^{(i)} \end{bmatrix}$$

takes the following form [Yang 1997]:

$$\hat{\mathbf{C}}_{\hat{\mathbf{X}}_i} = \hat{\sigma}_0^2 \left(\mathbf{A}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{A}_i\right)^{-1} = \hat{\sigma}_0^2 \mathbf{Q}_{\hat{\mathbf{X}}_i} = \hat{\sigma}_0^2 \mathbf{P}_{\hat{\mathbf{X}}_i}^{-1} \tag{3.49}$$

(if only we accept the equivalent covariance matrixes models $\tilde{\tilde{\mathbf{C}}}_{\boldsymbol{x}_i}$ and $\tilde{\tilde{\mathbf{C}}}_{\mathbf{X}_{P_i}^{DGPS}}$ presented in (4.45) and existing $\left(\mathbf{A}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{A}_i\right)^{-1}$). Taking into consideration a structure of the matrix $\mathbf{A}_i$ and $\tilde{\tilde{\mathbf{P}}}_i$, the interesting for us weights matrix $\mathbf{P}_{\hat{\mathbf{X}}_i}$ of the estimator $\hat{\mathbf{X}}_i$, can be expressed as follows:

$$\mathbf{P}_{\hat{\mathbf{X}}_i} = \mathbf{Q}_{\hat{\mathbf{X}}_i}^{-1} = \mathbf{A}_i^T \widetilde{\widetilde{\mathbf{P}}}_i \mathbf{A}_i = \begin{bmatrix} \mathbf{A}_{P_i}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} \mathbf{A}_{P_i} + \widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} & \vdots & \mathbf{A}_{P_i}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} \mathbf{A}_{R_i}^{(i)} \\ \cdots & \vdots & \cdots \\ \left(\mathbf{A}_{R_i}^{(i)}\right)^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} \mathbf{A}_{P_i} & \vdots & \left(\mathbf{A}_{R_i}^{(i)}\right)^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} \mathbf{A}_{R_i}^{(i)} \end{bmatrix} = \tag{3.50}$$

$$= \begin{bmatrix} \mathbf{P}_{\hat{\mathbf{X}}_{P_i}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{P_i}, \hat{\mathbf{X}}_{R_i}^{(i)}} \\ \cdots & \vdots & \cdots \\ \mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i)}, \hat{\mathbf{X}}_{P_i}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i)}} \end{bmatrix}$$

The process of resolving the task (4.46) (already mentioned) is of an iterative character. A start-step of the process is the classic adjustment applying the least squares method with the decisive weights matrix a priori

$$\mathbf{P}_i^{(0)} = Diag\left(\widetilde{\mathbf{P}}_{\boldsymbol{x}_i}, \widetilde{\mathbf{P}}_{\mathbf{X}_i^{DGPS}}\right) = \widetilde{\mathbf{P}}_i \tag{3.51}$$

Every following step

$$\mathbf{P}^{(l+1)} = Diag\left(\mathbf{P}_{\boldsymbol{x}_i}^{(l+1)}, \mathbf{P}_{\mathbf{X}_i^{DGPS}}^{(l+1)}\right)$$

where:

$$\mathbf{P}_{\boldsymbol{x}_i}^{(l+1)} = \widetilde{\mathbf{T}}_{sqr}\left(\boldsymbol{x}_i, \mathbf{V}_{\boldsymbol{x}_i}^{(l)}\right) \mathbf{P}_{\boldsymbol{x}_i}^{(l)} \widetilde{\mathbf{T}}_{sqr}\left(\boldsymbol{x}_i, \mathbf{V}_{\boldsymbol{x}_i}^{(l)}\right)$$

$$\mathbf{P}_{\mathbf{X}_i^{DGPS}}^{(l+1)} = \widetilde{\mathbf{T}}_{sqr}\left(\mathbf{X}_{P_i}^{DGPS}, \mathbf{V}_{\mathbf{X}_{P_i}^{DGPS}}^{(l)}\right) \mathbf{P}_{\mathbf{X}_{P_i}^{DGPS}}^{(l)} \widetilde{\mathbf{T}}_{sqr}\left(\mathbf{X}_{P_i}^{DGPS}, \mathbf{V}_{\mathbf{X}_{P_i}^{DGPS}}^{(l)}\right)$$

Let us assume, that the decisive matrixes (adopted before carrying out the adjustment) $\Im(\boldsymbol{x}_i)$ and $\Im(\mathbf{X}_{P_i}^{DGPS})$ are not subject to modification within the iteration process (no “information transfer” between the attenuation function and the decisive functions). Then

$$\mathbf{P}_{\boldsymbol{x}_i}^{(l+1)} = \Im(\boldsymbol{x}_i)\mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_i}^{(l)})\mathbf{P}_{\boldsymbol{x}_i}^{(l)}\mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_i}^{(l)})\Im(\boldsymbol{x}_i) = \mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_i}^{(l)})\mathbf{P}_{\boldsymbol{x}_i}^{(l)}\mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_i}^{(l)})$$

(if only in the start-step $l = 0$ the weights matrix is of decisive character (3.51)). Similarly

$$\mathbf{P}_{\mathbf{X}_i^{DGPS}}^{(l+1)} = \mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})\mathbf{P}_{\mathbf{X}_i^{DGPS}}^{(l)}\mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_i^{DGPS}})$$

The adjustment task solution (3.45) can get simplified, when interaction between the functions $\iota(x)$ and $t(\upsilon)$ is assumed. According to the traditional approach, introducing correction υ_j (corresponding to the unaccepted observation x_j) to the interval $\Delta^-_{\upsilon_j}$ can, in many cases, proceed relatively slowly. It results from a character of many attenuation functions, for which the υ axis is a horizontal asymptote. Then, even if $\upsilon_j \in \Delta^-_{\upsilon_j}$, still $t(\upsilon_j) > 0$, what may cause elongation of the iterative process (not applicable in case of "radical" and non continuous functions of attenuation, e.g.: the Hampel's, Huber's functions [Wiśniewski 2004, Yang 1997]). The suggested interaction can be presented in a form of the following expression:

$$\left.\begin{array}{l} \left\{t\left(\upsilon_j^{(l)}\right) < e\right\} \Rightarrow \left(\left\{\iota\left(x_j\right) := 0\right\} \Leftrightarrow \left\{\tilde{\tilde{P}}_j = \tilde{\tilde{P}}_j^{(l+1)} = 0\right\}\right) \\ \left\{\iota\left(x_j\right) = 0\right\} \Rightarrow \left(\left\{t\left(\upsilon_j^{(l)}\right) := 0\right\} \Leftrightarrow \left\{\tilde{\tilde{P}}_j = \tilde{\tilde{P}}_j^{(l+1)} = 0\right\}\right) \end{array}\right\} \tag{3.52}$$

In the iterative process of the adjustment task solution, the decisive weights matrix is a priori $\tilde{\mathbf{P}}_i$, converted into the decisive – equivalent form $\tilde{\tilde{\mathbf{P}}}_i$. Each stepwise weights matrix $\mathbf{P}^{(l+1)}$ refers to the increments vector $\mathbf{d}^{(l+1)}_{\mathbf{X}_i}$ and the corrections vector $\mathbf{V}_i^{(l+1)}$. Therefore, in essence, resolving the adjustment task consists in forming sequences.

$$\left(\mathbf{P}_i^{(0)} = \tilde{\mathbf{P}}_i\right) \rightarrow \mathbf{P}_i^{(l+1)} \xrightarrow{l=0,1,.....} \tilde{\mathbf{P}}_i$$

$$\mathbf{d}^{(l)}_{\mathbf{X}_i} \xrightarrow{l=0,1,.....} \hat{\mathbf{d}}_{\mathbf{X}_i}$$

$$\mathbf{V}_i^{(l)} \xrightarrow{l=0,1,.....} \mathbf{V}_i$$

The problem, substantial for practical solving the task (3.45), is selecting rational intervals Δ^+_υ, $\Delta^\pm_\upsilon$, Δ^-_υ. Generally, in similar cases, these intervals are substituted with intervals $\Delta^+_{\bar{\upsilon}}$, $\Delta^\pm_{\bar{\upsilon}}$ and $\Delta^-_{\bar{\upsilon}}$ referring to the standardized corrections $\bar{\upsilon} = \dfrac{\upsilon}{\hat{\sigma}_\upsilon}$ (for example: $\Delta^+_{\bar{\upsilon}} = \langle -k, k \rangle$, $\Delta^\pm_{\bar{\upsilon}} = \langle 2k, 5k \rangle$ and $\Delta^-_{\bar{\upsilon}} = \langle 5k, \infty \rangle$, for $k = 1$ or $k = 1{,}5$, or $k = 2$ etc.).

The estimator $\hat{\sigma}_\upsilon$ of the standard deviation σ_υ of the correction υ is also a root of the respective diagonal element of the estimator $\hat{\mathbf{C}}_{\mathbf{V}_i}$ of the corrections vector $\mathbf{V}_i$ covariance matrix $\mathbf{C}_{\mathbf{V}_i}$, it means $\hat{\sigma}_{\upsilon_j} = \sqrt{\left[\hat{\mathbf{C}}_{\mathbf{V}_i}\right]_{jj}}$. Anyhow, as (easy to prove)

$$\hat{\mathbf{C}}_{\mathbf{V}_i} = \hat{\sigma}_0^2 \left\{ \tilde{\tilde{\mathbf{P}}}_i - \mathbf{A}_i \left(\mathbf{A}_i^T \tilde{\tilde{\mathbf{P}}}_i \mathbf{A}_i \right)^{-1} \mathbf{A}_i^T \right\} \tag{3.53}$$

so in every $l-th$ iterative step

$$\sigma_{\upsilon_j}^{(l)} = \sigma_0^{(l)} \sqrt{\left[\tilde{\tilde{\mathbf{P}}}_i^{(l)} - \mathbf{A}_i \left(\mathbf{A}_i^T \tilde{\tilde{\mathbf{P}}}_i^{(l)} \mathbf{A}_i \right)^{-1} \mathbf{A}_i^T \right]_{jj}} \tag{3.54}$$

where: $\sigma_0^{(l)} \xrightarrow{l=0,1,\ldots} \hat{\sigma}_0$

Estimation of $\sigma_o^{(l)}$, especially for several first l values is substantially deformed by large values of the corrections, covering gross observations errors. It is caused by a fact, that in those first steps, the weights matrix $\mathbf{P}_i^{(l)}$ differs significantly from the equivalent weights matrix $\tilde{\tilde{\mathbf{P}}}$, and consequently, to an insufficient degree, there "runs out" an influence of large values $\upsilon_j^{(l)}$ on the value of $\sigma_o^{(l)} = \sqrt{\frac{\left\{ \left(\mathbf{V}_i^{(l)}\right)^T \mathbf{P}_i^{(l)} \mathbf{V}_i^{(l)} \right\}}{f}}$. The literature on the above subject matter there are suggested various solution of the task, as for example applying robust estimation of the coefficient σ_0 (VR – estimation, [Wiśniewski 1999]). However it is confirmed that (what has been applied by the author in some of his previous papers) that good results can be obtained when at every iterative step a theoretical value $\sigma_0 = 1$ is assumed. [Wiśniewski 2004]. In our situation it consists in making an interim assumption that $\mathbf{C}_{\boldsymbol{x}_i} = \mathbf{Q}_{\boldsymbol{x}_i} = \mathbf{P}_{\boldsymbol{x}_i}^{-1}$ and $\mathbf{C}_{\mathbf{X}_i^{DGPS}} = \mathbf{Q}_{\mathbf{X}_i^{DGPS}} = \mathbf{P}_{\mathbf{X}_i^{DGPS}}^{-1}$. The real value of the variance coefficient σ_0^2 is then determined basing on the formula (3.48) and on the grounds of the stabilized, equivalent weights matrix $\tilde{\tilde{\mathbf{P}}}_i$.

3.4. Special cases

A. Let's assume that in *IANS* element only the observations towards $\mathfrak{Z}$ points were carried out and they are a basis for determination of the watercraft position. Neither GPS survey are performed nor measurements which apply for determination of the new adjustment points $\Re_i$. The following decisive matrixes values correspond to such a basic task of terrestrial navigation:

without DGPS observation $\rightarrow$ $\Im(\mathbf{X}_{P_i}^{DGPS}) = \mathbf{0}$

without observation to $\Re_i$ - points $\rightarrow$ $\Im(\boldsymbol{x}_{R_i}^{(i)}) = \mathbf{0}$

Then, taking to consideration (3.36a), (3.37) and the internal structure of the weights matrix $\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_i} = Diag\left(\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}, \widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_{R_i}^{(i)}}\right)$, we may obtain

$$\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_i} = Diag\left(\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}, \widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_{R_i}^{(i)}}\right) = Diag\left\{\Im(\boldsymbol{x}_{Z_i})\widehat{\mathbf{P}}_{\boldsymbol{x}_{Z_i}}\Im(\boldsymbol{x}_{Z_i}), \Im(\boldsymbol{x}_{R_i}^{(i)})\widehat{\mathbf{P}}_{\boldsymbol{x}_{R_i}^{(i)}}\Im(\boldsymbol{x}_{R_i}^{(i)})\right\} =$$

$$= Diag\left\{\Im(\boldsymbol{x}_{Z_i})\widehat{\mathbf{P}}_{\boldsymbol{x}_{Z_i}}\Im(\boldsymbol{x}_{Z_i}), \mathbf{0}\right\} = Diag\left(\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}, \mathbf{0}\right)$$

$$\widetilde{\widehat{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} = \Im(\mathbf{X}_{P_i}^{DGPS})\widehat{\mathbf{P}}_{\mathbf{X}_i^{DGPS}}\Im(\mathbf{X}_{P_i}^{DGPS}) = \mathbf{0}$$

Thus

$$\mathbf{A}_{P_i}^T\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_i}\mathbf{A}_{P_i} + \mathbf{P}_{\mathbf{X}_i^{DGPS}} = \begin{bmatrix}\mathbf{A}_{Z_iP_i}^T & \mathbf{A}_{R_iP_i}^T\end{bmatrix}\begin{bmatrix}\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{A}_{Z_iP_i}\\ \mathbf{A}_{R_iP_i}\end{bmatrix} + \mathbf{P}_{\mathbf{X}_i^{DGPS}} = \mathbf{A}_{Z_iP_i}^T\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}\mathbf{A}_{Z_iP_i} + \mathbf{0}$$

$$\mathbf{A}_{P_i}^T\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_i}\mathbf{A}_{R_i}^{(i)} = \begin{bmatrix}\mathbf{A}_{Z_iP_i}^T & \mathbf{A}_{R_iP_i}^T\end{bmatrix}\begin{bmatrix}\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{A}_{Z_iR_i} = \mathbf{0}\\ \mathbf{A}_{R_i}^{(i)}\end{bmatrix} = \mathbf{0}$$

$$\left(\mathbf{A}_{R_i}^{(i)}\right)^T\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_i}\mathbf{A}_{R_i}^{(i)} = \begin{bmatrix}\mathbf{A}_{Z_iR_i}^T & \left(\mathbf{A}_{R_iR_i}^{(i)}\right)^T\end{bmatrix}\begin{bmatrix}\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{A}_{Z_iR_i} = \mathbf{0}\\ \mathbf{A}_{R_iR_i}^{(i)}\end{bmatrix} = \mathbf{0}$$

(having earlier proved, that $\mathbf{A}_{Z_iR_i} = \partial_{\mathbf{X}_{R_i}}\mathbf{F}_{Z_i}(\mathbf{X}_{P_i}^0, \mathbf{X}_{R_i}^0) = \mathbf{0}$), and basing on it

$$\mathbf{A}_i^T\widetilde{\widehat{\mathbf{P}}}_i\mathbf{A}_i = \begin{bmatrix}\mathbf{A}_{Z_iP_i}^T\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}\mathbf{A}_{Z_iP_i} & \vdots & \mathbf{0}\\ \cdots & \vdots & \cdots\\ \mathbf{0} & \vdots & \mathbf{0}\end{bmatrix}$$

Moreover, as

$$\mathbf{A}_i^T\widetilde{\widetilde{\mathbf{P}}}_i\mathbf{L}_i=\begin{bmatrix}\mathbf{A}_{P_i}^T & \mathbf{I}_{(2)}\\ \left(\mathbf{A}_{R_iR_i}^{(i)}\right)^T & \mathbf{0}\end{bmatrix}\begin{bmatrix}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} & \\ & \widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}}\end{bmatrix}\begin{bmatrix}\mathbf{L}_{\boldsymbol{x}_i}\\ \mathbf{X}_{P_i}^0-\mathbf{X}_{P_i}^{DGPS}\end{bmatrix}=\begin{bmatrix}\mathbf{A}_{P_i}^T\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i}\mathbf{L}_{\boldsymbol{x}_i}+\widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}}\left(\mathbf{X}_{P_i}^0-\mathbf{X}_{P_i}^{DGPS}\right)\\ \left(\mathbf{A}_{R_iR_i}^{(i)}\right)^T\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i}\mathbf{L}_{\boldsymbol{x}_i}\end{bmatrix}$$

and (considering the assumed simplifications)

$$\mathbf{A}_{P_i}^T\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i}\mathbf{L}_{\boldsymbol{x}_i}+\widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}}\left(\mathbf{X}_{P_i}^0-\mathbf{X}_{P_i}^{DGPS}\right)=\mathbf{A}_{Z_iP_i}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}\mathbf{L}_{Z_i}$$

$$\left(\mathbf{A}_{R_iR_i}^{(i)}\right)^T\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i}\mathbf{L}_{\boldsymbol{x}_i}=\mathbf{0}$$

so

$$\mathbf{A}_i^T\widetilde{\widetilde{\mathbf{P}}}_i\mathbf{L}_i=\begin{bmatrix}\mathbf{A}_{Z_iP_i}^T\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}\mathbf{L}_{Z_i}\\ \mathbf{0}\end{bmatrix}$$

As e.g. [Rao 1982]

$$\left(\mathbf{A}_i^T\widetilde{\widetilde{\mathbf{P}}}_i\mathbf{A}_i\right)^-=\begin{bmatrix}\mathbf{A}_{Z_iP_i}^T\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}\mathbf{A}_{Z_iP_i} & \vdots & \mathbf{0}\\ \cdots & \vdots & \cdots\\ \mathbf{0} & \vdots & \mathbf{0}\end{bmatrix}^-=\begin{bmatrix}\left(\mathbf{A}_{Z_iP_i}^T\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}\mathbf{A}_{Z_iP_i}\right)^{-1} & \vdots & \mathbf{0}\\ \cdots & \vdots & \cdots\\ \mathbf{0} & \vdots & \mathbf{0}\end{bmatrix}$$

so

$$\hat{\mathbf{d}}_{\mathbf{X}_i}=-\left(\mathbf{A}_i^T\widetilde{\widetilde{\mathbf{P}}}_i\mathbf{A}_i\right)^-\mathbf{A}_i^T\widetilde{\widetilde{\mathbf{P}}}_i\mathbf{L}_i=-\begin{bmatrix}\left(\mathbf{A}_{Z_iP_i}^T\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}\mathbf{A}_{Z_iP_i}\right)^{-1} & \vdots & \mathbf{0}\\ \cdots & \vdots & \cdots\\ \mathbf{0} & \vdots & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{A}_{Z_iP_i}^T\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}\mathbf{L}_{Z_i}\\ \mathbf{0}\end{bmatrix}= \qquad (3.55)$$

$$=\begin{bmatrix}-\left(\mathbf{A}_{Z_iP_i}^T\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}\mathbf{A}_{Z_iP_i}\right)^{-1}\mathbf{A}_{Z_iP_i}^T\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}\mathbf{L}_{Z_i}\\ \mathbf{0}\end{bmatrix}=\begin{bmatrix}\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)}\\ \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}\end{bmatrix}$$

(if $\left(\mathbf{A}_{Z_iP_i}^T\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}\mathbf{A}_{Z_iP_i}\right)^{-1}$ only exists).

The case described hereby, making additional simplifying assumptions:

$$\left.\begin{array}{l}\left.\begin{array}{l}\text{the adjustment comprises all the}\\ \text{observations of the set } \mathcal{O}_i^Z\end{array}\right\} \rightarrow \Im(\boldsymbol{x}_{Z_i}) = \mathbf{I}_{(2n_{Z_i})} \\ \left.\begin{array}{l}\text{the adjustment is neutral, what}\\ \text{means that no weights attenuation}\end{array}\right\} \rightarrow \mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_{Z_i}}) = \mathbf{I}_{(2n_{Z_i})}\end{array}\right\} \Leftrightarrow \left(\widetilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}} = \mathbf{P}_{\boldsymbol{x}_{Z_i}}\right)$$

is the most simple variant of the watercraft position adjustment, determined basing on the observations carried out toward the adjustment points $\mathfrak{Z}$. Practical examples of such an adjustment are presented in the basic literature [Urbański, Kopacz, Posiła 2000; Górski, Jackowski, Urbański 1990; Wiśniewski 2004]. Theoretical –numerical analyses, concerning such a task, formulated on grounds of the quotient navigational system (in that case pseudo-observations are quotients of the distances to $\mathfrak{Z}$ points) were the subject of publications [Czaplewski 1998, 1999; Kołaczyński 1995, Wiśniewski 2002]. An influence of the additional observations in the quotient navigational system on improvement of watercraft position survey accuracy was a subject of the analysis presented in the paper [Czaplewski, Wiśniewski 1999b].

Solution of the basic task, with application of robust estimation, thus applying the attenuation matrix, is presented in the works [Czaplewski 2004a; Czaplewski, Wiśniewski 2003b,c]. Making an assumption that the observations are mutually independent, in the above mentioned works there was applied the equivalent weights matrix, of the following form $\widehat{\mathbf{P}}_{\boldsymbol{x}_{Z_i}} = \mathbf{T}(\mathbf{V}_{\boldsymbol{x}_{Z_i}})\mathbf{P}_{\boldsymbol{x}_{Z_i}}$, what is a particular case of the matrix (3.38) $\left(at\left(\Im(\boldsymbol{x}_{Z_i}) = \mathbf{I}_{(2n_{Z_i})}\right) \Leftrightarrow \left(\widetilde{\mathbf{T}}(\boldsymbol{x}_{Z_i}, \mathbf{V}_{\boldsymbol{x}_{Z_i}}) = \mathbf{T}(\mathbf{V}_{\boldsymbol{x}_{Z_i}})\right)\right)$, having been generalized in this work. In the cited works, as the diagonal elements of the attenuation matrix, there is adopted the attenuation function of the following form [Krarup, Kubik 1982]:

$$t(\bar{\upsilon}) = \begin{cases} 1 & for \quad \bar{\upsilon} \in \Delta_{\bar{\upsilon}}^{+} \\ \exp\left\{-\left(l|\bar{\upsilon}| - k\right)^{g}\right\} & for \quad \bar{\upsilon} \notin \Delta_{\bar{\upsilon}}^{+} \end{cases}$$

(the so-called Danish Attenuation Function)

B. Let us assume that in the *IANS* element, apart from the observations towards the points $\mathfrak{Z}$, there also were DGPS measurements taken; thus obtaining the vector of coordinates $\mathbf{X}_{P_i}^{DGPS}$ of the covariance matrix $\mathbf{C}_{\mathbf{X}_i^{DGPS}} = \sigma_0^2 \mathbf{Q}_{\mathbf{X}_i^{DGPS}} = \sigma_0^2 \mathbf{P}_{\mathbf{X}_i^{DGPS}}^{-1}$. It is extremely important terrestrial navigation, essential for coasting trade, supported with the satellite system. The general model of the *IANS* element can be reduced to the described case, through acceptance of the following decisive matrix value:

no observations toward $\mathfrak{R}_i$ - points $\quad \rightarrow \mathfrak{I}(\boldsymbol{x}_{R_i}^{(i)}) = \mathbf{0}$

Then it remains as follows (as in the case $(\mathbf{A})$):

$$\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} = Diag\left(\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}, \mathbf{0}\right)$$

but this time, as $\mathfrak{I}(\mathbf{X}_i^{DGPS}) \neq \mathbf{0}$, then also $\widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} \neq \mathbf{0}$.

Therefore $\quad \mathbf{A}_{P_i}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} \mathbf{A}_{P_i} + \widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} = \mathbf{A}_{Z_i P_i}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}} \mathbf{A}_{Z_i P_i} + \widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}}$

$$\mathbf{A}_{P_i}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} \mathbf{A}_{R_i}^{(i)} = \mathbf{0}$$

$$\left(\mathbf{A}_{R_i}^{(i)}\right)^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} \mathbf{A}_{R_i}^{(i)} = \mathbf{0}$$

$$\mathbf{A}_{P_i}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} \mathbf{L}_{\boldsymbol{x}_i} + \widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} \left(\mathbf{X}_{P_i}^0 - \mathbf{X}_{P_i}^{DGPS}\right) = \mathbf{A}_{Z_i P_i}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}} \mathbf{L}_{Z_i} + \widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} \left(\mathbf{X}_{P_i}^0 - \mathbf{X}_{P_i}^{DGPS}\right)$$

$$\left(\mathbf{A}_{R_i}^{(i)}\right)^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_i} \mathbf{L}_{\boldsymbol{x}_i} = \mathbf{0}$$

and next

$$\mathbf{A}_i^T \widetilde{\widetilde{\mathbf{P}}}_i \mathbf{A}_i = \begin{bmatrix} \mathbf{A}_{Z_i P_i}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}} \mathbf{A}_{Z_i P_i} + \widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} & \vdots & \mathbf{0} \\ \cdots & \vdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{0} \end{bmatrix}$$

$$\mathbf{A}_i^T \widetilde{\widetilde{\mathbf{P}}}_i \mathbf{L}_i = \begin{bmatrix} \mathbf{A}_{Z_i P_i}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}} \mathbf{L}_{Z_i} + \widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} \left(\mathbf{X}_{P_i}^0 - \mathbf{X}_{P_i}^{DGPS}\right) \\ \mathbf{0} \end{bmatrix}$$

thus

$$\left.\begin{aligned} \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} &= -\left(\mathbf{A}^T_{Z_iP_i}\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}\mathbf{A}_{Z_iP_i} + \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}}\right)^{-1}\left\{\mathbf{A}^T_{Z_iP_i}\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}}\mathbf{L}_{Z_i} + \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}}\left(\mathbf{X}^0_{P_i} - \mathbf{X}^{DGPS}_{P_i}\right)\right\} \\ \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_R} &= \mathbf{0}\end{aligned}\right\} \tag{3.56}$$

Moreover, if it is assumed that

the adjustment is comprising all the observations of $\mho_i^Z$ set $\Big\}\rightarrow \Im(\boldsymbol{x}_{Z_i}) = \mathbf{I}(2n_{Z_i})$

the adjustment is neutral, what means that no weights attenuation $\Big\}\rightarrow \mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_{Z_i}}) = \mathbf{I}(2n_{Z_i})$

$$\Bigg\} \Leftrightarrow \left(\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_i}} = \mathbf{P}_{\boldsymbol{x}_{Z_i}}\right)$$

the adjustment comprises the both coordinates $X_{P_i}^{DGPS}, Y_{P_i}^{DGPS}$ $\Big\}\rightarrow \Im(\mathbf{X}_{P_i}^{DGPS}) = \mathbf{I}(2)$

no weights attenuation $\Big\}\rightarrow \mathbf{T}_{sqr}(\mathbf{V}_{\mathbf{X}_{P_i}^{DGPS}}) = \mathbf{I}(2)$

$$\Bigg\} \Leftrightarrow \left(\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} = \mathbf{P}_{\mathbf{X}_i^{DGPS}}\right)$$

Thus solution (3.56) is a solution of a classic sequential task [Sikorski 1979,1991, Baran 1999]. The task is presented in this paper in its most simple version [Baran 1999] which solution, in essence, is generalization of the arithmetic mean, thus in this case: coordinates obtained as a result of survey, obtained on carrying out measurements towards the station $\mathfrak{Z}$ and the coordinates obtained applying the GPS method.

4. DEVELOPMENT OF THE *IANS* CHAIN

4.1. The Assumptions

An intrinsic *IANS* element for $i = 1$ is the one which enables adjusting only the proper position P_i. The points which are included in $\Re_i$ can be in this case determined uniquely. Therefore let's assume (according to the assumptions made in Chapter 2) such a navigational situation, in which after dislocation of a vessel to the position P_{i+1} there are carried out surveys towards the adjustment points $\mathfrak{Z}_{i+1} \subset \mathfrak{Z}$, also DGPS surveys based on the reference stations $\mathfrak{Z}_{i+1}^{DGPS} \subset \mathfrak{Z}$, surveys to the points $\Re_i \subset \Re$, determined before and surveys to the new (for the P_{i+1} positions) points $\Re_{i+1} \subset \Re$ as well. We also assume that the route elements are known (a course, distance travelled). The more interesting alternative of such situation is an assumption, that P_i and P_{i+1} stand for positions of two vessels, carrying out a common navigational task, which is consisted in developing of *IANS* chain. Then the route vector elements may be substituted with mutual observations, characterized with accuracy as of the principal surveys.

Therefore let's assume that in the position P_{i+1} the following sets and observation vectors are available:

- the observations carried out at the point P_{i+1} toward the adjustment points $\mathfrak{Z}_i \subset \Re$

$$\mathfrak{O}_{i+1}^{Z} \rightarrow \boldsymbol{x}_{Z_{i+1}} \in \mathfrak{M}\left(2n_{Z_{i+1}},1\right) \begin{cases} \text{with weights matrix } \mathbf{P}_{\boldsymbol{x}_{Z_{i+1}}} \\ \text{with decisive matrix } \mathfrak{I}(\boldsymbol{x}_{Z_{i+1}}) \end{cases}$$

- the observations carried out at the point P_{i+1} towards the previously determined points $\Re_i \subset \Re$

$$\mathfrak{O}_{i+1}^{R(i)} \rightarrow \boldsymbol{x}_{R_i}^{(i+1)} \in \mathfrak{M}\left(2n_{R_i},1\right) \begin{cases} \text{with weights matrix } \mathbf{P}_{\boldsymbol{x}_{R_i}^{(i+1)}} \\ \text{with decisive matrix } \mathfrak{I}(\boldsymbol{x}_{R_i}^{(i+1)}) \end{cases}$$

- the observations carried out at the point P_{i+1} toward new points $\Re_{i+1} \subset \Re$

$$\mathcal{O}_{i+1}^{R(i+1)} \rightarrow \boldsymbol{x}_{R_{i+1}}^{(i+1)} \in \mathfrak{M}(2n_{R_{i+1}},1) \begin{cases} \text{with weights matrix } \mathbf{P}_{\boldsymbol{x}_{R_{i+1}}^{(i+1)}} \\ \text{with decisive matrix } \mathfrak{I}(\boldsymbol{x}_{R_{i+1}}^{(i+1)}) \end{cases}$$

- the route vector or the mutual observations vector

$$\mathcal{O}_{i+1}^{W} \rightarrow \boldsymbol{x}_{W_{i+1}} \in \mathfrak{M}(n_{W_{i+1}},1) \begin{cases} \text{with weights matrix } \mathbf{P}_{\boldsymbol{x}_{W_{i+1}}} \\ \text{with decisive matrix } \mathfrak{I}(\boldsymbol{x}_{W_{i+1}}) \end{cases}$$

(especially when $\mathcal{O}_{i+1}^{W}$ is the route vector $n_{W_{i+1}} = 2$ - the course and travelled route).

Basing on the previous determinations, the following pseudo-observations are also available:

- coordinates of the position P_i

$$\hat{\mathbf{X}}_{P_i}^{(i)} \begin{cases} \text{with weight matrix } \mathbf{P}_{\hat{\mathbf{X}}_{P_i}^{(i)}} = \mathbf{A}_{P_i}^{T} \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_i} \mathbf{A}_{P_i} + \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_i^{DGPS}} \\ \text{with decisive matrix } \mathfrak{I}(\hat{\mathbf{X}}_{P_i}^{(i)}) \end{cases}$$

- the obtained at the position P_i coordinates of the points $\Re_i \subset \Re$

$$\hat{\mathbf{X}}_{R_i}^{(i)} \begin{cases} \text{with weights matrix } \mathrm{P}_{\hat{\mathrm{X}}_{\mathrm{R_i}}^{(i)}} = \left(\mathrm{A}_{\mathrm{R_i}}^{(i)}\right)^{\mathrm{T}} \tilde{\tilde{\mathrm{P}}}_{\boldsymbol{x}_i} \mathrm{A}_{\mathrm{R_i}}^{(i)} \\ \text{with decisive matrix } \mathfrak{I}(\hat{\mathbf{X}}_{R_i}^{(i)}) \end{cases}$$

- DGPS coordinates of the position P_{i+1}

$$\mathbf{X}_{P_{i+1}}^{DGPS} \begin{cases} \text{with weights matrix } \mathbf{P}_{\mathbf{X}_{P_{i+1}}^{DGPS}} = \mathbf{Q}_{\mathbf{X}_{P_{i+1}}^{DGPS}}^{-1} \\ \text{with decisive matrix } \mathfrak{I}(\mathbf{X}_{P_{i+1}}^{DGPS}) \end{cases}$$

The accepted observational system is also explained in Fig. 4.1.

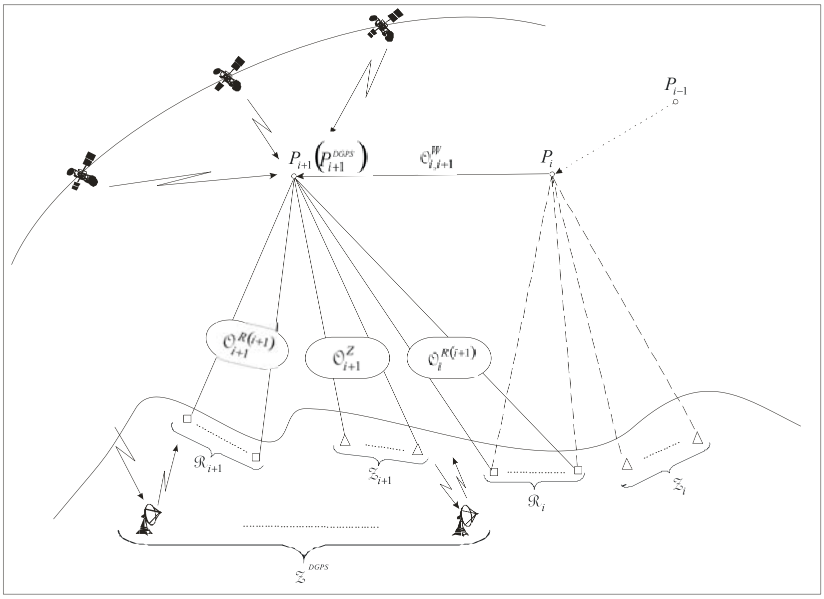

Fig. 4.1 *IANS* development stage at the $i+1$ moment

Let's agree that

$$\hat{\hat{\mathbf{x}}}_{Z_{i+1}} = \mathbf{F}_{Z_{i+1}}(\hat{\mathbf{X}}_{P_{i+1}}^{(i+1)}, \hat{\mathbf{X}}_{R_{i+1}}^{(i+1)}, \hat{\mathbf{X}}_{R_i}^{(i+1)}, \hat{\mathbf{X}}_{P_i}^{(i+1)}) \rightarrow \begin{cases} \partial_{\mathbf{x}_{P_{i+1}}} \mathbf{F}_{Z_{i+1}} = \mathbf{A}_{Z_{i+1}P_{i+1}} \in \mathfrak{M}\left(2n_{Z_{i+1}}, 2\right) \\ \partial_{\mathbf{x}_{R_{i+1}}} \mathbf{F}_{Z_{i+1}} = \mathbf{A}_{Z_{i+1}R_{i+1}} = \mathbf{0} \in \mathfrak{M}\left(2n_{Z_{i+1}}, 2n_{R_{i+1}}\right) \\ \partial_{\mathbf{x}_{R_i}} \mathbf{F}_{Z_{i+1}} = \mathbf{A}_{Z_{i+1}R_i} = \mathbf{0} \in \mathfrak{M}\left(2n_{Z_{i+1}}, 2n_{R_i}\right) \\ \partial_{\mathbf{x}_{P_i}} \mathbf{F}_{Z_{i+1}} = \mathbf{A}_{Z_{i+1}P_i} = \mathbf{0} \in \mathfrak{M}\left(2n_{Z_{i+1}}, 2\right) \end{cases}$$

$$\hat{\hat{\mathbf{x}}}_{R_{i+1}}^{(i+1)} = \mathbf{F}_{R_{i+1}}^{(i+1)}(\hat{\mathbf{X}}_{P_{i+1}}^{(i+1)}, \hat{\mathbf{X}}_{R_{i+1}}^{(i+1)}, \hat{\mathbf{X}}_{R_i}^{(i+1)}, \hat{\mathbf{X}}_{P_i}^{(i+1)}) \rightarrow \begin{cases} \partial_{\mathbf{x}_{P_{i+1}}} \mathbf{F}_{R_{i+1}}^{(i+1)} = \mathbf{A}_{R_{i+1}P_{i+1}} \in \mathfrak{M}\left(2n_{R_{i+1}}, 2\right) \\ \partial_{\mathbf{x}_{R_{i+1}}} \mathbf{F}_{R_{i+1}}^{(i+1)} = \mathbf{A}_{R_{i+1}R_{i+1}} \in \mathfrak{M}\left(2n_{R_{i+1}}, 2n_{R_{i+1}}\right) \\ \partial_{\mathbf{x}_{R_i}} \mathbf{F}_{R_{i+1}}^{(i+1)} = \mathbf{A}_{R_{i+1}R_i} = \mathbf{0} \in \mathfrak{M}\left(2n_{R_{i+1}}, 2n_{R_i}\right) \\ \partial_{\mathbf{x}_{P_i}} \mathbf{F}_{R_{i+1}}^{(i+1)} = \mathbf{A}_{R_{i+1}P_i} = \mathbf{0} \in \mathfrak{M}\left(2n_{R_{i+1}}, 2\right) \end{cases}$$

$$\hat{\hat{\mathbf{x}}}_{R_i}^{(i+1)} = \mathbf{F}_{R_i}^{(i+1)}(\hat{\mathbf{X}}_{P_{i+1}}^{(i+1)}, \hat{\mathbf{X}}_{R_{i+1}}^{(i+1)}, \hat{\mathbf{X}}_{R_i}^{(i+1)}, \hat{\mathbf{X}}_{P_i}^{(i+1)}) \rightarrow \begin{cases} \partial_{\mathbf{X}_{P_{i+1}}} \mathbf{F}_{R_i}^{(i+1)} = \mathbf{A}_{R_i P_{i+1}} \in \mathfrak{M}(2n_{R_{i+1}}, 2) \\ \partial_{\mathbf{X}_{R_{i+1}}} \mathbf{F}_{R_i}^{(i+1)} = \mathbf{A}_{R_i R_{i+1}} = \mathbf{0} \in \mathfrak{M}(2n_{R_i}, 2n_{R_{i+1}}) \\ \partial_{\mathbf{X}_{R_i}} \mathbf{F}_{R_i}^{(i+1)} = \mathbf{A}_{R_i R_i}^{(i+1)} \in \mathfrak{M}(2n_{R_i}, 2n_{R_i}) \\ \partial_{\mathbf{X}_{P_i}} \mathbf{F}_{R_i}^{(i+1)} = \mathbf{A}_{R_i P_i}^{(i+1)} = \mathbf{0} \in \mathfrak{M}(2n_{R_i}, 2) \end{cases}$$

$$\hat{\hat{\mathbf{x}}}_{W_{i+1}} = \mathbf{F}_{W_{i+1}}(\hat{\mathbf{X}}_{P_{i+1}}^{(i+1)}, \hat{\mathbf{X}}_{R_{i+1}}^{(i+1)}, \hat{\mathbf{X}}_{R_i}^{(i+1)}, \hat{\mathbf{X}}_{P_i}^{(i+1)}) \rightarrow \begin{cases} \partial_{\mathbf{X}_{P_{i+1}}} \mathbf{F}_{W_{i+1}} = \mathbf{A}_{W_{i+1} P_{i+1}} \in \mathfrak{M}(2n_{W_{i+1}}, 2) \\ \partial_{\mathbf{X}_{R_{i+1}}} \mathbf{F}_{W_{i+1}} = \mathbf{A}_{W_{i+1} R_{i+1}} = \mathbf{0} \in \mathfrak{M}(2n_{W_{i+1}}, 2n_{R_{i+1}}) \\ \partial_{\mathbf{X}_{R_i}} \mathbf{F}_{W_{i+1}} = \mathbf{A}_{W_{i+1} R_i} = \mathbf{0} \in \mathfrak{M}(2n_{W_{i+1}}, 2n_{R_i}) \\ \partial_{\mathbf{X}_{P_i}} \mathbf{F}_{W_{i+1}} = \mathbf{A}_{W_{i+1} P_i} \in \mathfrak{M}(2n_{W_{i+1}}, 2) \end{cases}$$

and

$\hat{\hat{\mathbf{x}}}_{Z_{i+1}} = \mathbf{x}_{Z_{i+1}} + \mathbf{V}_{Z_{i+1}}$, $\hat{\hat{\boldsymbol{x}}}_{R_{i+1}} = \boldsymbol{x}_{R_{i+1}} + \mathbf{V}_{R_{i+1}}^{(i+1)}$, $\hat{\hat{\boldsymbol{x}}}_{R_i}^{(i+1)} = \boldsymbol{x}_{R_i}^{(i+1)} + \mathbf{V}_{R_i}^{(i+1)}$,
$\hat{\hat{\boldsymbol{x}}}_{W_{i+1}} = \boldsymbol{x}_{W_{i+1}} + \mathbf{V}_{W_{i+1}}$

as we assume that at the position P_{i+1} there also are carried out DGPS surveys (conducting to obtain coordinates $\mathbf{X}_{P_{i+1}}^{DGPS}$ of the covariance matrix $\mathbf{C}_{\mathbf{X}_{P_{i+1}}^{DGPS}} = \sigma_0^2 \mathbf{Q}_{\mathbf{X}_{P_{i+1}}^{DGPS}}$, by way of analogy to (3.6) and (3.7) we can write down that:

$$\left. \begin{aligned} \mathbf{V}_{\mathbf{X}_{i+1}^{DGPS}} &= \hat{\mathbf{X}}_{P_{i+1}}^{(i+1)} - \mathbf{X}_{P_{i+1}}^{DGPS} \\ \hat{\mathbf{X}}_{P_{i+1}}^{(i+1)} &= \mathbf{X}_{P_{i+1}}^{0} + \hat{\mathbf{d}}_{\mathbf{X}_{P_{i+1}}}^{(i+1)} \end{aligned} \right\} \Rightarrow \mathbf{V}_{\mathbf{X}_{i+1}^{DGPS}} = \hat{\mathbf{d}}_{\mathbf{X}_{P_{i+1}}}^{(i+1)} + \mathbf{X}_{P_{i+1}}^{0} - \mathbf{X}_{P_{i+1}}^{DGPS} \tag{4.1}$$

The $\mathbf{X}_{P_i}$ coordinates of the position P_i have already been adjusted and in the *INS* chain being currently worked out, they are represented by $\hat{\mathbf{X}}_{P_i}^{(i)}$ estimator of the covariance matrix $\mathbf{C}_{\hat{\mathbf{X}}_{P_i}^{(i)}} = \sigma_0^2 \mathbf{Q}_{\hat{\mathbf{X}}_{P_i}^{(i)}} = \sigma_0^2 \mathbf{P}_{\hat{\mathbf{X}}_{P_i}^{(i)}}^{-1}$. However, the positions P_i and P_{i+1} are mutually related through the route vector elements (or mutual observations of two vessels), and also, indirectly, through the points, common for the both *INS* elements, and included to $\Re_i$. In this situation the $\hat{\mathbf{X}}_{P_i}^{(i)}$ estimator is not a final assessment of the earlier position P_i. Let us assume that $\hat{\mathbf{X}}_{P_i}^{(i+1)}$ is the assessment.

Then, considering the earlier made assessment as pseudo-observation, according to the general principles of sequence adjustment, we can state that:

$$\left.\begin{array}{r}\hat{\mathbf{X}}_{P_i}^{(i+1)}=\hat{\mathbf{X}}_{P_i}^{(i)}+\mathbf{V}_{\hat{\mathbf{X}}_{P_i}}\\ \mathbf{X}_{P_i}^{0}+\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i+1)}=\mathbf{X}_{P_i}^{0}+\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)}+\mathbf{V}_{\hat{\mathbf{X}}_{P_i}}\end{array}\right\}\Rightarrow \mathbf{V}_{\hat{\mathbf{X}}_{P_i}}=\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i+1)}-\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)} \qquad (4.2)$$

In navigation of the singular object, the coordinates $\hat{\mathbf{X}}_{P_i}^{(i+1)}$, at the current position P_{i+1}, are not of practical, serious significance, (however, basing on values of the corrections vector $\mathbf{V}_{\hat{\mathbf{X}}_{P_i}}$ there might be reached additional conclusions about navigation quality – it's a separate problem anyway, not to be considered in this paper). The corrected assessment of the earlier position is a result of working up jointly all the observations, available in current situation P_{i+1}. Joint working out is necessary anyhow to obtain the best assessment of coordinates of the points $\Re$ which are of peculiar interest for us. Importance of the current $\hat{\mathbf{X}}_{P_i}^{(i+1)}$ assessment is, however, significant also from the practical point of view, in situation when P_i and P_{i+1} are positions at two vessels, which carry on their common navigational (or hydrographic) task.

Also the assessment of $\hat{\mathbf{X}}_{R_i}^{(i+1)}$ coordinates of the points $\Re_i$, obtained at the P_i position we can considered as pseudo-observation. Assuming that a joint assessment of the position P_{i+1} dimension is the vector $\hat{\mathbf{X}}_{R_i}^{(i+1)}$, by analogy to (4.2) we may put as follows:

$$\left.\begin{array}{r}\hat{\mathbf{X}}_{R_i}^{(i+1)}=\hat{\mathbf{X}}_{R_i}^{(i)}+\mathbf{V}_{\hat{\mathbf{X}}_{R_i}}\\ \mathbf{X}_{R_i}^{0}+\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i+1)}=\mathbf{X}_{R_i}^{0}+\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}+\mathbf{V}_{\hat{\mathbf{X}}_{R_i}}\end{array}\right\}\Rightarrow \mathbf{V}_{\hat{\mathbf{X}}_{R_i}}=\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i+1)}-\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)}$$

Taking into consideration the presented assumptions, the following system of the functional-decisive models concerning the *IANS* chain, being a subject of analysis, can be formulated:

$$\begin{array}{l}\Im(\boldsymbol{x}_{Z_{i+1}})\cdot\ \mathbf{V}_{Z_{i+1}}=\mathbf{A}_{Z_{i+1}P_{i+1}}\hat{\mathbf{d}}_{\mathbf{X}_{P_{i+1}}}^{(i+1)}+\mathbf{A}_{Z_{i+1}R_{i+1}}\hat{\mathbf{d}}_{\mathbf{X}_{R_{i+1}}}^{(i+1)}+\mathbf{A}_{Z_{i+1}R_i}\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i+1)}+\mathbf{A}_{Z_{i+1}P_i}\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i+1)}+\mathbf{L}_{Z_{i+1}}\\ \Im(\boldsymbol{x}_{R_{i+1}}^{(i+1)})\cdot\ \mathbf{V}_{R_{i+1}}^{(i+1)}=\mathbf{A}_{R_{i+1}P_{i+1}}\hat{\mathbf{d}}_{\mathbf{X}_{P_{i+1}}}^{(i+1)}+\mathbf{A}_{R_{i+1}R_{i+1}}\hat{\mathbf{d}}_{\mathbf{X}_{R_{i+1}}}^{(i+1)}+\mathbf{A}_{R_{i+1}R_i}\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i+1)}+\mathbf{A}_{R_{i+1}P_i}\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i+1)}+\mathbf{L}_{R_{i+1}}\\ \Im(\boldsymbol{x}_{R_i}^{(i+1)})\cdot\ \mathbf{V}_{R_i}^{(i+1)}=\mathbf{A}_{R_iP_{i+1}}\hat{\mathbf{d}}_{\mathbf{X}_{P_{i+1}}}^{(i+1)}+\mathbf{A}_{R_iR_{i+1}}\hat{\mathbf{d}}_{\mathbf{X}_{R_{i+1}}}^{(i+1)}+\mathbf{A}_{R_iR_i}\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i+1)}+\mathbf{A}_{R_iP_i}\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i+1)}+\mathbf{L}_{R_i}^{(i+1)}\\ \Im(\boldsymbol{x}_{W_{i+1}})\cdot\ \mathbf{V}_{W_{i+1}}=\mathbf{A}_{W_{i+1}P_{i+1}}\hat{\mathbf{d}}_{\mathbf{X}_{P_{i+1}}}^{(i+1)}+\mathbf{A}_{W_{i+1}R_{i+1}}\hat{\mathbf{d}}_{\mathbf{X}_{R_{i+1}}}^{(i+1)}+\mathbf{A}_{W_{i+1}R_i}\hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i+1)}+\mathbf{A}_{W_{i+1}P_i}\hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i+1)}+\mathbf{L}_{W_{i+1}}\end{array}$$

$$\Im(\mathbf{X}_{P_{i+1}}^{DGPS})\cdot\ \mathbf{V}_{\mathbf{X}_{i+1}^{DGPS}} = \hat{\mathbf{d}}_{\mathbf{X}_{P_{i+1}}}^{(i+1)} + \mathbf{X}_{p_{i+1}}^{0} - \mathbf{X}_{P_{i+1}}^{DGPS}$$

$$\Im(\hat{\mathbf{X}}_{P_i}^{(i)})\cdot\ \mathbf{V}_{\hat{\mathbf{X}}_{P_i}} = \hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i+1)} - \hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i)}$$

$$\Im(\hat{\mathbf{X}}_{R_i}^{(i)})\cdot\ \mathbf{V}_{\hat{\mathbf{X}}_{R_i}} = \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i+1)} - \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i)}$$

where:

$$\mathbf{L}_{Z_{i+1}} = \mathbf{F}_{Z_{i+1}}(\mathbf{X}_{i+1}^{0}) - \boldsymbol{x}_{Z_{i+1}} \qquad \mathbf{L}_{R_{i+1}} = \mathbf{F}_{R_{i+1}}(\mathbf{X}_{i+1}^{0}) - \boldsymbol{x}_{R_{i+1}}$$

$$\mathbf{L}_{R_i}^{(i+1)} = \mathbf{F}_{R_i}^{(i+1)}(\mathbf{X}_{i+1}^{0}) - \boldsymbol{x}_{R_i}^{(i+1)} \qquad \mathbf{L}_{W_{i+1}} = \mathbf{F}_{W_{i+1}}(\mathbf{X}_{i+1}^{0}) - \boldsymbol{x}_{W_{i+1}}$$

and

$$\mathbf{X}_{i+1}^{0} = \left[\left(\mathbf{X}_{P_{i+1}}^{0}\right)^{T},\left(\mathbf{X}_{R_{i+1}}^{0}\right)^{T},\left(\mathbf{X}_{R_i}^{0}\right)^{T},\left(\mathbf{X}_{P_i}^{0}\right)^{T}\right]^{T}$$

By introducing additional designations

$$\mathbf{V}_{\boldsymbol{x}_{i+1}} = \begin{bmatrix}\mathbf{V}_{Z_{i+1}}\\ \mathbf{V}_{R_{i+1}}^{(i+1)}\\ \mathbf{V}_{R_i}^{(i+1)}\\ \mathbf{V}_{W_{i+1}}\end{bmatrix} \in \mathfrak{M}(n_{i+1},1), \qquad \mathbf{A}_{P_{i+1}} = \begin{bmatrix}\mathbf{A}_{Z_{i+1}P_{i+1}}\\ \mathbf{A}_{R_{i+1}P_{i+1}}\\ \mathbf{A}_{R_iP_{i+1}}\\ \mathbf{A}_{W_{i+1}P_{i+1}}\end{bmatrix} \in \mathfrak{M}(n_{i+1},2),$$

$$\mathbf{A}_{R_{i+1}}^{(i+1)} = \begin{bmatrix}\mathbf{A}_{Z_{i+1}R_{i+1}}\\ \mathbf{A}_{R_{i+1}R_{i+1}}\\ \mathbf{A}_{R_iR_{i+1}}\\ \mathbf{A}_{W_{i+1}R_{i+1}}\end{bmatrix} \in \mathfrak{M}(n_{i+1},n_{R_{i+1}}), \qquad \mathbf{A}_{R_i}^{(i+1)} = \begin{bmatrix}\mathbf{A}_{Z_{i+1}R_i}\\ \mathbf{A}_{R_{i+1}R_i}\\ \mathbf{A}_{R_iR_i}^{(i+1)}\\ \mathbf{A}_{W_{i+1}R_i}\end{bmatrix} \in \mathfrak{M}(n_{i+1},n_{R_i}),$$

$$\mathbf{A}_{P_i}^{(i+1)} = \begin{bmatrix}\mathbf{A}_{Z_{i+1}P_i}\\ \mathbf{A}_{R_{i+1}P_i}\\ \mathbf{A}_{R_iP_i}^{(i+1)}\\ \mathbf{A}_{W_{i+1}P_i}\end{bmatrix} \in \mathfrak{M}(n_{i+1},2), \qquad \mathbf{L}_{\boldsymbol{x}_{i+1}} = \left[\mathbf{L}_{Z_{i+1}}^{T},\mathbf{L}_{R_{i+1}}^{T},\left(\mathbf{L}_{R_i}^{(i+1)}\right)^{T},\mathbf{L}_{W_{i+1}}^{T}\right]^{T},$$

$$\Im(\boldsymbol{x}_{i+1}) = Diag\left\{\Im(\boldsymbol{x}_{Z_{i+1}}),\Im(\boldsymbol{x}_{R_{i+1}}),\Im(\boldsymbol{x}_{R_i}^{(i+1)}),\Im(\boldsymbol{x}_{W_{i+1}})\right\}$$

there can be obtained the following functional-decisive model:

$$\left.\begin{array}{ll}\Im(\boldsymbol{x}_{i+1})\cdot & \mathbf{V}_{\mathbf{x}_{i+1}} = \mathbf{A}_{P_{i+1}}\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_{i+1}}} + \mathbf{A}_{R_{i+1}}\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_{i+1}}} + \mathbf{A}_{R_i}\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_i}} + \mathbf{A}_{P_i}\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_i}} + \mathbf{L}_{\mathbf{x}_{i+1}} \\ \Im(\mathbf{X}^{DGPS}_{P_{i+1}})\cdot & \mathbf{V}_{\mathbf{X}^{DGPS}_{i+1}} = \hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_{i+1}}} + \mathbf{X}^{o}_{P_{i+1}} - \mathbf{X}^{DGPS}_{P_{i+1}} \\ \Im(\hat{\mathbf{X}}^{(i)}_{P_i})\cdot & \mathbf{V}_{\hat{\mathbf{X}}_{P_i}} = \hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_i}} - \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} \\ \Im(\hat{\mathbf{X}}^{(i)}_{R_i})\cdot & \mathbf{V}_{\hat{\mathbf{X}}_{R_i}} = \hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_i}} - \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}} \end{array}\right\} \tag{4.3}$$

($n_{i+1} = 2\left(n_{Z_{i+1}} + n_{R_{i+1}} n_{R_i}\right) + n_{W_i}$ - observations number, $r_{i+1} = 2\left(2 + n_{R_{i+1}} + n_{R_i}\right)$ - unknowns number).

The above model is connected with a statistical model of the following form:

$$\left.\begin{array}{l}\left.\begin{array}{l}\mathbf{C}_{\boldsymbol{x}_{Z_{i+1}}} = \sigma_0^2\mathbf{Q}_{\boldsymbol{x}_{Z_{i+1}}} = \sigma_0^2\mathbf{P}^{-1}_{\boldsymbol{x}_{Z_{i+1}}} \\ \mathbf{C}_{\boldsymbol{x}^{(i+1)}_{R_{i+1}}} = \sigma_0^2\mathbf{Q}_{\boldsymbol{x}^{(i+1)}_{R_{i+1}}} = \sigma_0^2\mathbf{P}^{-1}_{\boldsymbol{x}^{(i+1)}_{R_{i+1}}} \\ \mathbf{C}_{\boldsymbol{x}^{(i+1)}_{R_i}} = \sigma_0^2\mathbf{Q}_{\boldsymbol{x}^{(i+1)}_{R_i}} = \sigma_0^2\mathbf{P}^{-1}_{\boldsymbol{x}^{(i+1)}_{R_i}} \\ \mathbf{C}_{\boldsymbol{x}_{W_{i+1}}} = \sigma_0^2\mathbf{Q}_{\boldsymbol{x}_{W_{i+1}}} = \sigma_0^2\mathbf{P}^{-1}_{\boldsymbol{x}_{W_{i+1}}}\end{array}\right\} \Leftrightarrow \mathbf{C}_{\boldsymbol{x}_{i+1}} = \sigma_0^2\mathbf{Q}_{\boldsymbol{x}_{i+1}} = \sigma_0^2\mathbf{P}^{-1}_{\boldsymbol{x}_{i+1}} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \mathbf{C}_{\mathbf{X}^{DGPS}_{i+1}} = \sigma_0^2\mathbf{Q}_{\mathbf{X}^{DGPS}_{i+1}} = \sigma_0^2\mathbf{P}^{-1}_{\mathbf{X}^{DGPS}_{i+1}} \\ \mathbf{C}_{\hat{\mathbf{X}}^{(i)}_{P_i}} = \sigma_0^2\mathbf{Q}_{\hat{\mathbf{X}}^{(i)}_{P_i}} = \sigma_0^2\mathbf{P}^{-1}_{\hat{\mathbf{X}}^{(i)}_{P_i}} \\ \mathbf{C}_{\hat{\mathbf{X}}^{(i)}_{R_i}} = \sigma_0^2\mathbf{Q}_{\hat{\mathbf{X}}^{(i)}_{R_i}} = \sigma_0^2\mathbf{P}^{-1}_{\hat{\mathbf{X}}^{(i)}_{R_i}}\end{array}\right\} \tag{4.4}$$

where:

$$\mathbf{Q}_{\boldsymbol{x}_{i+1}} = Diag\left(\mathbf{Q}_{\boldsymbol{x}_{Z_{i+1}}}, \mathbf{Q}_{\boldsymbol{x}^{(i+1)}_{R_{i+1}}}, \mathbf{Q}_{\boldsymbol{x}^{(i+1)}_{R_i}}, \mathbf{Q}_{\boldsymbol{x}_{W_{i+1}}}\right) = Diag\left(\mathbf{P}^{-1}_{\boldsymbol{x}_{Z_{i+1}}}, \mathbf{P}^{-1}_{\boldsymbol{x}^{(i+1)}_{R_{i+1}}}, \mathbf{P}^{-1}_{\boldsymbol{x}^{(i+1)}_{R_i}}, \mathbf{P}^{-1}_{\boldsymbol{x}_{W_{i+1}}}\right) = \mathbf{P}^{-1}_{\boldsymbol{x}_{i+1}}$$

(the mutual independence of the observations, which belong to different $\mho$ sets remains in force).

4.2. Adjustment task and its solution

The decisive–robust adjustment task, concerning the *IANS* chain, will be resolved basing on the following target function:

$$\Phi^{D-R}\left(\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_{i+1}}},\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_{i+1}}},\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_i}},\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_i}}\right)=\Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)=\Phi^{D-R}_{\boldsymbol{x}}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)+\Phi^{D-R}_{DGPS}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)+ \\ +\Phi^{D-R}_{P_i}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)+\Phi^{D-R}_{R_i}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right) \tag{4.5}$$

where:

$$\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}=\left[\left(\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_{i+1}}}\right)^T,\left(\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_{i+1}}}\right)^T,\left(\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_i}}\right)^T,\left(\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_i}}\right)^T\right]^T$$

The function components $\Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)$ are of the following forms here:

$$\left.\begin{aligned}
\Phi^{D-R}_{\boldsymbol{x}}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)&=\mathbf{V}^T_{\boldsymbol{x}_{i+1}}\tilde{\widehat{\mathbf{P}}}_{\boldsymbol{x}_{i+1}}\mathbf{V}_{\boldsymbol{x}_{i+1}}\\
\Phi^{D-R}_{DGPS}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)&=\mathbf{V}^T_{\mathbf{X}^{DGPS}_{i+1}}\tilde{\widehat{\mathbf{P}}}_{\mathbf{X}^{DGPS}_{i+1}}\mathbf{V}_{\mathbf{X}^{DGPS}_{i+1}}\\
\Phi^{D-R}_{P_i}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)&=\mathbf{V}^T_{\hat{\mathbf{X}}_{P_i}}\tilde{\widehat{\mathbf{P}}}_{\hat{\mathbf{X}}^{(i)}_{P_i}}\mathbf{V}_{\hat{\mathbf{X}}_{P_i}}\\
\Phi^{D-R}_{R_i}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)&=\mathbf{V}^T_{\hat{\mathbf{X}}_{R_i}}\tilde{\widehat{\mathbf{P}}}_{\hat{\mathbf{X}}^{(i)}_{R_i}}\mathbf{V}_{\hat{\mathbf{X}}_{R_i}}
\end{aligned}\right\} \tag{4.6}$$

These functions are resulted from substitutions

$\mathbf{V}_{\boldsymbol{x}_{i+1}}:=\Im(\boldsymbol{x}_{i+1})\mathbf{V}_{\boldsymbol{x}_i},\ \ldots\ ,\mathbf{V}_{\hat{\mathbf{X}}_{R_i}}:=\Im(\hat{\mathbf{X}}^{(i)}_{R_i})\mathbf{V}_{\hat{\mathbf{X}}_{R_i}}$ applied for the components:

$$\Phi^{R}_{\boldsymbol{x}}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)=\mathbf{V}^T_{\boldsymbol{x}_{i+1}}\widehat{\mathbf{P}}_{\boldsymbol{x}_{i+1}}\mathbf{V}_{\boldsymbol{x}_{i+1}},\ \ldots\ ,\Phi^{R}_{R_i}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)=\mathbf{V}^T_{\hat{\mathbf{X}}_{R_i}}\widehat{\mathbf{P}}_{\hat{\mathbf{X}}^{(i)}_{R_i}}\mathbf{V}_{\hat{\mathbf{X}}_{R_i}}$$

where:

$$\widehat{\mathbf{P}}_{\boldsymbol{x}_{i+1}}=\mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_{i+1}})\mathbf{P}_{\boldsymbol{x}_{i+1}}\mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_{i+1}}),\ \ldots\ ,\widehat{\mathbf{P}}_{\hat{\mathbf{X}}^{(i)}_{R_i}}=\mathbf{T}_{sqr}(\mathbf{V}_{\hat{\mathbf{X}}_{R_i}})\mathbf{P}_{\hat{\mathbf{X}}^{(i)}_{R_i}}\mathbf{T}_{sqr}(\mathbf{V}_{\hat{\mathbf{X}}_{R_i}})$$

Moreover, if (the same as in (3.33) and (3.34))

$$\widetilde{\mathbf{T}}_{sqr}(\boldsymbol{x}_{i+1}, \mathbf{V}_{\boldsymbol{x}_{i+1}}) = \Im_{sqr}(\boldsymbol{x}_{i+1})\mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_{i+1}}) = \Im(\boldsymbol{x}_{i+1})\mathbf{T}_{sqr}(\mathbf{V}_{\boldsymbol{x}_{i+1}})$$

$$\vdots$$

$$\widetilde{\mathbf{T}}_{sqr}(\hat{\mathbf{X}}_{R_i}^{(i)}, \mathbf{V}_{\hat{\mathbf{X}}_{R_i}}) = \Im_{sqr}(\hat{\mathbf{X}}_{R_i}^{(i)})\mathbf{T}_{sqr}(\mathbf{V}_{\hat{\mathbf{X}}_{R_i}}) = \Im(\hat{\mathbf{X}}_{R_i}^{(i)})\mathbf{T}_{sqr}(\mathbf{V}_{\hat{\mathbf{X}}_{R_i}})$$

then

$$\left.\begin{aligned}
\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{i+1}} &= \widetilde{\mathbf{T}}_{sqr}(\boldsymbol{x}_{i+1}, \mathbf{V}_{\boldsymbol{x}_{i+1}})\mathbf{P}_{\boldsymbol{x}_{i+1}}\widetilde{\mathbf{T}}_{sqr}(\boldsymbol{x}_{i+1}, \mathbf{V}_{\boldsymbol{x}_{i+1}}) \\
\widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_{i+1}^{DGPS}} &= \widetilde{\mathbf{T}}_{sqr}(\mathbf{X}_{P_{i+1}}^{DGPS}, \mathbf{V}_{\mathbf{X}_{i+1}^{DGPS}})\mathbf{P}_{\mathbf{X}_{i+1}^{DGPS}}\widetilde{\mathbf{T}}_{sqr}(\mathbf{X}_{P_{i+1}}^{DGPS}, \mathbf{V}_{\mathbf{X}_{i+1}^{DGPS}}) \\
\widetilde{\widetilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{P_i}^{(i)}} &= \widetilde{\mathbf{T}}_{sqr}(\hat{\mathbf{X}}_{P_i}^{(i)}, \mathbf{V}_{\hat{\mathbf{X}}_{P_i}})\mathbf{P}_{\hat{\mathbf{X}}_{P_i}^{(i)}}\widetilde{\mathbf{T}}_{sqr}(\hat{\mathbf{X}}_{P_i}^{(i)}, \mathbf{V}_{\hat{\mathbf{X}}_{P_i}}) \\
\widetilde{\widetilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{R_i}^{(i)}} &= \widetilde{\mathbf{T}}_{sqr}(\hat{\mathbf{X}}_{R_i}^{(i)}, \mathbf{V}_{\hat{\mathbf{X}}_{R_i}})\mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i)}}\widetilde{\mathbf{T}}_{sqr}(\hat{\mathbf{X}}_{R_i}^{(i)}, \mathbf{V}_{\hat{\mathbf{X}}_{R_i}})
\end{aligned}\right\} \quad (4.7)$$

For independent variables (or by neglecting dependences) it may be stated as follows:

$$\mathbf{P}_{\boldsymbol{x}_{i+1}} = Diag\left(\mathbf{P}_{\boldsymbol{x}_{i+1}}\right) \Leftrightarrow \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{i+1}} = \widetilde{\mathbf{T}}(\boldsymbol{x}_{i+1}, \mathbf{V}_{\boldsymbol{x}_{i+1}})\mathbf{P}_{\boldsymbol{x}_{i+1}}$$

$$\vdots$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i)}} = Diag\left(\mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i)}}\right) \Leftrightarrow \widetilde{\widetilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{R_i}^{(i)}} = \widetilde{\mathbf{T}}(\hat{\mathbf{X}}_{R_i}^{(i)}, \mathbf{V}_{\hat{\mathbf{X}}_{R_i}})\mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i)}}$$

and for each of $\widetilde{\mathbf{T}}$ matrixes, the equation $\widetilde{\mathbf{T}} = \widetilde{\mathbf{T}}_{sqr}\widetilde{\mathbf{T}}_{sqr}$ is occurring.

(Note, that $\widetilde{\mathbf{T}}_{sqr}$, the same as $\widetilde{\mathbf{T}}$ and $\mathbf{T}$, are diagonal matrixes).

Taking advantage of the above presented settlements concerning the function of target and the formulated before functional models and statistical models, the following decisive- robust adjustment task can be suggested:

$$\left.\begin{array}{l}
\mathbf{V}_{\boldsymbol{x}_{i+1}} = \mathbf{A}_{P_{i+1}}\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_{i+1}}} + \mathbf{A}_{R_{i+1}}\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_{i+1}}} + \mathbf{A}^{(i+1)}_{R_i}\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_i}} + \mathbf{A}^{(i+1)}_{P_i}\hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_i}} + \mathbf{L}_{\boldsymbol{x}_{i+1}} \\
\mathbf{V}_{\mathbf{X}^{DGPS}_{i+1}} = \hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_{i+1}}} + \mathbf{X}^0_{P_{i+1}} - \mathbf{X}^{DGPS}_{P_{i+1}} \\
\mathbf{V}_{\hat{\mathbf{X}}_{P_i}} = \hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{P_i}} - \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} \\
\mathbf{V}_{\hat{\mathbf{X}}_{R_i}} = \hat{\mathbf{d}}^{(i+1)}_{\mathbf{X}_{R_i}} - \hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\mathbf{C}_{\boldsymbol{x}_{i+1}} = \sigma_0^2\mathbf{P}^{-1}_{\boldsymbol{x}_{i+1}} \quad \rightarrow \tilde{\tilde{\mathbf{C}}}_{\boldsymbol{x}_{i+1}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{\boldsymbol{x}_{i+1}} \\
\mathbf{C}_{\mathbf{X}^{DGPS}_{i+1}} = \sigma_0^2\mathbf{P}^{-1}_{\mathbf{X}^{DGPS}_{i+1}} \quad \rightarrow \tilde{\tilde{\mathbf{C}}}_{\mathbf{X}^{DGPS}_{i+1}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{\mathbf{X}^{DGPS}_{i+1}} \\
\mathbf{C}_{\hat{\mathbf{X}}^{(i)}_{P_i}} = \sigma_0^2\mathbf{P}^{-1}_{\hat{\mathbf{X}}^{(i)}_{P_i}} \quad \rightarrow \tilde{\tilde{\mathbf{C}}}_{\hat{\mathbf{X}}^{(i)}_{P_i}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{\hat{\mathbf{X}}^{(i)}_{P_i}} \\
\mathbf{C}_{\hat{\mathbf{X}}^{(i)}_{R_i}} = \sigma_0^2\mathbf{P}^{-1}_{\hat{\mathbf{X}}^{(i)}_{R_i}} \quad \rightarrow \tilde{\tilde{\mathbf{C}}}_{\hat{\mathbf{X}}^{(i)}_{R_i}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}_{\hat{\mathbf{X}}^{(i)}_{R_i}} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\min_{\mathbf{d}_{\mathbf{X}_{i+1}}} \Phi^{D-R}\left(\mathbf{d}_{\mathbf{X}_{i+1}}\right) = \Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right) = \\
= \Phi^{D-R}_{\boldsymbol{x}}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right) + \Phi^{D-R}_{DGPS}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right) + \Phi^{D-R}_{P_i}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right) + \Phi^{D-R}_{R_i}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right)
\end{array}\right\} \quad (4.8)$$

with equivalent covariance matrixes $\tilde{\tilde{\mathbf{C}}} = \sigma_0^2\tilde{\tilde{\mathbf{P}}}^{-1}$ which substitute the original matrixes $\mathbf{C} = \sigma_0^2\mathbf{P}^{-1}$. By introducing the designations:

$$\mathbf{V}_{i+1} = \begin{bmatrix} \mathbf{V}_{\boldsymbol{x}_{i+1}} \\ \mathbf{V}_{\mathbf{X}^{DGPS}_{i+1}} \\ \mathbf{V}_{\hat{\mathbf{X}}_{P_i}} \\ \mathbf{V}_{\hat{\mathbf{X}}_{R_i}} \end{bmatrix},\ \mathbf{A}_{i+1} = \begin{bmatrix} \mathbf{A}_{P_{i+1}} & \mathbf{A}_{R_{i+1}} & \mathbf{A}^{(i+1)}_{R_i} & \mathbf{A}^{(i+1)}_{P_i} \\ \mathbf{I}_{(2)} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}_{(2)} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{(2n_{R_i})} & \mathbf{0} \end{bmatrix},\ \mathbf{L}_i = \begin{bmatrix} \mathbf{L}_{\boldsymbol{x}_{i+1}} \\ \mathbf{X}^0_{P_{i+1}} - \mathbf{X}^{DGPS}_{P_{i+1}} \\ -\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{P_i}} \\ -\hat{\mathbf{d}}^{(i)}_{\mathbf{X}_{R_i}} \end{bmatrix}$$

$$\tilde{\tilde{\mathbf{C}}}_{i+1} = Diag\left(\tilde{\tilde{\mathbf{C}}}_{\boldsymbol{x}_{i+1}}, \tilde{\tilde{\mathbf{C}}}_{\mathbf{X}^{DGPS}_{i+1}}, \tilde{\tilde{\mathbf{C}}}_{\hat{\mathbf{X}}^{(i)}_{P_i}}, \tilde{\tilde{\mathbf{C}}}_{\hat{\mathbf{X}}^{(i)}_{R_i}}\right), \tilde{\tilde{\mathbf{P}}}_{i+1} = Diag\left(\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{i+1}}, \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}^{DGPS}_{i+1}}, \tilde{\tilde{\mathbf{P}}}_{\hat{\mathbf{X}}^{(i)}_{P_i}}, \tilde{\tilde{\mathbf{P}}}_{\hat{\mathbf{X}}^{(i)}_{R_i}}\right)$$

the task (4.8) can be presented in the following form

$$\left.\begin{aligned} &\mathbf{V}_{i+1} = \mathbf{A}_{i+1}\hat{\mathbf{d}}_{\mathbf{X}_{i+1}} + \mathbf{L}_{i+1} \\ &\tilde{\tilde{\mathbf{C}}}_{i+1} = \sigma_0^2 \tilde{\tilde{\mathbf{P}}}_{i+1}^{-1} \\ &\min_{\mathbf{d}_{\mathbf{X}_{i+1}}} \Phi^{D-R}\left(\mathbf{d}_{\mathbf{X}_{i+1}}\right) = \Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}_{i+1}}\right) \end{aligned}\right\} \tag{4.9}$$

Its solution is the estimator as follows

$$\hat{\mathbf{d}}_{\mathbf{X}_{i+1}} = -\left(\mathbf{A}_{i+1}^T \tilde{\tilde{\mathbf{P}}}_{i+1} \mathbf{A}_{i+1}\right)^{-} \mathbf{A}_{i+1}^T \tilde{\tilde{\mathbf{P}}}_{i+1} \mathbf{L}_{i+1} \tag{4.10}$$

or

$$\hat{\mathbf{d}}_{\mathbf{X}_{i+1}} = -\left(\mathbf{A}_{i+1}^T \tilde{\tilde{\mathbf{P}}}_{i+1} \mathbf{A}_{i+1}\right)^{-1} \mathbf{A}_{i+1}^T \tilde{\tilde{\mathbf{P}}}_{i+1} \mathbf{L}_{i+1} \tag{4.11}$$

if

$$rank\left(\mathbf{A}_{i+1}^T \tilde{\tilde{\mathbf{P}}}_{i+1} \mathbf{A}_{i+1}\right) = r_{i+1} = 2\left(2 + n_{R_{i+1}} + n_{R_i}\right) \tag{4.12}$$

Moreover, (if $\left(\mathbf{A}_{i+1}^T \tilde{\tilde{\mathbf{P}}}_{i+1}^{-1} \mathbf{A}_{i+1}\right)^{-1}$ does exist)

$$\hat{\mathbf{C}}_{\hat{\mathbf{X}}_{i+1}} = \hat{\mathbf{C}}_{\hat{\mathbf{d}}\mathbf{X}_{i+1}} = \hat{\sigma}_0^2\left(\mathbf{A}_{i+1}^T \tilde{\tilde{\mathbf{P}}}_{i+1} \mathbf{A}_{i+1}\right)^{-1} \tag{4.13}$$

oraz

$$\hat{\sigma}_0^2 = \frac{1}{f_{i+1}} \mathbf{V}_{i+1}^T \tilde{\tilde{\mathbf{P}}}_{i+1} \mathbf{V}_{i+1} \tag{4.14}$$

where:

$$f_{i+1} = \underset{\text{number of direct observartions}}{n_{i+1}} \underbrace{+ n_{R_i} + 4}_{\text{number of pseudo-observations}} - r_{i+1} = 2n_{Z_{i+1}} + n_{W_{i+1}} + n_{R_i}$$

In more general case, the matrix $\hat{\mathbf{C}}_{\hat{\mathbf{X}}_{i+1}}$ form is as follows:

$$\hat{\mathbf{C}}_{\hat{\mathbf{X}}_{i+1}} = \hat{\sigma}_o^2 \mathbf{D}_{i+1} \mathbf{Q}_{i+1} \mathbf{D}_{i+1}^T$$

where

$$\mathbf{D}_{i+1} = \left(\mathbf{A}_{i+1}^T \tilde{\tilde{\mathbf{P}}}_{i+1} \mathbf{A}_{i+1}\right)^{-} \mathbf{A}_{i+1}^T \tilde{\tilde{\mathbf{P}}}_{i+1}$$

The weights matrix of the all adjusted coordinates, it means

$$\hat{\mathbf{X}}_{i+1} = \mathbf{X}_{i+1}^0 + \hat{\mathbf{d}}_{\mathbf{X}_{i+1}} = \begin{bmatrix} \mathbf{X}_{P_{i+1}}^0 \\ \mathbf{X}_{R_{i+1}}^0 \\ \mathbf{X}_{R_i}^0 \\ \mathbf{X}_{P_i}^0 \end{bmatrix} + \begin{bmatrix} \hat{\mathbf{d}}_{\mathbf{X}_{P_{i+1}}}^{(i+1)} \\ \hat{\mathbf{d}}_{\mathbf{X}_{R_{i+1}}}^{(i+1)} \\ \hat{\mathbf{d}}_{\mathbf{X}_{R_i}}^{(i+1)} \\ \hat{\mathbf{d}}_{\mathbf{X}_{P_i}}^{(i+1)} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{X}}_{P_{i+1}}^{(i+1)} \\ \hat{\mathbf{X}}_{R_{i+1}}^{(i+1)} \\ \hat{\mathbf{X}}_{R_i}^{(i+1)} \\ \hat{\mathbf{X}}_{P_i}^{(i+1)} \end{bmatrix}$$

is of the form as follows

$$\mathbf{P}_{\hat{\mathbf{X}}_{i+1}} = \mathbf{A}_{i+1}^T \tilde{\tilde{\mathbf{P}}}_{i+1} \mathbf{A}_{i+1} = \begin{bmatrix} \mathbf{P}_{\hat{\mathbf{X}}_{P_{i+1}}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{P_{i+1}} \hat{\mathbf{X}}_{R_{i+1}}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{P_{i+1}} \hat{\mathbf{X}}_{R_i}^{(i+1)}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{P_{i+1}} \hat{\mathbf{X}}_{P_i}^{(i+1)}} \\ \cdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots \\ \mathbf{P}_{\hat{\mathbf{X}}_{R_{i+1}} \hat{\mathbf{X}}_{P_{i+1}}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{R_{i+1}}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{R_{i+1}} \hat{\mathbf{X}}_{R_i}^{(i+1)}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{R_{i+1}} \hat{\mathbf{X}}_{P_i}^{(i+1)}} \\ \cdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots \\ \mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i+1)} \hat{\mathbf{X}}_{P_{i+1}}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i+1)} \hat{\mathbf{X}}_{R_{i+1}}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i+1)}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i+1)} \hat{\mathbf{X}}_{P_i}^{(i+1)}} \\ \cdots & \vdots & \cdots & \vdots & \cdots & \vdots & \cdots \\ \mathbf{P}_{\hat{\mathbf{X}}_{P_i}^{(i+1)} \hat{\mathbf{X}}_{P_{i+1}}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{P_i}^{(i+1)} \hat{\mathbf{X}}_{R_{i+1}}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{P_i}^{(i+1)} \hat{\mathbf{X}}_{R_i}^{(i+1)}} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{P_i}^{(i+1)}} \end{bmatrix}$$

where:

$$\mathbf{P}_{\hat{\mathbf{X}}_{P_{i+1}}} = \mathbf{A}_{P_{i+1}}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{i+1}} \mathbf{A}_{P_{i+1}} + \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{i+1}^{DGPS}}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{R_{i+1}}} = \mathbf{A}_{R_{i+1}}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{i+1}} \mathbf{A}_{R_{i+1}}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i+1)}} = \left(\mathbf{A}_{R_i}^{(i+1)}\right)^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{i+1}} \mathbf{A}_{R_i}^{(i+1)} + \tilde{\tilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{R_i}^{(i)}}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{P_i}^{(i+1)}} = \left(\mathbf{A}_{P_i}^{(i+1)}\right)^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{i+1}} \mathbf{A}_{P_i}^{(i+1)} + \tilde{\tilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{P_i}^{(i)}}$$

and

$$\mathbf{P}_{\hat{\mathbf{X}}_{P_{i+1}} \hat{\mathbf{X}}_{R_{i+1}}} = \mathbf{P}_{\hat{\mathbf{X}}_{R_{i+1}} \hat{\mathbf{X}}_{P_{i+1}}}^T = \mathbf{A}_{P_{i+1}}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{i+1}} \mathbf{A}_{R_{i+1}}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{P_{i+1}} \hat{\mathbf{X}}_{R_i}^{(i+1)}} = \mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i+1)} \hat{\mathbf{X}}_{P_{i+1}}}^T = \mathbf{A}_{P_{i+1}}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{i+1}} \mathbf{A}_{R_i}^{(i+1)}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{P_{i+1}} \hat{\mathbf{X}}_{P_i}^{(i+1)}} = \mathbf{P}_{\hat{\mathbf{X}}_{P_i}^{(i+1)} \hat{\mathbf{X}}_{P_{i+1}}}^T = \mathbf{A}_{P_{i+1}}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{i+1}} \mathbf{A}_{P_i}^{(i+1)}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{R_{i+1}} \hat{\mathbf{X}}_{R_i}^{(i+1)}} = \mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i+1)} \hat{\mathbf{X}}_{R_{i+1}}}^T = \mathbf{A}_{R_{i+1}}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{i+1}} \mathbf{A}_{R_i}^{(i+1)}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{R_{i+1}} \hat{\mathbf{X}}_{P_i}^{(i+1)}} = \mathbf{P}_{\hat{\mathbf{X}}_{P_i}^{(i+1)} \hat{\mathbf{X}}_{R_{i+1}}}^T = \mathbf{A}_{R_{i+1}}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{i+1}} \mathbf{A}_{P_i}^{(i+1)}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{R_i}^{(i+1)} \hat{\mathbf{X}}_{P_i}^{(i+1)}} = \mathbf{P}_{\hat{\mathbf{X}}_{P_i}^{(i+1)} \hat{\mathbf{X}}_{R_i}^{(i+1)}}^T = \left(\mathbf{A}_{R_i}^{(i+1)}\right)^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{i+1}} \mathbf{A}_{P_i}^{(i+1)}$$

The solution presented above may be transferred to the further stages of *IANS* developing, thus for the successive $k = i+2, i+3, \ldots\ldots$. The general effect of such developments, according to the assumptions made in chapter 4, there are points $\Re$ qualified to the set $\mathfrak{Z}$. Let's agree that the basic, essential (although not the only one) qualification criterion is accuracy of the point positioning. The accuracy, when referred to the points $\Re_k$, determined in the $k+1$-stage, may be represented with confidence ellipses, determined on the basis of weights matrix $\mathbf{P}_{\hat{\mathbf{X}}^{k+1}_{R_k}}$, a value of the square form $\mathbf{V}^T_{k+1}\tilde{\tilde{\mathbf{P}}}_{k+1}\mathbf{V}_{k+1}$ and accepted confidence level γ (e.g.:[Baran 1999, Wiśniewski 2000, 2004]). With acceptance of some certain simplifications having theoretical character, the qualification process may also be conducted on the grounds of a point's position error value. For the point R_l including to the set $\Re_k$ and determined at the $k+1$ stage, a value of this parameter can be settled basing on the following formula:

$$M^{(k+1)}_{R_l \in \Re_k} = \sqrt{Tr\left[\hat{\mathbf{C}}_{\hat{\mathbf{X}}^{(k+1)}_{R_k}}\right]_{R_l}} = \sqrt{\left[\hat{\mathbf{C}}_{\hat{\mathbf{X}}^{(k+1)}_{R_k}}\right]_{2l-1,2l-1} + \left[\hat{\mathbf{C}}_{\hat{\mathbf{X}}^{(k+1)}_{R_k}}\right]_{2l,2l}} \qquad (4.16)$$

(Tr - matrix trace, $[\ ]_{R_l}$ - block of the matrix, corresponding to the point R_l, $[\ ]_{i,i}$ - diagonal element of matrix).

If M_{dop} is a value acceptable, allowing to qualify a point to be included into the set $\mathfrak{Z}$, then

$$\left(R_l \in R_k\right) \rightarrow \mathfrak{Z} \qquad \text{if} \qquad M^{(k+1)}_{R_l \in \Re_k} < M_{dop} \qquad (4.17)$$

Non fulfilment of this criterion demands either conducting the observations towards the point R_l in the subsequent stages of *IANS* developing or entire resignation thereof, as it may appear that due to the geometrical structure, survey accuracy etc., reduction of the location error value, obtained at specific stages, doesn't allow to expect for fulfilment of the (4.17) condition. The detailed analysis of the additional observations effect on the point position error can be carried out taking advantage of the method mentioned in the papers [Czaplewski 1999, 2000a, Czaplewski, Wiśniewski 1999a, 1999b]. Even if the method presented in the above paper referred to the proper position, the general principle remains unchanged (sequential increments of the respective covariance matrix).

4.3. Basic technologies of *IANS* developments; the peculiar cases.

Let us assume that *IANS* development is carried out by a singular watercraft, which, at each position, performs DGPS surveys and measurements towards the points $\Re$. Due to low accuracies, the route vector elements are not taken into consideration. In processing the data of stage $k+1$, the assessment $\hat{\mathbf{X}}_{P_k}^{(k)}$ of the previous, $k-th$ position is not considered (for the optional $k = i, i+1, i+2, ...$). In specific stages of *IANS* development, coordinates of the points $\Re$ are subject to corrections. A range of information transmitted from the position P_k to P_{k+1} as well of the information obtained at the positions mentioned is show in Fig. 4.2. below.

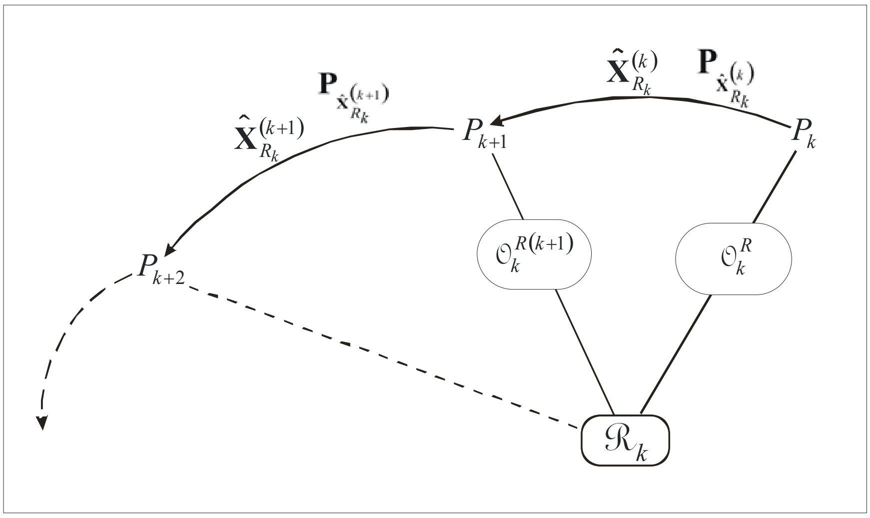

Fig. 4.2 The peculiar case of *IANS* development

The generalizations presented in this work may be brought to the described here, basic technology of *INS* development through acceptance of the following values of the decisive matrixes:

$$\mathcal{O}_{k+1}^{W} \text{ - empty set} \qquad \rightarrow \Im(\boldsymbol{x}_{W_{k+1}}) = \mathbf{0}$$

$$\text{Assessment } \hat{\mathbf{X}}_{P_k}^{(k)} \text{ is not subject to corrections at the } k+1 \text{ stage} \rightarrow \Im(\hat{\mathbf{X}}_{P_k}^{(k)}) = \mathbf{0} \tag{4.18}$$

Then

$$\Im(\boldsymbol{x}_{k+1}) = Diag\left\{\Im(\boldsymbol{x}_{Z_{k+1}}),\Im(\boldsymbol{x}_{R_{k+1}}^{(k+1)}),\Im(\boldsymbol{x}_{R_k}^{(k+1)}),\mathbf{0}\right\} \Leftrightarrow \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} = Diag\left\{\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_{k+1}}},\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{R_{k+1}}^{(k+1)}},\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{R_k}^{(k+1)}},\mathbf{0}\right\}$$

$$\Im(\hat{\mathbf{X}}_{P_k}^{(k)}) = \mathbf{0} \quad \Leftrightarrow \quad \widetilde{\widetilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{P_k}^{(k)}} = \mathbf{0}$$

and next (in reference to the block elements of the matrix $\mathbf{P}_{\boldsymbol{x}_{k+1}} = \mathbf{A}_{k+1}^T \widetilde{\widetilde{\mathbf{P}}}_{k+1}\mathbf{A}_{k+1}$ having (4.15) structure).

$$\mathbf{P}_{\hat{\mathbf{X}}_{P_{k+1}}} = \mathbf{A}_{P_{k+1}}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{P_{k+1}} + \widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_{k+1}^{DGPS}} = \left[\mathbf{A}_{Z_{k+1}P_{k+1}}^T, \mathbf{A}_{R_{k+1}P_{k+1}}^T, \mathbf{A}_{R_kP_{k+1}}^T, \mathbf{A}_{W_{k+1}P_{k+1}}^T\right] \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{P_{k+1}} +$$

$$+ \widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_{k+1}^{DGPS}} = \mathbf{A}_{Z_{k+1}P_{k+1}}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_{k+1}}} \mathbf{A}_{Z_{k+1}P_{k+1}} + \mathbf{A}_{R_{k+1}P_{k+1}}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{R_{k+1}}^{(k+1)}} \mathbf{A}_{R_{k+1}P_{k+1}} + \mathbf{A}_{R_kP_{k+1}}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{R_k}^{(k+1)}} \mathbf{A}_{R_kP_{k+1}} + \widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_{k+1}^{DGPS}}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{R_{k+1}}} = \mathbf{A}_{R_{k+1}}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{R_{k+1}} = \left[\underset{\overset{\uparrow}{\mathbf{0}}}{\mathbf{A}_{Z_{k+1}R_{k+1}}^T}, \mathbf{A}_{R_{k+1}R_{k+1}}^T, \underset{\overset{\uparrow}{\mathbf{0}}}{\mathbf{A}_{R_kR_{k+1}}^T}, \underset{\overset{\uparrow}{\mathbf{0}}}{\mathbf{A}_{W_{k+1}R_{k+1}}^T}\right] \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{R_{k+1}} =$$

$$= \mathbf{A}_{R_{k+1}R_{k+1}}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{R_{k+1}}^{(k+1)}} \mathbf{A}_{R_{k+1}R_{k+1}}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{R_k}^{(k+1)}} = \left(\mathbf{A}_{R_k}^{(k+1)}\right)^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{R_k}^{(k+1)} + \widetilde{\widetilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{R_k}^{(k)}} = \left[\underset{\overset{\uparrow}{\mathbf{0}}}{\mathbf{A}_{Z_{k+1}R_k}^T}, \underset{\overset{\uparrow}{\mathbf{0}}}{\mathbf{A}_{R_{k+1}R_k}^T}, \left(\mathbf{A}_{R_kR_k}^{(k+1)}\right)^T, \underset{\overset{\uparrow}{\mathbf{0}}}{\mathbf{A}_{W_{k+1}R_k}^T}\right] \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{R_k}^{(k+1)}$$

$$+ \widetilde{\widetilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{R_k}^{(k)}} = \left(\mathbf{A}_{R_kR_k}^{(k+1)}\right)^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{R_{k+1}}^{(k+1)}} \mathbf{A}_{R_kR_k}^{(k+1)} + \widetilde{\widetilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{R_k}^{(k)}}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{P_k}^{(k+1)}} = \left(\mathbf{A}_{P_k}^{(k+1)}\right)^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{P_k}^{(k+1)} + \widetilde{\widetilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{P_k}^{(k)}} = \left[\underset{\overset{\uparrow}{\mathbf{0}}}{\mathbf{A}_{Z_{k+1}P_k}^T}, \underset{\overset{\uparrow}{\mathbf{0}}}{\mathbf{A}_{R_{k+1}P_k}^T}, \underset{\overset{\uparrow}{\mathbf{0}}}{\left(\mathbf{A}_{R_kP_k}^{(k+1)}\right)^T}, \underset{\overset{\uparrow}{\mathbf{0}}}{\mathbf{A}_{W_{k+1}P_k}^T}\right] \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{P_k}^{(k+1)} + \underset{\overset{\uparrow}{\mathbf{0}}}{\widetilde{\widetilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{P_k}^{(k)}}} = \mathbf{0}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{P_{k+1}}\hat{\mathbf{X}}_{R_{k+1}}} = \mathbf{A}_{P_{k+1}}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{R_{k+1}} = \mathbf{A}_{R_{k+1}P_{k+1}}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{R_{k+1}}^{(k+1)}} \mathbf{A}_{R_{k+1}R_{k+1}}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{P_{k+1}}\hat{\mathbf{X}}_{R_k}^{(k+1)}} = \mathbf{A}_{P_{k+1}}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{R_k}^{(k+1)} = \mathbf{A}_{R_kP_{k+1}}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{R_k}^{(k+1)}} \mathbf{A}_{R_kR_k}^{(k+1)}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{P_{k+1}}\hat{\mathbf{X}}_{P_k}^{(i+1)}} = \mathbf{A}_{P_{k+1}}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{P_k}^{(i+1)} = \mathbf{0}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{R_{k+1}}\hat{\mathbf{X}}_{R_k}^{(k+1)}} = \mathbf{A}_{R_{k+1}}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{R_k}^{(k+1)} = \mathbf{0}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{R_{k+1}}\hat{\mathbf{X}}_{P_k}^{(k+1)}} = \mathbf{A}_{R_{k+1}}^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{P_k}^{(k+1)} = \mathbf{0}$$

$$\mathbf{P}_{\hat{\mathbf{X}}_{R_k}^{(k+1)}\hat{\mathbf{X}}_{P_k}^{(k+1)}} = \left(\mathbf{A}_{R_k}^{(k+1)}\right)^T \widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} \mathbf{A}_{P_k}^{(k+1)} = \mathbf{0}$$

Moreover

$$\mathbf{A}_{k+1}^{T}\widetilde{\widetilde{\mathbf{P}}}_{k+1}\mathbf{L}_{k+1}=\begin{bmatrix}\mathbf{A}_{P_{k+1}}^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\mathbf{x}_{k+1}}+\widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}_{k+1}^{DGPS}}\left(\mathbf{X}_{P_{k+1}}^{0}-\mathbf{X}_{P_{k+1}}^{DGPS}\right)\\ \mathbf{A}_{R_{k+1}}^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\mathbf{x}_{k+1}}\\ \left(\mathbf{A}_{R_k}^{(k+1)}\right)^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\mathbf{x}_{k+1}}-\widetilde{\widetilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{R_k}^{(k)}}\hat{\mathbf{d}}_{\mathbf{X}_{R_k}}^{(k)}\\ \left(\mathbf{A}_{P_k}^{(k+1)}\right)^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\mathbf{x}_{k+1}}-\widetilde{\widetilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{P_k}^{(k)}}\hat{\mathbf{d}}_{\mathbf{X}_{P_k}}^{(k)}\end{bmatrix}$$

where:

$$\mathbf{A}_{P_{k+1}}^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\boldsymbol{x}_{k+1}}=\mathbf{A}_{Z_{k+1}P_{k+1}}^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_{k+1}}}\mathbf{L}_{Z_{k+1}}+\mathbf{A}_{R_{k+1}P_{k+1}}^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{R_{k+1}}^{(k+1)}}\mathbf{L}_{R_{k+1}}+\mathbf{A}_{R_kP_{k+1}}^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{R_k}^{(k+1)}}\mathbf{L}_{R_k}^{(k+1)}$$

$$\mathbf{A}_{R_{k+1}}^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\boldsymbol{x}_{k+1}}=\mathbf{A}_{R_{k+1}R_{k+1}}^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{R_{k+1}}^{(k+1)}}\mathbf{L}_{R_{k+1}}$$

$$\left(\mathbf{A}_{R_k}^{(k+1)}\right)^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\boldsymbol{x}_{k+1}}=\left(\mathbf{A}_{R_kR_k}^{(k+1)}\right)^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{R_k}^{(k+1)}}\mathbf{L}_{R_k}^{(k+1)}$$

$$\left(\mathbf{A}_{P_k}^{(k+1)}\right)^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\boldsymbol{x}_{k+1}}=\mathbf{0}$$

thus

$$\hat{\mathbf{d}}_{\mathbf{X}_{k+1}}=-\left(\mathbf{A}_{k+1}^{T}\widetilde{\widetilde{\mathbf{P}}}_{k+1}\mathbf{A}_{k+1}\right)^{-}\mathbf{A}_{k+1}^{T}\widetilde{\widetilde{\mathbf{P}}}_{k+1}\mathbf{L}_{k+1}=$$

$$=-\begin{bmatrix}\mathbf{P}_{\hat{\mathbf{X}}_{P_{k+1}}} & \mathbf{P}_{\hat{\mathbf{X}}_{P_{k+1}}\hat{\mathbf{X}}_{R_{k+1}}} & \mathbf{P}_{\hat{\mathbf{X}}_{P_{k+1}}\mathbf{X}_{R_k}^{(k+1)}} & \vdots & \mathbf{0}\\ & \mathbf{P}_{\hat{\mathbf{X}}_{R_{k+1}}} & \mathbf{0} & \vdots & \mathbf{0}\\ symmetry & & \mathbf{P}_{\mathbf{X}_{R_k}^{(k+1)}} & \vdots & \mathbf{0}\\ \cdots & \cdots & \cdots & \vdots & \cdots\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \vdots & \mathbf{0}\end{bmatrix}^{-}\begin{bmatrix}\mathbf{A}_{P_{k+1}}^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\boldsymbol{x}_{k+1}}\\ \mathbf{A}_{R_{k+1}}^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\boldsymbol{x}_{k+1}}\\ \left(\mathbf{A}_{R_k}^{(k+1)}\right)^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\boldsymbol{x}_{k+1}}\\ \mathbf{0}\end{bmatrix}= \qquad (4.19)$$

$$=-\begin{bmatrix}\mathbf{P}_{\hat{\mathbf{X}}_{P_{k+1}}} & \mathbf{P}_{\hat{\mathbf{X}}_{P_{k+1}}\hat{\mathbf{X}}_{R_{k+1}}} & \mathbf{P}_{\hat{\mathbf{X}}_{P_{k+1}}\mathbf{X}_{R_k}^{(k+1)}}\\ & \mathbf{P}_{\hat{\mathbf{X}}_{R_{k+1}}} & \mathbf{0}\\ symmetry & & \mathbf{P}_{\mathbf{X}_{R_k}^{(k+1)}}\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{A}_{P_{k+1}}^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\boldsymbol{x}_{k+1}}\\ \mathbf{A}_{R_{k+1}}^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\boldsymbol{x}_{k+1}}\\ \left(\mathbf{A}_{R_k}^{(k+1)}\right)^{T}\widetilde{\widetilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}}\mathbf{L}_{\boldsymbol{x}_{k+1}}\end{bmatrix}=\begin{bmatrix}\hat{\mathbf{d}}_{\mathbf{X}_{P_{k+1}}}\\ \hat{\mathbf{d}}_{\mathbf{X}_{R_{k+1}}}\\ \hat{\mathbf{d}}_{\mathbf{X}_{R_k}}^{(k+1)}\end{bmatrix}$$

The determined above peculiar case ($\Im(\boldsymbol{x}_{W_{k+1}}) = \mathbf{0}$ and $\Im(\boldsymbol{x}_{P_k}^{(k)}) = \mathbf{0}$) has already been a subject of the previous analyses, carried out by the author. However, in the analyses there were assumed more serious simplifications. .

Developing *IANS* by a singular watercraft, working up its positions $P_k, k = i, i+1, i+2,...$ with terrestrial surveys and DGPS measurements, and also, by stages, making corrections of the point R coordinates were a subject of the works [Czaplewski 2002a, 2003b; Czaplewski, Wiśniewski 2003b,c]. The observations towards the point R were carried out at each $k-th$ - stage, however in these stages, there were not generated any new points, which may extend the set $\Re$ (here always $R \equiv \Re$ remains). No possibility of attenuation of any observations weights, biased with large errors, have also been taken into account. To such peculiar simplifications there is referred the general theory variant, resulting from acceptance of the following values of the decisive matrixes and also the attenuation matrixes:

$$\left.\begin{array}{l} \Im(\boldsymbol{x}_{W_{k+1}}) = \mathbf{0} \\ \Im(\boldsymbol{x}_{R_{k+1}}) = \mathbf{0} \end{array}\right\} \Leftrightarrow \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} = Diag\left(\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_{k+1}}}, \mathbf{0}, \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{R_k}^{(k+1)}}, \mathbf{0}\right)$$

$$\Im(\hat{\mathbf{X}}_{P_k}^{(k)}) = \mathbf{0} \Leftrightarrow \tilde{\tilde{\mathbf{P}}}_{\hat{\mathbf{X}}_{P_k}^{(k)}} = \mathbf{0}$$

$$\left.\begin{array}{l} \mathrm{T}(\mathbf{V}_{Z_{k+1}}) = \mathbf{I}_{\left(n_{Z_{k+1}}\right)} \\ \mathrm{T}(\mathbf{V}_{R_k}^{(k+1)}) = \mathbf{I}_{\left(n_{R_k}^{(k+1)}\right)} \end{array}\right\} \Leftrightarrow \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} = \tilde{\mathbf{P}}_{\boldsymbol{x}_{k+1}}$$

In the cited work, similarly as in other, earlier papers of the author, there has been accepted the entire, agreed as regards initial assumptions, observations programme. In the presented study, with the general theory there corresponds the assumption, that $\Im(\boldsymbol{x}_{Z_{k+1}}) = \mathbf{I}_{\left(n_{Z_{k+1}}\right)}; \Im(\boldsymbol{x}_{R_k}^{(k+1)}) = \mathbf{I}_{\left(n_{R_k}^{(k+1)}\right)}$.

Therefore

$$\underbrace{\underbrace{\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{k+1}} = \tilde{\mathbf{P}}_{\boldsymbol{x}_{k+1}}}_{\textit{no attenuation}} = \mathbf{P}_{\boldsymbol{x}_{k+1}}}_{\textit{no decision about observation elimination}}$$

The more serious simplification was, however, accepted in the work [Czaplewski 2002b]. The *IANS* developing vessel determines its positions basing only on terrestrial surveys (bearings to navigational on-shore marks).

Apart from all the previously made assumptions concerning the values of decisive and attenuation matrixes, also the following additional assumption corresponds

$$\mathfrak{I}(\mathbf{X}_k^{DGPS}) = \mathbf{0} \qquad \text{for every } k = i, i+1, i+2, ...$$

Development of *IANS* through a singular vessel leads to the simplest technologies, from both - the practical and theoretical point of view. And even more, if the route vector is not taken into consideration. Considering this vector's elements, due to its low accuracies, possible to achieve, can cause certain problems. The problems consist mainly in a fact, that the mean errors of a course and travelled route, in their actual measurement, can really be grosser than the mean ones of other determinations. Due to the accepted statistical models (common variance coefficient σ_0^2), demanding accuracy homogeneity of the jointly worked up observations sets, joining the course and the route with other observations, may result in serious disturbances of the final assessments. In spite of the above reservations, in some previous publications, the author treated $\mathcal{O}_{k+1}^{W}$ set as elements of the route vector [Czaplewski 2002a, 2003a, 2004a; Czaplewski, Wiśniewski 2002, 2003b,c]. In reference to the general solutions suggested in this paper, the set $\mathcal{O}_{k+1}^{W}$ can be, as said before, treated as a set of mutual observations carried out by two vessels, commonly developing *IANS*. Such technology is especially interesting, mainly because all the observations accuracies may be at his point similar, also due to a chance for rapid obtaining good assessments of the point $\mathfrak{R}$ coordinates.
It results from a possibility of transferring between the vessels, by radio (or some other way), information about the effected by stages assessments of the coordinates (with their weight matrix) we are interested in. Therefore let us assume that two vessels, $\langle 1 \rangle$ and $\langle 2 \rangle$, carrying out the described, common navigational task, are generating the following observations sets:

$$\langle 1 \rangle : \qquad \left\{ \mathcal{O}_k^{Z}, \mathcal{O}_k^{R}, \mathcal{O}_{k+1}^{W\langle 1 \rangle} \right\}$$

$$\langle 2 \rangle : \qquad \left\{ \mathcal{O}_{k+1}^{Z}, \mathcal{O}_{k+1}^{R(k+1)}, \mathcal{O}_{k+1}^{R(k)}, \mathcal{O}_{k+1}^{W\langle 2 \rangle} \right\}$$

with $\mathcal{O}_{k+1}^{W\langle 1 \rangle} \cup \mathcal{O}_{k+1}^{W\langle 2 \rangle} = \mathcal{O}_{k+1}^{W}$ ($\mathcal{O}_{k+1}^{W\langle 1 \rangle}$ - observations carried out at the vessel $\langle 1 \rangle$ towards the vessel $\langle 2 \rangle$, $\mathcal{O}_{k+1}^{W\langle 2 \rangle}$ - observations carried out at the vessel $\langle 2 \rangle$ towards the $\langle 1 \rangle$).

Basing on the above sets, with application of the theoretical solutions suggested in this paper, there are determined partial assessments (by vessel $\langle 1 \rangle$) and final assessments (by vessel $\langle 2 \rangle$). Determination of the final coordinates' assessments made at vessel $\langle 2 \rangle$ (with the accuracy analysis) requires not only carrying out suitable observations, but also transferring information about the partial assessments of the set $\mho_{k+1}^{W\langle 1 \rangle}$ from vessel $\langle 1 \rangle$ to vessel $\langle 2 \rangle$. Moreover, taking into consideration a fact, that in the entire observational programme, on the both vessels the DGPS surveys are performed, and also at vessel $\langle 2 \rangle$ the additional measurements $\mho_{k+1}^{R}$ towards the new points $\Re_{k+1}$, the described task can be presented in a form of the following scheme:

$$\langle 1 \rangle : \left\{ \mho_k^Z, \mho_k^R, \mho_{k+1}^{W\langle 1 \rangle}, \mathbf{X}_{P_k}^{DGPS} \right\} \rightarrow \boxed{\hat{\mathbf{X}}_{P_k}^{(k)}, \hat{\mathbf{X}}_{R_k}^{(k)}}, \mathbf{P}_{\hat{\mathbf{X}}_{P_k}^{(k)}}, \mathbf{P}_{\hat{\mathbf{X}}_{R_k}^{(k)}}$$

$$\langle 2 \rangle : \left\{ \mho_{k+1}^Z, \mho_k^{R(k+1)}, \mho_{k+1}^R, \mho_{k+1}^{W\langle 2 \rangle}, \mathbf{X}_{P_k}^{DGPS} \right\} \cup \left\{ \mho_{k+1}^{W\langle 1 \rangle}, \hat{\mathbf{X}}_{P_k}^{(k)}, \hat{\mathbf{X}}_{R_k}^{(k)}, \mathbf{P}_{\hat{\mathbf{X}}_{P_k}^{(k)}}, \mathbf{P}_{\hat{\mathbf{X}}_{R_k}^{(k)}} \right\} \rightarrow$$

$$\rightarrow \hat{\mathbf{X}}_{P_{k+1}}^{(k+1)}, \boxed{\hat{\mathbf{X}}_{P_k}^{(k+1)}, \hat{\mathbf{X}}_{R_k}^{(k+1)},} \quad \hat{\mathbf{X}}_{R_{k+1}}^{(k+1)}$$

A range of interactions (exchange of information) between the vessels is also explained in Fig. 4.3.

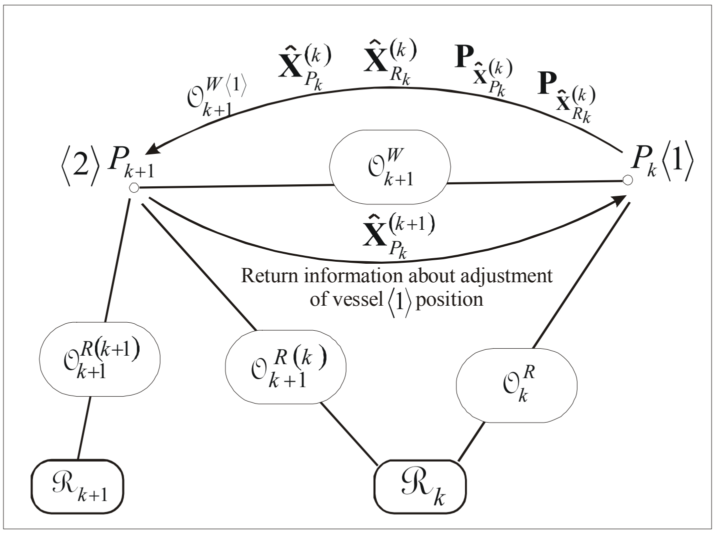

Fig. 4.3. Exchange of information between the vessels which commonly develop *IANS*

5. NAVIGATION BASED ON POINTS OF THE SET $\Re$ (hybrid *M*-estimation)

Location of some $\Re$ set points may, in practice, be determined directly on *IANS* developing. In spite of high carefulness, taken for physical positioning and specification (sailing directions) of the points, due to their character, eventualities of mistakes in later identification thereof are not out of the question. In special cases, a point identification error not necessarily affects the survey result (e.g. a bearing toward a point situated in a line of another point bearing). In another cases, mistaken point identification may anyhow, throughout a change in the survey result, suggest an effect of gross survey errors. Combining "pseudo-outlier" observations and the observations factually biased with major errors may considerably weaken a significance of robust estimation, defined in a classic way. It refers to the estimation, for which attenuation of the original observations weights is the basis (for the version accepted in this paper). An interesting conception in the situation on issue could be elimination of an influence on the ultimate position determinations (position coordinates estimators) not only the gross errors biased observations, but also elimination of an influence of outlier adjustment points, in this particular case, the points which are covered by set $\Re$. Such hybrid robust estimation will be of special importance for navigation, carried out basing on the points of this set only, thus basing on a set of the adjustment points, which are especially at risk of mistaken identification.

A problem of geodetic observations adjustment, taking into consideration the out-lying adjustment points, has been in details analyzed in the publication [Kamiński 2000]. A basis for the analysis and the adjustment methods, suggested in the work were rules of robust Bayes's estimation.

A problem of identification of incorrect adjustment points and elimination of their affecting the basic navigational positions determinations was for the first time formulated in the paper worked out by [Wiśniewski 2002]. The presented conceptions were later verified (in respect of remark system) in the publications [Szubrycht 2002; Szubrycht, Wiśniewski 2003]. A basis of this conception is applying the incorrect points of free adjustment in the identification process. In such adjustment, the geometrical navigational structure is a free structure (of non zero freedom degree value *sw* in relation to the coordinates system). In such structure adjustment process, there are determined increments not only to approximated coordinates of newly determined points (for example of the proper position), but also to coordinates of the points, which are traditionally considered fixed (adjustment points).

It is assumed also that the increments to the incorrect points coordinates (i.e. misidentified or of improper coordinates) will take the values, exceeding the acceptable ones. Observing the principles similar to those as in *M*-estimation, referred to the observations sets, such the increments (after standardization) can be later on "inserted" to admissible intervals, thus eliminating an influence of the outstanding adjustment points on the basic estimators values.

As said before, from the viewpoint of navigation safety, carried out basing on the set $\Re$ points, the essential is joining *M*-estimation, robust to survey gross errors (as it has been till now) with robust free adjustment, creating a chance for identification, and next, elimination of the outlier adjustment points. The suggestion of applying hybrid (outlier observations and outlier adjustment points), robust *M*-estimation, is the basic contents of this Chapter.

Let us assume, that a watercraft is carrying out navigation on the basis of some points of the set $\Re$. Let's accept that (only for convenience of further consideration), that $\mathfrak{Z}^{(k)} = \mathfrak{Z}^{(k+1)} := \left\{ \Re \right\}$ (at each stage of the travel, the adjustment points set is thus the same set $\Re$). To determine the proper positions P_k, P_{k+1} there are carried out observations (bearings and radar measured ranges etc.). Moreover, assuming that approximated coordinates of the position (for example the reckoned) and treating the points $\Re$ coordinates as quantities which may be subject to further changes, there is obtained in this way the navigational survey structure with two degrees of freedom in relation to the coordinates system ($sw = 2$ if there is measured at least one bearing, which stabilizes twisting the survey structure towards *X* axis), Fig. 5.1.

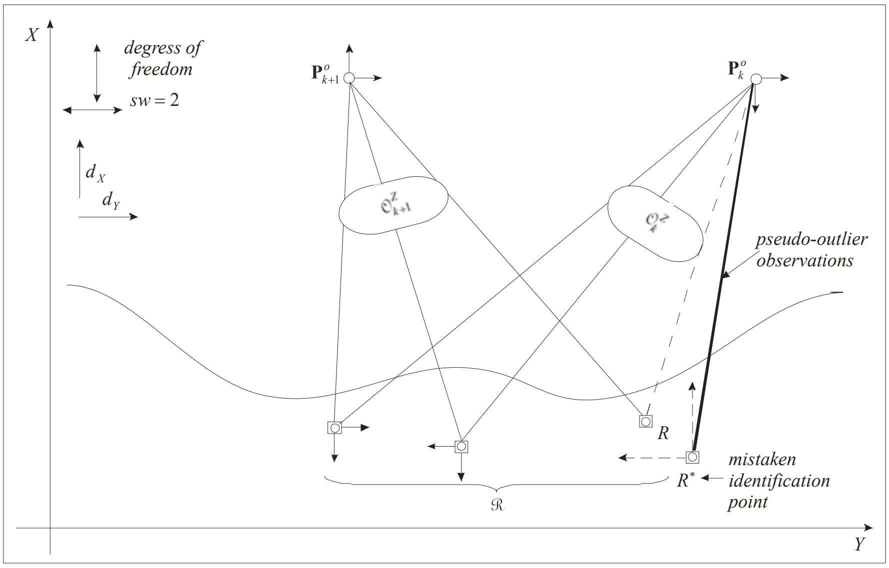

Fig. 5.1. Navigation based on points of the set $\Re$

Basing on the set of observations $\mho_k^Z \to \boldsymbol{x}_{Z_k}; \mho_{k+1}^Z \to \boldsymbol{x}_{Z_{k+1}}$ (as in the example of the structure presented in Fig.5.1), according to the principles applied up to the present, we shall form a system of two matrix corrections equations:

$$\begin{array}{lllll} \hat{\hat{\boldsymbol{x}}}_{Z_k} = \mathbf{F}_{Z_k}(\hat{\boldsymbol{\Gamma}}) & \to & \mathbf{v}_{Z_k} = \mathbf{A}_{Z_kR}\hat{\mathbf{d}}_{\hat{\mathbf{X}}_R} & \vdots + \mathbf{A}_{Z_kP_k}\hat{\mathbf{d}}_{\mathbf{X}_{P_k}} & + \mathbf{L}_{Z_k} \\ \hat{\hat{\boldsymbol{x}}}_{Z_{k+1}} = \mathbf{F}_{Z_{k+1}}(\hat{\boldsymbol{\Gamma}}) & \to & \mathbf{v}_{Z_{k+1}} = \underbrace{\mathbf{A}_{Z_{k+1}R}\hat{\mathbf{d}}_{\hat{\mathbf{X}}_R}}_{\Re} & \vdots \underbrace{\quad + \mathbf{A}_{Z_{k+1}P_{k+1}}\hat{\mathbf{d}}_{\mathbf{X}_{P_{k+1}}}}_{\wp=\{P_k, P_{k+1}\}} & + \mathbf{L}_{Z_{k+1}} \end{array} \qquad (5.1)$$

where:

$$\mathbf{A}_{Z_kR} = \partial_{\mathbf{X}_R}\mathbf{F}_{Z_k}(\boldsymbol{\Gamma}^0), \quad \mathbf{A}_{Z_kP_k} = \partial_{\mathbf{X}_{P_k}}\mathbf{F}_{Z_k}(\boldsymbol{\Gamma}^0), \quad \mathbf{A}_{Z_{k+1}R} = \partial_{\mathbf{X}_R}\mathbf{F}_{Z_{k+1}}(\boldsymbol{\Gamma}^0),$$

$$\mathbf{A}_{Z_{k+1}P_{k+1}} = \partial_{\mathbf{X}_{P_{k+1}}}\mathbf{F}_{Z_{k+1}}(\boldsymbol{\Gamma}^0), \quad \mathbf{L}_{Z_k} = \mathbf{F}_{Z_k}(\boldsymbol{\Gamma}^0) - \mathbf{x}_{Z_k}, \quad \mathbf{L}_{Z_{k+1}} = \mathbf{F}_{Z_{k+1}}(\boldsymbol{\Gamma}^0) - \mathbf{x}_{Z_{k+1}}$$

and

$$\hat{\boldsymbol{\Gamma}} = \begin{bmatrix} \hat{\hat{\mathbf{X}}}_R \\ \hat{\mathbf{X}}_{P_k} \\ \hat{\mathbf{X}}_{P_{k+1}} \end{bmatrix}, \qquad \boldsymbol{\Gamma}^0 = \begin{bmatrix} \hat{\mathbf{X}}_R \\ \mathbf{X}_{P_k}^0 \\ \mathbf{X}_{P_{k+1}}^0 \end{bmatrix}, \qquad \begin{array}{l} \hat{\hat{\mathbf{X}}}_R = \hat{\mathbf{X}}_R + \hat{\mathbf{d}}_{\hat{\mathbf{X}}_R} \\ \hat{\mathbf{X}}_{P_k} = \mathbf{X}_{P_k}^0 + \hat{\mathbf{d}}_{\mathbf{X}_{P_k}} \\ \hat{\mathbf{X}}_{P_{k+1}} = \mathbf{X}_{P_{k+1}}^0 + \hat{\mathbf{d}}_{\mathbf{X}_{P_{k+1}}} \end{array}.$$

Suppose, that coordinates of the points $\Re$ were adjusted before (i.e. as in Chapter 3) and are represented by the estimator $\hat{\mathbf{X}}_R$ of the weights matrix $\mathbf{P}_{\hat{\mathbf{X}}_R}$. In the free adjustment, suggested in this paper, thus after "releasing" navigational survey structure, the coordinates $\hat{\mathbf{X}}_R$ of the point $\Re$ obtain an additional increment $\hat{\mathbf{d}}_{\hat{\mathbf{X}}_R}$ resulting from the postulate of optimal fitting in the survey structure $\left\{ \hat{\Re}, P_k^0, P_{k+1}^0 \right\}$ into the adjusted structure $\left\{ \hat{\hat{\Re}}, \hat{P}_k, \hat{P}_{k+1} \right\}$ (then finally $\hat{\hat{\mathbf{X}}}_R = \hat{\mathbf{X}}_R + \hat{\mathbf{d}}_{\hat{\mathbf{X}}_R}$). Coordinates $\mathbf{X}_{P_k}^0, \mathbf{X}_{P_{k+1}}^0$ are the approximated coordinates of the position P_k and P_{k+1}, obtained, for example, through reckoning the route, or, determined on the basis of survey carried out towards the points $\Re$. To those coordinates the weights matrixes $\mathbf{P}_{\mathbf{X}_{P_k}^0}, \mathbf{P}_{\mathbf{X}_{P_{k+1}}^0}$ should be subordinated, of values justified with the survey technology and the primary evaluation of the obtained results accuracy.

The system of equations (5.1) may also be presented in the following form:

$$\mathbf{V} = \mathbf{A}\hat{\mathbf{d}}_{\mathbf{X}} + \mathbf{L} \tag{5.2}$$

where:

$$\mathbf{V} = \left[\mathbf{V}_{Z_k}^T , \mathbf{V}_{Z_{k+1}}^T \right]^T ,$$

$$\hat{\mathbf{d}}_{\mathbf{X}} = \left[\hat{\mathbf{d}}_{\hat{\mathbf{X}}_R}^T , \hat{\mathbf{d}}_{\mathbf{X}_{P_k}}^T , \hat{\mathbf{d}}_{\mathbf{X}_{P_{k+1}}}^T \right]^T$$

and

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}_{Z_k} \\ \mathbf{L}_{Z_{k+1}} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{F}_{Z_k}\left(\mathbf{\Gamma}^0\right) \\ \mathbf{F}_{Z_{k+1}}\left(\mathbf{\Gamma}^0\right) \end{bmatrix}}_{\mathbf{F}\left(\mathbf{\Gamma}^0\right)} - \underbrace{\begin{bmatrix} \boldsymbol{x}_{Z_k} \\ \boldsymbol{x}_{Z_{k+1}} \end{bmatrix}}_{\boldsymbol{x}} = \mathbf{F}\left(\mathbf{\Gamma}^0\right) - \boldsymbol{x} \tag{5.3}$$

The free terms vector $\mathbf{L} = \mathbf{F}\left(\mathbf{\Gamma}^0\right) - \boldsymbol{x}$ shall be of peculiar importance in the process of establishing covariance matrixes, obtained in free adjustment of the estimators. Besides, let the following designation be introduced:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{Z_k R} & \vdots & \mathbf{A}_{Z_k P_k} & \mathbf{0} \\ \mathbf{A}_{Z_{k+1} R} & \vdots & \mathbf{0} & \mathbf{A}_{Z_{k+1} P_{k+1}} \end{bmatrix} \in \mathfrak{M}_{(n,r)}$$

where generally:

$n = 4n_R$ - at the points P_k and P_{k+1} there are measured bearings and distances to: n_R - points of the set $\mathfrak{Z} = \mathfrak{R}$

$r = 2\left(2 + n_R\right)$

To the functional model (5.2) there is subordinated the decisive matrix:

$$\mathfrak{I}(\boldsymbol{x}) = Diag\left\{ \mathfrak{I}(\boldsymbol{x}_{Z_k}), \mathfrak{I}(\boldsymbol{x}_{Z_{k+1}}) \right\}$$

and the attenuation matrix

$$\mathbf{T}(\mathbf{V}) = Diag\left\{ \mathbf{T}(\mathbf{V}_{Z_k}), \mathbf{T}(\mathbf{V}_{Z_{k+1}}) \right\} \rightarrow \mathbf{T}_{sqr}(\mathbf{V}) = Diag\left\{ \mathbf{T}_{sqr}(\mathbf{V}_{Z_k}), \mathbf{T}_{sqr}(\mathbf{V}_{Z_{k+1}}) \right\}$$

Basing on those matrixes and the original weights matrix of the observations

$$\mathbf{P}_{\boldsymbol{x}} = Diag\left(\mathbf{P}_{\boldsymbol{x}_{Z_k}}, \mathbf{P}_{\boldsymbol{x}_{Z_{k+1}}} \right)$$

there can be established the following decisive – equivalent weights matrix:

$$\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} = \tilde{\mathbf{T}}_{sqr}(\boldsymbol{x}, \mathbf{V})\mathbf{P}_{\boldsymbol{x}}\tilde{\mathbf{T}}_{sqr}(\boldsymbol{x}, \mathbf{V})$$

where (the same as before):

$$\tilde{\mathbf{T}}_{sqr}(\boldsymbol{x}, \mathbf{V}) = \Im(\boldsymbol{x})\mathbf{T}_{sqr}(\mathbf{V})$$

(for the mutually independent observations: $\mathbf{P}_{\boldsymbol{x}} = Diag(\mathbf{P}_{\boldsymbol{x}})$ and $\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} = \Im(\boldsymbol{x})\mathbf{T}(\mathbf{V})\mathbf{P}_{\boldsymbol{x}}$).

Due to the none-zero number of the analyzed navigational structure's freedom degrees $(sw = 2)$, matrix $\mathbf{A} \in \mathfrak{M}_{(n,r)}$ of the model (5.2) is not matrix of column full rank, thus:

$$(rank(\mathbf{A}) = r_{\mathbf{A}}) < r$$

(the matrix of column full rank is the system coefficient matrix, where $\hat{\mathbf{d}}_{\hat{\mathbf{X}}} = \mathbf{0}$ what can be noticed in classic adjustment of the proper positions on the basis of so-called elementary navigational structures, [Czaplewski 2003a,c; Czaplewski, Wąż 2004]). In the matrix algebra theory, applied for working out free geometrical measurement structures it has been proved, (e.g.: [Pelermuter 1980; Wolf 1972, 1979; Świątek, Wiśniewski 1983]), that in such cases, the matrix $\mathbf{A}$ rank is deducted by its defect (d). While the matrix defect is equal to a number of freedom degrees, thus:

$$rank(\mathbf{A}) = r - d = r - sw = r - 2 = r_{\mathbf{A}}$$

Also the matrix rank $\mathbf{A}^T\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}\mathbf{A} \in \mathfrak{M}_{(r,r)}$ is equal to value $r_{\mathbf{A}} = r - d = r - 2$ (if only $rank\left(\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}\right) \geq r_{\mathbf{A}}$).

From the property as follows

$$\left(rank(\mathbf{A}) = rank\left(\mathbf{A}^T\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}\mathbf{A}\right) = r_{\mathbf{A}} \right) < r$$

it results, that the estimator $\hat{\mathbf{d}}_{\mathbf{X}}$ which solves the adjustment task:

$$\left.\begin{aligned} &\mathbf{V} = \mathbf{A}\hat{\mathbf{d}}_{\mathbf{X}} + \mathbf{L} \\ &\mathbf{C}_{x} = \sigma_0^2 \mathbf{P}_{x} \rightarrow \mathbf{C}_{x} = \sigma_0^2 \widetilde{\widetilde{\mathbf{P}}}_{x} \\ &\min_{\mathbf{d}_{\mathbf{X}}} \Phi^{D-R}(\mathbf{d}_{\mathbf{X}}) = \Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}}\right) = \mathbf{V}^T \widetilde{\widetilde{\mathbf{P}}}_{x} \mathbf{V} \end{aligned}\right\}_{rank(\mathbf{A}) = r_{\mathbf{A}} < r} \tag{5.4}$$

may be the vector

$$\hat{\mathbf{d}}_{\mathbf{X}} = -\mathbf{A}^{+}_{\mathbf{MN}}\mathbf{L} \tag{5.5}$$

where $\mathbf{A}^{+}_{\mathbf{MN}}$ is g - inverse of minimal standard in the least squares method in relation to the matrixes $\mathbf{M}$ and $\mathbf{N}$ ([Rao 1982, Wiśniewski 2000, 2004]). Applying such g-inverse causes, that $\hat{\mathbf{d}}_{\mathbf{X}} = -\mathbf{A}^{+}_{\mathbf{MN}}\mathbf{L}$ not only minimizes the function $\Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}}\right) = \mathbf{V}^T \widetilde{\widetilde{\mathbf{P}}}_{x} \mathbf{V}$ (unless $\mathbf{M} = \widehat{\mathbf{P}}_{x}$), but also fulfils an additional optimization criterion:

$$\min_{\mathbf{d}_{\mathbf{X}}} \Psi(\mathbf{d}_{\mathbf{X}}) = \Psi\left(\hat{\mathbf{d}}_{\mathbf{X}}\right) = \hat{\mathbf{d}}_{\mathbf{X}}^T \mathbf{N} \hat{\mathbf{d}}_{\mathbf{X}} \tag{5.6}$$

The additional criterion is of specific importance for identification of the outlier adjustment points, because it results there from that in the free adjustment process, what had been emphasized in the study [Wiśniewski 2002], there are determined not only optimal (in a sense of the criterion $\min_{\mathbf{d}_{\mathbf{X}}} \Phi(\mathbf{d}_{\mathbf{X}}) = \Phi\left(\hat{\mathbf{d}}_{\mathbf{X}}\right)$) values of the corrections $\mathbf{V}$, but also there occurs (in a sense of the criterion $\min_{\mathbf{d}_{\mathbf{X}}} \Psi(\mathbf{d}_{\mathbf{X}}) = \Psi\left(\hat{\mathbf{d}}_{\mathbf{X}}\right)$) fitting in the adjusted structure into the approximated structure. As fitting in is performed in relation to all the points, including also the points $\mathfrak{Z} = \Re$, one may expect relatively large values of increments to the outlier points coordinates, such as R^* in Fig. 5.1. Conception of the outlier points identification on the basis of the increments values is in direct relationship with the applied in robust M-estimation principle of identification of the outlier observations, based on values of the corrections (with all known limitations in this range, including, mainly, "blurring" by square optimization criterions an influence of deterministic disturbances – survey gross errors and inadequate coordinates of the outlier points – to values of corrections and increments.). By analogy to that estimation it is also possible to weaken an influence of the outlier points on the estimator $\hat{\mathbf{d}}_{\mathbf{X}}$ values, including, first of all, (from the navigation safety point of view) on those its elements, which refer to the position P_k and P_{k+1}. Making the solution robust to nonadequate coordinates of the points can be obtained by substitutian at the $\mathbf{N}$ matrix occurring in the function $\Psi\left(\hat{\mathbf{d}}_{\mathbf{X}}\right) = \hat{\mathbf{d}}_{\mathbf{X}}^T \mathbf{N} \hat{\mathbf{d}}_{\mathbf{X}}$, decisive-equivalent weights matrix (in analogy to the function $\Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}}\right)=\mathbf{V}^{T}\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}\mathbf{V}$) of the following form:

$$\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}=\tilde{\mathbf{T}}_{sqr}(\mathbf{\Gamma}^{0},\hat{\mathbf{d}}_{\mathbf{X}})\mathbf{P}_{\mathbf{X}}\tilde{\mathbf{T}}_{sqr}(\mathbf{\Gamma}^{0},\hat{\mathbf{d}}_{\mathbf{X}}) \tag{5.7}$$

where:

$\mathbf{P}_{\mathbf{X}}=Diag\left(\mathbf{P}_{\hat{\mathbf{X}}_{R}},\mathbf{P}_{\mathbf{X}^{0}_{P_k}},\mathbf{P}_{\mathbf{X}^{0}_{P_{k+1}}}\right)=\mathbf{P}_{\mathbf{\Gamma}^{0}}$ - is original, generally quasi-diagonal is matrix of the points $\Re, P_k, P_{k+1}$ coordinates $\hat{\mathbf{X}}_{R},\mathbf{X}^{0}_{P_k},\mathbf{X}^{0}_{P_{k+1}}$

The matrix $\tilde{\mathbf{T}}_{sqr}(\mathbf{\Gamma}^{0},\hat{\mathbf{d}}_{\mathbf{X}})$ in the expression (5.7) is the following decisive-attenuation matrix:

$$\tilde{\mathbf{T}}_{sqr}(\mathbf{\Gamma}^{0},\hat{\mathbf{d}}_{\mathbf{X}})=\Im(\mathbf{\Gamma}^{0})\mathbf{T}_{sqr}(\hat{\mathbf{d}}_{\mathbf{X}}) \tag{5.8}$$

where:

$$\Im(\mathbf{\Gamma}^{0})=Diag\left\{\Im(\hat{\mathbf{X}}_{R}),\Im(\mathbf{X}^{0}_{P_k}),\Im(\mathbf{X}^{0}_{P_{k+1}})\right\}$$

$$\mathbf{T}(\hat{\mathbf{d}}_{\mathbf{X}})=Diag\left\{\mathbf{T}(\hat{\mathbf{d}}_{\hat{\mathbf{X}}_{R}}),\mathbf{T}(\hat{\mathbf{d}}_{\mathbf{X}_{P_k}}),\mathbf{T}(\hat{\mathbf{d}}_{\mathbf{X}_{P_{k+1}}})\right\}\rightarrow$$

$$\mathbf{T}_{sqr}(\hat{\mathbf{d}}_{\mathbf{X}})=Diag\left\{\mathbf{T}_{sqr}(\hat{\mathbf{d}}_{\hat{\mathbf{X}}_{R}}),\mathbf{T}_{sqr}(\hat{\mathbf{d}}_{\mathbf{X}_{P_k}}),\mathbf{T}_{sqr}(\hat{\mathbf{d}}_{\mathbf{X}_{P_{k+1}}})\right\}=$$

$$=Diag\left\{\sqrt{t(\hat{d}_{X_{R_1}})},\sqrt{t(\hat{d}_{Y_{R_1}})},\ldots,\sqrt{t(\hat{d}_{X_{P_{k+1}}})},\sqrt{t(\hat{d}_{Y_{P_{k+1}}})}\right\}$$

($t(\hat{d})$ - function of attenuation).

If justified (at least from the practical point of view), the assumption concerning diagonal structure of the weights matrix $\mathbf{P}_{\mathbf{X}}$ (e.g. if mutual relations between the adjusted coordinates of the points $\Re$ are neglected), then:

$$\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}=\tilde{\mathbf{T}}_{sqr}(\mathbf{\Gamma}^{0},\hat{\mathbf{d}}_{\mathbf{X}})\mathbf{P}_{\mathbf{X}}\tilde{\mathbf{T}}_{sqr}(\mathbf{\Gamma}^{0},\hat{\mathbf{d}}_{\mathbf{X}})=\tilde{\mathbf{T}}(\mathbf{\Gamma}^{0},\hat{\mathbf{d}}_{\mathbf{X}})\mathbf{P}_{\mathbf{X}}=\Im(\mathbf{\Gamma}^{0})\mathbf{T}(\hat{\mathbf{d}}_{\mathbf{X}})\mathbf{P}_{\mathbf{X}} \tag{5.9}$$

In consequence of the assumptions made, g-inverse $\mathbf{A}^{+}_{\mathbf{MN}}$ to the matrix $\mathbf{M}=\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}$ and $\mathbf{N}=\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}$ has taken the following form [Rao 1982, Wiśniewski 2000]:

$$\mathbf{A}^{+}_{\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}}=\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}^{-1}\mathbf{\Theta}\left(\mathbf{\Theta}\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}^{-1}\mathbf{\Theta}\right)^{-}\mathbf{A}^{T}\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \tag{5.10}$$

where $\mathbf{\Theta}=\mathbf{A}^{T}\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}\mathbf{A}$

To determine the generalized inverse $\left(\mathbf{\Theta}\widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}}^{-1}\mathbf{\Theta}\right)^{-}$ the matrix $\mathbf{A}\in\mathfrak{M}_{(n,r)}$ of $rank(\mathbf{A})=r_{\mathbf{A}}$, $r_{\mathbf{A}}=r-d$, $d=2$ rank, in conformity with the general principles presented in the paper [Wiśniewski 2004], will be presented in the following block form:

$$\mathbf{A}=\left[\mathbf{A}_r\in\mathbf{M}_{(n,r_{\mathbf{A}})}\ \vdots\ \mathbf{A}_d\in\mathbf{M}_{(n,d)}\right] \tag{5.11}$$

and with $rank(\mathbf{A}_r)=r_{\mathbf{A}}$

(because in our problem $d=2$ block $\mathbf{A}_d$ is created by the last two columns of the matrix $\mathbf{A}$). In reference to the model (5.1) and the resulting therefrom matrix $\mathbf{A}$ of structure (5.3), one can notice that the two columns include coefficients which stand at increments to the point P_k coordinates. In such a form, the matrix $\mathbf{A}$ can also be presented in the following, specific form:

$$\mathbf{A}=\underbrace{\begin{bmatrix}\mathbf{A}_{Z_kR} & \mathbf{A}_{Z_kP_k} \\ \mathbf{A}_{Z_{k+1}R} & \mathbf{0}\end{bmatrix}}_{\mathbf{A}_{..}}\begin{matrix}\vdots\\ \vdots\end{matrix}\underbrace{\begin{bmatrix}\mathbf{0} \\ \mathbf{A}_{Z_{k+1}P_{k+1}}\end{bmatrix}}_{\mathbf{A}_d}=\left[\mathbf{A}_r\ \vdots\ \mathbf{A}_d\right] \tag{5.12}$$

Such structure of the matrix $\mathbf{A}$ is corresponding to the presented below vector $\hat{\mathbf{d}}_{\mathbf{X}}$ structure and the weights matrix $\mathbf{P}_{\mathbf{X}}$ (and thereby to the matrix $\widetilde{\widetilde{\mathbf{P}}}_{\mathbf{X}}$ as well):

$$\hat{\mathbf{d}}_{\mathbf{X}}=\begin{bmatrix}\hat{\mathbf{d}}_{\hat{\mathbf{X}}_R}\\ \hat{\mathbf{d}}_{\mathbf{X}_{P_k}}\\ \cdots\\ \hat{\mathbf{d}}_{\mathbf{X}_{P_{k+1}}}\end{bmatrix}=\begin{bmatrix}\hat{\mathbf{d}}_{\mathbf{X}_r}\\ \cdots\\ \hat{\mathbf{d}}_{\mathbf{X}_d}=\hat{\mathbf{d}}_{\mathbf{X}_{P_{k+1}}}\end{bmatrix}\begin{matrix}\rightarrow\\ \rightarrow\\ \rightarrow\end{matrix}\begin{bmatrix}\mathbf{P}_{\hat{\mathbf{X}}_R} & & \vdots & \\ & \mathbf{P}_{\mathbf{X}^0_{P_k}} & \vdots & \\ \cdots & \cdots & \vdots & \cdots\\ & & \vdots & \mathbf{P}_{\mathbf{X}^0_{P_{k+1}}}\end{bmatrix}=$$

$$=\begin{bmatrix}\mathbf{P}_{\mathbf{X}_r} & \vdots & \\ \cdots & \vdots & \cdots\\ & \vdots & \mathbf{P}_{\mathbf{X}_d}=\mathbf{P}_{\mathbf{X}^0_{P_{k+1}}}\end{bmatrix}=\mathbf{P}_{\mathbf{X}}$$

where:

$$\hat{\mathbf{d}}_{\mathbf{X}_r}=\left[\hat{\mathbf{d}}^T_{\hat{\mathbf{X}}_R},\hat{\mathbf{d}}^T_{\hat{\mathbf{X}}_{P_k}}\right]^T\in\mathfrak{M}_{(r_{\mathbf{A}},1)},\ \hat{\mathbf{d}}_{\mathbf{X}_d}=\hat{\mathbf{d}}^T_{\mathbf{X}_{P_{k+1}}}\in\mathfrak{M}_{(d,1)}$$

and

$$\mathbf{P}_{\mathbf{X}_r} = Diag\left(\mathbf{P}_{\hat{\mathbf{X}}_R}, P_{\mathbf{X}^0_{P_k}}\right) \in \mathfrak{M}_{(r_\mathbf{A}, r_\mathbf{A})},\ \mathbf{P}_{\mathbf{X}_d} = \mathbf{P}_{\mathbf{X}^0_{P_{k+1}}} \in \mathfrak{M}_{(d,d)}$$

Thus

$$\mathbf{\Theta} = \mathbf{A}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \mathbf{A} = \begin{bmatrix} \mathbf{\Theta}_{rr} & \mathbf{\Theta}_{rd} \\ \mathbf{\Theta}_{rd}^T & \mathbf{\Theta}_{dd} \end{bmatrix}$$

and

$$\mathbf{\Theta} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}^{-1} \mathbf{\Theta} = \begin{bmatrix} \overline{\mathbf{\Theta}}_{rr} & \overline{\mathbf{\Theta}}_{rd} \\ \overline{\mathbf{\Theta}}_{rr}^T & \overline{\mathbf{\Theta}}_{dd} \end{bmatrix}$$

where:

$$\mathbf{\Theta}_{rr} = \mathbf{A}_r^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \mathbf{A}_r \in \mathfrak{M}_{(r_\mathbf{A}, r_\mathbf{A})}, \quad rank(\mathbf{\Theta}_{rr}) = r_\mathbf{A}$$

$$\mathbf{\Theta}_{dd} = \mathbf{A}_d^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \mathbf{A}_d$$

$$\mathbf{\Theta}_{rd} = \mathbf{A}_r^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \mathbf{A}_d$$

$$\overline{\mathbf{\Theta}}_{rr} = \mathbf{\Theta}_{rr} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_r}^{-1} \mathbf{\Theta}_{rr} + \mathbf{\Theta}_{rd} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_d}^{-1} \mathbf{\Theta}_{rd}^T \in \mathfrak{M}_{(r_\mathbf{A}, r_\mathbf{A})}, \quad rank(\overline{\mathbf{\Theta}}_{rr}) = r_\mathbf{A}$$

$$\overline{\mathbf{\Theta}}_{rd} = \mathbf{\Theta}_{rr} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_r}^{-1} \mathbf{\Theta}_{rd} + \mathbf{\Theta}_{rd} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_d}^{-1} \mathbf{\Theta}_{dd}$$

$$\overline{\mathbf{\Theta}}_{dd} = \mathbf{\Theta}_{rd}^T \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_r}^{-1} \mathbf{\Theta}_{rd} + \mathbf{\Theta}_{dd} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_d}^{-1} \mathbf{\Theta}_{dd}$$

As $rank\left(\mathbf{\Theta} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}^{-1} \mathbf{\Theta}\right) = rank(\overline{\mathbf{\Theta}}_{rr}) = r_\mathbf{A}$

and $\overline{\mathbf{\Theta}}_{rr} \in \mathfrak{M}_{(r_\mathbf{A}, r_\mathbf{A})}$ (the matrix $\overline{\mathbf{\Theta}}_{rr}$ is the full rank matrix), so the generalized inverse $\left(\overline{\mathbf{\Theta}} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}^{-1} \overline{\mathbf{\Theta}}\right)^-$ in the version already applied in this paper is of the following form:

$$\left(\overline{\mathbf{\Theta}} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}^{-1} \overline{\mathbf{\Theta}}\right)^- = \begin{bmatrix} \overline{\mathbf{\Theta}}_{rr} & \vdots & \overline{\mathbf{\Theta}}_{rd} \\ \cdots & \vdots & \cdots \\ \overline{\mathbf{\Theta}}_{rd}^T & \vdots & \overline{\mathbf{\Theta}}_{dd} \end{bmatrix}^- = \begin{bmatrix} \overline{\mathbf{\Theta}}_{rr}^{-1} & \vdots & \mathbf{0} \\ \cdots & \vdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{0} \end{bmatrix}$$

Therefore

$$\mathbf{A}^+_{\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}} = \begin{bmatrix} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_r}^{-1} & \vdots & \\ \cdots & \vdots & \cdots \\ & \vdots & \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_d}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{\Theta}_{rr} & \mathbf{\Theta}_{rd} \\ \mathbf{\Theta}_{rd}^T & \mathbf{\Theta}_{dd} \end{bmatrix} \begin{bmatrix} \overline{\mathbf{\Theta}}_{rr}^{-1} & \vdots & \mathbf{0} \\ \cdots & \vdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{A}_r^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \\ \mathbf{A}_d^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \end{bmatrix} = \begin{bmatrix} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_r}^{-1} \mathbf{\Theta}_{rr} \overline{\mathbf{\Theta}}_{rr}^{-1} \mathbf{A}_r^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \\ \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_d}^{-1} \mathbf{\Theta}_{rd}^T \overline{\mathbf{\Theta}}_{rr}^{-1} \mathbf{A}_r^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \end{bmatrix} \quad (5.13)$$

and next

$$\hat{\mathbf{d}}_{\mathbf{X}} = -\mathbf{A}^{+}_{\tilde{\tilde{\mathbf{P}}}_x \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}} \mathbf{L} = -\begin{bmatrix} \tilde{\tilde{\mathbf{P}}}^{-1}_{\mathbf{X}_r} \mathbf{\Theta}_{rr} \overline{\mathbf{\Theta}}^{-1}_{rr} \mathbf{A}^T_r \tilde{\tilde{\mathbf{P}}}_x \mathbf{L} \\ \tilde{\tilde{\mathbf{P}}}^{-1}_{\mathbf{X}_d} \mathbf{\Theta}^T_{rd} \overline{\mathbf{\Theta}}^{-1}_{rr} \mathbf{A}^T_r \tilde{\tilde{\mathbf{P}}}_x \mathbf{L} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{d}}_{\mathbf{X}_r} \\ \cdots \\ \hat{\mathbf{d}}_{\mathbf{X}_d} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{d}}_{\hat{\mathbf{X}}_R} \\ \hat{\mathbf{d}}_{\mathbf{X}_{P_k}} \\ \cdots \\ \hat{\mathbf{d}}_{\mathbf{X}_{P_{k+1}}} \end{bmatrix} \tag{5.14}$$

Taking into account practical applications of the suggestions on issue and the resulting therefrom detailed structure of the matrix **A**, it worth to be noticed that:

$$\mathbf{\Theta}_{rr} = \mathbf{A}^T_r \tilde{\tilde{\mathbf{P}}}_x \mathbf{A}_r = \begin{bmatrix} \mathbf{Q}_{Z_k R} + \mathbf{Q}_{Z_{k+1} R} & \vdots & \mathbf{Q}_{Z_k R P_k} \\ \cdots & \cdots & \cdots \\ \mathbf{Q}^T_{Z_k R P_k} & \vdots & \mathbf{Q}_{Z_k P_k} \end{bmatrix},$$

$$\mathbf{\Theta}^T_{rd} = \mathbf{A}^T_d \tilde{\tilde{\mathbf{P}}}_x \mathbf{A}_r = \begin{bmatrix} \mathbf{Q}_{Z_{k+1} P_{k+1} R} & \vdots & \mathbf{0} \end{bmatrix}$$

gdzie:

$$\mathbf{Q}_{Z_k R} = \mathbf{A}^T_{Z_k R} \tilde{\tilde{\mathbf{P}}}_{x Z_k} \mathbf{A}_{Z_k R}$$

$$\mathbf{Q}_{Z_k R P_k} = \mathbf{A}^T_{Z_k R} \tilde{\tilde{\mathbf{P}}}_{x Z_k} \mathbf{A}_{Z_k P_k}$$

$$\mathbf{Q}_{Z_{k+1} P_{k+1} R} = \mathbf{A}^T_{Z_{k+1} P_{k+1}} \tilde{\tilde{\mathbf{P}}}_{x Z_{k+1}} \mathbf{A}_{Z_{k+1} R}$$

$$\mathbf{Q}_{Z_{k+1} R} = \mathbf{A}^T_{Z_{k+1} R} \tilde{\tilde{\mathbf{P}}}_{x Z_{k+1}} \mathbf{A}_{Z_{k+1} R}$$

$$\mathbf{Q}_{Z_k P_k} = \mathbf{A}^T_{Z_k P_k} \tilde{\tilde{\mathbf{P}}}_{x Z_k} \mathbf{A}_{Z_k P_k}$$

Now let it be as follows:

$$\overline{\mathbf{\Theta}}^{-1}_{rr} = \left(\mathbf{\Theta}_{rr} \tilde{\tilde{\mathbf{P}}}^{-1}_{\mathbf{X}_r} \mathbf{\Theta}_{rr} + \mathbf{\Theta}_{rd} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_d} \mathbf{\Theta}^T_{rd} \right)^{-1} = \begin{bmatrix} [\overline{\mathbf{\Theta}}_{rr}]_{11} & [\overline{\mathbf{\Theta}}_{rr}]_{12} \\ [\overline{\mathbf{\Theta}}_{rr}]^T_{12} & [\overline{\mathbf{\Theta}}_{rr}]_{22} \end{bmatrix}^{-1} = \begin{bmatrix} \overline{\overline{\mathbf{\Theta}}}_{11} & \overline{\overline{\mathbf{\Theta}}}_{12} \\ \overline{\overline{\mathbf{\Theta}}}^T_{12} & \overline{\overline{\mathbf{\Theta}}}_{22} \end{bmatrix}$$

where:

$$[\overline{\mathbf{\Theta}}_{rr}]_{11} = (\mathbf{Q}_{Z_k R} + \mathbf{Q}_{Z_{k+1} R}) \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_R} (\mathbf{Q}_{Z_k R} + \mathbf{Q}_{Z_{k+1} R}) + \mathbf{Q}_{Z_k R P_k} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{P_k}} \mathbf{Q}^T_{Z_k R P_k} + \mathbf{Q}^T_{Z_{k+1} P_{k+1} R} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_d} \mathbf{Q}_{Z_{k+1} P_{k+1} R}$$

$$[\overline{\mathbf{\Theta}}_{rr}]_{12} = (\mathbf{Q}_{Z_k R} + \mathbf{Q}_{Z_{k+1} R}) \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_R} \mathbf{Q}_{Z_k R P_k} + \mathbf{Q}_{Z_k R P_k} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{P_k}} \mathbf{Q}_{Z_k P_k}$$

$$[\overline{\mathbf{\Theta}}_{rr}]_{22} = \mathbf{Q}_{Z_k R P_k} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_R} \mathbf{Q}_{Z_k R P_k} + \mathbf{Q}_{Z_k P_k} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_d} \mathbf{Q}_{Z_k P_k}$$

($[\overline{\mathbf{\Theta}}_{rr}]_{ij}$ - i, j - part of matrix $\overline{\mathbf{\Theta}}_{rr}$).

Then $\begin{bmatrix} \hat{\mathbf{d}}_{\hat{\mathbf{X}}_R} \\ \hat{\mathbf{d}}_{\mathbf{X}_{P_k}} \end{bmatrix} = -\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_r}^{-1} \mathbf{\Theta}_{rr} \overline{\mathbf{\Theta}}_{rr}^{-1} \mathbf{A}_r^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \mathbf{L} \quad \Rightarrow$

$$\hat{\mathbf{d}}_{\hat{\mathbf{X}}_R} = -\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_R}^{-1} \left\{ \left(\mathbf{Q}_{Z_k R} + \mathbf{Q}_{Z_{k+1} R} \right) \xi_1 + \mathbf{Q}_{Z_k R P_k} \xi_2 \right\} \tag{5.15}$$

$$\hat{\mathbf{d}}_{\mathbf{X}_{P_k}} = -\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{P_k}}^{-1} \left(\mathbf{Q}_{Z_k R P_k} \xi_1 + \mathbf{Q}_{Z_k P_k} \xi_2 \right) \tag{5.16}$$

and

$$\hat{\mathbf{d}}_{\mathbf{X}_d} = \hat{\mathbf{d}}_{\mathbf{X}_{P_{k+1}}} = -\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_d}^{-1} \mathbf{\Theta}_{rd}^T \overline{\mathbf{\Theta}}_{rr}^{-1} \mathbf{A}_r^T \mathbf{P}_{\boldsymbol{x}} \mathbf{L} = -\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{P_{k+1}}}^{-1} \mathbf{Q}_{Z_{k+1} P_{k+1} R} \xi_1 \tag{5.17}$$

where

$$\xi_1 = \overline{\overline{\mathbf{\Theta}}}_{11} \left(\mathbf{\Lambda}_{Z_k R} + \mathbf{\Lambda}_{Z_{k+1} R} \right) + \overline{\overline{\mathbf{\Theta}}}_{12} \mathbf{\Lambda}_{Z_k P}$$

$$\xi_2 = \overline{\overline{\mathbf{\Theta}}}_{12}^T \left(\mathbf{\Lambda}_{Z_k R} + \mathbf{\Lambda}_{Z_{k+1} R} \right) + \overline{\overline{\mathbf{\Theta}}}_{22} \mathbf{\Lambda}_{Z_k P}$$

$$\mathbf{\Lambda}_{Z_k R} = \mathbf{A}_{Z_k R}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x} Z_k} \mathbf{L}_{Z_k}$$

$$\mathbf{\Lambda}_{Z_{k+1} R} = \mathbf{A}_{Z_{k+1} R}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x} Z_{k+1}} \mathbf{L}_{Z_{k+1}}$$

$$\mathbf{\Lambda}_{Z_k P} = \mathbf{A}_{Z_k P_k}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x} Z_k} \mathbf{L}_{Z_k}$$

The vector $\hat{\mathbf{d}}_{\mathbf{X}} = -\mathbf{A}_{\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}}^{+} \mathbf{L}$ (or with more detailed developments (5.15÷5.17) is a solution of the decisive and hybrid adjustment task - from the robustness viewpoint – taking the following form:

$$\left. \begin{aligned} & \mathbf{V} = \mathbf{A}\hat{\mathbf{d}}_{\mathbf{X}} + \mathbf{L} = \begin{bmatrix} \mathbf{A}_r & \vdots & \mathbf{A}_d \end{bmatrix} \begin{bmatrix} \hat{\mathbf{d}}_{\mathbf{X}_r} \\ \hat{\mathbf{d}}_{\mathbf{X}_d} = \hat{\mathbf{d}}_{\mathbf{X}_{P_{k+1}}} \end{bmatrix} \\ & \mathbf{C}_{\boldsymbol{x}} = \sigma_0^2 \mathbf{P}_{\boldsymbol{x}}^{-1} \quad \rightarrow \quad \mathbf{C}_{\boldsymbol{x}} = \sigma_0^2 \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}^{-1} \\ & \min_{\mathbf{d}_{\mathbf{X}}} \Phi^{D-R}\left(\mathbf{d}_{\mathbf{X}}\right) = \Phi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}}\right) = \mathbf{V}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \mathbf{V} \\ & \min_{\mathbf{d}_{\mathbf{X}}} \Psi^{D-R}\left(\mathbf{d}_{\mathbf{X}}\right) = \Psi^{D-R}\left(\hat{\mathbf{d}}_{\mathbf{X}}\right) = \hat{\mathbf{d}}_{\mathbf{X}}{}^T \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}} \hat{\mathbf{d}}_{\mathbf{X}} \\ & \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}} = Diag\left(\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_R}, \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{P_k}^0} \; \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_{P_{k+1}}^0} \right) \end{aligned} \right\} \tag{5.18}$$

The course of the solution (of iterative character) is connected with the identification process, and next with attenuation of the influence, both the outlier observations and, what is of special significance, the identification and attenuation of the outlier points coordinates effect. The process, what has been mentioned before, is carried out much easier, if standardized quantities are applied. In the suggested hybrid *M*-estimation, it is necessary to determine both: the standardized corrections $\left(\upsilon_i = [\mathbf{V}]_i\right)$

$$\bar{\upsilon}_i = \frac{\upsilon_i}{\hat{\sigma}_{\upsilon_i}} = \frac{\upsilon_i}{\sqrt{\left[\hat{\mathbf{C}}_{\mathbf{V}}\right]_{ii}}}, \quad i = 1,2,\ldots,n$$

and the standardized increments $\left(\hat{d}_{X_j} = \left[\hat{d}_X\right]_j\right)$

$$\bar{\hat{d}}_{X_j} = \frac{\hat{d}_{X_j}}{\hat{\sigma}_{\hat{d}_{X_j}}} = \frac{\hat{d}_{X_j}}{\sqrt{\left[\hat{\mathbf{C}}_{\hat{\mathbf{d}}_{\mathbf{X}}}\right]_{jj}}}, \quad j = 1,2,\ldots,r$$

Aiming at the above it is necessary to establish forms of the covariance matrixes $\hat{\mathbf{C}}_{\mathbf{V}}$ of the corrections vector $\mathbf{V}$ and of the covariance matrixes $\hat{\mathbf{C}}_{\hat{\mathbf{d}}_{\mathbf{X}}}$ of the estimator $\hat{\mathbf{d}}_{\mathbf{X}}$, obtained at free hybrid adjustment.

To determine the estimator $\hat{\mathbf{C}}_{\mathbf{V}}$ of the corrections vector covariance matrix $\mathbf{V}$, we can write down as follows:

$$\mathbf{V} = \mathbf{A}\hat{\mathbf{d}}_{\mathbf{X}} + \mathbf{L} = -\mathbf{A}\mathbf{A}^{+}_{\tilde{\tilde{\mathbf{P}}}_x \tilde{\mathbf{P}}_{\mathbf{X}}}\mathbf{L} + \mathbf{L} = \left(\mathbf{I}_{(n)} - \mathbf{A}\mathbf{A}^{+}_{\tilde{\tilde{\mathbf{P}}}_x \tilde{\mathbf{P}}_{\mathbf{X}}}\right)\mathbf{L} = \mathbf{M}_{+}\mathbf{L} \quad (5.19)$$

where $\mathbf{M}_{+} = \mathbf{I}_{(n)} - \mathbf{A}\mathbf{A}^{+}_{\tilde{\tilde{\mathbf{P}}}_x \tilde{\mathbf{P}}_{\mathbf{X}}}$

Then, taking advantage of the covariance matrix propagation principle, the following is obtained:

$$\mathbf{C}_{\mathbf{V}} = \mathbf{M}_{+}\tilde{\tilde{\mathbf{C}}}_{\mathbf{L}}\mathbf{M}_{+}^{T}$$

The covariance matrix $\tilde{\tilde{\mathbf{C}}}_{\mathbf{L}}$ of the free terms vector $\mathbf{L}$ can be determined with the relationship (5.3):

$$\mathbf{L} = \mathbf{F}\left(\mathbf{\Gamma}^0\right) - \mathbf{x}$$

on assumption, that

$$\tilde{\tilde{\mathbf{C}}}_{\mathbf{\Gamma}^0} = \sigma_0^2 \tilde{\tilde{\mathbf{P}}}_{\mathbf{\Gamma}^0}^{-1} = \sigma_0^2 \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}^{-1}$$

$$\tilde{\tilde{\mathbf{C}}}_{\boldsymbol{x}} = \sigma_0^2 \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}^{-1}$$

Thus (also on the basis of the covariance matrix propagation principle)

$$\tilde{\tilde{\mathbf{C}}}_{\mathbf{L}} = \partial_{\mathbf{\Gamma}} \mathbf{F}\left(\mathbf{\Gamma}^0\right) \tilde{\tilde{\mathbf{C}}}_{\mathbf{\Gamma}^0} \left[\partial_{\mathbf{\Gamma}} \mathbf{F}\left(\mathbf{\Gamma}^0\right) \right]^T + \tilde{\tilde{\mathbf{C}}}_{\boldsymbol{x}}$$

However, because

$$\partial_{\mathbf{\Gamma}} \mathbf{F}\left(\mathbf{\Gamma}^0\right) = \partial_{\mathbf{\Gamma}} \mathbf{F}\left(\hat{\mathbf{X}}_R, \mathbf{X}_{P_k}^0, \mathbf{X}_{P_{k+1}}^0\right) = \mathbf{A}$$

so

$$\tilde{\tilde{\mathbf{C}}}_{\mathbf{L}} = \mathbf{A} \tilde{\tilde{\mathbf{C}}}_{\mathbf{\Gamma}^0} \mathbf{A}^T + \tilde{\tilde{\mathbf{C}}}_{\boldsymbol{x}} = \sigma_0^2 \left(\mathbf{A} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}^{-1} \mathbf{A}^T + \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}^{-1} \right)$$

Therefore

$$\mathbf{C}_{\mathbf{V}} = \mathbf{M}_{+} \mathbf{C}_{\mathbf{L}} \mathbf{M}_{+}^T = \sigma_0^2 \mathbf{M}_{+} \left(\mathbf{A} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}^{-1} \mathbf{A}^T + \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}^{-1} \right) \mathbf{M}_{+}^T \tag{5.20}$$

and there from

$$\hat{\mathbf{C}}_{\mathbf{V}} = \hat{\sigma}_0^2 \mathbf{M}_{+} \left(\mathbf{A} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}^{-1} \mathbf{A}^T + \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}^{-1} \right) \mathbf{M}_{+}^T \tag{5.21}$$

In classic, robust adjustment, it means when $rank(\mathbf{A}) = rank\left(\mathbf{A}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \mathbf{A} \right) = r$, $(d = 0)$ g-inverse $\mathbf{A}^{+}_{\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}}$ is of the following, specific form (e.g.: [Wiśniewski 2004]):

$$\mathbf{A}^{+}_{\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}} = \left(\mathbf{A}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \mathbf{A} \right)^{-1} \mathbf{A}^T \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}$$

In this specific case $\mathbf{C}_{\mathbf{L}} = \mathbf{C}_{\boldsymbol{x}}$ (vector $\mathbf{\Gamma}^0$ is then not the random one) and

$$\mathbf{M}_{+} = \mathbf{I}_{(n)} - \mathbf{A}^{+}_{\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}} \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}}} = \mathbf{I}_{(n)} - \mathbf{A}\left(\mathbf{A}^T \mathbf{P}_{\boldsymbol{x}} \mathbf{A} \right)^{-1} \mathbf{A}^T \mathbf{P}_{\boldsymbol{x}}$$

$$\mathbf{C}_{\mathbf{L}} = \mathbf{C}_{\boldsymbol{x}} = \sigma_0^2 \mathbf{P}_{\boldsymbol{x}}^{-1}$$

Then

$$\mathbf{C_V} = \mathbf{M}_+\mathbf{C_L}\mathbf{M}_+^T = \mathbf{M}_+\mathbf{C}_x\mathbf{M}_+^T = \sigma_0^2\mathbf{M}_+\widetilde{\widetilde{\mathbf{P}}}_x^{-1}\mathbf{M}_+^T = \sigma_0^2\left[\widetilde{\widetilde{\mathbf{P}}}_x^{-1} - \mathbf{A}\left(\mathbf{A}^T\widetilde{\widetilde{\mathbf{P}}}_x\mathbf{A}\right)^{-1}\mathbf{A}^T\right] \Rightarrow$$

$$\Rightarrow \hat{\mathbf{C}}_\mathbf{V} = \hat{\sigma}_0^2\left[\widetilde{\widetilde{\mathbf{P}}}_x^{-1} - \mathbf{A}\left(\mathbf{A}^T\widetilde{\widetilde{\mathbf{P}}}_x\mathbf{A}\right)^{-1}\mathbf{A}^T\right]$$

what stands for the result which is well known and applied already before in this study (expression (3.53)).

The covariance matrix $\mathbf{C}_{\hat{\mathbf{d}}_\mathbf{X}}$ obtained in adjustment of free geometrical survey structures is a subject of the analyses, described in publications [Wolf 1972, 1979; Mittermayer 1972; Świątek, Wiśniewski 1983]. Taking into consideration (as above) in the covariance matrix of free terms vector $\mathbf{L}$ of the points coordinates covariance matrix was applied anyhow in the paper [Wiśniewski 2004]. In reference to the results and assumptions formulated in this Chapter, we may put down as follows:

$$\hat{\mathbf{d}}_\mathbf{X} = -\mathbf{A}^+_{\widetilde{\widetilde{\mathbf{P}}}_x\widetilde{\mathbf{P}}_\mathbf{X}}\mathbf{L} \quad \Rightarrow$$
$$\Rightarrow \mathbf{C}_{\hat{\mathbf{d}}_\mathbf{X}} = \mathbf{A}^+_{\widetilde{\widetilde{\mathbf{P}}}_x\widetilde{\mathbf{P}}_\mathbf{X}}\mathbf{C_L}\left(\mathbf{A}^+_{\widetilde{\widetilde{\mathbf{P}}}_x\widetilde{\mathbf{P}}_\mathbf{X}}\right)^T = \sigma_0^2\mathbf{A}^+_{\widetilde{\widetilde{\mathbf{P}}}_x\widetilde{\mathbf{P}}_\mathbf{X}}\left(\mathbf{A}\widetilde{\widetilde{\mathbf{P}}}_\mathbf{X}^{-1}\mathbf{A}^T + \widetilde{\widetilde{\mathbf{P}}}_x^{-1}\right)\left(\mathbf{A}^+_{\widetilde{\widetilde{\mathbf{P}}}_x\widetilde{\mathbf{P}}_\mathbf{X}}\right)^T \tag{5.22}$$

and on this basis

$$\hat{\mathbf{C}}_{\hat{\mathbf{d}}_\mathbf{X}} = \hat{\sigma}_0^2\mathbf{A}^+_{\widetilde{\widetilde{\mathbf{P}}}_x\widetilde{\mathbf{P}}_\mathbf{X}}\left(\mathbf{A}\widetilde{\widetilde{\mathbf{P}}}_\mathbf{X}^{-1}\mathbf{A}^T + \widetilde{\widetilde{\mathbf{P}}}_x^{-1}\right)\left(\mathbf{A}^+_{\widetilde{\widetilde{\mathbf{P}}}_x\widetilde{\mathbf{P}}_\mathbf{X}}\right)^T \tag{5.23}$$

Bringing the obtained expression for the case of robust classic adjustment, we may say that $\left(\mathbf{C_L} = \mathbf{C}_x = \sigma_0^2\widetilde{\widetilde{\mathbf{P}}}_x^{-1}\right)$

$$\mathbf{C}_{\hat{\mathbf{d}}_\mathbf{x}} = \sigma_o^2\underbrace{\left(\mathbf{A}^T\widetilde{\widetilde{\mathbf{P}}}_x\mathbf{A}\right)^{-1}\mathbf{A}^T\widetilde{\widetilde{\mathbf{P}}}_x}_{\mathbf{A}^+_{\widetilde{\widetilde{\mathbf{P}}}_x\widetilde{\mathbf{P}}_\mathbf{X}}}\widetilde{\widetilde{\mathbf{P}}}_x^{-1}\underbrace{\widetilde{\widetilde{\mathbf{P}}}_x\mathbf{A}\left(\mathbf{A}^T\widetilde{\widetilde{\mathbf{P}}}_x\mathbf{A}\right)^{-1}}_{\left(\mathbf{A}^+_{\widetilde{\widetilde{\mathbf{P}}}_x\widetilde{\mathbf{P}}_\mathbf{X}}\right)^T} = \sigma_o^2\left(\mathbf{A}^T\widetilde{\widetilde{\mathbf{P}}}_x\mathbf{A}\right)^{-1}$$

what is a well known result as well.

6. NUMERICAL TESTS

This Chapter presents two numerical tests. The tests illustrate theoretical solutions, described in the paper. They concern those of the author's research elements, which have not been a subject of previous analyses. The first of them shows a possibility of the Interactive Navigational Structure common development carried out by a team of two hydrographical vessels. The second describes an opportunity of using the existing navigational structure and hybrid *M*-estimation, suggested in Chapter 5, in classic navigational task performance.

In the both tests the vessels are navigating basing on an optional Cartesian coordinate system. The tasks under performance are hypothetic; therefore their results cannot fully refer to reality. Anyhow, such simplification allows presenting clearly the possibilities of implementing in practice the questions suggested in this paper.

6.1. Determining objects' positions by a team of vessels

Let us assume that a team of two hydrographical vessels is performing a common hydrographical task or any kind of other special work of a similar scope within a coastal water area. The team has determined the proper positions taking advantage of the set $\mathfrak{Z}$ and the set $\mathfrak{Z}^{DGPS}$ points. While analysing a number of the navigational signs, there has been found a necessity of increasing the set $\mathfrak{Z}$ by new elements $\Re$ (in compliance with the assumptions mentioned in Chapter 2). Therefore *INS* development was decided, to be carried out on the basis of the observed sign $Z_1 \in \mathfrak{Z}$, i – th proper positions $\left(P_i^{\langle 1\rangle}, P_i^{\langle 2\rangle}\right) \in \wp$ and two visible onshore objects $\left(R_1, R_2\right) \in \Re$. Positions of vessels $\langle 1\rangle$ and $\langle 2\rangle$ have been determined with a use of GPS system and through reckoning. The determination tasks were divided into two stages (Fig. 6.1.: I stage – red colour, II stage – green colour), realised on grounds of considerations presented in point 4.3. (two vessels cooperation).

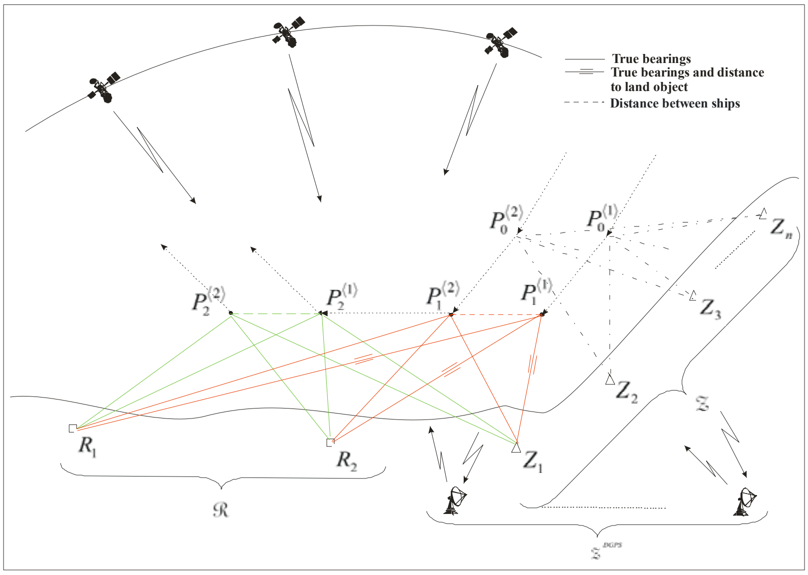

Fig. 6.1. Graphical interpretation of the Test No 1

STAGE I

With a use of DGPS system, there has been carried out determination of proper positions $\left(X^{DGPS}, Y^{DGPS}\right)$ and also the route travelled reckoning, to find out the positions reckoned $\left(X^0, Y^0\right)$. For the test purpose there was assumed that the coordinates' values were equal; the determinations are specified in Table. 6.1.

Table 6.1. Positions of the vessels accepted for computations of the stage I

Vessels' positions	Cartesian coordinates	
	X^{DGPS}	Y^{DGPS}
$P_1^{\langle 1 \rangle}$	1400 [m]	800 [m]
$P_1^{\langle 2 \rangle}$	1100 [m]	800 [m]

Next, from both the vessels the observations toward Z_1 sign and the selected objects R_1, R_2 were carried out. Simulated results of the observations are specified in the Table 6.2. It was assumed, that the mean error of bearing determination was $m_N = 0{,}1^{\circ}$, and of the distance to the onshore objects survey was $m_d = 2$ [m].

Whereas an error of survey of the distance between the vessels was $m_{d^W} = 0{,}5$ [m]. The coordinates of the navigational sign are specified in Table 6.3.

Table 6.2. The observations obtained in the I stage of determination

Positions of vessels	Observations
$P_1^{\langle 1\rangle}$	$N_{1,1} = 196{,}0^\circ$
	$d_{1,1} = 728$ [m]
	$NR_{1,1}^{\langle 1\rangle} = 239{,}7^\circ$
	$d_{1,1}^{\langle 1\rangle} = 1390$ [m]
	$NR_{2,1}^{\langle 1\rangle} = 220{,}5^\circ$
	$d_{2,1}^{\langle 1\rangle} = 922$ [m]
$P_1^{\langle 2\rangle}$	$N_{1,2} = 172{,}0^\circ$
	$NR_{1,2}^{\langle 2\rangle} = 232{,}0^\circ$
	$NR_{2,2}^{\langle 2\rangle} = 203{,}0^\circ$
	$d_1^{W\langle 2\rangle} = 300$ [m]

Table 6.3.Sign Z_1 coordinates

Sign name	Cartesian coordinates	
	X	Y
Z_1	1200 **[m]**	100 **[m]**

For the observations obtained on vessel $\langle 1\rangle$ the following observational equations system can be formulated:

$$\bar{\boldsymbol{x}}_1^{\langle 1\rangle} = \mathbf{F}_{P_1}^{\langle 1\rangle}(\mathbf{X}_{P_1}^{\langle 1\rangle}, \mathbf{X}_{R_1}, \mathbf{X}_{R_2}) = \mathbf{F}_{P_1}^{\langle 1\rangle}(\mathbf{X})$$

where $\mathbf{X}_{P_1}^{\langle 1\rangle}$ is true coordinates of vessel $\langle 1\rangle$ on P_1 - position, $\mathbf{X}_{R_1}, \mathbf{X}_{R_2}$ is true coordinates of points $(R_1, R_2) \in \Re$, however $\bar{\boldsymbol{x}}_1^{\langle 1\rangle} = \left[\underbrace{\bar{N}_{1,1}, \bar{d}_{1,1}}_{\bar{\boldsymbol{x}}_{Z_1}^T}, \underbrace{\bar{N}R_{1,1}^{\langle 1\rangle}, \bar{d}R_{1,1}^{\langle 1\rangle}}_{\bar{\boldsymbol{x}}_{R_1}^T}, \underbrace{\bar{N}R_{2,1}^{\langle 1\rangle}, \bar{d}R_{2,1}^{\langle 1\rangle}}_{\bar{\boldsymbol{x}}_{R_2}^T} \right]^T$ is the measured quantities vector

($\overline{N}_{i,j}$ - *j*-th true bearing to the i-th navigational sign, $\overline{d}_{i,j}$ - *j*-th distance to the *i*-th navigational sign, $\overline{NR}_{i,j}^{\langle 1\rangle}$ - *j*-th true bearing to the *i*-th object *R* taken from vessel $\langle 1\rangle$, $\overline{dR}_{i,j}^{\langle 1\rangle}$ - *j*-th distance to the *i*-th object *R* taken from vessel $\langle 1\rangle$).

To this way formulated observational equations system, completed GPS observations, there refers the following functional model (basing on (3.8)):

$$\begin{cases} \mathrm{V}_{Z_1} = \mathrm{A}_{Z_1,\mathrm{P}_1^{\langle 1\rangle}} \hat{\mathrm{d}}_{\mathrm{X}_{\mathrm{P}_1^{\langle 1\rangle}}} + \mathrm{A}_{Z_1,\mathrm{R}_1} \hat{\mathrm{d}}^{\langle 1\rangle}_{\mathrm{X}_{\mathrm{R}_1}} + \mathrm{A}_{Z_1,\mathrm{R}_2} \hat{\mathrm{d}}^{\langle 1\rangle}_{\mathrm{X}_{\mathrm{R}_2}} + \mathrm{L}_{Z_1} \\ \mathrm{V}^{\langle 1\rangle}_{\mathrm{R}_1} = \mathrm{A}_{\mathrm{R}_1,\mathrm{P}_1^{\langle 1\rangle}} \hat{\mathrm{d}}_{\mathrm{X}_{\mathrm{P}_1^{\langle 1\rangle}}} + \mathrm{A}_{\mathrm{R}_1,\mathrm{R}_1} \hat{\mathrm{d}}^{\langle 1\rangle}_{\mathrm{X}_{\mathrm{R}_1}} + \mathrm{A}_{\mathrm{R}_1,\mathrm{R}_2} \hat{\mathrm{d}}^{\langle 1\rangle}_{\mathrm{X}_{\mathrm{R}_2}} + \mathrm{L}_{\mathrm{R}_1} \\ \mathrm{V}^{\langle 1\rangle}_{\mathrm{R}_2} = \mathrm{A}_{\mathrm{R}_2,\mathrm{P}_1^{\langle 1\rangle}} \hat{\mathrm{d}}_{\mathrm{X}_{\mathrm{P}_1^{\langle 1\rangle}}} + \mathrm{A}_{\mathrm{R}_2,\mathrm{R}_1} \hat{\mathrm{d}}^{\langle 1\rangle}_{\mathrm{X}_{\mathrm{R}_1}} + \mathrm{A}_{\mathrm{R}_2,\mathrm{R}_2} \hat{\mathrm{d}}^{\langle 1\rangle}_{\mathrm{X}_{\mathrm{R}_2}} + \mathrm{L}_{\mathrm{R}_2} \\ \mathrm{V}^{\mathrm{DGPS}}_{\mathrm{X}_1^{\langle 1\rangle}} = \hat{\mathrm{d}}_{\mathrm{X}_{\mathrm{P}_1^{\langle 1\rangle}}} + \left(\mathrm{X}^{0}_{\mathrm{P}_1^{\langle 1\rangle}} - \mathrm{X}^{\mathrm{DGPS}}_{\mathrm{P}_1^{\langle 1\rangle}} \right) \end{cases}$$

The following decisive matrixes refer to the assumed observation structure (observations toward the $\mathfrak{Z}$, $\mathfrak{R}$ set elements and GPS observations):

$$\Im(\boldsymbol{x}_{z_1}) = \mathbf{I}_{(2)}, \Im(\boldsymbol{x}_{R_1}^{\langle 1\rangle}) = \mathbf{I}_{(2)}, \Im(\boldsymbol{x}_{R_2}^{\langle 1\rangle}) = \mathbf{I}_{(2)} \rightarrow \Im(\boldsymbol{x}_1^{\langle 1\rangle}) = Diag\left\{ \Im(\boldsymbol{x}_{z_1}), \Im(\boldsymbol{x}_{R_1}^{\langle 1\rangle}), \Im(\boldsymbol{x}_{R_2}^{\langle 1\rangle}) \right\} = \mathbf{I}_{(6)}$$

$$\Im(\mathbf{X}^{GPS}_{P_1^{\langle 1\rangle}}) = \mathbf{I}_{(2)}$$

With reference to the *M*-estimation principles we assume also the attenuation matrix, having the form as follows:

$$\mathbf{T}\left(\mathbf{V}_{\boldsymbol{x}_1^{\langle 1\rangle}} \right) = Diag\left\{ \mathbf{T}\left(\mathbf{V}_{Z_1}\right), \mathbf{T}\left(\mathbf{V}_{R_1}\right), \mathbf{T}\left(\mathbf{V}_{R_2}\right) \right\}$$

Then the decisive – equivalent weight matrixes assume the forms as follows:

$$\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_1^{\langle 1\rangle}} = Diag\left(\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{Z_1}}, \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{R_1}^{\langle 1\rangle}}, \tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}_{R_2}^{\langle 1\rangle}} \right) = \tilde{\mathbf{T}}\left(x_1^{\langle 1\rangle}, \mathbf{V}_{\boldsymbol{x}_1^{\langle 1\rangle}} \right) \cdot \mathbf{P}_{\boldsymbol{x}_1^{\langle 1\rangle}}$$

$$\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_1^{\langle 1\rangle DGPS}} = \tilde{\mathbf{T}}\left(\mathbf{X}^{DGPS}_{P_1^{\langle 1\rangle}}, \mathbf{V}^{DGPS}_{\mathbf{X}_1^{\langle 1\rangle}} \right) \cdot \mathbf{P}_{\mathbf{X}_1^{\langle 1\rangle DGPS}}$$

where: $$\widetilde{\mathbf{T}}\left(x_1^{\langle 1\rangle}, \mathbf{V}_{x_1^{\langle 1\rangle}}\right) = \mathfrak{I}(x_1^{\langle 1\rangle})\mathbf{T}\left(\mathbf{V}_{x_1^{\langle 1\rangle}}\right),$$

$$\widetilde{\mathbf{T}}\left(\mathbf{X}_{P_1^{\langle 1\rangle}}^{DGPS}, \mathbf{V}_{\mathbf{X}_1^{\langle 1\rangle}}^{DGPS}\right) = \mathfrak{I}(\mathbf{X}_{P_1^{\langle 1\rangle}}^{DGPS})\mathbf{T}\left(\mathbf{V}_{\mathbf{X}_1^{\langle 1\rangle}}^{DGPS}\right)$$

as a result of making adjustment in accordance with the rules presented in this paper and binding for *IANS* element determined in this way, the following estimators were obtained:

$$\hat{\mathbf{X}}^{\langle 1\rangle} = \mathbf{X}^0 + \hat{\mathbf{d}}_{\mathbf{x}_1^{\langle 1\rangle}} = \begin{bmatrix} 1400 \\ 800 \\ 200 \\ 100 \\ 800 \\ 100 \end{bmatrix} + \begin{bmatrix} 0,0 \\ 0,0 \\ -1,2 \\ 0,5 \\ -1,3 \\ 1,0 \end{bmatrix} = \begin{bmatrix} 1400,0 \\ 800,0 \\ 198,8 \\ 100,5 \\ 798,7 \\ 101,0 \end{bmatrix}_{[m]} = \begin{bmatrix} \hat{X}_{P_1}^{\langle 1\rangle} \\ \hat{Y}_{P_1}^{\langle 1\rangle} \\ \hat{X}_{R_1} \\ \hat{Y}_{R_1} \\ \hat{X}_{R_2} \\ \hat{Y}_{R_2} \end{bmatrix} \to \mathbf{V}_{\mathbf{x}_1^{\langle 1\rangle}} = \begin{bmatrix} -0,1^\circ \\ 0,1_{[m]} \\ 0,0^\circ \\ 0,0_{[m]} \\ 0,0^\circ \\ 0,0_{[m]} \\ 0,0_{[m]} \\ 0,0_{[m]} \end{bmatrix} \begin{matrix} \left.\right\}\mathbf{V}_{Z_1} \\ \left.\right\}\mathbf{V}_{R_1}^{\langle 1\rangle} \\ \left.\right\}\mathbf{V}_{R_2}^{\langle 1\rangle} \\ \left.\right\}\mathbf{V}_{\mathbf{X}_1^{\langle 1\rangle}}^{DGPS} \end{matrix}$$

Let's settle an interval, acceptable for standardized corrections $\bar{v} : \Delta\bar{v} = \langle -2,0;2,0 \rangle$ (the interval corresponds with the confidence level $\gamma = 0,95$ generally assumed in navigation), and let's also determine $\bar{v}_1 = -0,54$, $\bar{v}_2 = 0,0$, $\bar{v}_3 = 0,0$, $\bar{v}_4 = 0,0$, $\bar{v}_5 = 0,0$, $\bar{v}_6 = 0,0$, $\bar{v}_7 = -0,5$, $\bar{v}_8 = 0,2$, where: $\bar{v}_i = \dfrac{v_i}{\sqrt{\left[\hat{\mathbf{C}}_{\mathrm{V}}\right]_{ii}}}$; $v_i = \left[\mathbf{V}_{x_1^{\langle 1\rangle}}\right]_i$.

It appears that $\forall \bar{v}_i : \bar{v}_i \in \Delta\bar{v}$, thus it means that none of the simulated observations was gross error biased (in compliance with the assumptions).

On carrying out simulation of such situation, in which one of the observations is out-lying, let's assume that a bearing toward Z_1 (the first observation) is of $N_{1,1} = 210,0^\circ$ value ($N_{1,1} = 196,0^\circ$ before, gross error was 14º). On solving the adjustment task once more time, we obtain the standardized corrections' values. Corrections $\bar{v}_1, \bar{v}_7, \bar{v}_8$ are not covered by the accepted interval $\Delta\bar{v}$. Therefore the adjustment process is to be continued applying the decisive–attenuation function (we accept Danish function as the attenuation function). The above process is of iterative character (Table 6.4).

Table 6.4. Robust estimation results at $P_1^{\langle 1\rangle}$ position (course of iterative process)

Step "0"		Step "1"		Step "2"		Step "3"		Step "4"	
Values of standardized corrections									
$\bar{v}_1$	**-140,1**	$\bar{v}_1$	**-36,7**	$\bar{v}_1$	**-4,6**	$\bar{v}_1$	**-2,1**	$\bar{v}_1$	-1,9
$\bar{v}_2$	0,0	$\bar{v}_2$	-0,1	$\bar{v}_2$	-0,1	$\bar{v}_2$	0,1	$\bar{v}_2$	0,0
$\bar{v}_3$	0,0	$\bar{v}_3$	0,0	$\bar{v}_3$	0,0	$\bar{v}_3$	0,0	$\bar{v}_3$	0,0
$\bar{v}_4$	0,0	$\bar{v}_4$	0,0	$\bar{v}_4$	0,0	$\bar{v}_4$	0,0	$\bar{v}_4$	0,0
$\bar{v}_5$	0,0	$\bar{v}_5$	0,0	$\bar{v}_5$	0,0	$\bar{v}_5$	0,0	$\bar{v}_5$	0,0
$\bar{v}_6$	0,0	$\bar{v}_6$	0,0	$\bar{v}_6$	0,0	$\bar{v}_6$	0,0	$\bar{v}_6$	0,0
$\bar{v}_7$	**-137,8**	$\bar{v}_7$	**-30,2**	$\bar{v}_7$	-0,8	$\bar{v}_7$	-0,2	$\bar{v}_7$	0,1
$\bar{v}_8$	**57,37**	$\bar{v}_8$	**4,4**	$\bar{v}_8$	0,2	$\bar{v}_8$	0,1	$\bar{v}_8$	0,1
Parameters of the attenuation function									
1	1,0	1	1,0	1	1,0	1	1,0	1	--
g	0,2	g	0,4	g	0,5	g	0,6	g	--
Values of the decisive-attenuation function									
$\tilde{t}(\bar{v}_1)$	0,1	$\tilde{t}(\bar{v}_1)$	0,1	$\tilde{t}(\bar{v}_1)$	0,2	$\tilde{t}(\bar{v}_1)$	0,8	$\tilde{t}(\bar{v}_1)$	1
$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{v}_2)$	1
$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{v}_3)$	1
$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\bar{v}_4)$	1
$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\bar{v}_5)$	1
$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{v}_6)$	1
$\tilde{t}(\bar{v}_7)$	0,1	$\tilde{t}(\bar{v}_7)$	0,1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{v}_7)$	1
$\tilde{t}(\bar{v}_8)$	0,1	$\tilde{t}(\bar{v}_8)$	0,1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{v}_8)$	1

The final results of the adjustment carried out on vessel $\langle 1\rangle$ are of the following values:

$$\hat{\mathbf{X}}^{\langle 1\rangle}=\mathbf{X}^0+\hat{\mathbf{d}}_{\mathbf{x}_1^{\langle 1\rangle}}=\begin{bmatrix}1400\\800\\200\\100\\800\\100\end{bmatrix}+\begin{bmatrix}0,0\\0,0\\-1,2\\0,5\\-1,3\\1,0\end{bmatrix}\begin{matrix}\big\}\hat{\mathbf{d}}_{\mathbf{x}_{P_1^{\langle 1\rangle}}}\\\big\}\hat{\mathbf{d}}^{\langle 1\rangle}_{\mathbf{x}_{R_1}}\\\big\}\hat{\mathbf{d}}^{\langle 1\rangle}_{\mathbf{x}_{R_2}}\end{matrix}=\begin{bmatrix}1400,0\\800,0\\198,8\\100,5\\798,7\\101,0\end{bmatrix}_{[m]}=\begin{bmatrix}\hat{X}_{P_1}^{\langle 1\rangle}\\\hat{Y}_{P_1}^{\langle 1\rangle}\\\hat{X}_{R_1}\\\hat{Y}_{R_1}\\\hat{X}_{R_2}\\\hat{Y}_{R_2}\end{bmatrix}\rightarrow\mathbf{V}_{\mathbf{x}_1^{\langle 1\rangle}}=\begin{bmatrix}14,1^\circ\\0,1\,[m]\\0,0^\circ\\0,0\,[m]\\0,0^\circ\\0,0\,[m]\\-0,1\,[m]\\0,0\,[m]\end{bmatrix}$$

According to the assumptions made in 4.3, the adjusted coordinates of the proper position $\left(P_1^{\langle 1\rangle}\right)$ and the determined ones of the $\Re$ set elements coordinates have been transferred by radio to vessel$\langle 2\rangle$. The weights matrixes were included into the information transferred as well (to simplify the into transfer, the relations between the adjusted coordinates of various points were neglected):

$$\mathbf{P}_{\hat{\mathbf{x}}_{P_1}^{\langle 1\rangle}} = \begin{bmatrix} 0{,}73 & 0{,}03 \\ 0{,}03 & 3{,}34 \end{bmatrix}, \mathbf{P}_{\hat{\mathbf{x}}_{R}^{\langle 1\rangle}} = \begin{bmatrix} 0{,}23 & 0{,}03 & \vdots & 0 & 0 \\ 0{,}03 & 0{,}19 & \vdots & 0 & 0 \\ \cdots & \cdots & \vdots & \cdots & \cdots \\ 0 & 0 & \vdots & 0{,}33 & -0{,}07 \\ 0 & 0 & \vdots & -0{,}07 & 0{,}31 \end{bmatrix} = \begin{bmatrix} \mathbf{P}_{\hat{\mathbf{x}}_{R_1}^{\langle 1\rangle}} & \vdots & \mathbf{0} \\ \cdots & \vdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{P}_{\hat{\mathbf{x}}_{R_2}^{\langle 1\rangle}} \end{bmatrix}$$

At the I-st stage, on vessel$\langle 2\rangle$, the following observational equations system can be formulated:

$$\bar{\boldsymbol{x}}_1^{\langle 2\rangle} = \mathbf{F}_{P_1}^{\langle 2\rangle}(\mathbf{X}_{P_1}^{\langle 2\rangle}, \mathbf{X}_{R_1}, \mathbf{X}_{R_2}) = \mathbf{F}_{P_1}^{\langle 2\rangle}(\mathbf{X})$$

where $\mathbf{X}_{P_1}^{\langle 2\rangle}$ is true coordinates of vessel $\langle 2\rangle$ on P_1 - position , $\mathbf{X}_{R_1}, \mathbf{X}_{R_2}$ is true coordinates of points $\left(R_1, R_2\right) \in \Re$, however $\bar{\boldsymbol{x}}_1^{\langle 2\rangle} = \left[\underbrace{\overline{N}_{1,2}}_{\bar{x}_{Z_1}}, \underbrace{\overline{N}R_{1,1}^{\langle 2\rangle}, \overline{N}R_{2,1}^{\langle 2\rangle}}_{\bar{x}_R^T}, \underbrace{\bar{d}_1^{W\langle 2\rangle}}_{\bar{x}_W} \right]^T$ is the measured quantities vector ($\overline{N}_{i,j}$ - *j*-th true bearing toward the *i*-th navigational sign, $\overline{N}R_{i,j}^{\langle 2\rangle}$ - *j*-th true bearing toward the *i*-th object R taken from vessel $\langle 2\rangle$, $\bar{d}_1^{W\langle 2\rangle}$ - distance between chips).

Moreover, taking into consideration the adjustment results ($\hat{\mathbf{d}}_{\mathbf{X}_{P_1^{\langle 1\rangle}}}, \hat{\mathbf{d}}_{\mathbf{X}_{R_1}}^{\langle 1\rangle}, \hat{\mathbf{d}}_{\mathbf{X}_{R_2}}^{\langle 1\rangle}$ with respective weights matrixes), transferred from vessel $\langle 1\rangle$ and results of GPS survey carried out on vessel $\langle 2\rangle$, also the route reckoning, for this watercraft we obtain the following corrections equation system:

$$\begin{cases} \mathbf{V}_{Z_1} = \mathbf{A}_{Z_1,P_1^{\langle 2\rangle}}\hat{\mathbf{d}}_{\mathbf{X}_{P_1^{\langle 2\rangle}}} + \mathbf{A}_{Z_1,R_1}\hat{\mathbf{d}}_{\mathbf{X}_{R_1}}^{\langle 2\rangle} + \mathbf{A}_{Z_1,R_2}\hat{\mathbf{d}}_{\mathbf{X}_{R_2}}^{\langle 2\rangle} + \mathbf{A}_{Z_1,P_1^{\langle 1\rangle}}\hat{\mathbf{d}}_{\mathbf{X}_{P_1^{\langle 1\rangle}}}^{\langle 2\rangle} + \mathbf{L}_{Z_1} \\ \mathbf{V}_{R_1}^{\langle 2\rangle} = \mathbf{A}_{R_1,P_1^{\langle 2\rangle}}\hat{\mathbf{d}}_{\mathbf{X}_{P_1^{\langle 2\rangle}}} + \mathbf{A}_{R_1,R_1}\hat{\mathbf{d}}_{\mathbf{X}_{R_1}}^{\langle 2\rangle} + \mathbf{A}_{R_1,R_2}\hat{\mathbf{d}}_{\mathbf{X}_{R_2}}^{\langle 2\rangle} + \mathbf{A}_{R_1,P_1^{\langle 1\rangle}}\hat{\mathbf{d}}_{\mathbf{X}_{P_1^{\langle 1\rangle}}}^{\langle 2\rangle} + \mathbf{L}_{R_1} \\ \mathbf{V}_{R_2}^{\langle 2\rangle} = \mathbf{A}_{R_2,P_1^{\langle 2\rangle}}\hat{\mathbf{d}}_{\mathbf{X}_{P_1^{\langle 2\rangle}}} + \mathbf{A}_{R_2,R_1}\hat{\mathbf{d}}_{\mathbf{X}_{R_1}}^{\langle 2\rangle} + \mathbf{A}_{R_2,R_2}\hat{\mathbf{d}}_{\mathbf{X}_{R_2}}^{\langle 2\rangle} + \mathbf{A}_{R_2,P_1^{\langle 1\rangle}}\hat{\mathbf{d}}_{\mathbf{X}_{P_1^{\langle 1\rangle}}}^{\langle 2\rangle} + \mathbf{L}_{R_2} \\ \mathbf{V}_{W_1}^{\langle 2\rangle} = \mathbf{A}_{W_1,P_1^{\langle 2\rangle}}\hat{\mathbf{d}}_{\mathbf{X}_{P_1^{\langle 2\rangle}}} + \mathbf{A}_{W_1,R_1}\hat{\mathbf{d}}_{\mathbf{X}_{R_1}}^{\langle 2\rangle} + \mathbf{A}_{W_1,R_2}\hat{\mathbf{d}}_{\mathbf{X}_{R_2}}^{\langle 2\rangle} + \mathbf{A}_{W_1,P_1^{\langle 1\rangle}}\hat{\mathbf{d}}_{\mathbf{X}_{P_1^{\langle 1\rangle}}}^{\langle 2\rangle} + \mathbf{L}_{W_1} \\ \overset{DGPS}{\mathbf{V}}_{\mathbf{X}_1^{\langle 2\rangle}} = \hat{\mathbf{d}}_{\mathbf{X}_{P_1^{\langle 2\rangle}}} + \left(\mathbf{X}_{P_1^{\langle 2\rangle}}^0 - \overset{DGPS}{\mathbf{X}}_{P_1^{\langle 2\rangle}}\right) \\ \mathbf{v}_{R_1}^{\langle 2\rangle} = \hat{\mathbf{d}}_{\mathbf{X}_{R_1}}^{\langle 2\rangle} - \hat{\mathbf{d}}_{\mathbf{X}_{R_1}}^{\langle 1\rangle} \\ \mathbf{v}_{R_2}^{\langle 2\rangle} = \hat{\mathbf{d}}_{\mathbf{X}_{R_2}}^{\langle 2\rangle} - \hat{\mathbf{d}}_{\mathbf{X}_{R_2}}^{\langle 1\rangle} \\ \mathbf{v}_{P_1^{\langle 1\rangle}}^{\langle 2\rangle} = \hat{\mathbf{d}}_{\mathbf{X}_{P_1^{\langle 1\rangle}}}^{\langle 2\rangle} - \hat{\mathbf{d}}_{\mathbf{X}_{P_1^{\langle 1\rangle}}} \end{cases}$$

By determining for vessel $\langle 2\rangle$ the decisive matrixes, corresponding with the survey structure we obtain:

$$\Im(\boldsymbol{x}_{z_1}) = 1,\ \Im(\boldsymbol{x}_{R_1}^{\langle 2\rangle}) = 1,\ \Im(\boldsymbol{x}_{R_2}^{\langle 2\rangle}) = 1,\ \Im(\boldsymbol{x}_{W_1}^{\langle 2\rangle}) = 1 \rightarrow$$

$$\rightarrow \Im(\boldsymbol{x}_1^{\langle 2\rangle}) = Diag\left\{\Im(\boldsymbol{x}_{z_1}), \Im(\boldsymbol{x}_{R_1}^{\langle 2\rangle}), \Im(\boldsymbol{x}_{R_2}^{\langle 2\rangle}), \Im(\boldsymbol{x}_{W_1}^{\langle 2\rangle})\right\} = \mathbf{I}_{(4)}$$

$$\Im(\overset{DGPS}{\mathbf{X}}_{P_1^{\langle 2\rangle}}) = \mathbf{I}_{(2)},\ \Im(\mathbf{X}_{R_1}^{\langle 2\rangle}) = \mathbf{I}_{(2)},\ \Im(\mathbf{X}_{R_2}^{\langle 2\rangle}) = \mathbf{I}_{(2)}$$

We assume, that position of vessel $\langle 1\rangle$ is not to be corrected (justified from navigation standpoint). Thus, let $\Im(\mathbf{X}_{P_1}^{\langle 1\rangle}) = \mathbf{0}$, what means that the last line of the corrections equations system in further determinations becomes “ignored”.

With the covariance matrix model applied for the presented functional model and the decisive matrixes:

$$\tilde{\tilde{\mathbf{C}}}_1^{\langle 2\rangle} = \sigma_0^2\left(\tilde{\tilde{\mathbf{P}}}_1^{\langle 2\rangle}\right)^{-1},\ \tilde{\tilde{\mathbf{P}}}_1^{\langle 2\rangle} = Diag\left(\tilde{\tilde{\mathbf{P}}}_{\mathbf{x}_1^{\langle 2\rangle}}, \tilde{\tilde{\mathbf{P}}}_{\mathbf{X}_1^{\langle 2\rangle GPS}}\right)$$

we obtain following solution of adjustment task (with reference to the observations presented in Table 6.2.):

$$\mathbf{V}_{\mathbf{x}_1^{\langle 2\rangle}} = \begin{bmatrix} \mathbf{V}_{Z_1} \\ \mathbf{V}_{R_1}^{\langle 2\rangle} \\ \mathbf{V}_{R_2}^{\langle 2\rangle} \\ \mathbf{V}_{W}^{\langle 2\rangle} \\ \cdots \\ \overset{DGPS}{\mathbf{V}}_{\mathbf{x}_1^{\langle 2\rangle}} \\ \cdots \\ \mathbf{v}_{R_1}^{\langle 2\rangle} \\ \cdots \\ \mathbf{v}_{R_2}^{\langle 2\rangle} \end{bmatrix} = \begin{bmatrix} -0{,}1 \\ 0{,}1 \\ 0{,}1 \\ 0{,}0 \\ \cdots \\ 0{,}0 \\ 0{,}0 \\ \cdots \\ -0{,}8 \\ 1{,}2 \\ \cdots \\ -1{,}5 \\ 0{,}4 \end{bmatrix}, \ \hat{\mathbf{d}}_{\mathbf{x}_1^{\langle 2\rangle}} = \begin{bmatrix} \hat{\mathbf{d}}_{\mathbf{X}_{P_1}^{\langle 2\rangle}} \\ \hat{\mathbf{d}}_{\mathbf{X}_{R_1}}^{\langle 2\rangle} \\ \hat{\mathbf{d}}_{\mathbf{X}_{R_2}}^{\langle 2\rangle} \end{bmatrix} = \begin{bmatrix} 0{,}0 \\ 0{,}0 \\ -2{,}1 \\ 1{,}7 \\ -2{,}9 \\ 1{,}4 \end{bmatrix}, \ \hat{\mathbf{X}}_{P_1}^{\langle 2\rangle} = \begin{bmatrix} 1100{,}0 \\ 800{,}0 \\ 196{,}7 \\ 102{,}2 \\ 795{,}9 \\ 102{,}5 \end{bmatrix} = \begin{bmatrix} \hat{X}_{P_1}^{\langle 2\rangle} \\ \hat{Y}_{P_1}^{\langle 2\rangle} \\ \hat{X}_{R_1} \\ \hat{Y}_{R_1} \\ \hat{X}_{R_2} \\ \hat{Y}_{R_2} \end{bmatrix}$$

While limits of the interval, acceptable for random corrections, remain in force, we may state, that $\forall \bar{v}_i : \bar{v}_i \in \Delta\bar{v} = \langle -2;2 \rangle$, and $\bar{v}_1 = -1{,}3$, $\bar{v}_2 = 0{,}8$, $\bar{v}_3 = 1{,}2$, $\bar{v}_4 = 0{,}0$, $\bar{v}_5 = 0{,}2$, $\bar{v}_6 = -1{,}7$, $\bar{v}_7 = -0{,}8$, $\bar{v}_8 = 0{,}8$, $\bar{v}_9 = -1{,}2$, $\bar{v}_{10} = 1{,}2$.

The same as for the previous set of observations (vessel $\langle 1 \rangle$), let us assume now another variant of observations carried out on vessel $\langle 2 \rangle$, in which the bearing toward point Z_1 (the first observation) is 7º gross error biased. Such disturbed bearing is of the value $N_{1,2} = 165{,}0^\circ$ ($N_{1,2} = 172{,}0^\circ$ before). When the adjustment is being carried out again, we obtain standardized corrections of the values presented in Table 6.5. (in individual iterative steps).

Table 6.5. Results of robust estimation - position $P_1^{\langle 2 \rangle}$ (course of iterative process)

Step"0"		Step"1"		Step"2"		Step"3"		Step"4"	
Values of standardized corrections									
$\bar{v}_1$	**68,5**	$\bar{v}_1$	**24,1**	$\bar{v}_1$	**10,4**	$\bar{v}_1$	**4,8**	$\bar{v}_1$	1,9
$\bar{v}_2$	0,7	$\bar{v}_2$	0,8	$\bar{v}_2$	0,8	$\bar{v}_2$	0,8	$\bar{v}_2$	0,8
$\bar{v}_3$	0,9	$\bar{v}_3$	1,1	$\bar{v}_3$	1,1	$\bar{v}_3$	1,1	$\bar{v}_3$	1,1
$\bar{v}_4$	-1,1	$\bar{v}_4$	-0,4	$\bar{v}_4$	-0,2	$\bar{v}_4$	-0,1	$\bar{v}_4$	-0,1
$\bar{v}_5$	**24,7**	$\bar{v}_5$	**3,4**	$\bar{v}_5$	0,8	$\bar{v}_5$	0,4	$\bar{v}_5$	0,3
$\bar{v}_6$	**21,2**	$\bar{v}_6$	1,6	$\bar{v}_6$	-0,7	$\bar{v}_6$	-1,2	$\bar{v}_6$	-1,3
$\bar{v}_7$	-0,7	$\bar{v}_7$	0,8	$\bar{v}_7$	-0,8	$\bar{v}_7$	-0,8	$\bar{v}_7$	0,8
$\bar{v}_8$	0,7	$\bar{v}_8$	0,8	$\bar{v}_8$	0,8	$\bar{v}_8$	0,8	$\bar{v}_8$	0,8
$\bar{v}_9$	-1,0	$\bar{v}_9$	1,5	$\bar{v}_9$	-1,1	$\bar{v}_9$	-1,1	$\bar{v}_9$	-1,1
$\bar{v}_{10}$	1,3	$\bar{v}_{10}$	1,1	$\bar{v}_{10}$	1,2	$\bar{v}_{10}$	1,1	$\bar{v}_{10}$	1,1
Parameters of the attenuation function									
1	0,6	1	0,6	1	1,0	1	1,0	1	--
g	0,2	g	0,2	g	0,2	g	0,6	g	--
Values of the decision-attenuation function									
$\tilde{t}(\bar{v}_1)$	0,1	$\tilde{t}(\bar{v}_1)$	0,2	$\tilde{t}(\bar{v}_1)$	0,2	$\tilde{t}(\bar{v}_1)$	0,2	$\tilde{t}(\bar{v}_1)$	1
$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{v}_2)$	1
$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{v}_3)$	1
$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\bar{v}_4)$	1
$\tilde{t}(\bar{v}_5)$	0,3	$\tilde{t}(\bar{v}_5)$	0,6	$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\bar{v}_5)$	1
$\tilde{t}(\bar{v}_6)$	0,4	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{v}_6)$	1
$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{v}_7)$	1
$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{v}_8)$	1
$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{v}_9)$	1
$\tilde{t}(\bar{v}_{10})$	1	$\tilde{t}(\bar{v}_{10})$	1	$\tilde{t}(\bar{v}_{10})$	1	$\tilde{t}(\bar{v}_{10})$	1	$\tilde{t}(\bar{v}_{10})$	1

The final results of the robust, sequential adjustment, introduced on vessel $\langle 2 \rangle$ are as follows:

$$\mathbf{V}_{\mathbf{x}_1^{\langle 2\rangle}} = \begin{bmatrix} 6{,}8 \\ -0{,}1 \\ 0{,}1 \\ 0{,}1 \\ 0{,}0 \\ 0{,}0 \\ -0{,}7 \\ 1{,}1 \\ -1{,}4 \\ 0{,}6 \end{bmatrix} \begin{matrix} \left.\begin{matrix} \\ \end{matrix}\right\} \mathbf{V}_{Z_1} \\ \rightarrow \mathbf{V}_{R_1}^{\langle 2\rangle} \\ \rightarrow \mathbf{V}_{R_2}^{\langle 2\rangle} \\ \left.\begin{matrix} \\ \end{matrix}\right\} \mathbf{V}_{\mathbf{x}_1^{\langle 2\rangle}}^{DGPS} \\ \left.\begin{matrix} \\ \end{matrix}\right\} \mathbf{v}_{R_1}^{\langle 2\rangle} \\ \left.\begin{matrix} \\ \end{matrix}\right\} \mathbf{v}_{R_2}^{\langle 2\rangle} \end{matrix}, \quad \hat{\mathbf{d}}_{\mathbf{x}_1^{\langle 2\rangle}} = \begin{bmatrix} 0{,}0 \\ -0{,}1 \\ -2{,}0 \\ 1{,}6 \\ -2{,}8 \\ 1{,}7 \end{bmatrix} \quad \rightarrow \quad \hat{\mathbf{X}}_{P_1}^{\langle 2\rangle} = \begin{bmatrix} 1100{,}0 \\ 799{,}9 \\ 196{,}8 \\ 102{,}1 \\ 795{,}9 \\ 102{,}7 \end{bmatrix} = \begin{bmatrix} \hat{X}_{P_1}^{\langle 2\rangle} \\ \hat{Y}_{P_1}^{\langle 2\rangle} \\ \hat{X}_{R_1} \\ \hat{Y}_{R_1} \\ \hat{X}_{R_2} \\ \hat{Y}_{R_2} \end{bmatrix}$$

STAGE II

While navigation was proceeding, the vessels reached position $P_2^{\langle 1\rangle}$ and $P_2^{\langle 2\rangle}$ (position coordinates are presented in Table 6.6). The second stage (marked in Fig. 6.1. green) consists in correcting the determinations obtained in stage I based on new observations included to elements of the set $\mathfrak{Z}$, $\mathfrak{R}$ and on the grounds of DGPS set $\overset{DGPS}{\mathfrak{Z}}$. Therefore, apart from determining DGPS position on the both vessels, there have been carried out observations toward the navigational sign and $\mathfrak{R}$ objects under observation. There has also been taken measurement of the distance from vessel $\langle 2\rangle$ to vessel $\langle 1\rangle$.

Results of those observations are specified in Table 6.7.

Table 6.6. Positions of the vessels accepted for computations at stage II

Positions of the vessels	Cartesian coordinates	
	$\overset{DGPS}{X}$	$\overset{DGPS}{Y}$
$P_2^{\langle 1\rangle}$	700 [m]	800 [m]
$P_2^{\langle 2\rangle}$	400 [m]	800 [m]

Table 6.7. Observations obtained at stage II

Positions of the vessels	Observations
$P_2^{\langle 1\rangle}$	$N_{1,3} = 145{,}0^\circ$
	$NR_{1,3}^{\langle 1\rangle} = 215{,}5^\circ$
	$NR_{2,3}^{\langle 2\rangle} = 172{,}0^\circ$
$P_2^{\langle 2\rangle}$	$N_{1,4} = 131{,}0^\circ$
	$NR_{1,4}^{\langle 2\rangle} = 196{,}0^\circ$
	$NR_{2,4}^{\langle 2\rangle} = 150{,}0^\circ$
	$d_2^{w\langle 2\rangle} = 300$ [m]

We assume that the same as at stage I, the bearing toward the point Z_1 are gross error biased. The simulated survey results are specified in Table 6.7., while their gross error biased values, equal to 15º and 14º are $N_{1,3} = 130{,}0^\circ$ and $N_{1,4} = 145{,}0^\circ$ respectively. With the computations carried out in accordance with principles suggested in this work (developing *IANS* by two vessels) and applying the decisive-attenuation function, the same way as at stage I, the following final results were obtained:

$$\underbrace{\hat{\mathbf{X}}_{P_2}^{\langle 1\rangle} = \begin{bmatrix} 699{,}8 \\ 800{,}0 \\ 194{,}9 \\ 103{,}7 \\ 793{,}2 \\ 104{,}4 \end{bmatrix}_{[\mathrm{m}]}, \mathbf{P}_{\hat{\mathbf{x}}_{R_1}^{\langle 1\rangle}} = \begin{bmatrix} 0{,}25 & -0{,}03 \\ -0{,}03 & 0{,}24 \end{bmatrix}, \mathbf{P}_{\hat{\mathbf{x}}_{R_2}^{\langle 1\rangle}} = \begin{bmatrix} 0{,}72 & 0{,}03 \\ 0{,}03 & 0{,}25 \end{bmatrix},}_{\text{Information transferred to vessel } \langle 2\rangle \text{ at stage}}$$

$$\underbrace{\hat{\mathbf{X}}_{P_2}^{\langle 2\rangle} = \begin{bmatrix} 400{,}1 \\ 800{,}3 \\ 192{,}3 \\ 104{,}7 \\ 790{,}5 \\ 103{,}2 \end{bmatrix}_{[\mathrm{m}]} = \begin{bmatrix} \hat{X}_{P_2}^{\langle 2\rangle} \\ \hat{Y}_{P_2}^{\langle 2\rangle} \\ \hat{X}_{R_1} \\ \hat{Y}_{R_1} \\ \hat{X}_{R_2} \\ \hat{Y}_{R_2} \end{bmatrix}}_{\text{Final results of the adjustment}}$$

In this case, an assessment of the final determinations' accuracies is limited to a parameter, the most often applied in navigation. The parameter is the position error $M = \sqrt{\sigma_{\hat{X}}^2 + \sigma_{\hat{Y}}^2}$. The assessment shall be described for the consideration variants presented before, with the following designations of the positions errors put up with:

- M^{clean} - variant without the out-laying observations;
- M^{D-R} - the set includes out-lying observations, the adjustment was carried out applying the solutions suggested in this work;
- $M^{classic}$ - the set includes out-lying observations, the adjustment was carried out with the classic least squares method.

The positions' errors, determined at each of *IANS* developing stages, are presented in Table 6.8.

Table 6.8. Errors of determination of the vessels proper positions and the new *IANS* elements

Stage	Vessel	Point	M^{clean}	M^{D-R}	$M^{classic}$
I	$\langle 1 \rangle$	$P_1^{\langle 1 \rangle}$	0,1	2,7	1407,9
		R_1	1,9	0,9	64,2
		R_2	1,6	1,1	79,0
	$\langle 2 \rangle$	$P_1^{\langle 2 \rangle}$	0,1	0,3	4,8
		R_1	2,4	3,1	86,6
		R_2	2,1	2,5	75,9
II	$\langle 1 \rangle$	$P_2^{\langle 1 \rangle}$	0,1	2,3	11,4
		R_1	2,5	2,9	235,1
		R_2	2,0	2,4	188,4
	$\langle 2 \rangle$	$P_2^{\langle 2 \rangle}$	0,2	2,3	9,6
		R_1	3,8	3,0	163,6
		R_2	3,3	4,2	141,1

One may state that position errors, both of the proper and the $\Re$ points, determined on the basis of the observations unbiased with gross errors as well as for the error biased observations but adjustment in conformity with the principles suggested in the paper are within the same interval of values. Adjustment of the observations sets, comprising out-lying observations, applying the classic least squares method only, results in positions errors of a number of times higher values.

6.2. Navigation with Free Adjustment Implementation

Let us assume that a singular vessel is sailing along the sea coast and surveying its positions P_k, taking advantage of on-shore signs. While taking P_0 position it was found out, that there was a necessity of making use of three points, included in $\Re$, of which the adjusted coordinates $\hat{\mathbf{X}}_R$ of weights matrix $\mathbf{P}_{\hat{\mathbf{X}}_R}$ were determined before, in *IANS* developing process. Having in mind a risk of misidentification of any of $\Re$ set points or a risk of biasing such point's coordinates with gross errors, the observational system adjustment process was carried out, applying the hybrid M - estimation, as advised in Chapter 6. Such an adjustment enables also, if necessary, correcting the set $\Re$ adjustment points existing coordinates and minimization of an observation gross errors influence on the final determinations. In addition, it was also assumed that the positions P_1 and P_2 would be adjusted commonly at the second position, taken by the ship. At the moment of achieving the position P_1 by the ship, five observations toward three elements of $\Re$ set were surveyed (bearings and distances), and after travelling a certain route distance (position P_2), the next five observations, also towards all three elements of the same set were measured as well. The described navigating situation is presented in Fig. 6.2. The reckoned positions of the vessel and also the simulated survey results are specified in Tables 6.9. and 6.10. respectively. To demonstrate various possibilities of applying hybrid *M*-estimation in maritime navigation, as advised in this paper, the task shall be resolved in four different variants:

- variant I: observations and coordinates are not gross errors biased;
- variant II: one observation is gross error biased;
- variant III: two observations are gross error biased;
- variant IV: the gross error biased is one observation and coordinates of one of the set $\Re$ points.

Table 6.9. Reckoned positions of the vessel

Subsequent positions	Cartesian coordinates	
	X^0	Y^0
P_1	1550 **[m]**	800 **[m]**
P_2	500 **[m]**	800 **[m]**

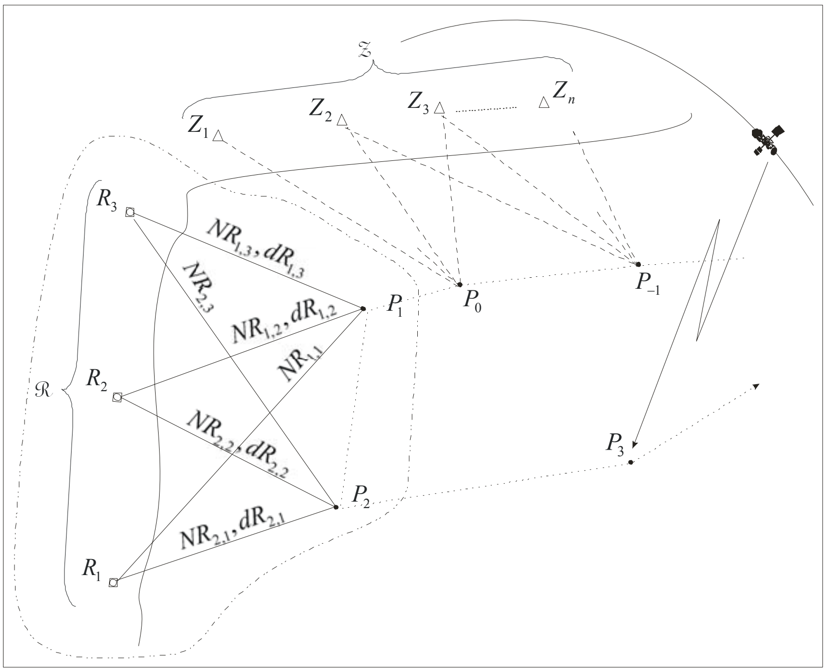

Fig. 6.2. Trajectory of the simulated ship motion

Table 6.10. Survey results

Vessels positions	Observation type	Navigational sign	Observation value
P_1	bearing	R_1	205,6º
		R_2	237,5º
		R_3	302,6º
	distance	R_2	832,8 [m]
		R_3	832,0 [m]
P_2	bearing	R_1	240,4º
		R_2	310,4º
		R_3	335,0º
	distance	R_1	806,7 [m]
		R_2	921,2 [m]

Coordinates of the points covered by the set $\Re$ are specified in Table 6.11.

Table 6.11. The adjusted coordinates of the $\Re$ set points

Sign name	Cartesian coordinates	
	$\hat{X}$	$\hat{Y}$
R_1	100 **[m]**	100 **[m]**
R_2	1100 **[m]**	100 **[m]**
R_3	2000 **[m]**	100 **[m]**

For the variants of the entire task there was assumed that the mean error of each of the bearings is $m_{NR} = 0{,}15^\circ$, whereas the mean error of each of the distance survey results is $m_d = 2{,}0$ [m].

Now let's make an assumption that coordinates of the $\Re$ set points were adjusted before, what means that the values are mutually dependent (the weights matrix $\mathbf{P}_{\hat{\mathbf{X}}_R}$ is not a diagonal one). For the test purpose let's assume also that the above dependence is expressed by the correlation coefficient $\rho_{\hat{X},\hat{Y}} = 0{,}25$. In addition, let's suppose that the mean errors (standard deviations estimators) of coordinates of the set $\Re$ points are of the following common values: $m_{\hat{X}_\Re} = m_{\hat{Y}_\Re} = 0{,}1$ [m].

The assumed values enable determination of the following weights matrix $\mathbf{P}_{\hat{\mathbf{X}}_R}$ of the $\Re$ set points' coordinates (to simplify the problem, relations between the coordinates of different points may be omitted – hence $\mathbf{P}_{\hat{\mathbf{X}}_R}$ is a quasi diagonal matrix):

$$\mathbf{P}_{\hat{\mathbf{X}}_R} = \begin{bmatrix} \mathbf{P}_{\hat{\mathbf{X}}_{R_1}} & \vdots & \mathbf{0} & \vdots & \mathbf{0} \\ \cdots & \vdots & \cdots & \vdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{R_2}} & \vdots & \mathbf{0} \\ \cdots & \vdots & \cdots & \vdots & \cdots \\ \mathbf{0} & \vdots & \mathbf{0} & \vdots & \mathbf{P}_{\hat{\mathbf{X}}_{R_3}} \end{bmatrix} =$$

$$= \begin{bmatrix} m^2_{\hat{X}_{R_1}} & \mathrm{cov}\left(\hat{X}_{R_1}, \hat{Y}_{R_1}\right) & \vdots & 0 & 0 & \vdots & 0 & 0 \\ \mathrm{cov}\left(\hat{Y}_{R_1}, \hat{X}_{R_1}\right) & m^2_{\hat{Y}_{R_1}} & \vdots & 0 & 0 & \vdots & 0 & 0 \\ \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots \\ 0 & 0 & \vdots & m^2_{\hat{X}_{R_2}} & \mathrm{cov}\left(\hat{X}_{R_2}, \hat{Y}_{R_2}\right) & \vdots & 0 & 0 \\ 0 & 0 & \vdots & \mathrm{cov}\left(\hat{Y}_{R_2}, \hat{X}_{R_2}\right) & m^2_{\hat{Y}_{R_2}} & \vdots & 0 & 0 \\ \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots \\ 0 & 0 & \vdots & 0 & 0 & \vdots & m^2_{\hat{X}_{R_3}} & \mathrm{cov}\left(\hat{X}_{R_3}, \hat{Y}_{R_3}\right) \\ 0 & 0 & \vdots & 0 & 0 & \vdots & \mathrm{cov}\left(\hat{Y}_{R_3}, \hat{X}_{R_3}\right) & m^2_{\hat{Y}_{R_3}} \end{bmatrix}^{-1} =$$

$$= \begin{bmatrix} 0{,}01 & 0{,}002 & \vdots & 0 & 0 & \vdots & 0 & 0 \\ 0{,}002 & 0{,}01 & \vdots & 0 & 0 & \vdots & 0 & 0 \\ \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots \\ 0 & 0 & \vdots & 0{,}01 & 0{,}002 & \vdots & 0 & 0 \\ 0 & 0 & \vdots & 0{,}002 & 0{,}01 & \vdots & 0 & 0 \\ \cdots & \cdots & \vdots & \cdots & \cdots & \vdots & \cdots & \cdots \\ 0 & 0 & \vdots & 0 & 0 & \vdots & 0{,}01 & 0{,}002 \\ 0 & 0 & \vdots & 0 & 0 & \vdots & 0{,}002 & 0{,}01 \end{bmatrix}^{-1} =$$

$$= \begin{bmatrix} 104{,}2 & -20{,}8 & 0 & 0 & 0 & 0 \\ -20{,}8 & 104{,}2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 104{,}2 & -20{,}8 & 0 & 0 \\ 0 & 0 & -20{,}8 & 104{,}2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 104{,}2 & -20{,}8 \\ 0 & 0 & 0 & 0 & -20{,}8 & 104{,}2 \end{bmatrix}$$

Moreover, taking into consideration that the mean errors of the reckoned positions coordinates (mutually independent) are of the values: $m_{X_{\wp}} = m_{Y_{\wp}} = 1\,[\mathrm{m}]$, one may finally set the following weights matrix:

$$\mathbf{P_X}=\begin{bmatrix}\mathbf{P}_{\hat{\mathbf{X}}_R} & \vdots & \\ \cdots & \vdots & \cdots \\ & \vdots & \mathbf{P}_{\mathbf{X}_P}\end{bmatrix}=\begin{bmatrix}106{,}7 & -26{,}7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ & 106{,}7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ & & 106{,}7 & -26{,}7 & 0 & 0 & 0 & 0 & 0 & 0\\ & & & 106{,}7 & 0 & 0 & 0 & 0 & 0 & 0\\ & & & & 106{,}7 & -26{,}7 & 0 & 0 & 0 & 0\\ & & & & & 106{,}7 & 0 & 0 & 0 & 0\\ & & & \mathit{symetry} & & & 1 & 0 & 0 & 0\\ & & & & & & & 1 & 0 & 0\\ & & & & & & & & 1 & 0\\ & & & & & & & & & 1\end{bmatrix}$$

The following functional model corresponds with the geometrical navigational structure, accepted in this example:

$$\begin{matrix}\hat{\boldsymbol{x}}_{\boldsymbol{Z_1}}=\mathbf{F}_{Z_1}(\hat{\boldsymbol{\Gamma}})\rightarrow\\ \hat{\boldsymbol{x}}_{\boldsymbol{Z_2}}=\mathbf{F}_{Z_2}(\hat{\boldsymbol{\Gamma}})\rightarrow\end{matrix}\begin{cases}\mathbf{v}_{Z_1}=\mathbf{A}_{Z_1R}\hat{\mathbf{d}}_{\hat{\mathbf{X}}_R}+\mathbf{A}_{Z_1P_1}\hat{\mathbf{d}}_{\mathbf{X}_{P_1}} & +\mathbf{L}_{Z_1}\\ \mathbf{v}_{Z_2}=\mathbf{A}_{Z_2R}\hat{\mathbf{d}}_{\hat{\mathbf{X}}_R} & +\mathbf{A}_{Z_2P_2}\hat{\mathbf{d}}_{\mathbf{X}_{P_2}}+\mathbf{L}_{Z_2}\end{cases}\Leftrightarrow\mathbf{V}=\mathbf{A}\hat{\mathbf{d}}_{\mathbf{X}}+\mathbf{L}$$

where:

$$\boldsymbol{x}_{\boldsymbol{Z_1}}=\left[\overline{NR}_{1,1},\overline{NR}_{1,2},\overline{dR}_{1,2},\overline{NR}_{1,3},\overline{dR}_{1,3}\right]^T,\ \boldsymbol{x}_{\boldsymbol{Z_2}}=\left[\overline{NR}_{2,1},\overline{dR}_{2,1},\overline{NR}_{2,2},\overline{dR}_{2,2},\overline{NR}_{2,3}\right]^T$$

and

$$\hat{\boldsymbol{\Gamma}}=\left[\hat{\mathbf{X}}_{R_1}^T,\hat{\mathbf{X}}_{R_2}^T,\hat{\mathbf{X}}_{R_3}^T,\hat{\mathbf{X}}_{P_1}^T,\hat{\mathbf{X}}_{P_2}^T\right]^T,\ \mathbf{V}=\begin{bmatrix}\mathbf{v}_{Z_1}\\ \mathbf{v}_{Z_2}\end{bmatrix},\ \mathbf{A}=\begin{bmatrix}\mathbf{A}_{Z_kR} & \mathbf{A}_{Z_kP_k} & \mathbf{0}\\ \mathbf{A}_{Z_{k+1}R} & \mathbf{0} & \mathbf{A}_{Z_{k+1}P_{k+1}}\end{bmatrix},$$

$$\hat{\mathbf{d}}_{\mathbf{X}}=\begin{bmatrix}\hat{\mathbf{d}}_{\hat{\mathbf{X}}_R}\\ \hat{\mathbf{d}}_{\mathbf{X}_{P_1}}\\ \hat{\mathbf{d}}_{\mathbf{X}_{P_2}}\end{bmatrix},\ \mathbf{L}=\begin{bmatrix}\mathbf{L}_{Z_k}\\ \mathbf{L}_{Z_{k+1}}\end{bmatrix}.$$

Due to the assumption regarding mutual independence of direct observations, the decisive –equivalent weights matrix of those values is of the form as follows:

$$\tilde{\tilde{\mathbf{P}}}_{\boldsymbol{x}}=\tilde{\mathbf{T}}(\boldsymbol{x},\mathbf{V})\mathbf{P}_{\boldsymbol{x}}=\Im(\boldsymbol{x})\mathbf{T}(\mathbf{V})\mathbf{P}_{\boldsymbol{x}}$$

where the weights matrix, a priori, stands for the following matrix:

$$\mathbf{P}_{\boldsymbol{x}}=Diag(m_{NR_{1,1}}^{-2},m_{NR_{1,2}}^{-2},m_{dR_{1,2}}^{-2},m_{NR_{1,3}}^{-2},m_{dR_{1,3}}^{-2},m_{NR_{2,1}}^{-2},m_{dR_{2,1}}^{-2},m_{NR_{2,2}}^{-2},m_{dR_{2,2}}^{-2},m_{NR_{2,3}}^{-2}=$$
$$=Diag\left(44{,}4\quad 44{,}4\quad 4\quad 44{,}4\quad 4\quad 44{,}4\quad 4\quad 44{,}4\quad 4\quad 44{,}4\right)$$

The decisive-equivalent weights matrix of the points $\mathfrak{R}$ and $\mathfrak{P}$ coordinates is of the following form:

$$\tilde{\tilde{\mathbf{P}}}_{\mathbf{X}} = \tilde{\mathbf{T}}_{sqr}(\mathbf{\Gamma}^0, \hat{\mathbf{d}}_{\mathbf{X}})\mathbf{P}_{\mathbf{X}}\tilde{\mathbf{T}}_{sqr}(\mathbf{\Gamma}^0, \hat{\mathbf{d}}_{\mathbf{X}})$$

where $$\tilde{\mathbf{T}}_{sqr}(\mathbf{\Gamma}^0, \hat{\mathbf{d}}_{\mathbf{X}}) = \mathfrak{I}(\mathbf{\Gamma}^0)\mathbf{T}_{sqr}(\hat{\mathbf{d}}_{\mathbf{X}})$$

(the weights matrix was a priori $\mathbf{P}_{\mathbf{X}}$ determined before).

As in test 1, the interval, admissible for the standardized corrections: $\Delta\bar{v} = \langle -2,0;2,0 \rangle$ was accepted. The same interval was also accepted for random, standardized increments $\Delta\bar{\hat{d}} = \langle -2,0;2,0 \rangle$. The computations were carried out in compliance with the principles, presented in Chapter 5.

Variant I. Adjustment was connected with the set of observations, unbiased with gross errors. (Table 6.10). The basic result of the adjustment is vector

$$\hat{\mathbf{d}}_{\mathbf{X}} = \left[\overbrace{\underbrace{2{,}5;\, 0{,}3}_{R_1};\underbrace{-0{,}1;-0{,}6}_{R_2};\underbrace{-2{,}4;\, 0{,}3}_{R_3}}^{\hat{\mathbf{d}}_{\mathbf{x}R}} \;\vdots\; \overbrace{\underbrace{-1{,}2;\, 0{,}8}_{P_1};\underbrace{2{,}6;\, 0{,}8}_{P_2}}^{\hat{\mathbf{d}}_{\mathbf{X}P}} \right]^T .$$

As a result of determining the vector's covariance matrix and carrying out standardization, the following was obtained:

$$\bar{\hat{d}}_{X_{R_1}} = 1{,}4,\ \bar{\hat{d}}_{Y_{R_1}} = 0{,}3,\ \bar{\hat{d}}_{X_{R_2}} = -0{,}1,\ \bar{\hat{d}}_{Y_{R_2}} = -0{,}7,\ \bar{\hat{d}}_{X_{R_3}} = -1{,}4,\ \bar{\hat{d}}_{Y_{R_3}} = 0{,}3,$$
$$\bar{\hat{d}}_{X_{P_1}} = -0{,}7,\ \bar{\hat{d}}_{Y_{P_1}} = 0{,}6,\ \bar{\hat{d}}_{X_{P_2}} = 1{,}4,\ \bar{\hat{d}}_{Y_{P_2}} = 0{,}5$$

So it became proved that $\forall \bar{\hat{d}}_X, \bar{\hat{d}}_Y : \left(\bar{\hat{d}}_X \in \Delta\bar{\hat{d}}\right) \wedge \left(\bar{\hat{d}}_Y \in \Delta\bar{\hat{d}}\right)$. Thus, it seems that there is no grounds for a statement that any of coordinates were gross error biased (according with the assumptions). In navigating practice it would indicate, for example, that identification of new adjustment points (the points covered by $\mathfrak{R}$) was carried out properly. However, the standardized values of corrections are as follows:

$$\bar{v}_1 = 1{,}7,\ \bar{v}_2 = -1{,}6,\ \bar{v}_3 = -0{,}1,\ \bar{v}_4 = 0{,}8,\ \bar{v}_5 = -0{,}8,$$

$$\bar{v}_6 = -1{,}7,\ \bar{v}_7 = -1{,}7,\ \bar{v}_8 = 0{,}3,\ \bar{v}_9 = 1{,}3,\ \bar{v}_{10} = -0{,}8,$$

As $\forall v : v \in \Delta\bar{v}$, there is also no grounds for statement, that any of the observations is an out-lying one. Therefore $\hat{\mathbf{d}}_{\mathbf{X}}$ estimator, indicated before, is a final solution for the adjustment task, thus:

$$\hat{\mathbf{X}} = \mathbf{X}^0 + \hat{\mathbf{d}}_{\mathbf{X}} = \begin{bmatrix} 100 \\ 100 \\ 1100 \\ 100 \\ 2000 \\ 100 \\ 1550 \\ 800 \\ 500 \\ 800 \end{bmatrix} + \begin{bmatrix} 2,5 \\ 0,3 \\ -0,1 \\ -0,6 \\ -2,4 \\ 0,3 \\ -1,2 \\ 0,8 \\ 2,6 \\ 0,8 \end{bmatrix} = \begin{bmatrix} 102,5 \\ 100,3 \\ 1099,9 \\ 99,4 \\ 1997,6 \\ 100,3 \\ 1549,8 \\ 800,8 \\ 502,6 \\ 800,8 \end{bmatrix}_{[m]} = \begin{bmatrix} \hat{\hat{X}}_{R_1} \\ \hat{\hat{Y}}_{R_1} \\ \hat{\hat{X}}_{R_2} \\ \hat{\hat{Y}}_{R_2} \\ \hat{\hat{X}}_{R_3} \\ \hat{\hat{Y}}_{R_3} \\ \hat{X}_{P_1} \\ \hat{Y}_{P_1} \\ \hat{X}_{P_2} \\ \hat{Y}_{P_2} \end{bmatrix}$$

and

$$\mathbf{V} = \begin{bmatrix} 0,2^\circ & -0,1^\circ & -0,1[\mathrm{m}] & 0,1^\circ & -0,1[\mathrm{m}] & -0,1^\circ & -0,1[\mathrm{m}] & 0,1^\circ & 0,1[\mathrm{m}] & -0,1^\circ \end{bmatrix}^T .$$

In addition, on determining the position errors, (basing on the covariance matrix $\hat{\mathbf{C}}_{\hat{\mathbf{X}}}$) the following was obtained:

$$M_{R_1} = 2,8\,[\mathrm{m}],\ M_{R_2} = 2,0\,[\mathrm{m}],\ M_{R_3} = 2,7\,[\mathrm{m}],\ M_{P_1} = 3,0\,[\mathrm{m}],\ M_{P_2} = 3,1\,[\mathrm{m}]$$

Variant II. Let us assume that the true bearing, taken at P_2 towards R_3 (the tenth observation) is gross error biased and of $NR_{2,3} = 335,8^\circ$ value (whereas before it was $NR_{2,3} = 335,0^\circ$). On resolving the adjustment task in this particular case, the values of standardized corrections and increments obtained, were not ranged within the intervals acceptable therefor. So the adjustment task resolving process was at this point of an iterative character. Results of the gross errors identifying step (Step "0"), as well as a course of the entire process, with implementation of the Danish attenuation function (both for the coordinates' weights and observations' weights) are related to in Tables below.

Table 6.12.a The iterative process course

Step "0"				Step "1"				Step "2"			
Values of corrections and standardized increments											
$\bar{v}_1$	2,0	$\bar{\hat{d}}_{X_{R_1}}$	-0,4	$\bar{v}_1$	1,9	$\bar{\hat{d}}_{X_{R_1}}$	0,0	$\bar{v}_1$	1,8	$\bar{\hat{d}}_{X_{R_1}}$	0,5
$\bar{v}_2$	-1,3	$\bar{\hat{d}}_{Y_{R_1}}$	-1,8	$\bar{v}_2$	-1,6	$\bar{\hat{d}}_{Y_{R_1}}$	-1,0	$\bar{v}_2$	-1,6	$\bar{\hat{d}}_{Y_{R_1}}$	-0,3
$\bar{v}_3$	**3,6**	$\bar{\hat{d}}_{X_{R_2}}$	0,2	$\bar{v}_3$	1,5	$\bar{\hat{d}}_{X_{R_2}}$	-0,2	$\bar{v}_3$	-0,1	$\bar{\hat{d}}_{X_{R_2}}$	-0,6
$\bar{v}_4$	**5,1**	$\bar{\hat{d}}_{Y_{R_2}}$	-0,5	$\bar{v}_4$	**3,5**	$\bar{\hat{d}}_{Y_{R_2}}$	-0,7	$\bar{v}_4$	**2,2**	$\bar{\hat{d}}_{Y_{R_2}}$	-0,9
$\bar{v}_5$	**-5,1**	$\bar{\hat{d}}_{X_{R_3}}$	0,3	$\bar{v}_5$	**-3,5**	$\bar{\hat{d}}_{X_{R_3}}$	0,1	$\bar{v}_5$	**-2,2**	$\bar{\hat{d}}_{X_{R_3}}$	0,0
$\bar{v}_6$	-20	$\bar{\hat{d}}_{Y_{R_3}}$	**2,3**	$\bar{v}_6$	-1,9	$\bar{\hat{d}}_{Y_{R_3}}$	1,5	$\bar{v}_6$	-1,8	$\bar{\hat{d}}_{Y_{R_3}}$	0,8
$\bar{v}_7$	**-2,1**	$\bar{\hat{d}}_{X_{P_1}}$	-0,6	$\bar{v}_7$	-1,9	$\bar{\hat{d}}_{X_{P_1}}$	-0,7	$\bar{v}_7$	-1,8	$\bar{\hat{d}}_{X_{P_1}}$	-0,9
$\bar{v}_8$	**4,4**	$\bar{\hat{d}}_{Y_{P_1}}$	0,9	$\bar{v}_8$	**2,5**	$\bar{\hat{d}}_{Y_{P_1}}$	0,5	$\bar{v}_8$	0,8	$\bar{\hat{d}}_{Y_{P_1}}$	0,1
$\bar{v}_9$	0,0	$\bar{\hat{d}}_{X_{P_2}}$	-0,2	$\bar{v}_9$	0,1	$\bar{\hat{d}}_{X_{P_2}}$	0,1	$\bar{v}_9$	1,4	$\bar{\hat{d}}_{X_{P_2}}$	0,5
$\bar{v}_{10}$	**-5,1**	$\bar{\hat{d}}_{Y_{P_2}}$	-1,2	$\bar{v}_{10}$	**-3,5**	$\bar{\hat{d}}_{Y_{P_2}}$	-0,6	$\bar{v}_{10}$	**-2,2**	$\bar{\hat{d}}_{Y_{P_2}}$	0,1
Attenuation function parameters											
l	0,3	l	0,3	l	0,7	l	--	l	1,4	l	--
g	2,0	g	2,0	g	2,0	g	--	g	2,0	g	--
Decisive-attenuation function values											
$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1	$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1	$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1
$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1
$\tilde{t}(\bar{v}_3)$	0,8	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1
$\tilde{t}(\bar{v}_4)$	0,4	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	1	$\tilde{t}(\bar{v}_4)$	0,4	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	1	$\tilde{t}(\bar{v}_4)$	0,9	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	1
$\tilde{t}(\bar{v}_5)$	0,4	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1	$\tilde{t}(\bar{v}_5)$	0,4	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1	$\tilde{t}(\bar{v}_5)$	0,9	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1
$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	0,9	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	1
$\tilde{t}(\bar{v}_7)$	0,9	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1
$\tilde{t}(\bar{v}_8)$	0,6	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1	$\tilde{t}(\bar{v}_8)$	0,9	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1
$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1
$\tilde{t}(\bar{v}_{10})$	0,4	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1	$\tilde{t}(\bar{v}_{10})$	0,4	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1	$\tilde{t}(\bar{v}_{10})$	0,9	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1

Table 6.12.b The iterative process course

Step "3"				Step "4"				Step "5"			
Values of corrections and standardized increments											
$\bar{v}_1$	1,8	$\bar{\hat{d}}_{X_{R_1}}$	0,5	$\bar{v}_1$	1,8	$\bar{\hat{d}}_{X_{R_1}}$	0,5	$\bar{v}_1$	1,8	$\bar{\hat{d}}_{X_{R_1}}$	0,5
$\bar{v}_2$	-1,6	$\bar{\hat{d}}_{Y_{R_1}}$	-0,3	$\bar{v}_2$	-1,6	$\bar{\hat{d}}_{Y_{R_1}}$	-0,3	$\bar{v}_2$	-1,6	$\bar{\hat{d}}_{Y_{R_1}}$	-0,3
$\bar{v}_3$	-0,2	$\bar{\hat{d}}_{X_{R_2}}$	-0,6	$\bar{v}_3$	-0,2	$\bar{\hat{d}}_{X_{R_2}}$	-0,5	$\bar{v}_3$	-0,2	$\bar{\hat{d}}_{X_{R_2}}$	-0,5
$\bar{v}_4$	**2,1**	$\bar{\hat{d}}_{Y_{R_2}}$	-0,9	$\bar{v}_4$	2,0	$\bar{\hat{d}}_{Y_{R_2}}$	-0,9	$\bar{v}_4$	2,0	$\bar{\hat{d}}_{Y_{R_2}}$	-0,9
$\bar{v}_5$	**2,1**	$\bar{\hat{d}}_{X_{R_3}}$	0,0	$\bar{v}_5$	2,0	$\bar{\hat{d}}_{X_{R_3}}$	-0,1	$\bar{v}_5$	2,0	$\bar{\hat{d}}_{X_{R_3}}$	0,0
$\bar{v}_6$	-1,8	$\bar{\hat{d}}_{Y_{R_3}}$	0,8	$\bar{v}_6$	-1,8	$\bar{\hat{d}}_{Y_{R_3}}$	0,8	$\bar{v}_6$	-1,8	$\bar{\hat{d}}_{Y_{R_3}}$	0,8
$\bar{v}_7$	-1,8	$\bar{\hat{d}}_{X_{P_1}}$	-1,0	$\bar{v}_7$	-1,8	$\bar{\hat{d}}_{X_{P_1}}$	-0,9	$\bar{v}_7$	-1,8	$\bar{\hat{d}}_{X_{P_1}}$	-0,9
$\bar{v}_8$	0,7	$\bar{\hat{d}}_{Y_{P_1}}$	0,1	$\bar{v}_8$	0,6	$\bar{\hat{d}}_{Y_{P_1}}$	0,1	$\bar{v}_8$	0,6	$\bar{\hat{d}}_{Y_{P_1}}$	-0,1
$\bar{v}_9$	1,4	$\bar{\hat{d}}_{X_{P_2}}$	0,5	$\bar{v}_9$	1,4	$\bar{\hat{d}}_{X_{P_2}}$	0,5	$\bar{v}_9$	1,4	$\bar{\hat{d}}_{X_{P_2}}$	0,5
$\bar{v}_{10}$	**-2,1**	$\bar{\hat{d}}_{Y_{P_2}}$	0,0	$\bar{v}_{10}$	-2,1	$\bar{\hat{d}}_{Y_{P_2}}$	0,0	$\bar{v}_{10}$	-2,0	$\bar{\hat{d}}_{Y_{P_2}}$	0,0
Attenuation function parameters											
l	2,1	l	--	l	0,03	l	--	l	--	l	--
g	2,0	g	--	g	1	g	--	g	--	g	--
Decisive-attenuation function values											
$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1	$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1	$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1
$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1
$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1
$\tilde{t}(\bar{v}_4)$	0,9	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	1	$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	1	$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	1
$\tilde{t}(\bar{v}_5)$	0,9	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1	$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1	$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1
$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	1
$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1
$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1
$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1
$\tilde{t}(\bar{v}_{10})$	0,9	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1	$\tilde{t}(\bar{v}_{10})$	0,9	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1	$\tilde{t}(\bar{v}_{10})$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1

Finally, on finishing the iterative process (in step “5”), the following was obtained $\left\{\forall v : v \in \Delta\bar{v}\right\} \Rightarrow \left\{\tilde{\mathbf{T}}(\mathbf{V}) = \mathbf{I}_{(10)}\right\}$ and $\left\{\forall \bar{\hat{d}}_X, \bar{\hat{d}}_Y : \bar{\hat{d}}_X, \bar{\hat{d}}_Y \in \Delta\bar{\hat{d}}\right\} \Rightarrow \left\{\tilde{\mathbf{T}}\left(\bar{\hat{d}}_{\mathbf{X}}\right) = \mathbf{I}_{(10)}\right\}$. Thus:

$$\hat{\mathbf{X}} = \mathbf{X}^0 + \hat{\mathbf{d}}_{\mathbf{X}} = \begin{bmatrix} 100 \\ 100 \\ 1100 \\ 100 \\ 2000 \\ 100 \\ 1550 \\ 800 \\ 500 \\ 800 \end{bmatrix} + \begin{bmatrix} 1,2 \\ -0,5 \\ -1,1 \\ -1,3 \\ -0,1 \\ 1,8 \\ -2,2 \\ 0,2 \\ 1,3 \\ -0,1 \end{bmatrix} = \begin{bmatrix} 101,2 \\ 99,5 \\ 1098,9 \\ 98,7 \\ 1999,9 \\ 101,8 \\ 1547,8 \\ 800,2 \\ 501,3 \\ 799,9 \end{bmatrix}_{[\mathrm{m}]} = \begin{bmatrix} \hat{\hat{X}}_{R_1} \\ \hat{\hat{Y}}_{R_1} \\ \hat{\hat{X}}_{R_2} \\ \hat{\hat{Y}}_{R_2} \\ \hat{\hat{X}}_{R_3} \\ \hat{\hat{Y}}_{R_3} \\ \hat{X}_{P_1} \\ \hat{Y}_{P_1} \\ \hat{X}_{P_2} \\ \hat{Y}_{P_2} \end{bmatrix}$$

In variant II of determinations, the position errors, determined on the basis of the covariance $\hat{\mathbf{C}}_{\hat{\mathbf{X}}}$ matrix are as follows, respectively:

$$M_{R_1} = 5{,}6\,[\mathrm{m}],\ M_{R_2} = 4{,}5\,[\mathrm{m}],\ M_{R_3} = 7{,}4\,[\mathrm{m}],\ M_{P_1} = 5{,}5[\mathrm{m}],\ M_{P_2} = 5{,}9[\mathrm{m}]$$

Variant III. Two observations are the error biased: $NR_{2,3} = 335{,}8^\circ$ (the tenth observation, the same as in variant II) and $NR_{1,3} = 302{,}8^\circ$ (the fourth observation, $NR_{1,3} = 302{,}5^\circ$). For this test variant the iterative process outcomes are presented in Table 6.13.

Table 6.13.a The iterative process course

Step “0”				Step “1”				Step “2”			
Values of corrections and standardized increments											
$\bar{v}_1$	2,0	$\bar{\hat{d}}_{X_{R_1}}$	-0,7	$\bar{v}_1$	1,9	$\bar{\hat{d}}_{X_{R_1}}$	-0,2	$\bar{v}_1$	1,8	$\bar{\hat{d}}_{X_{R_1}}$	0,1
$\bar{v}_2$	-1,3	$\bar{\hat{d}}_{Y_{R_1}}$	-2,0	$\bar{v}_2$	-1,6	$\bar{\hat{d}}_{Y_{R_1}}$	-1,0	$\bar{v}_2$	-1,6	$\bar{\hat{d}}_{Y_{R_1}}$	-0,7
$\bar{v}_3$	**3,1**	$\bar{\hat{d}}_{X_{R_2}}$	-0,7	$\bar{v}_3$	1,0	$\bar{\hat{d}}_{X_{R_2}}$	-0,9	$\bar{v}_3$	0,3	$\bar{\hat{d}}_{X_{R_2}}$	-1,0
$\bar{v}_4$	**4,5**	$\bar{\hat{d}}_{Y_{R_2}}$	-1,2	$\bar{v}_4$	**2,9**	$\bar{\hat{d}}_{Y_{R_2}}$	-1,1	$\bar{v}_4$	**2,3**	$\bar{\hat{d}}_{Y_{R_2}}$	-1,2
$\bar{v}_5$	**-4,5**	$\bar{\hat{d}}_{X_{R_3}}$	1,2	$\bar{v}_5$	**-2,9**	$\bar{\hat{d}}_{X_{R_3}}$	0,8	$\bar{v}_5$	**-2,3**	$\bar{\hat{d}}_{X_{R_3}}$	0,6
$\bar{v}_6$	-2,0	$\bar{\hat{d}}_{Y_{R_3}}$	**3,1**	$\bar{v}_6$	-1,9	$\bar{\hat{d}}_{Y_{R_3}}$	2,0	$\bar{v}_6$	-1,8	$\bar{\hat{d}}_{Y_{R_3}}$	1,6
$\bar{v}_7$	-2,0	$\bar{\hat{d}}_{X_{P_1}}$	-1,2	$\bar{v}_7$	-1,9	$\bar{\hat{d}}_{X_{P_1}}$	-1,2	$\bar{v}_7$	-1,8	$\bar{\hat{d}}_{X_{P_1}}$	-1,3
$\bar{v}_8$	**3,8**	$\bar{\hat{d}}_{Y_{P_1}}$	0,4	$\bar{v}_8$	1,9	$\bar{\hat{d}}_{Y_{P_1}}$	0,2	$\bar{v}_8$	1,1	$\bar{\hat{d}}_{Y_{P_1}}$	0,1
$\bar{v}_9$	0,2	$\bar{\hat{d}}_{X_{P_2}}$	-0,5	$\bar{v}_9$	1,0	$\bar{\hat{d}}_{X_{P_2}}$	-0,1	$\bar{v}_9$	1,3	$\bar{\hat{d}}_{X_{P_2}}$	0,1
$\bar{v}_{10}$	**-4,5**	$\bar{\hat{d}}_{Y_{P_2}}$	-1,4	$\bar{v}_{10}$	**-2,9**	$\bar{\hat{d}}_{Y_{P_2}}$	-0,6	$\bar{v}_{10}$	**-2,3**	$\bar{\hat{d}}_{Y_{P_2}}$	-0,3
Attenuation function parameters											
l	0,4	l	0,5	l	0,8	l	--	l	1,5	l	--
g	20	g	2,0	g	2,0	g	--	g	2,0	g	--
Decisive-attenuation function values											
$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1	$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1	$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1
$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1
$\tilde{t}(\bar{v}_3)$	0,8	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1
$\tilde{t}(\bar{v}_4)$	0,4	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	1	$\tilde{t}(\bar{v}_4)$	0,6	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	1	$\tilde{t}(\bar{v}_4)$	0,8	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	1
$\tilde{t}(\bar{v}_5)$	0,4	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1	$\tilde{t}(\bar{v}_5)$	0,6	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1	$\tilde{t}(\bar{v}_5)$	0,8	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1
$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	0,7	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	1
$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1
$\tilde{t}(\bar{v}_8)$	0,6	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1
$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1
$\tilde{t}(\bar{v}_{10})$	0,5	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1	$\tilde{t}(\bar{v}_{10})$	0,6	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1	$\tilde{t}(\bar{v}_{10})$	0,1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1

Table 6.13.b The iterative process course

Step „3”				Step „4”				Step „5”			
Values of corrections and standardized increments											
$\bar{v}_1$	1,8	$\bar{\hat{d}}_{X_{R_1}}$	0,1	$\bar{v}_1$	1,7	$\bar{\hat{d}}_{X_{R_1}}$	0,1	$\bar{v}_1$	1,8	$\bar{\hat{d}}_{X_{R_1}}$	0,1
$\bar{v}_2$	-1,6	$\bar{\hat{d}}_{Y_{R_1}}$	-0,6	$\bar{v}_2$	-1,6	$\bar{\hat{d}}_{Y_{R_1}}$	-0,61	$\bar{v}_2$	-1,6	$\bar{\hat{d}}_{Y_{R_1}}$	-0,5
$\bar{v}_3$	0,1	$\bar{\hat{d}}_{X_{R_2}}$	-1,0	$\bar{v}_3$	0,0	$\bar{\hat{d}}_{X_{R_2}}$	-1,0	$\bar{v}_3$	-0,1	$\bar{\hat{d}}_{X_{R_2}}$	-1,0
$\bar{v}_4$	**2,1**	$\bar{\hat{d}}_{Y_{R_2}}$	-1,2	$\bar{v}_4$	**2,1**	$\bar{\hat{d}}_{Y_{R_2}}$	1,2	$\bar{v}_4$	2,0	$\bar{\hat{d}}_{Y_{R_2}}$	-1,2
$\bar{v}_5$	**-2,0**	$\bar{\hat{d}}_{X_{R_3}}$	0,6	$\bar{v}_5$	-2,0	$\bar{\hat{d}}_{X_{R_3}}$	0,5	$\bar{v}_5$	-2,0	$\bar{\hat{d}}_{X_{R_3}}$	0,5
$\bar{v}_6$	-1,8	$\bar{\hat{d}}_{Y_{R_3}}$	1,5	$\bar{v}_6$	-1,8	$\bar{\hat{d}}_{Y_{R_3}}$	1,5	$\bar{v}_6$	-1,8	$\bar{\hat{d}}_{Y_{R_3}}$	1,4
$\bar{v}_7$	-1,8	$\bar{\hat{d}}_{X_{P_1}}$	-1,3	$\bar{v}_7$	-1,8	$\bar{\hat{d}}_{X_{P_1}}$	-1,3	$\bar{v}_7$	-1,8	$\bar{\hat{d}}_{X_{P_1}}$	-1,3
$\bar{v}_8$	0,8	$\bar{\hat{d}}_{Y_{P_1}}$	0,0	$\bar{v}_8$	0,8	$\bar{\hat{d}}_{Y_{P_1}}$	0,0	$\bar{v}_8$	0,8	$\bar{\hat{d}}_{Y_{P_1}}$	0,0
$\bar{v}_9$	1,4	$\bar{\hat{d}}_{X_{P_2}}$	0,2	$\bar{v}_9$	1,4	$\bar{\hat{d}}_{X_{P_2}}$	0,2	$\bar{v}_9$	1,4	$\bar{\hat{d}}_{X_{P_2}}$	0,2
$\bar{v}_{10}$	**-2,1**	$\bar{\hat{d}}_{Y_{P_2}}$	-0,2	$\bar{v}_{10}$	**-2,1**	$\bar{\hat{d}}_{Y_{P_2}}$	-0,2	$\bar{v}_{10}$	-2,0	$\bar{\hat{d}}_{Y_{P_2}}$	-0,2
Attenuation function parameters											
l	1,6	l	--	l	1,6	l	--	l	--	l	--
g	2,0	g	--	g	2,0	g	--	g	--	g	--
Decisive-attenuation function values											
$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1	$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1	$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1
$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1
$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1
$\tilde{t}(\bar{v}_4)$	0,9	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	1	$\tilde{t}(\bar{v}_4)$	0,9	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	1	$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	1
$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1	$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1	$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1
$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	1
$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1
$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1
$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1
$\tilde{t}(\bar{v}_{10})$	0,9	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1	$\tilde{t}(\bar{v}_{10})$	0,9	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1	$\tilde{t}(\bar{v}_{10})$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1

In result of the iterative process (in Step "5") the following was finally obtained $\left\{\forall v : v \in \Delta\overline{v}\right\} \Rightarrow \left\{\widetilde{\mathbf{T}}(\mathbf{V}) = \mathbf{I}_{(10)}\right\}$ and $\left\{\forall \overline{\hat{d}}_X, \overline{\hat{d}}_Y : \overline{\hat{d}}_X, \overline{\hat{d}}_Y \in \Delta\overline{\hat{d}}\right\} \Rightarrow \left\{\widetilde{\mathbf{T}}\left(\overline{\hat{d}}_{\mathbf{X}}\right) = \mathbf{I}_{(10)}\right\}$.

Thus:

$$\hat{\mathbf{X}} = \mathbf{X}^0 + \hat{\mathbf{d}}_{\mathbf{X}} = \begin{bmatrix} 100 \\ 100 \\ 1100 \\ 100 \\ 2000 \\ 100 \\ 1550 \\ 800 \\ 500 \\ 800 \end{bmatrix} + \begin{bmatrix} 0,3 \\ -0,8 \\ -1,9 \\ -1,5 \\ 1,7 \\ 3,2 \\ -3,0 \\ -0,1 \\ 0,4 \\ 0,4 \end{bmatrix} = \begin{bmatrix} 100,3 \\ 99,2 \\ 1098,1 \\ 98,5 \\ 1998,3 \\ 103,2 \\ 1547,0 \\ 799,9 \\ 500,4 \\ 800,4 \end{bmatrix}_{[\mathrm{m}]} = \begin{bmatrix} \hat{\hat{X}}_{R_1} \\ \hat{\hat{Y}}_{R_1} \\ \hat{\hat{X}}_{R_2} \\ \hat{\hat{Y}}_{R_2} \\ \hat{\hat{X}}_{R_3} \\ \hat{\hat{Y}}_{R_3} \\ \hat{X}_{P_1} \\ \hat{Y}_{P_1} \\ \hat{X}_{P_2} \\ \hat{Y}_{P_2} \end{bmatrix}$$

In variant III of determinations, the position errors, determined on the basis of the covariance $\hat{\mathbf{C}}_{\hat{\mathbf{X}}}$ matrix, are, respectively, as follows:

$$M_{R_1} = 5,2\,[\mathrm{m}],\; M_{R_2} = 4,1[\mathrm{m}],\; M_{R_3} = 6,9[\mathrm{m}],\; M_{P_1} = 5,3[\mathrm{m}],\; M_{P_2} = 5,5[\mathrm{m}]$$

Variant IV. The error biased observation is again $NR_{2,3} = 335,8^\circ$. However this time, there were assumed incorrect coordinates of sign R_2: $\hat{X}_{R_2} = 1104,5\,[\mathrm{m}]$, $\hat{Y}_{R_2} = 103\,[\mathrm{m}]$ (previously: $\hat{X}_{R_2} = 1100\,[\mathrm{m}]$, $\hat{Y}_{R_2} = 100\,[\mathrm{m}]$). The iterative process results for IV test variant are specified in Table 6.14.

Table 6.14.a The iterative process course

Step "0"				Step "1"				Step "2"			
Values of corrections and standardized increments											
$\overline{v}_1$	2,0	$\overline{\hat{d}}_{X_{R_1}}$	0,5	$\overline{v}_1$	1,8	$\overline{\hat{d}}_{X_{R_1}}$	0,9	$\overline{v}_1$	1,8	$\overline{\hat{d}}_{X_{R_1}}$	0,9
$\overline{v}_2$	1,3	$\overline{\hat{d}}_{Y_{R_1}}$	-1,0	$\overline{v}_2$	-1,6	$\overline{\hat{d}}_{Y_{R_1}}$	0,0	$\overline{v}_2$	-1,6	$\overline{\hat{d}}_{Y_{R_1}}$	0,1
$\overline{v}_3$	**3,6**	$\overline{\hat{d}}_{X_{R_2}}$	**-2,3**	$\overline{v}_3$	0,3	$\overline{\hat{d}}_{X_{R_2}}$	**-2,1**	$\overline{v}_3$	0,1	$\overline{\hat{d}}_{X_{R_2}}$	**-2,1**
$\overline{v}_4$	**5,0**	$\overline{\hat{d}}_{Y_{R_2}}$	**-2,8**	$\overline{v}_4$	**2,5**	$\overline{\hat{d}}_{Y_{R_2}}$	**-2,3**	$\overline{v}_4$	**2,3**	$\overline{\hat{d}}_{Y_{R_2}}$	**-2,2**
$\overline{v}_5$	**-5,0**	$\overline{\hat{d}}_{X_{R_3}}$	1,1	$\overline{v}_5$	**-2,5**	$\overline{\hat{d}}_{X_{R_3}}$	0,6	$\overline{v}_5$	**2,3**	$\overline{\hat{d}}_{X_{R_3}}$	0,5
$\overline{v}_6$	-2,0	$\overline{\hat{d}}_{Y_{R_3}}$	**3,1**	$\overline{v}_6$	-1,8	$\overline{\hat{d}}_{Y_{R_3}}$	1,6	$\overline{v}_6$	-1,8	$\overline{\hat{d}}_{Y_{R_3}}$	1,4
$\overline{v}_7$	-2,0	$\overline{\hat{d}}_{X_{P_1}}$	0,2	$\overline{v}_7$	-1,8	$\overline{\hat{d}}_{X_{P_1}}$	-0,2	$\overline{v}_7$	-1,8	$\overline{\hat{d}}_{X_{P_1}}$	-0,2
$\overline{v}_8$	**4,4**	$\overline{\hat{d}}_{Y_{P_1}}$	1,6	$\overline{v}_8$	1,3	$\overline{\hat{d}}_{Y_{P_1}}$	0,9	$\overline{v}_8$	1,0	$\overline{\hat{d}}_{Y_{P_1}}$	0,8
$\overline{v}_9$	0,0	$\overline{\hat{d}}_{X_{P_2}}$	0,6	$\overline{v}_9$	1,3	$\overline{\hat{d}}_{X_{P_2}}$	0,9	$\overline{v}_9$	1,4	$\overline{\hat{d}}_{X_{P_2}}$	1,0
$\overline{v}_{10}$	**-5,1**	$\overline{\hat{d}}_{Y_{P_2}}$	-0,5	$\overline{v}_{10}$	**-2,6**	$\overline{\hat{d}}_{Y_{P_2}}$	0,2	$\overline{v}_{10}$	**-2,4**	$\overline{\hat{d}}_{Y_{P_2}}$	0,3
Attenuation function parameters											
l	0,4	l	0,1	l	0,8	l	0,2	l	1,2	l	1,8
g	2,0	g	2	g	2,0	g	2,0	g	2,0	g	2,0
Decisive-attenuation function values											
$\tilde{t}(\overline{v}_1)$	1	$\tilde{t}(\overline{\hat{d}}_{X_{R_1}})$	1	$\tilde{t}(\overline{v}_1)$	1	$\tilde{t}(\overline{\hat{d}}_{X_{R_1}})$	1	$\tilde{t}(\overline{v}_1)$	1	$\tilde{t}(\overline{\hat{d}}_{X_{R_1}})$	1
$\tilde{t}(\overline{v}_2)$	1	$\tilde{t}(\overline{\hat{d}}_{Y_{R_1}})$	1	$\tilde{t}(\overline{v}_2)$	1	$\tilde{t}(\overline{\hat{d}}_{Y_{R_1}})$	1	$\tilde{t}(\overline{v}_2)$	1	$\tilde{t}(\overline{\hat{d}}_{Y_{R_1}})$	1
$\tilde{t}(\overline{v}_3)$	0,7	$\tilde{t}(\overline{\hat{d}}_{X_{R_2}})$	0,9	$\tilde{t}(\overline{v}_3)$	1	$\tilde{t}(\overline{\hat{d}}_{X_{R_2}})$	0,9	$\tilde{t}(\overline{v}_3)$	1	$\tilde{t}(\overline{\hat{d}}_{X_{R_2}})$	0,9
$\tilde{t}(\overline{v}_4)$	0,3	$\tilde{t}(\overline{\hat{d}}_{Y_{R_2}})$	0,9	$\tilde{t}(\overline{v}_4)$	0,8	$\tilde{t}(\overline{\hat{d}}_{Y_{R_2}})$	0,9	$\tilde{t}(\overline{v}_4)$	0,8	$\tilde{t}(\overline{\hat{d}}_{Y_{R_2}})$	0,8
$\tilde{t}(\overline{v}_5)$	0,3	$\tilde{t}(\overline{\hat{d}}_{X_{R_3}})$	1	$\tilde{t}(\overline{v}_5)$	0,8	$\tilde{t}(\overline{\hat{d}}_{X_{R_3}})$	1	$\tilde{t}(\overline{v}_5)$	0,8	$\tilde{t}(\overline{\hat{d}}_{X_{R_3}})$	1
$\tilde{t}(\overline{v}_6)$	1	$\tilde{t}(\overline{\hat{d}}_{Y_{R_3}})$	0,9	$\tilde{t}(\overline{v}_6)$	1	$\tilde{t}(\overline{\hat{d}}_{Y_{R_3}})$	1	$\tilde{t}(\overline{v}_6)$	1	$\tilde{t}(\overline{\hat{d}}_{Y_{R_3}})$	1
$\tilde{t}(\overline{v}_7)$	1	$\tilde{t}(\overline{\hat{d}}_{X_{P_1}})$	1	$\tilde{t}(\overline{v}_7)$	1	$\tilde{t}(\overline{\hat{d}}_{X_{P_1}})$	1	$\tilde{t}(\overline{v}_7)$	1	$\tilde{t}(\overline{\hat{d}}_{X_{P_1}})$	1
$\tilde{t}(\overline{v}_8)$	0,4	$\tilde{t}(\overline{\hat{d}}_{Y_{P_1}})$	1	$\tilde{t}(\overline{v}_8)$	1	$\tilde{t}(\overline{\hat{d}}_{Y_{P_1}})$	1	$\tilde{t}(\overline{v}_8)$	1	$\tilde{t}(\overline{\hat{d}}_{Y_{P_1}})$	1
$\tilde{t}(\overline{v}_9)$	1	$\tilde{t}(\overline{\hat{d}}_{X_{P_2}})$	1	$\tilde{t}(\overline{v}_9)$	1	$\tilde{t}(\overline{\hat{d}}_{X_{P_2}})$	1	$\tilde{t}(\overline{v}_9)$	1	$\tilde{t}(\overline{\hat{d}}_{X_{P_2}})$	1
$\tilde{t}(\overline{v}_{10})$	0,6	$\tilde{t}(\overline{\hat{d}}_{Y_{P_2}})$	1	$\tilde{t}(\overline{v}_{10})$	0,8	$\tilde{t}(\overline{\hat{d}}_{Y_{P_2}})$	1	$\tilde{t}(\overline{v}_{10})$	0,8	$\tilde{t}(\overline{\hat{d}}_{Y_{P_2}})$	1

Table 6.14.b The iterative process course

Step "3"				Step "4"				Step "5"			
Values of corrections and standardized increments											
$\bar{v}_1$	1,8	$\hat{\bar{d}}_{X_{R_1}}$	1,0	$\bar{v}_1$	1,8	$\hat{\bar{d}}_{X_{R_1}}$	1,0	$\bar{v}_1$	1,7	$\hat{\bar{d}}_{X_{R_1}}$	0,9
$\bar{v}_2$	-1,6	$\hat{\bar{d}}_{Y_{R_1}}$	0,1	$\bar{v}_2$	-1,7	$\hat{\bar{d}}_{Y_{R_1}}$	0,0	$\bar{v}_2$	-1,6	$\hat{\bar{d}}_{Y_{R_1}}$	-0,1
$\bar{v}_3$	-0,1	$\hat{\bar{d}}_{X_{R_2}}$	-2,0	$\bar{v}_3$	-0,2	$\hat{\bar{d}}_{X_{R_2}}$	-1,9	$\bar{v}_3$	-0,2	$\hat{\bar{d}}_{X_{R_2}}$	-1,9
$\bar{v}_4$	2,2	$\hat{\bar{d}}_{Y_{R_2}}$	**-2,2**	$\bar{v}_4$	-2,0	$\hat{\bar{d}}_{Y_{R_2}}$	-2,2	$\bar{v}_4$	2,0	$\hat{\bar{d}}_{Y_{R_2}}$	-2,2
$\bar{v}_5$	-2,2	$\hat{\bar{d}}_{X_{R_3}}$	0,4	$\bar{v}_5$	-2,0	$\hat{\bar{d}}_{X_{R_3}}$	0,4	$\bar{v}_5$	2,0	$\hat{\bar{d}}_{X_{R_3}}$	0,4
$\bar{v}_6$	-1,8	$\hat{\bar{d}}_{Y_{R_3}}$	1,2	$\bar{v}_6$	-1,8	$\hat{\bar{d}}_{Y_{R_3}}$	1,1	$\bar{v}_6$	-1,7	$\hat{\bar{d}}_{Y_{R_3}}$	1,0
$\bar{v}_7$	-1,8	$\hat{\bar{d}}_{X_{P_1}}$	-0,3	$\bar{v}_7$	-1,8	$\hat{\bar{d}}_{X_{P_1}}$	-0,3	$\bar{v}_7$	-1,7	$\hat{\bar{d}}_{X_{P_1}}$	-0,3
$\bar{v}_8$	0,8	$\hat{\bar{d}}_{Y_{P_1}}$	0,6	$\bar{v}_8$	0,6	$\hat{\bar{d}}_{Y_{P_1}}$	0,4	$\bar{v}_8$	0,6	$\hat{\bar{d}}_{Y_{P_1}}$	0,3
$\bar{v}_9$	1,4	$\hat{\bar{d}}_{X_{P_2}}$	1,0	$\bar{v}_9$	1,4	$\hat{\bar{d}}_{X_{P_2}}$	1,0	$\bar{v}_9$	1,4	$\hat{\bar{d}}_{X_{P_2}}$	1,0
$\bar{v}_{10}$	-2,2	$\hat{\bar{d}}_{Y_{P_2}}$	0,3	$\bar{v}_{10}$	-2,1	$\hat{\bar{d}}_{Y_{P_2}}$	0,2	$\bar{v}_{10}$	-2,1	$\hat{\bar{d}}_{Y_{P_2}}$	0,1
Attenuation function parameters											
l	2,0	l	2,5	l	2,0	l	3,5	l	2,0	l	12
g	2,0	g	2,0	g	2,0	g	2,0	g	2,0	g	2,0
Decisive-attenuation function values											
$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{R_1}})$	1	$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{R_1}})$	1	$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{R_1}})$	1
$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\hat{\bar{d}}_{Y_{R_1}})$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\hat{\bar{d}}_{Y_{R_1}})$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\hat{\bar{d}}_{Y_{R_1}})$	1
$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{R_2}})$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{R_2}})$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{R_2}})$	1
$\tilde{t}(\bar{v}_4)$	0,9	$\tilde{t}(\hat{\bar{d}}_{Y_{R_2}})$	0,8	$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\hat{\bar{d}}_{Y_{R_2}})$	0,7	$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\hat{\bar{d}}_{Y_{R_2}})$	0,1
$\tilde{t}(\bar{v}_5)$	0,9	$\tilde{t}(\hat{\bar{d}}_{X_{R_3}})$	1	$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{R_3}})$	1	$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{R_3}})$	1
$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\hat{\bar{d}}_{Y_{R_3}})$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\hat{\bar{d}}_{Y_{R_3}})$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\hat{\bar{d}}_{Y_{R_3}})$	1
$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{P_1}})$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{P_1}})$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{P_1}})$	1
$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\hat{\bar{d}}_{Y_{P_1}})$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\hat{\bar{d}}_{Y_{P_1}})$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\hat{\bar{d}}_{Y_{P_1}})$	1
$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{P_2}})$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{P_2}})$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\hat{\bar{d}}_{X_{P_2}})$	1
$\tilde{t}(\bar{v}_{10})$	0,9	$\tilde{t}(\hat{\bar{d}}_{Y_{P_2}})$	1	$\tilde{t}(\bar{v}_{10})$	0,9	$\tilde{t}(\hat{\bar{d}}_{Y_{P_2}})$	1	$\tilde{t}(\bar{v}_{10})$	0,8	$\tilde{t}(\hat{\bar{d}}_{Y_{P_2}})$	1

Table 6.14.c The iterative process course

Step "6"				Step "7"			
Values of corrections and standardized increme							
$\bar{v}_1$	1,7	$\bar{\hat{d}}_{X_{R_1}}$	1,0	$\bar{v}_1$	1,7	$\bar{\hat{d}}_{X_{R_1}}$	1,0
$\bar{v}_2$	-1,6	$\bar{\hat{d}}_{Y_{R_1}}$	-0,4	$\bar{v}_2$	-1,7	$\bar{\hat{d}}_{Y_{R_1}}$	-0,4
$\bar{v}_3$	-0,2	$\bar{\hat{d}}_{X_{R_2}}$	-1,9	$\bar{v}_3$	-1,9	$\bar{\hat{d}}_{X_{R_2}}$	-1,9
$\bar{v}_4$	2,0	$\bar{\hat{d}}_{Y_{R_2}}$	-2,1	$\bar{v}_4$	2,0	$\bar{\hat{d}}_{Y_{R_2}}$	-2,0
$\bar{v}_5$	-2,0	$\bar{\hat{d}}_{X_{R_3}}$	0,4	$\bar{v}_5$	-2,0	$\bar{\hat{d}}_{X_{R_3}}$	0,4
$\bar{v}_6$	-1,7	$\bar{\hat{d}}_{Y_{R_3}}$	0,7	$\bar{v}_6$	-1,7	$\bar{\hat{d}}_{Y_{R_3}}$	0,8
$\bar{v}_7$	-1,7	$\bar{\hat{d}}_{X_{P_1}}$	-0,3	$\bar{v}_7$	-1,7	$\bar{\hat{d}}_{X_{P_1}}$	-0,2
$\bar{v}_8$	0,6	$\bar{\hat{d}}_{Y_{P_1}}$	0,0	$\bar{v}_8$	0,7	$\bar{\hat{d}}_{Y_{P_1}}$	0,0
$\bar{v}_9$	1,4	$\bar{\hat{d}}_{X_{P_2}}$	1,1	$\bar{v}_9$	-0,1	$\bar{\hat{d}}_{X_{P_2}}$	1,1
$\bar{v}_{10}$	-2,0	$\bar{\hat{d}}_{Y_{P_2}}$	-0,1	$\bar{v}_{10}$	-2,0	$\bar{\hat{d}}_{Y_{P_2}}$	-0,1
Attenuation function parameters							
l	--	l	17,9	l	--	l	--
g	--	g	2,0	g	--	g	--
Decisive-attenuation function values							
$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1	$\tilde{t}(\bar{v}_1)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_1}})$	1
$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1	$\tilde{t}(\bar{v}_2)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_1}})$	1
$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1	$\tilde{t}(\bar{v}_3)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_2}})$	1
$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	0,1	$\tilde{t}(\bar{v}_4)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_2}})$	1
$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1	$\tilde{t}(\bar{v}_5)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{R_3}})$	1
$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	1	$\tilde{t}(\bar{v}_6)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{R_3}})$	1
$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1	$\tilde{t}(\bar{v}_7)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_1}})$	1
$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1	$\tilde{t}(\bar{v}_8)$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_1}})$	1
$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1	$\tilde{t}(\bar{v}_9)$	1	$\tilde{t}(\bar{\hat{d}}_{X_{P_2}})$	1
$\tilde{t}(\bar{v}_{10})$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1	$\tilde{t}(\bar{v}_{10})$	1	$\tilde{t}(\bar{\hat{d}}_{Y_{P_2}})$	1

On ending the iterative process (in Step "7"), it has finally been obtained as follows $\left\{\forall v : v \in \Delta\overline{v}\right\} \Rightarrow \left\{\widetilde{\mathbf{T}}(\mathbf{V}) = \mathbf{I}_{(10)}\right\}$ and $\left\{\forall \overline{\hat{d}}_X, \overline{\hat{d}}_Y : \overline{\hat{d}}_X, \overline{\hat{d}}_Y \in \Delta\overline{\hat{d}}\right\} \Rightarrow \left\{\widetilde{\mathbf{T}}\left(\overline{\hat{d}}_{\mathbf{X}}\right) = \mathbf{I}_{(10)}\right\}$.

Thus:

$$\hat{\mathbf{X}} = \mathbf{X}^0 + \hat{\mathbf{d}}_{\mathbf{X}} = \begin{bmatrix} 100 \\ 100 \\ 1100 \\ 100 \\ 2000 \\ 100 \\ 1550 \\ 800 \\ 500 \\ 800 \end{bmatrix} + \begin{bmatrix} 2,7 \\ -0,7 \\ -4,0 \\ -4,3 \\ 1,4 \\ 1,6 \\ 0,6 \\ 0,0 \\ 2,8 \\ -0,3 \end{bmatrix} = \begin{bmatrix} 102,7 \\ 99,3 \\ 1096,0 \\ 95,7 \\ 2001,4 \\ 101,6 \\ 1550,6 \\ 800,0 \\ 502,8 \\ 800,3 \end{bmatrix}_{[m]} = \begin{bmatrix} \hat{\hat{X}}_{R_1} \\ \hat{\hat{Y}}_{R_1} \\ \hat{\hat{X}}_{R_2} \\ \hat{\hat{Y}}_{R_2} \\ \hat{\hat{X}}_{R_3} \\ \hat{\hat{Y}}_{R_3} \\ \hat{X}_{P_1} \\ \hat{Y}_{P_1} \\ \hat{X}_{P_2} \\ \hat{Y}_{P_2} \end{bmatrix}$$

In IV determinations variant, the position errors, determined on the basis of the covariance $\hat{\mathbf{C}}_{\hat{\mathbf{X}}}$ matrix are respectively:

$$M_{R_1} = 5,9\,[\mathrm{m}],\ M_{R_2} = 4,7\,[\mathrm{m}],\ M_{R_3} = 7,3\,[\mathrm{m}],\ M_{P_1} = 5,7\,[\mathrm{m}],\ M_{P_2} = 6,2\,[\mathrm{m}]$$

Let's resolve the position determination errors for each of the variants. Applying the following designations for the position errors:

- M^{clean} - variant with neither out-lying coordinates nor out-lying observations;
- M^{D-R} - when the set includes the out-lying observations or out-lying coordinates and observations and the adjustment was carried out using the concepts suggested in this paper;
- $M^{classic}$ - the set includes the out-lying observations or out-lying coordinates and observations, and the adjustment was carried out applying the classical least squares method.

We obtain the values presented in Table 6.15.

Table 6.15. Determination errors related to the points R_1, R_2, R_3 positions coordinates and the positions P_1, P_2 for specific test variants

	Variant I	Variant II		Variant III		Variant IV	
	M^{clean}	M^{D-R}	$M^{classic}$	M^{D-R}	$M^{classic}$	M^{D-R}	$M^{classic}$
R_1	2,8 [m]	5,6 [m]	9,6 [m]	5,2 [m]	11,5 [m]	5,9 [m]	10,3 [m]
R_2	2,0 [m]	4,5 [m]	7,0 [m]	4,1 [m]	9,3 [m]	4,7 [m]	9,5 [m]
R_3	2,7 [m]	7,4 [m]	10,1 [m]	6,9 [m]	16,4 [m]	7,3 [m]	13,6 [m]
P_1	3,0 [m]	5,5 [m]	9,7 [m]	5,3 [m]	12,9 [m]	5,7 [m]	11,6 [m]
P_2	3,1 [m]	5,9 [m]	10,3 [m]	5,5 [m]	12,0 [m]	6,2 [m]	11,2 [m]

The Table 6.16 presents (for each of the variants) the final, adjusted coordinates of the points included in the set $\Re$ and $\wp$.

Table 6.16. The adjusted coordinates of the set $\Re$ and $\wp$ points for each test variant

	Points of set $\Re$						Points of set $\wp$			
	R_1		R_2		R_3		P_1		P_2	
	$\hat{\hat{X}}_{R_1}$	$\hat{\hat{Y}}_{R_1}$	$\hat{\hat{X}}_{R_2}$	$\hat{\hat{Y}}_{R_2}$	$\hat{\hat{X}}_{R_3}$	$\hat{\hat{Y}}_{R_3}$	$\hat{X}_{P_1}$	$\hat{Y}_{P_1}$	$\hat{X}_{P_2}$	$\hat{Y}_{P_2}$
Variant I	102,5	100,3	1099,9	99,9	1997,6	100,3	1549,8	800,8	502,6	800,8
Variant II	101,2	99,5	1098,9	98,7	1999,9	101,8	1547,8	800,2	501,3	799,9
Variant III	100,3	99,2	1098,1	98,5	1998,3	103,2	1547,0	799,9	500,4	800,4
Variant IV	102,7	99,3	1096,0	95,7	2001,4	101,6	1550,6	800,0	502,8	800,3

Properties of the suggested method were also analysed on grounds of other ways of biasing observations and coordinates with gross errors. Variant II would be solved, for example, assuming consecutively, that every observation was gross error biased. Variant III was analyzed for all the observation pairs with an assumption made that one of them was the bearing $NR_{2,3}$. Variant IV instead, is an analysis of biasing the bearing $NR_{2,3}$, each of signs R_1 and R_2 coordinates and their specific points coordinates' pairs with gross errors. The simulated gross errors' values were in each variant the same ($5m_{NR}$ - for the bearings, $10m_{dR}$ - for the distances, $45m_{X_\Re}$, $30m_{Y_\Re}$ - for the coordinates).

Results of each displayed variant of biasing appeared similar to those obtained at the points presented before. Diagrams which show the courses of position determination errors' values for any specific Structure points of the chosen versions of each of the variants are shown below.

The structure points are as follows: 1 - R_1, 2 - R_2, 3 - R_3, 4 - P_1, 5 - P_2

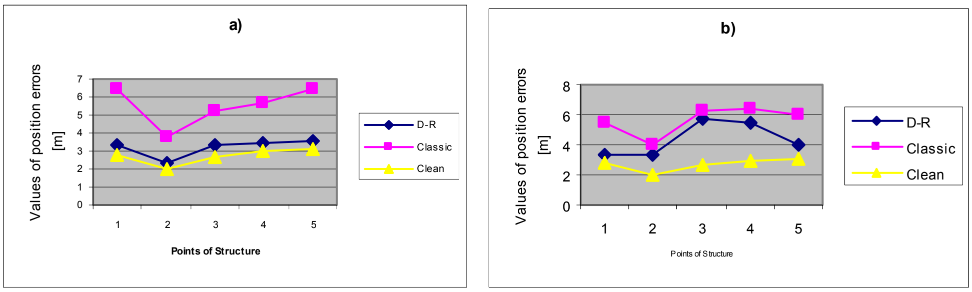

Fig. 6.3. The observations a) $NR_{2,1}$ and b) $NR_{1,2}$ are gross error biased

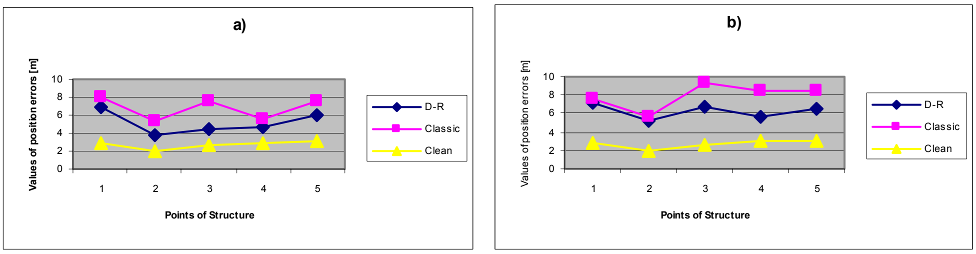

Fig. 6.4. The gross error biased are the observations:
a) $NR_{2,3}$, $NR_{2,2}$ and b) $NR_{2,3}$, $dR_{1,2}$

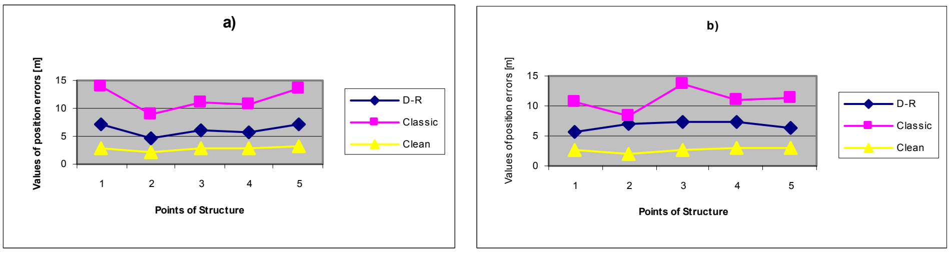

Fig. 6.5. The gross error biased is the observation $NR_{2,3}$ and also:
a) X_{R_1} and b) $\left(X_{R_1}, Y_{R_1}\right)$

Having considered the above tables and diagrams one may find out as follows:

- accuracy of determining points of the Structure (for all the variants) is similar and depends on the method of determinations;
- implementation of hybrid *M*-estimation enables improving the final determinations standard significantly - if compared to the adjustments, carried out with traditional methods;
- the ideas described and suggested in this paper, allow to obtain a fairly accurate proper position, nevertheless observations and coordinates are gross error biased;
- the most wide-ranging application of hybrid *M*-estimation, carried out with a use of *IANS*, can take place in case a navigator is not sure about a accuracy of his observations, and correctness of coordinates of the observed navigational signs.

SUMMARY

The most essential outcome of theoretical studies and selected numerical analyses, presented in this work, are the suggestions related to the technology of producing and working out the results of observations, carried out in the Interactive Navigational Structures. Implementing such a Structure into navigation practice will enable supporting the positioning process by taking use of objects, which until present, in classic navigation, have been omitted due to a lack of information about their coordinates. Establishing and dynamic developing the Interactive Navigational Structures is of special importance, in case the available positioning systems appear insufficient (e.g. in submarine navigation or any work of special character).

IANS can be based on various systems and navigational observations, including the satellite GPS systems. Selection of an observational model, accommodated to any current navigational situation, is simplified owing to the decisive functions, recommended in the work; applying the above functions, in conjunction with the functions of attenuation, has resulted in making more efficient the process of estimation, robust for out-lying observations. Due to diversity of the observations sets and the “by stages” way of working them out, the estimation is of sequential character (sequential robust estimation). The adjustment task, formulated and resolved in this work, has been related to the method and adjusted to the *IANS* chain being under development. The fundamental elements of this task’s function of target are, the suggested in the work, the equivalent-decisive covariance matrix and equivalent-decisive weights matrix, both connected by statistical model.

The method of identification of not only out-lying observations but also out-lying adjustment points and neutralization of an influence thereof is also advised in the work; the method has been resulted from free adjustment and *M*-estimation principles.

The method, named hybrid *M*-estimation, can make a great difference specially in extreme navigation conditions, carried out basing intensely on points of the set $\Re$.

The numerical tests, described in Chapter 6, refer to simulation of elementary navigational situations, connected with developing *IANS* and using thereof. They are to illustrate principal properties of the suggested conceptions. The results of the first of the tests have confirmed a possibility of taking use of the recommended structures in some special tasks of navigation at sea. Extremely interesting properties (the second test), first of all those of robust character, were revealed by hybrid *M*-estimation. The obtained results have proved that there is a chance to carry out reliable navigation, being compelled only to applying of *IANS*.

The presented conceptions have been basically completed in respect of the theory concerning establishing, developing and mathematical working out the Interactive Navigational Structures. However, the work is incomprehensive in regard to practical implementation of the displayed models. For example, there is a possibility to use the described structures in submarine navigation. A lack of any classic navigational systems in sea depth has been forcing to seek new solutions, as the Interactive Navigational Structure is. The above solutions may also be extensively employed in radar navigation. Radar observations are often biased with gross errors, caused by radar echo generation technique. The robust estimation, if applied in the version presented in this work, may significantly improve final determinations' standard.

The recommended mathematical models and methods of their parameters evaluation are applicable to maritime navigation under a certain condition. The available at present nautical information about navigational signing elements should be complemented. All the suggested solutions may not be in today's situation fully exercised (especially hybrid *M*-estimation) due to a lack of information about covariance matrixes or at least about errors of determining coordinates of stable and floating navigational signs, embraced by any optional navigational system.

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