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SPPED AS A VESSEL TRAFFIC OPTIMIZATION CRITERION

ABSTRACT Problems of vessel traffic organization in approach channels are solved by Vessel Traffic Management Services (VTMS). One essential task for VTMS is to ensure a required level of traffic safety. As often vessel traffic is intensive, attempts are made to optimize it. This author aims to find a relevant solution based on a mathematical model, in which vessel speeds at particular fairway sections are varied. The model belongs to the class of mixed integer linear programming. The branch-and-bound method has been adapted to the proposed solution of the problem. The results are shown in tables, while certain aspects of solutions are illustrated graphically.

INTRODUCTION

Vessel traffic in the fairway is subject to various restrictions. On the one hand, these result from the relationship between the ship size and the hydro-technical parameters of the fairway. On the other hand, the restrictions are due to other traffic in the fairway, hydro-meteorological conditions, the cargoes carried by ships on the fairway and other factors. The principles of fairway traffic are defined by local regulations. The monitoring of the compliance with these principles is performed by the Vessel Traffic Management Services.

Those institutions are authorized, among others, to allow a ship to enter the fairway. If such a decision is made, it is preceded by an assessment of the situation, i.e. if a new ship permitted to enter the fairway might be an obstruction for other traffic or if it violates the restrictions imposed by the regulations [Port Regulations, 1993]. If such a situation were to occur, the ship has to wait to pass the fairway. The natural criterion of traffic optimization is the minimization of total times of waiting and passing the fairway by all ships involved. Due to hydro-technical conditions, the regulations restrict admissible speeds at particular fairway sections (v_{min}, v_{max}). These speeds depend on the ship size (length and draft) and on the particular fairway section.

This article attempts at optimizing the selection of vessel speeds so that, with all the restrictions being complied with, the total time of waiting for and passing the fairway is minimized for all vessels. a mathematical model was built that belongs to the class of mixed integer linear programming [Chudy, 2001][Ignasiak, 1997].

Solving this type of problems is not easy, especially if the problem is of large dimensions class. This article presents an adaptation of the branch-and-bound method and the results obtained from its application.

A MATHEMATICAL MODEL

The following assumptions have been made for the fairway [Port Regulations, 1993]:

- the fairway is divided into sections where the constant rules are in force: these refer to the admissible speeds (minimum and maximum), vessel passing and overtaking,
- admissible speed values depend on vessel parameters (length and draft),
- criteria for allowed vessel passing and overtaking depend on the mutual relations between vessel parameters (length and draft),
- fairway vessel traffic is determined (i.e. it cannot be optimized; it is only taken into account as a constraint for other traffic).

In [Uchacz, 2001] [Uchacz et al.,2000] a mathematical model of vessel traffic was presented. This model additionally assumes that vessels move at constant maximum speeds (admitted by the port regulations). The minimization of vessel total waiting time was assumed as an optimization criterion.

With the following notations:

T_i, T_j - real ready-to-enter vessels *i,j* waiting time

t_i, t_j - waiting time of vesels *i,j* to enter the fairway

m,n - numbers of vessels waiting for fairway passage: $i=1,...,n$, $j=1,...,m$

r - number of fairway sections: $k=1,...,r$

v_{ip}, v_{jp} - speeds of vessels *i,j* at *p-th* section (constant, equal to the maximum values set forth by Port Regulations [2]),

$f_{p_1}^m$ - function defining the time of reaching the closer limit of *p-th* section of passing

$f_{p_2}^m$ - function defining the time of reaching the farther limit of *p-th* section of passing

$f^{w}_{p_1}$ - function defining the time of reaching the closer limit of *p-th* section of overtaking

$f^{w}_{p_2}$ - function defining the time of reaching the farther limit of *p-th* section of overtaking

the system of constraints for vessel traffic can be written as follows:

$$t_i - t_j \leq f^{m}_{p_2}(v_i, v_j, p, T_i, T_j)$$

$$t_i - t_j \geq f^{m}_{p_1}(v_i, v_j, p, T_i, T_j)$$

for each pair (i,j): i=1,...,n; j=1,...,m; for each section $p \in P^{m}_{ij}$, gdzie P^{m}_{ij} – a set of sections where two ships (i,j) may pass each other.

$$t_i - t_j \leq f^{w}_{p_2}(v_i, v_j, p, T_i, T_j)$$

$$t_i - t_j \geq f^{w}_{p_1}(v_i, v_j, p, T_i, T_j)$$

for each pair (i,j): i=1,...,n; j=1,...,n; i≠j; i=1,...,m; j=1,...,m; i≠j; for each section $p \in P^{w}_{ij}$, where P^{w}_{ij} – a set of sections where one ship can overtake another (ships i,j)

$$t_i \leq C^{1}_{ijp}$$

$$t_i \geq C^{2}_{ijp}$$

where: C^{1}_{ijp}, C^{2}_{ijp} - constants, dependent on the pair *(i,j)* and on the section *p,* ($C^{1}_{ijp} \geq C^{2}_{ijp}$)

The minimization of total waiting times of all ships has been assumed as the optimization criterion.

$$FC = \min\left(\sum_{i=1}^{n} c_i t_i + \sum_{j=1}^{m} c_j t_j\right)$$

The system of constraints is supplemented with obvious constraints

$$t_i, t_j \geq 0 \qquad i=1,...,n; j=1,...,n$$

which leads to the problem of linear programming. However, a system thus written has internal contradictions, as it will contain constraints permitting (forcing) simultaneous passing (overtaking) of the ships at all admissible sections of the fairway.

In order to remove those dichotomies, artificial binary variables are introduced. Then, the system of constraints will have this form:

$$t_i - t_j - Mx_{ijk} \leq f_{p_2}^{m} (v_i, v_j, p, T_i, T_j)$$

$$t_i - t_j + Mx_{ikj} \geq f_{p_1}^{m} (v_i, v_j, p, T_i, T_j)$$

$$\sum_{k=1}^{r} x_{ijk} = r - 1$$

$$t_i - t_j - My_{ijk} \leq f_{p_2}^{w} (v_i, v_j, p, T_i, T_j)$$

$$t_i - t_j + My_{ijk} \geq f_{p_1}^{w} (v_i, v_j, p, T_i, T_j)$$

$$\sum_{k=1}^{r} y_{ijk} = r - 1$$

$$t_i \leq C_{ijp}^{1}$$

$$t_i \geq C_{ijp}^{2}$$

$$t_i, t_j \geq 0$$

where *x, y* – binary variables, *M* – sufficiently great number. Thanks to this, the pair of appropriate inequalities is non-trivial for only one *p*. The model described in this way belongs to the class of problems of mathematical integer linear mixed programming (PCLM). Solving such problems generally belongs to the class of NP-difficult problems. Methods of obtaining solutions and the discussion of the results are presented in [Uchacz et al., 2000].

THE MODEL WITH VARIABLE SPEEDS FOR PARTICULAR FAIRWAY SECTIONS

By introducing additional notations:

l_p - length of *p-th* section,

τ_{ip} , τ_{jp} - time of passing *p-th* section by ships *i,j*,

additionally assuming that v_{ip}, v_{jp} are variables, one can write the following constraints in the following form:

a pair of ships *i,j* passing each other at the *p* section of the fairway:

$$T_i + t_i + \sum_{k=1}^{p-1} \tau_{ik} \le T_j + t_j + \sum_{k=p}^{r} \tau_{jk}$$

$$T_i + t_i + \sum_{k=1}^{p} \tau_{ik} \ge T_j + t_j + \sum_{k=p+1}^{r} \tau_{jk}$$

overtaking of the ship i by the ship *j* at the section *p*:

$$T_i + t_i + \sum_{k=1}^{p-1} \tau_{ik} \le T_j + t_j + \sum_{k=1}^{p-1} \tau_{jk}$$

$$T_i + t_i + \sum_{k=1}^{p} \tau_{ik} \ge T_j + t_j + \sum_{k=1}^{p} \tau_{jk}$$

the constraints imposed by the *j-th* ship are as follows:

$$T_i + t_i + \sum_{k=1}^{p-1} \tau_{ik} \le C_{jp}^1$$

$$T_i + t_i + \sum_{k=1}^{p} \tau_{ik} \ge C_{jp}^2$$

In addition:

$$\tau_{ip} = l_p v_{ip}^{-1}$$
$$\tau_{jp} = l_p v_{jp}^{-1}$$
$$\min v_{ip} \le v_{ip} \le \max v_{ip}$$
$$\min v_{jp} \le v_{jp} \le \max v_{jp}$$
$$t_i, t_j, v_{ip}, v_{jp} \ge 0$$

while the objective function will take this form:

$$FC = \min(\sum_{i=1}^{n} c_i (t_i + \sum_{k=1}^{r} \tau_{ik}) + \sum_{j=1}^{m} c_j (t_j + \sum_{k=1}^{r} \tau_{jk}))$$

Having removed the dichotomy, the system of constraints will finally be written thus:

$$T_i + t_i + \sum_{k=1}^{p-1} \tau_{ik} - Mx_{ijk} \le T_j + t_j + \sum_{k=p}^{r} \tau_{jk}$$

$$T_i + t_i + \sum_{k=1}^{p} \tau_{ik} + Mx_{ijk} \ge T_j + t_j + \sum_{k=p+1}^{r} \tau_{jk}$$

$$\sum_{k=1}^{r} x_{ijk} = r - 1$$

$$T_i + t_i + \sum_{k=1}^{p-1} \tau_{ik} - My_{ijk} \le T_j + t_j + \sum_{k=1}^{p-1} \tau_{jk}$$

$$T_i + t_i + \sum_{k=1}^{p} \tau_{ik} + My_{ijk} \ge T_j + t_j + \sum_{k=1}^{p} \tau_{jk}$$

$$\sum_{k=1}^{r} y_{ijk} = r - 1$$

$$T_i + t_i + \sum_{k=1}^{p-1} \tau_{ik} \le C_{jp}^{1}$$

$$T_i + t_i + \sum_{k=1}^{p} \tau_{ik} \ge C_{jp}^{2}$$

$$\tau_{ip} = l_p v_{ip}^{-1}$$

$$\tau_{jp} = l_p v_{jp}^{-1}$$

$$\min v_{ip} \le v_{ip} \le \max v_{ip}$$

$$\min v_{jp} \le v_{jp} \le \max v_{jp}$$

$$t_i, t_j, v_{ip}, v_{jp} \ge 0$$

THE METHOD OF COMPUTING

A dichotomy in the system of constraints make it very difficult to find an optimal solution to the problem defined as presented above [Uchacz et al., 2000]. The branch-and-bound method has been adapted for solving the above mentioned problem. The basic concept of the algorithm can be expressed in the following steps:

1. The construction of an acceptable convex area of a linear programming problem, including all disjoints acceptable areas of the source problem.
2. The computation of an optimal solution for the linear programming problem.
3. If the obtained optimal solution is an acceptable solution of the source problem – it is the optimal solution, stop.
4. Otherwise, the convex area is divided into sub-areas (vertexes) so that to exclude the area comprising the obtained solution. The branching creates two convex areas of constraints.
5. The procedure set forth in points 2-4 is continued until acceptable solutions of all partial problems are found.
6. In order to limit the branching of convex sets of constraints in the case when acceptable solutions are not found, a method of estimating the prospective values of the objective functions relative to the current objective function value (sub-optimal solution) is used. Besides, heuristics is applied which utilizes the specific nature of the problem to limit the division of vertexes.

The computing algorithm for the above problem of vessel traffic optimization is given below:

1. Construct the most general problem, i.e. for each pair of vessels, determine the area of constraints encompassing all dichotomic constraints (this corresponds to a situation in which the fairway does not hamper vessel traffic in any way). Solve the problem by the simplex method, as a general linear programming problem. Bound the vertex. If the solution is acceptable, it is the optimal solution, stop. If not, return to step 2.
2. Branching. Find another group of constraints, for which the obtained solution is not acceptable. Determine two nearest constraints, between which the solution is found. Divide the set of constraints into two subsets so that the subset containing the unacceptable solution is excluded. Place the two problems in the list of problems to be solved (make both vertexes open).
3. Estimation. Estimate the open vertexes according to the assumed criteria (FC_{subopt}, existence of cycles, contradiction etc.). Bound the vertexes whose

branching excludes a possibility of finding a better value of the objective function.

4. The end of computing criterion. Are there any open vertexes? No – stop. Yes – solve it.
5. Solution. Choose an open vertex. Solve it as a linear programming problem. Return to step 2.

The prepared algorithm was implemented in the Visual Basic language. Heuristics was used as criteria for bounding the vertexes:

- Estimation of the objective function value obtained in the i-th step relative to the currently best objective function - FC_{subopt}: as each branching leads to the limitation of the acceptable area, each subsequent solution obtained through further branching can only be worse. Therefore, if $FC_i > FC_{subopt}$ branching should not be continued (the vertex should be bounded), as further divisions of the vertex will not improve FC_{subopt}.
- Bounding due to the formation of 'cycles': divisions of various branches may result in a vertex that had been divided before. In such cases the vertex is bounded, because continued branching will result in a division previously analysed.
- Bounding, if the problem was contradictory: if a solution for the examined area of constraints does not exist, then it does not exist for sub-area of these constraints.
- Bounding, when the vertex was used for branching before.

COMPUTATION RESULTS

Seven problems of varied degree of difficulty were chosen. The problems differ in the number of constraint groups and in the number of constraints in particular groups. The values of particular constraints also affected the results (length and position of fairway sections, maximum fairway speeds).

Each problem was solved in four variants:

1. A variant in which the objective function is affected by two components: one that is due to vessel waiting times to enter, the other which is related to the vessel speed at various fairway sections:

$$FC = \min\left(\sum_{i=1}^{n} c_i (t_i + \sum_{k=1}^{r} \tau_{ik}) + \sum_{j=1}^{m} c_j (t_j + \sum_{k=1}^{r} \tau_{jk})\right),$$

gdzie: $\tau_{ik} = l_k v_{ik}^{-1}$, $\tau_{jk} = l_k v_{jk}^{-1}$

2. A variant similar to that in p. 1, in which heuristics is additionally applied to limit the survey of vertexes, thus accelerating the determination of optimal solution.
3. A variant, in which the objective function is significantly affected by a component related to vessel waiting times to enter (coefficient placed by the component connected with vessel speeds makes them tend towards maximum allowed speeds):

$$FC = \min(\sum_{i=1}^{n} c_i (t_i + M \sum_{k=1}^{r} \tau_{ik}) + \sum_{j=1}^{m} c_j (t_j + M \sum_{k=1}^{r} \tau_{jk})) \quad M - \text{large number}$$

4. A variant in which the objective function is significantly affected by the component related to vessel speeds (coefficient placed by the component connected with waiting times to enter makes these time tend towards zero):

$$FC = \min(\sum_{i=1}^{n} c_i (Mt_i + \sum_{k=1}^{r} \tau_{ik}) + \sum_{j=1}^{m} c_j (Mt_j + \sum_{k=1}^{r} \tau_{jk})) \quad M - \text{large number}$$

In the analyzed case the number of vessels proceeding in one direction is n=2 (x_1, x_2), for the opposite direction it is m=3 (y_1, y_2, y_3). The fairway is divided into 13 sections with various admissible speeds. For simplification, it was assumed that all the vessels belong to the same group of vessels selected by the maximum speed criterion. Speed limit values are presented in Table 1.

Table 1. Speed limit values at subsequent fairway sections

Section no	Length [km]	v_{min}[knots]	v_{max}[knots]
1	*2.2*	*3*	*6*
2	*1.4*	*3*	*6*
3	*1.7*	*3*	*6*
4	*5.2*	*3*	*7*
5	*6.2*	*3*	*7*
6	*18.3*	*3*	*12*
7	*2*	*3*	*8*
8	*6.1*	*3*	*12*
9	*2.2*	*3*	*12*
10	*3.1*	*3*	*8*
11	*5.7*	*3*	*12*
12	*9.5*	*3*	*6*
13	*4.1*	*3*	*6*

The tables below present the results for a chosen problem. Table 2 shows computed speeds at subsequent fairway sections, whereas Table 3 gives the waiting times and times of passing fairway sections.

Two moments can be seen when the number of open vertexes drops sharply. That results from the fact that a new, significantly better objective function FC_{subopt} was found. Consequently, all the open vertexes valued $FC \geq FC_{subopt}$ are bounded.

Table 2. Vessel speeds at particular fairway sections

		Fairway sections												
		o1	o2	o3	o4	o5	o6	o7	o8	o9	o10	o11	o12	o13
Vessels	X_1	6	6	6	7	7	12	8	12	12	8	12	6	6
	X_2	3	3,82	6	7	7	12	8	12	12	8	12	6	6
	Y_1	6	6	6	7	7	12	8	12	12	8	12	6	3
	Y_2	6	6	6	7	7	12	8	6,26	12	8	12	6	6
	Y_3	6	6	6	7	7	12	8	12	6,9	8	12	6	6

Table 3. Times of waiting and passing through particular fairway sections

		Fairway sections													Waiting time
		o1	o2	O3	o4	O5	O6	o7	o8	o9	o10	o11	o12	o13	
Vessels	X_1	0.37	0.23	0.28	0.74	0.89	1.53	0.25	0.51	0.18	0.39	0.48	1.58	0.68	0.00
	X_2	0.73	0.37	0.28	0.74	0.89	1.53	0.25	0.51	0.18	0.39	0.48	1.58	0.68	0.00
	Y_1	0.37	0.23	0.28	0.74	0.89	1.53	0.25	0.51	0.18	0.39	0.48	1.58	1.37	0.28
	Y_2	0.37	0.23	0.28	0.74	0.89	1.53	0.25	0.97	0.18	0.39	0.48	1.58	0.68	0.00
	Y_3	0.37	0.23	0.28	0.74	0.89	1.53	0.25	0.51	0.32	0.39	0.48	1.58	0.68	1.85

Figure 1 illustrates the number of simultaneously open vertexes as the function of subsequent computing steps.

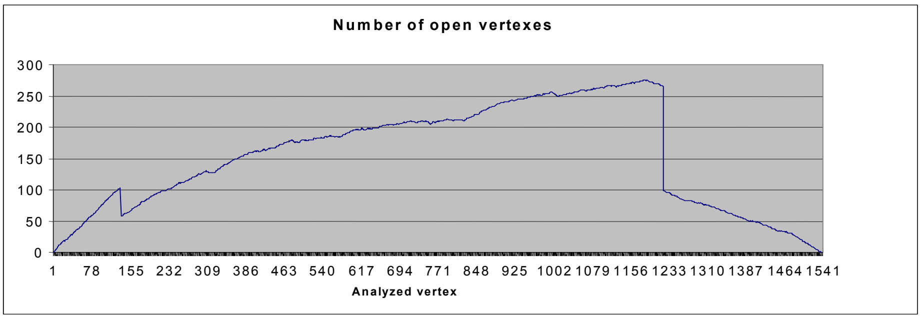

Fig. 1. Number of open vertexes as the function of subsequent computing steps (case: problem 2 variant 2)

UchaczTable 4.pdf

Table 4 contains the results of seven selected cases, analyzed in four variants. The columns ”Criteria for vertex bounding ” contain numbers of bounded vertexes with the use of:

- *FC*: a vertex is bounded, when the value $FC_{akt} \geq FC_{subopt}$. Further branching may only worsen the value *FC*.
- No solution: the problem has no acceptable solution or is contradictory. Therefore, further branching will lead to the construction of problems, whose solutions are contradictory or will not be acceptable.
- Cycles: a vertex is closed because this variant of branching has already occurred and has been analyzed.

The column $\sum t$ contains the time of total delay in vessel traffic as a result of waiting for fairway entry. The column $\sum \tau$ is the total delay in vessel traffic caused by the instruction to reduce the speed relative to the maximum speed.

The difference between the variants 1 and 2 is that heuristics was used in the variant 2, limiting the survey of vertexes. The application of heuristics shortens the computation time to 20%. The weight coefficients c_i by the objective function components connected with the times of waiting for the entry and the fairway passage time (speed optimization) are equal to each other in the variants 1 and 2.

The speed maximization was optimized in the variant 3 (in the mathematical model it corresponded to the minimization of fairway passage time), whereas in the variant 4 waiting times were optimized. The results obtained, presented in the columns $\sum t$, $\sum \tau$, confirm that there is a strong dependence of those FC component values on the analyzed variant. It is obvious that, while optimizing the problem in which one of the two optimization factors varies, a worse value of the objective function can be obtained. On the other hand, the results show that in such cases the number of analyzed vertexes is smaller (shorter time of solving the problem). Substantial differences between individual problems, (particularly in the number of analyzed vertexes), are due to a strong influence of the conditions chosen for the analyzed problem: number of sections where vessels may pass each other, vessel length, etc. Such differences occur due to port regulations, which make these conditions dependent on vessel length (vessels that are shorter and draw less have less constraints for passing on the fairway).

CONCLUSIONS

Vessel traffic in the fairway is subject to some restrictions. The decision that a vessel can enter the fairway is based on an assessment of the fairway traffic situation in view of the compliance with the Port Regulations concerning vessels passing or overtaking [Port Regulations, 1993]. The main factor decisive for the compliance is the delayed entry of a vessel into the fairway.

This article has attempted at traffic control by another factor – fairway speed control. To this end a mathematical model representing the class of linear mixed programming problems. The branch-and-bound method was selected for the solution of the problem. The results justify the choice of fairway vessel speed regulation. The best effects are obtained for a model in which the optimization criterion consists of the minimized total of waiting times of vessels to enter the fairway and fairway passing times (which is equivalent to fairway speed maximization time) – with some additional constraints:

$$\min v_i \leq v_{ip} \leq \max v_i$$

The results show that the objective function values improve only a little as compared with the solutions obtained for models with fixed vessel speeds. The vessel speed regulation is particularly useful when a new vessel enters the fairway.

The applied heuristics has a substantial effect on the time of solving the problems. It limits the survey of the tree of solutions cutting down the time of solution by as much as several percent.

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Received April 2003
Reviewed August 2003