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OPTIMISATION OF VESSEL TRAFFIC USING FUZZY LINEAR PROGRAMMING

ABSTRACT The optimisation of vessel traffic in narrow channels is concerned with the optimisation of vessel passage times with the regulations in force being complied with. These types of problems are normally solved by classical methods of linear programming. However, the implementation of solutions thus obtained is practically hardly feasible. This is due to difficulties in observing exact times of vessel entries into the fairway, maintaining exactly the expected times of passage along particular fairway sections. Therefore, it becomes necessary to account for inaccurate vessel entry and times of passing particular sections. Consequently, an optimisation problem in this context naturally fits in the format of problems of linear programming with fuzzy coefficients. This approach enables a more flexible formulation of an optimisation problem. From the point of view of the solutions obtained, the interpretation of fuzzy inequalities in the system of constraints is of much importance. The article presents solutions to the vessel traffic optimisation problem with the use of various interpretations of fuzzy inequalities. The calculation results are shown. These refer to vessel traffic in the Szczecin-Swinoujscie fairway. The results have been interpreted and conclusions have been drawn.

INTRODUCTION

One of the methods of enhancing the effectiveness in sea transport is vessel traffic optimisation. This also refers to harbour approach areas. The reduction of the time of waiting for harbour entry brings savings to both carriers and port managers.

As port approach areas are often narrow fairways, the optimisation of vessel traffic in narrow channels is essential. It consists in the optimisation of vessel passage times with the regulations in force being complied with. These types of problems are normally solved by classical methods of linear programming. The application of fuzzy linear programming enables a more flexible formulation of an optimisation problem. The introduction of fuzziness is a consequence of inaccuracies, or varied times of ships' entering the fairway and times of passing along particular fairway sections. That is why fuzzy linear programming can be an alternative for a classical linear programming problem.

DESCRIPTION OF A VESSEL TRAFFIC MODEL

Principal assumptions of a linear mathematical model are presented in [6]. The fairway is divided into sections, at which the regulations in force are concerned with: permitted minimum and maximum speeds, vessels passing and overtaking. Values of admissible speeds depend on vessel parameters (length and draft). Criteria for allowed passing and overtaking depend on mutual relations between vessel parameters (lengths and drafts).

Vessel traffic on the fairway is determined (i.e. does not undergo optimisation, it is only taken into account as a limitation for other vessels).

The following notation has been applied:

T_i, T_j – real time of readiness of vessels *i,j* to enter the fairway,
t_i, t_j – time of vessels *i,j* waiting for fairway entry,
m,n   – numbers of vessels waiting for fairway passage: i=1,...,n, j=1,...,m
r   – number of fairway sections: k=1,...,r,
$f_{k_1}^m$ – function determining the time of reaching the closer limit of k-th passing section,
$f_{k_2}^m$ – function determining the time of reaching the farther limit of the k-th passing section,
$f_{k_1}^w$ – function determining the time of reaching the closer limit of the k-th overtaking section,
$f_{k_2}^w$ – function determining the time of reaching the farther limit of the k-th overtaking section.

The problem of vessel traffic optimisation can be brought down to a linear programming (LP) problem, which in a standard form can be presented as follows:

$$\min c^T x \tag{1}$$

with the constraints:

$$\begin{aligned} Ax \le b \\ x \ge 0 \end{aligned} \tag{2}$$

When we assume the minimization of the total waiting time of all ships as an optimisation criterion:

$$OF = \min \left(\sum_{i=1}^{n} c_i t_i + \sum_{j=1}^{m} c_j t_j \right) \tag{3}$$

and a system of constraints:

$$t_i - t_j - Mx_{ijk} \leq f_{k_2}^m(v_i, v_j, k, T_i, T_j)$$

$$t_i - t_j + Mx_{ijk} \geq f_{k_1}^m(v_i, v_j, k, T_i, T_j)$$

$$\sum_{k=1}^{r} x_{ijk} = r - 1$$

$$t_i - t_j - My_{ijk} \leq f_{k_2}^w(v_i, v_j, k, T_i, T_j)$$

$$t_i - t_j + My_{ijk} \geq f_{k_1}^w(v_i, v_j, k, T_i, T_j)$$

$$\sum_{k=1}^{r} y_{ijk} = r - 1$$

$$t_i \leq C_{ijk}^1$$

$$t_i \geq C_{ijk}^2$$

$$t_i, t_j \geq 0$$

(4)

where: x, y – binary variables,
M – sufficiently large number,

a model is obtained that belongs to a class of mathematical mixed linear integer programming problems. Generally, solving such problems is considered as an NP difficult problem.

In the analysed model, the vector *t* is the vector of the times of vessels waiting for entering the fairway and the times of vessel passing along particular fairway sections. The vector c includes weight coefficients, while the matrix a and the vector b include coefficients resulting from fairway traffic constraints – e.g. harbour regulations [3]. The works [7, 8] present an algorithm of solving the problem by the branch-and-bound method.

The presented mathematical model of vessel traffic optimisation contains a system of constraints, which accounts for conditions to be fulfilled while vessels pass or overtake each other and constraints resulting from vessels on the fairway.

One of the alternative areas admissible for n=2, m=3, i.e. numbers of vessels on the opposite course has been chosen as an example of an optimisation problem with fuzzy coefficients.

$$OF = \min \left(\sum_{i=1}^{2} t_i + \sum_{j=1}^{3} t_j \right) \tag{5}$$

$$2{,}28 \le t_1 - t_3 \le 4{,}0719$$
$$-0{,}216 \le t_1 - t_4 \le 1{,}4161$$
$$1{,}3361 \le t_1 - t_5 \le 2{,}553 \tag{6}$$
$$1{,}553 \le t_2 - t_3 \le 2{,}888$$
$$-2{,}1038 \le t_2 - t_4 \le -0{,}553$$
$$0{,}1161 \le t_2 - t_5 \le 1{,}4161$$

$t_i \ge 0$, $i=1,\dots,5$ – variables: times of waiting for entering the fairway.

The system has this optimal solution: $t_1=2{,}28$, $t_2=1{,}553$, $t_3=0$, $t_4=2{,}106$, $t_5=0{,}1369$.

Such a solution means that vessels will be passing at the limits of fairway sections where vessel passing is permitted. In reality, a vessel may not strictly observe the fairway entry time prescribed or move along with the accuracy that may result from calculations based on admissible vessel speeds and fairway limits set forth in harbour regulations. If this is so, a model utilizing fuzzy coefficients of constraints will render the situation more realistically. This calls for solving a problem of linear programming with fuzzy coefficients.

A PROBLEM OF LINEAR PROGRAMMING WITH FUZZY COEFFICIENTS

An optimization problem formulated in its classical form features strict and sharp relationships concerning both constraints and the objective function. Making these relationships less strict, thus allowing a slight non-fulfillment of the constraints enables us to find 'satisfactory' solutions. This is possible by introducing fuzzy coefficients into a classical linear programming problem [2, 4]. a problem of linear programming with fuzzy coefficients has this form:

$$\min \tilde{c}^T x \tag{7}$$
$$\tilde{A}x \lesseqgtr \tilde{b} \tag{8}$$
$$x \ge 0$$

This problem for n-element vector x (j=1,..., n) and for m constraints (i=1,..., m) can be written as follows:

$$\widetilde{Min}\ \tilde{c}_1 x_1 \mp \tilde{c}_2 x_2 \mp \dots \mp \tilde{c}_n x_n \tag{9}$$
$$\tilde{a}_{i1} x_1 \mp \tilde{a}_{i2} x_2 \mp \dots \mp \tilde{a}_{in} x_n \lesseqgtr \tilde{b} \tag{10}$$
$$x_j \ge 0$$

Each of the coefficients in relations (9) and (10) can be presented in the form of a fuzzy number in the L-R representation [1]:

$$\tilde{a}_{ij} = (\underline{a}_{ij}, \overline{a}_{ij}, \underline{\alpha}_{ij}, \overline{\alpha}_{ij})_{LR}$$
$$\tilde{b}_i = (\underline{b}_i, \overline{b}_i, \underline{\beta}_i, \overline{\beta}_i)_{LR}$$
$$\tilde{c}_j = (\underline{c}_j, \overline{c}_j, \underline{\chi}_j, \overline{\chi}_j)_{LR}$$

(11)

Then the left-hand side of the inequality (10) can be written as:

$$\tilde{a}_{i1}x_1 \tilde{+} \tilde{a}_{i2}x_2 \tilde{+} ... \tilde{+} \tilde{a}_{in}x_n = (\underline{a}_i(x), \overline{a}_i(x), \underline{\alpha}_i(x), \overline{\alpha}_i(x))_{LR} \quad (12)$$

where:

$$\underline{a}_i(x) = \sum_{j=1}^{n} \underline{a}_{ij}\, x_j \qquad \overline{a}_i(x) = \sum_{j=1}^{n} \overline{a}_{ij}\, x_j$$

$$\underline{\alpha}_i(x) = \sum_{j=1}^{n} \underline{\alpha}_{ij}\, x_j \qquad \overline{\alpha}_i(x) = \sum_{j=1}^{n} \overline{\alpha}_{ij}\, x_j$$

(13)

Then the conditions (constraints) for n-element vector x (j=1,..., n) and m constraints (i=1,..., m) take this form:

$$(\underline{a}_i(x), \overline{a}_i(x), \underline{\alpha}_i(x), \overline{\alpha}_i(x))_{LR} \tilde{\leq} (\underline{b}_i, \overline{b}_i, \underline{\beta}_i, \overline{\beta}_i)_{LR}$$
$$x_j \geq 0$$

(14)

Simplifying the shape of the membership function of right-hand sides of constraints by determining their shape only to the right from the core, we can write the fuzzy number $\tilde{b}_i$ in this form:

$$\tilde{b}_i = (b_i, 0, \overline{\beta_i})_{LR}$$

(15)

The interpretation of a fuzzy inequality is an essential problem (14). There exist a number of concepts of interpreting this type of inequalities in order to transform it into a system of non-fuzzy conditions. Thus it is possible to solve an optimisation problem by classical methods, e.g. by the SIMPLEX method.

INTERPRETATION OF FUZZY INEQUALITIES

In most approaches, the constraints of an optimisation problem having the form of fuzzy inequalities are replaced by one or two crisp linear constraints. At the same time, it is possible to control the degree of satisfying the inequality by a finite number of parameters.

A review of methods of comparing fuzzy numbers is included in [5]. The order of presented interpretations corresponds to passing from the pessimistic variant of comparing two fuzzy numbers to the optimistic variant.

For the fuzzy numbers

$$
\tilde{A}_i = (\underline{a}_i(x), \overline{a}_i(x), \underline{\alpha}_i(x), \overline{\alpha}_i(x))_{LR}
$$
$$
\tilde{B}_i = (\overline{b}_i, 0, \overline{\beta}_i)_{LR}
$$

$$(16)$$

the fuzzy inequality $\tilde{A}_i \tilde{\leq} \tilde{B}_i$ has this form:

1st method: Tanaka and Asai

$$
\overline{a}_i(x) + \overline{\alpha}_i(x)\,\varepsilon \leq \overline{b}_i
$$

$$(17)$$

2nd method: Ramik and Rimanek, Rommelfanger

$$
\begin{cases} \overline{a}_i(x) \leq \overline{b}_i \\ \overline{a}_i(x) + \overline{\alpha}_i(x)\,\varepsilon \leq \overline{b}_i + \overline{\beta}_i(x)\,\varepsilon \end{cases}
$$

$$(18)$$

3th method: Carlsson and Korhonen

$$
\overline{a}_i(x) + \overline{\alpha}_i(x)\,\varepsilon \leq \overline{b}_i + \overline{\beta}_i(x)\,\varepsilon
$$

$$(19)$$

4th method: Slowinski
pessimistic index

$$
\overline{a}_i(x) + \overline{\alpha}_i(x)\,\varepsilon \leq \overline{b}_i + \overline{\beta}_i(x)\,\varepsilon
$$

$$(20)$$

optimistic index

$$\underline{a}_i(x) - \underline{\alpha}_i(x)\,\varepsilon \le \overline{b}_i + \overline{\beta}_i(x)\,\varepsilon \tag{21}$$

5th method: Ramik and Rommelfanger

$$\begin{cases} \overline{a}_i(x) \le \overline{b}_i + \delta\,\overline{\beta}_i \\ \overline{a}_i(x) + (\varepsilon + \delta)\,\underline{\alpha}_i(x) \le \overline{b}_i + \varepsilon\,\overline{\beta}_i(x) \end{cases} \tag{22}$$

6th method: Sakawa and Yano

$$\underline{a}_i(x) - \underline{\alpha}_i\varepsilon \le \overline{b}_i + \overline{\beta}_i(x)\,\varepsilon \tag{23}$$

with ε - aspiration level and δ - distance of the ordinates of the points whose abscissae are being compared, while $(\delta + \varepsilon) \in [0,1]$, $\delta, \varepsilon \ge 0$.

The parameters ε *and* δ can be used to control the degree of decision maker's satisfaction with fuzzy constraints in an interactive way. All these crisp inequalities are obviously linear ones.

RESEARCH

The presented method of fuzzy linear programming was used in the optimisation problem of the minimisation of port entry time.

The L-R representation was used for the description of coefficients. The vector x in the analysed model is a vector of waiting times of ships awaiting the entry into the fairway and vessel passage times for particular fairway sections. The vector c contains weight coefficients, the matrix a and the vector b, coefficients resulting from limitations in vessel traffic, e.g. imposed by harbour regulations.

The fuzziness of absolute terms of constraints means the fuzziness of the time of vessel's arrival at the limit of passing section. The fuzziness of matrix a coefficients means the fuzziness of the time of a ship entry into the fairway. The fuzziness of these coefficients results from the fact that in reality certain deviations are allowed from the fixed values of constraint coefficients and variables.

Various interpretations of inequalities occurring in fuzzy constraints of an optimisation problem were applied.

Solutions to optimisation problems were analysed for various values of the level of aspiration ε, and the scatters of coefficients a and constraints B. Figures 1a,b,c – 5a,b,c present the results obtained for selected values of parameters $\alpha, \underline{a}, \overline{a}, \delta$. In all the cases an increase in α causes an increase in the values of *OF* (due to the limiting of the area of admissible solutions). The fuzziness of the free terms vector B (increase in β) extends the area of admissible solutions – this results in decreasing the value of the objective function. For small values β and large values of the coefficient ε (level of aspiration) no solutions are observed. The examined methods are presented in the following order: from maximum pessimistic to maximum optimistic methods.

This is confirmed by the obtained results (values of the objective function *OF*).

The differences in extreme cases amount to eighty percent. However, it should be emphasised that these differences are only quantitative; for all the methods the trends remain dependent on the examined parameters.

The fuzziness of inequality coefficients of the vessel traffic optimisation model (3), (4) corresponds to the fuzziness of the limits of passing and overtaking fairway sections, as well as the fuzziness of the area admissible for variable times of ship entry into the fairway. The choice of a particular method of fuzziness of a linear programming problem should depend on the assessment of actual conditions on the fairway. In this case, hydro-meteorological conditions as well as hydro-technical and other factors should be considered.
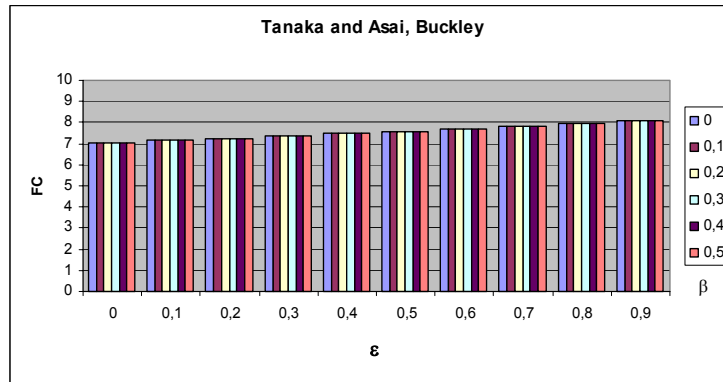
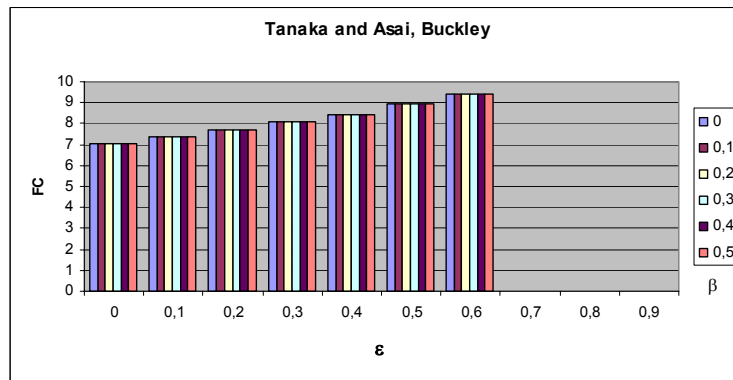**Fig. 1a.** A case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.9$, $\overline{a} = 1.1$, $\underline{\alpha} = 0.1$, $\overline{\alpha} = 0.1$

Fig. 1b. *A c*Case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.9$, $\overline{a} = 1.1$, $\underline{\alpha} = 0.3$, $\overline{\alpha} = 0.3$
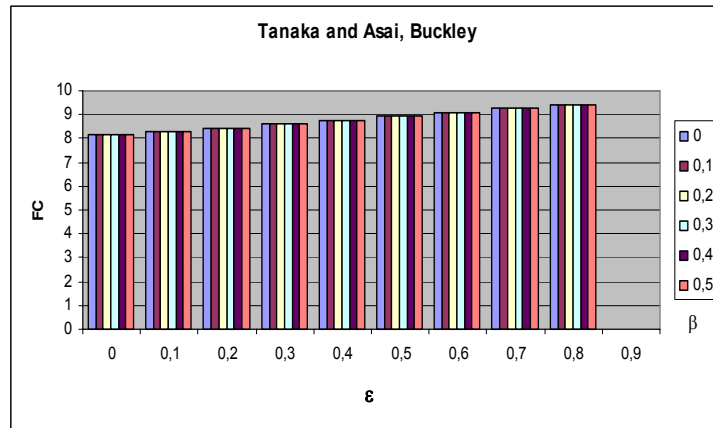

Fig. 1c. A case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.8$, $\overline{a} = 1.2$, $\underline{\alpha} = 0.1$, $\overline{\alpha} = 0.1$

Ramik and Rimanek, Rommelfanger

Fig. 2a. A case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.9$, $\overline{a} = 1.1$, $\underline{\alpha} = 0.1$, $\overline{\alpha} = 0.1$

Ramik and Rimanek, Rommelfanger

Fig. 2b. A case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.9$, $\overline{a} = 1.1$, $\underline{\alpha} = 0.3$, $\overline{\alpha} = 0.3$

Ramik and Rimanek, Rommelfanger
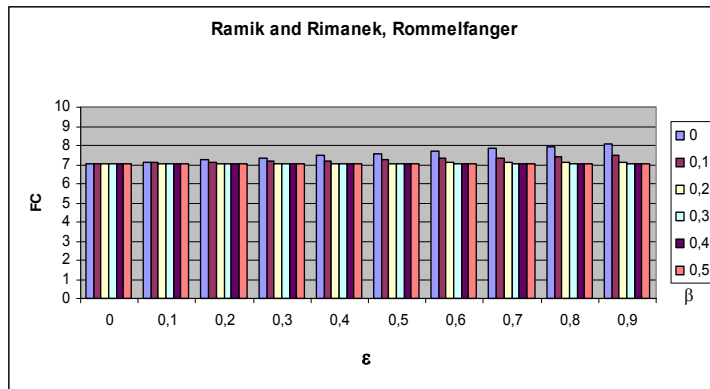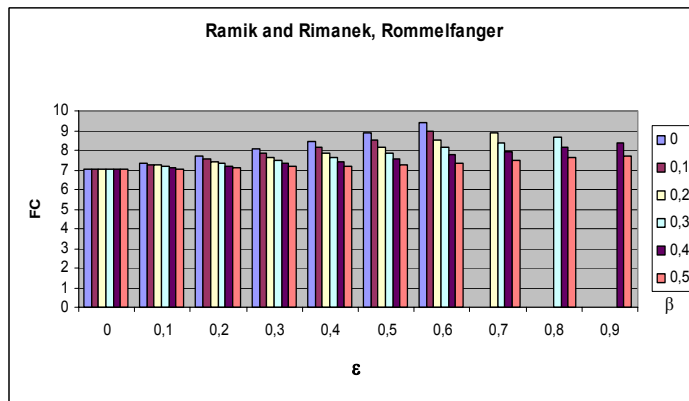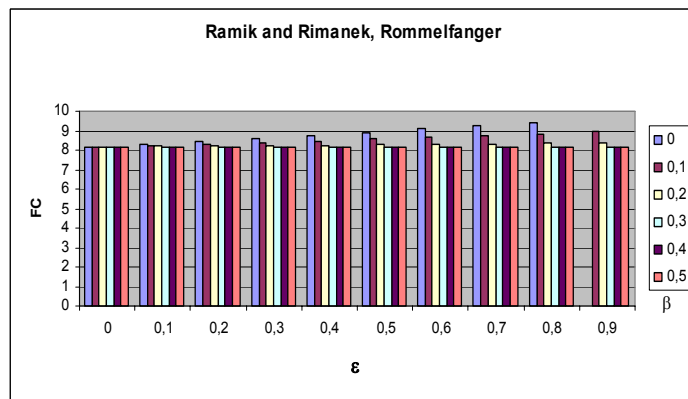
Fig. 2c. Case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.8$, $\overline{a} = 1.2$, $\underline{\alpha} = 0.1$, $\overline{\alpha} = 0.1$
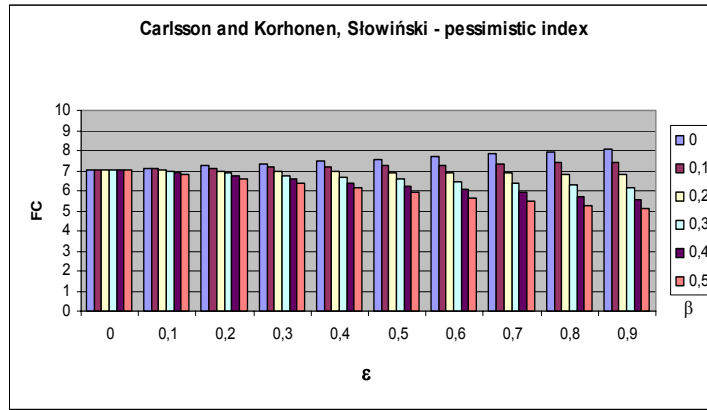
Fig. 3a. A case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.9$, $\overline{a} = 1.1$, $\underline{\alpha} = 0.1$, $\overline{\alpha} = 0.1$
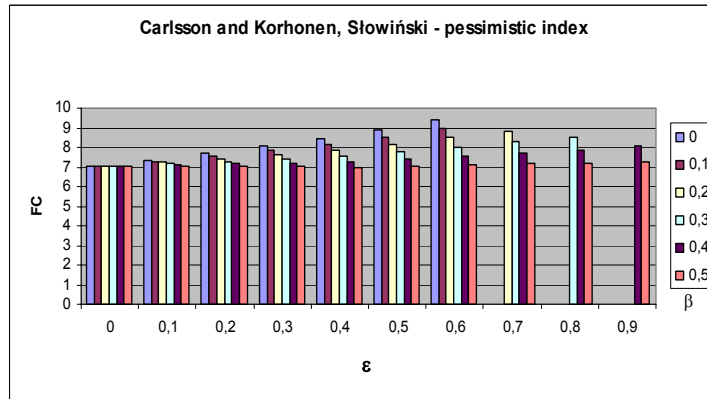
Fig. 3b. A case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.9$, $\overline{a} = 1.1$, $\underline{\alpha} = 0.3$, $\overline{\alpha} = 0.3$
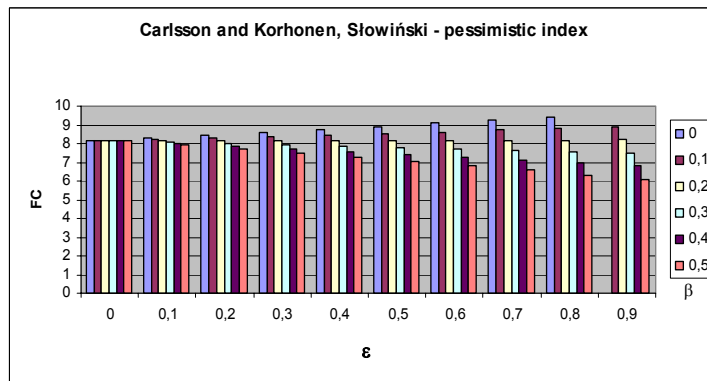
Fig. 3c. A case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.8$, $\overline{a} = 1.2$, $\underline{\alpha} = 0.1$, $\overline{\alpha} = 0.1$
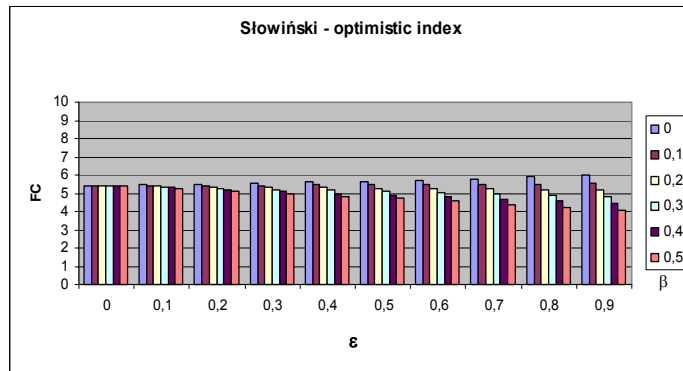
Fig. 4a. A case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.9$, $\overline{a} = 1.1$, $\underline{\alpha} = 0.1$, $\overline{\alpha} = 0.1$
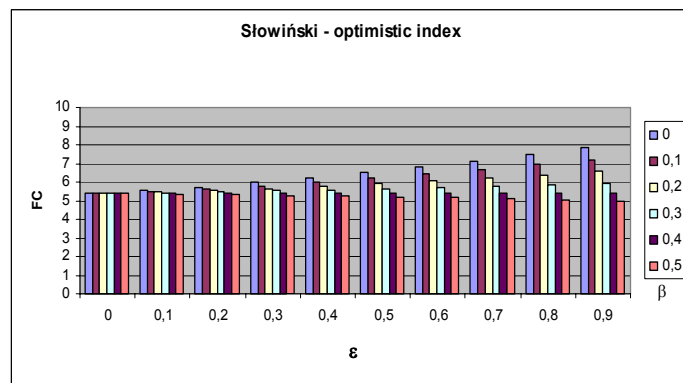
Fig. 4b. A case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.9$, $\overline{a} = 1.1$, $\underline{\alpha} = 0.3$, $\overline{\alpha} = 0.3$
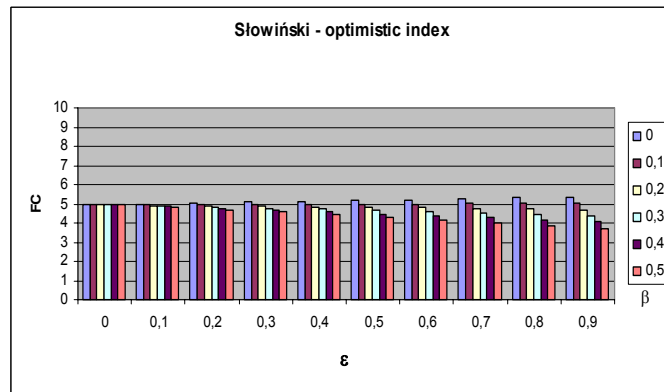
Fig. 4c. A case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.8$, $\overline{a} = 1.2$, $\underline{\alpha} = 0.1$, $\overline{\alpha} = 0.1$
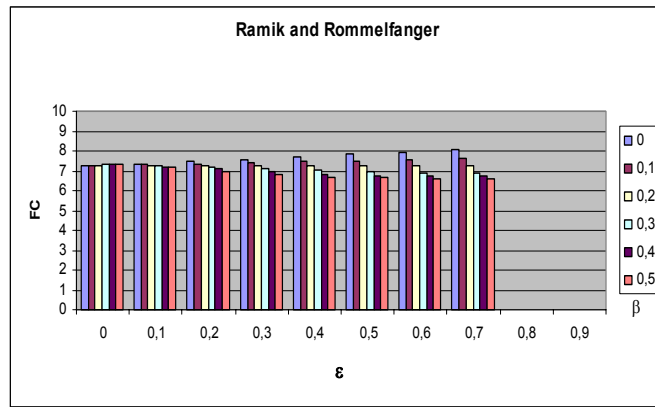
Fig. 5a. A case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.9$, $\overline{a} = 1.1$, $\underline{\alpha} = 0.1$, $\overline{\alpha} = 0.1$, $\delta = 0.2$
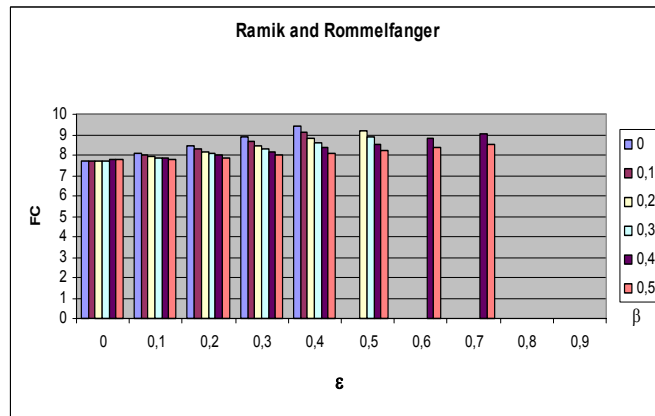
Fig. 5b. A case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.9$, $\overline{a} = 1.1$, $\underline{\alpha} = 0.3$, $\overline{\alpha} = 0.3$, $\delta = 0.2$
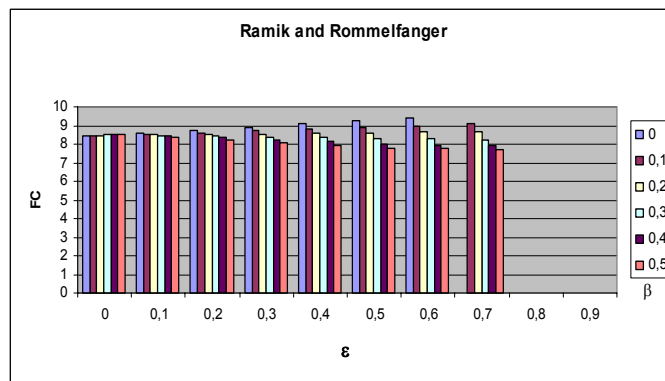
Fig. 5c. A case for $OF = f(\varepsilon, \beta)$, $\underline{a} = 0.8$, $\overline{a} = 1.2$, $\underline{\alpha} = 0.1$, $\overline{\alpha} = 0.1$, $\delta = 0.2$

CONCLUSIONS

The paper presents a problem of vessel traffic optimisation using a method of linear programming with fuzzy coefficients. The case of Szczecin-Swinoujscie fairway has been discussed.

The L-R representation of fuzzy numbers has been used for the description of constraint coefficients. The form of fuzzy constraints for linear programming problems is necessary in the case when accurate values of constraint coefficients are not known or certain deviations from crisp values are allowed. By transforming fuzzy constraints into crisp (non-fuzzy) linear constraints we can use conventional methods for solving a linear programming problem.

The results will depend on the degree of aspiration of fulfilling the fuzzy inequalities and on the assumed interpretation of fuzzy inequalities in the conditions (constraints) system of an optimisation problem. From the point of view of the solutions obtained, the interpretation of fuzzy inequalities in the system of constraints is of much importance.

The article presents solutions to the vessel traffic optimisation problem with the use of various interpretations of fuzzy inequalities. The calculation results are shown.

This approach enables a more flexible formulation of an optimisation problem. It can be an alternative to solutions based on methods of classical linear programming problem.

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