

Jaroslav Artyszuk
Szczecin Maritime University

ANCHOR FORCES IN SHIP MANOEUVRING MATHEMATICAL MODEL

ABSTRACT A compact and practical mathematical model of the anchor-related manoeuvring forces is developed. The most useful and original are non-dimensional lookup tables of the anchor cable horizontal tension, which are valid for any geometrical relationship between the ship hawse-pipe and the anchor itself. The anchor and/or cable failure aspects due to an excessive ship anchoring velocity are also raised.

INTRODUCTION

The catenary, a curve which is assumed by a heavy and flexible cable (chain), is one of the well recognised optimisation problems in the variational calculus. The catenary equation, though apparently relatively simple, poses some difficulties in identifying its parameters based on given boundary conditions as imposed by many engineering design tasks.

Since its great practical advantages and implications, the catenary has been an interest of mathematical and engineering sciences for years (besides pure geometrical relationships much concern is put on the cable tensions as well). The marine and/or nautical aspects of the catenary are seen during anchoring, mooring, or towing operations - e.g. [NFEC, 1985], [Makin, 1977, 1988], [Hong, 1983], [Shipp, 1977], [Polderdijk, 1985], [Liensdorf, 1986], [Gatzer et al., 1987].

The present study is devoted to a development of anchor manoeuvring force sub-model, which is essential from the practical point of view in the general ship manoeuvring mathematical model. The efforts are focused upon a compact formula derivation as expressing such excitations for an arbitrary water depth, cable length, and a horizontal distance between the anchor and the hawse-pipe. The obtained below non-dimensional relationships in form of a lookup table are valid for any ship.

CATENARY EQUATION

The catenary curve is described in a general form by the following expression:

$$y = a \cdot \cosh\left(\frac{x - c_1}{a}\right) + c_2 \quad (1)$$

where: a - catenary shape main parameter ([m], positive); c_1, c_2 - other parameters.

All the above three parameters shall be identified based on given boundary conditions. However, this is often not an analytical but numerical task. In view of further derivations, it is appreciable to introduce a simpler (more famous) relationship for the catenary:

$$y = a \cdot \left(\cosh \frac{x}{a} - 1 \right) \quad (2)$$

where the origin of coordinates (0,0) is assumed to be at the catenary extremum (the slope angle is zero) and the catenary length is written by:

$$l = a \cdot \sinh\left(\frac{x}{a}\right) \quad (3)$$

In ship manoeuvring the most interesting things are the horizontal component F_{ANR} of the catenary (anchor cable) tension at the top (i.e. referring to the hawse-pipe) and the cable slope angle α_D at the seabed. In case of an underwater current absence, the horizontal tension component is the same at any point of catenary and described by:

$$F_{ANR} = qag \quad (4)$$

where q is the unit weight of anchor cable [kg/m] in the water and g stands for the gravity acceleration (9.81[m/s²]).

TWO-DIMENSIONAL (2D) CONCEPT OF ANCHOR FORCES

Looking at the anchor cable in a side view, three distinct cases (ranges) can be specified, see Fig. 1, namely:

A- when the cable is leading vertically downwards from the hawse-pipe and the some of it is lying loose and chaotically and on the seabed, there is no tension in the cable (i.e. no force transmission),

- B- when the cable is hanging from the hawse-pipe at angle other than 90[°] and its part is stretched over the bottom, the slope angle of the hanging piece at the seabed equals zero,
 C- when the whole length of cable is hanging and the bottom slope angle is non-zero.

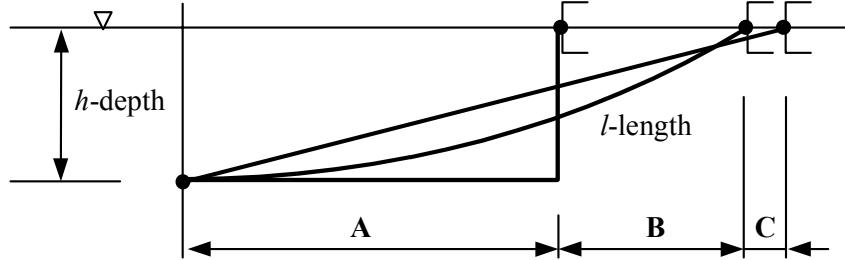


Fig. 1. Ranges of anchor cable behaviour

Defining the following variables (h - water depth, l - cable length):

$$x_{\min} = l - h = h \left(\frac{l}{h} - 1 \right) \quad (5)$$

$$x_{\text{med}} = a_{\text{med}} \sinh^{-1} \left(\frac{l}{a_{\text{med}}} \right), \text{ where } a_{\text{med}} = \frac{l^2 - h^2}{2h} = 0.5h \left[\left(\frac{l}{h} \right)^2 - 1 \right] \quad (6)$$

$$x_{\max} = \sqrt{l^2 - h^2} = h \sqrt{\left(\frac{l}{h} \right)^2 - 1} \quad (7)$$

the above ranges A , B , and C are established as below (the water depth is considered at a first approximation also as the hawse-pipe/seabed distance):

Range a ($0 \leq x \leq x_{\min}$):

$$\begin{cases} a[m] = 0 \\ \alpha_D[^\circ] = 0 \\ F_{ANR}[N] = qag = 0 \end{cases} \quad (8)$$

Range B ($x_{\min} < x \leq x_{\text{med}}$):

Let l_1 and l_2 be the lengths of the cable lying and hanging appropriately ($l_1 < x_{\min}$, $l_2 > h$). The cable statics is then represented by:

$$\left\{ \begin{array}{l} a \left[\cosh\left(\frac{x-l_1}{a}\right) - 1 \right] = h \\ a \sinh\left(\frac{x-l_1}{a}\right) = l_2 \\ l_1 + l_2 = l \end{array} \right. \quad \text{or} \quad \left\{ \begin{array}{l} a \cosh\left[\left(\frac{x-l+l_2}{a}\right) - 1\right] = h \\ a = \frac{l_2^2 - h^2}{2h} \\ l_1 + l_2 = l \end{array} \right. \quad (9)$$

The main catenary parameter a could be numerically solved now from the following non-explicit equation:

$$a \cosh^{-1}\left(\frac{h}{a} + 1\right) - (x-l) - \sqrt{2ah + h^2} = 0 \quad (10)$$

Adopting non-dimensional variables:

$$\bar{a} = \frac{a}{a_{\text{med}}} \in \langle 0,1 \rangle, \quad \text{and} \quad \bar{x}_1 = \frac{x - x_{\min}}{x_{\text{med}} - x_{\min}} \in \langle 0,1 \rangle \quad (11)$$

the solution of (10) is given graphically in Fig. 2. as being solely dependent upon the l/h ratio (so-called cable scope) irrespectively of the absolute values either of water depth h or cable length l . This chart is thus universal for all ships and is saved in a ship manoeuvring model kernel as a lookup table speeding up the real-time calculations.

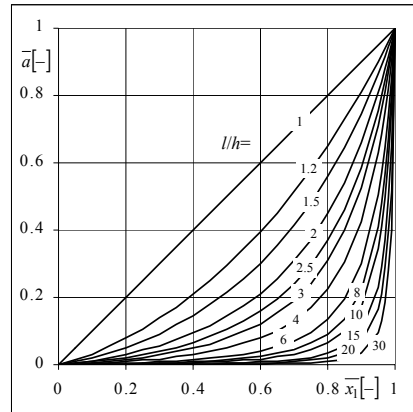


Fig. 2. Anchor forces for the partially lying cable (range B)

In a compact form one gets finally:

$$\begin{cases} a[m] = a_{\text{med}} \bar{a}(\bar{x}_1, l/h) \\ \alpha_D [^\circ] = 0 \\ F_{ANR} [N] = qag \end{cases} \quad (12)$$

Range C ($x_{\text{med}} < x < x_{\text{max}}$):

The shape of anchor cable in this zone is governed according to:

$$\begin{cases} \sinh\left(\frac{x_2}{a}\right) - \sinh\left(\frac{x_1}{a}\right) = \frac{l}{a} \\ \cosh\left(\frac{x_2}{a}\right) - \cosh\left(\frac{x_1}{a}\right) = \frac{h}{a} \\ x_2 - x_1 = x \end{cases} \quad (13)$$

where x_1 and x_2 are the anchor and hawse-pipe abscissas from a virtual catenary extremum (somewhere to the left and below the anchor location). The relationships (13) can be simplified to:

$$\begin{cases} \tanh\left(\frac{x_2 + x_1}{2a}\right) = \frac{h}{l} \\ 2 \sinh\left(\frac{x_2 - x_1}{2a}\right) \cosh\left(\frac{x_2 + x_1}{2a}\right) = \frac{l}{a} \\ x_2 - x_1 = x \end{cases} \quad (14)$$

which in turn leads again to a non-explicit equation of a as the unknown:

$$2a \sinh^{-1}\left(\frac{x_{\text{max}}}{2a}\right) - x = 0 \quad (15)$$

It appears that solution of (15) can be presented in an analogic manner to Fig. 2. The non-dimensional ship horizontal displacement is determined nevertheless by:

$$\bar{x}_2 = \frac{x - x_{\text{med}}}{x_{\text{max}} - x_{\text{med}}} \in \langle 0,1 \rangle \quad (16)$$

The equation (15) results in a family of curves (l/h dependent ones) as displayed in Fig. 3.

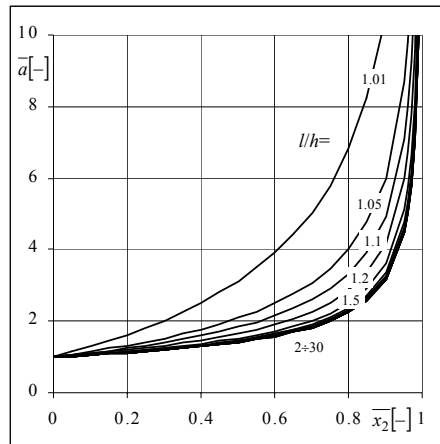


Fig. 3. Anchor forces for the wholly hanging cable (range C)

However both \bar{a} and a are unlimited in the range C (and thus in Fig. 3) while x is approaching x_{\max} . For a more refined algorithm, Fig. 2 and 3 can be easily combined together. The bottom slope angle and the anchor force read ultimately as follows:

$$\begin{cases} a[m] = a_{\text{med}} \bar{a}(x_2, l/h) \\ \alpha_D [^\circ] = \tan^{-1} \left[\sinh \left(0.5c - \sinh^{-1} \left(\frac{h}{2a \sinh(0.5c)} \right) \right) \right] , \quad c = 2 \tanh^{-1} \left(\frac{h}{l} \right) \\ F_{ANR} [N] = qag \end{cases} \quad (17)$$

It should be mentioned at the end that sizes of ranges A, B, and C are very contrasting to each other (as outlined in Fig. 1)- e.g. when the water depth is 40[m] and the cable scope l/h equals 3:

- A: $x_{\min} = 80[m]$,
- B: $x_{\text{med}} - x_{\min} \approx 30.9[m]$,
- C: $x_{\max} - x_{\text{med}} \approx 2.2[m]$.

ANCHOR DRAGGING RELATED TOPICS

So far it has been assumed that the horizontal component of the anchor cable tension F_{ANR} is within the anchor holding force F_{HP} modelled as:

$$F_{HP} = gm_{ANR}c_{HP}(\text{'soil type', } \alpha_D) \quad (18)$$

where: m_{ANR} - anchor mass [kg],
 c_{HP} - non-dimensional constant expressing the anchor holding force in anchor weight units (usually referring to the weight in water), e.g. in case of the Hall anchor and null bottom slope angle α_D the c_{HP} is about 3.5.

More exact relationships for c_{HP} can be found in literature, especially with regard to a very important effect of the bottom slope angle (as reducing drastically the anchor holding power).

In logic terms, two cases of ship behaviour shall be actually distinguished:

$$\begin{cases} F_{ANR} \leq F_{HP} & - \text{ anchor holding} \\ F_{ANR} > F_{HP} & - \text{ anchor dragging} \end{cases} \quad (19)$$

The ship movement while the anchor is holding the ground is restricted to a relatively small area determined by the water depth, anchor cable length and tension in the latter. The anchor cable works in such circumstances like follows.

Under external excitations (wind, current, waves, propeller thrust) the ship is travelling according to her dynamics (inertia) and thus changing the hawse-pipe (bow) position. The cable begins to stretch and develop a horizontal counter-force. At last at some point the force equilibrium is achieved and the ship is steady.

Concerning the anchor dragging, if the anchor holding force is overcome the anchor itself starts to move as well. Many more or less approximate methods are available to simulate the anchor (and the captured soil) dynamics in such circumstances. It seems that the anchor inertia maybe disregarded in view of relatively high anchor cable tensions and thus the anchor position in each numerical integration step is evaluated by the force balance condition (the static approach):

$$F_{ANR} = F_{HP} \quad (20)$$

which means that F_{HP} is the upper limit for the horizontal projection of the anchor cable tension F_{ANR} . Other models of the anchor dragging can be found e.g. in [Brook and Byrne, 1984].

THREE-DIMENSIONAL (3D) CONCEPT OF ANCHOR FORCES

Let's take a ship-fixed right handed cartesian system of coordinates as in Fig. 4 (z-axis points downwards).

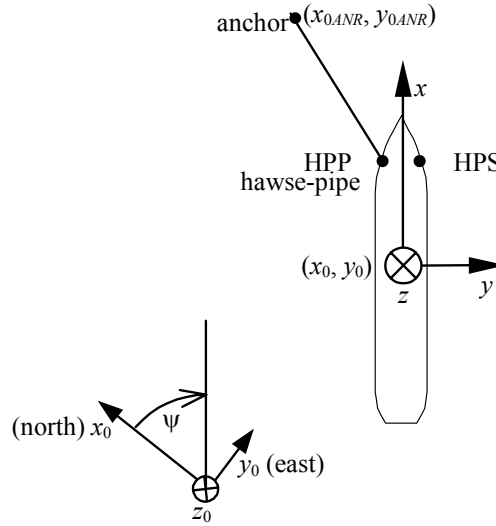


Fig. 3. Earth and ship body coordinates

The port and starboard hawse-pipe are marked in Fig. 4 by 'HPP' and 'HPS' correspondingly. Their locations in the ship reference system are:

$$\mathbf{r}_{HPP} = [x_{HPP}, y_{HPP}]^T, \quad \mathbf{r}_{HPS} = [x_{HPS}, y_{HPS}]^T \quad (21)$$

Both the ship and anchor positions on the earth are denoted by:

$$\mathbf{r}_0 = [x_0, y_0]^T, \quad \mathbf{r}_{0,ANR} = [x_{0,ANR}, y_{0,ANR}]^T \quad (22)$$

In the below derivations only the port hawse-pipe is considered as the starboard case is quite identical here ($y_{HPS} = -y_{HPP}$).

The anchor placement on the seabed is now expressed in ship body axes by:

$$\mathbf{r}_{ANR} = \begin{bmatrix} \cos \psi & + \sin \psi \\ - \sin \psi & \cos \psi \end{bmatrix} (\mathbf{r}_{0,ANR} - \mathbf{r}_0) \quad (23)$$

where ψ is the ship heading.

The horizontal distance between the anchor and the port hawse-pipe, being substantially the 'x-coordinate' of the previously analysed 2D concept, and the cable horizontal direction are written according to:

$$\mathbf{r}_{ANR-HPP} = \mathbf{r}_{ANR} - \mathbf{r}_{HPP} \quad (24)$$

$$x = |\mathbf{r}_{ANR-HPP}| \quad (25)$$

$$\gamma_{ANR} = \tan^{-1} \left(\frac{(\mathbf{r}_{ANR-HPP})_y}{(\mathbf{r}_{ANR-HPP})_x} \right) \in \langle -180^\circ, 180^\circ \rangle \quad (26)$$

Finally, the anchor cable produced surge/sway forces and yaw moment are:

$$\begin{cases} \mathbf{F}_{ANR} = \begin{bmatrix} F_{xANR} \\ F_{yANR} \end{bmatrix} = \begin{bmatrix} \cos \gamma_{ANR} \\ \sin \gamma_{ANR} \end{bmatrix} \underbrace{F_{ANR} \left(x, \frac{l}{h} \right)}_{\text{see 2D concept}} \\ \mathbf{M}_{ANR} = \mathbf{r}_{HPP} \times \mathbf{F}_{ANR} \end{cases} \quad (27)$$

ENERGY ABSORPTION BY ANCHOR CABLE - CASE STUDY

The current practise of designing both anchor and anchor cable for merchant ships (see e.g. [OCIMF, 1982], [IACS, 1999]) is that the anchor holding power is much less than the cable breaking load. This way under rough weather conditions the anchor is allowed intentionally to drag before the cable will break. Taking a closer look at the [IACS, 1999] standards, concerning among others the anchor strength data, there is one more issue- the anchor should sustain a damage first, just before 'something' happens to the cable. The outcome is that at least under static (slowly rising) external excitations and rather normal conditions of the bottom soil the anchor cable and the anchor itself may not be broken. The situation changes if the anchor encounters a hard object (e.g. a rock) and gets clutched firmly into it. However, the latter is somewhat rare occurrence, where the widely reported experimental data on the sea bottom holding power (in terms of anchor weight units) are not applied. A real-time simulation of such a phenomenon should be addressed stochastically by means of a risk (reliability) concept, also with regard to the fact that the anchor and the cable are generally considered improperly as new ones i.e. without noticeable wear and tear.

For a difficulty, the accident statistics of the anchoring equipment generally reports a distribution of failure causes (a major one is always the 'human factor') in the total sum of emergencies, rarely stating the whole number of successful operations, e.g. [OCIMF, 1982], [King, Ojo, 1984]. The very important circumstances of accidents are also rarely available (even to surveyors or superintendents) to perform a detailed analysis and draw sound conclusions.

Moreover, a bit unclear though very important case is when the ship is subject to suddenly developing (dynamic) loads e.g. wind gusts, wave action, rapid alterations in a direction of wind/wave/current. Due to the huge ship inertia involved in any ship motion it is purposeful to examine directly the resulting motions rather than the external disturbances. The latter approach encompasses also an adequate treatment of those cable dynamic loads (tensions) as being e.g. a result of 'quick' arresting an excessive yaw and/or surge velocity even in a calm weather. After all, the strength requirements of [IACS, 1999] are generally valid for static proof tests and there is surprisingly little amount of data in open literature concerning the dynamic strength of anchor equipment components. Moreover, the most of studies have dealt so far (for the sake of solution availability) with the anchor/cable breaking based on more or less justified quasi-static assumptions.

The anchor and/or cable breaking is a part of the ship manoeuvring mathematical model which requires anyhow a further research.

Below is given an example of the ship kinetic energy absorption during the absence (for simplicity and soundness) of wind/wave/current/shallow water effects when the whole length of cable is already paid out. The anchor and/or cable breaking possibilities are also investigated at the same time- nevertheless in the quasi-static way. The anchor cable is deemed initially slack and the anchor believed to be 'fixed to the ground' (i.e. no dragging is allowed which could safely put down the ship speed without endangering the anchor equipment). Figures 2 and 3 are used to calculate the cable potential energy increase (as product of force and horizontal displacement) to which the ship energy is transformed, of course within limits of the anchor maximum loads. Tab. 1 summarises data of three real-world tankers being different in deadweight and analysed hereafter. If some anchor-related data are missing for a particular ship model, due to different reasons, a reference is recommended to e.g. [Gurovic et al., 1975], [IACS, 1999] (world-wide standards rather narrowly complied with), or other design books of ship deck equipment.

Table 1. Ship particulars.

deadweight[t]- DWT	6k	87k	135k
mass[t]	9000	104000	155000
anchor weight in air[t]	3.3	11.1	12.0
anchor holding factor - average (in anchor weight units) [-]	3.5	3.5	10
anchor type [-]	Hall (SS)	Hall (SS)	AC14
anchor proof load [t]	51	107	130
anchor break load [t]-' BL_{ANR} ', estimated	73	153	186
cable link dia.[mm]	50	81	97
cable type [-]	Gr. 2	Gr. 3	Gr. 3
cable max. length [m]	250	343	370
cable proof load [t]	98	338	468
cable break load [t]	137	482	669
cable unit weight in air [kg/m] - ' q_{air} '	53.9	145.5	210

Taking the anchor breaking load as a more critical one, the maximum allowable value of the catenary parameter a (also in relative measures) can be specified (see Tab. 1):

$$a_{BL} = \frac{'BL_{ANR}'}{0.87'q_{air}'}, \quad \overline{a_{BL}} = \frac{a_{BL}}{a_{med}} \quad (28)$$

For the water depth 40[m] and three representative cable scopes i.e. the cable length equal to 1.5, 3, and 6 times the water depth, the outcome of eqs. (28) is showed in Tab. 2. It appears that the anchor failure is possible only just in the range C of the cable behaviour (Fig. 3)- $\overline{a_{BL}}$ is greater than 1.

Table 2. Non-dimensional maximum allowable a parameter- $\overline{a_{BL}}$.

DWT		6k	87k	135k
a_{BL}		1557	1209	1018
$h/l= 1.5$	$a_{med}= 25$	≤ 2	≤ 8	≤ 1
3.0	160	10	7.6	6.4
6.0	700	2.2	1.7	1.5

The initial kinetic energy according to two characteristic ship forward velocities, namely 0.25 and 0.50 [m/s], is computed in Tab. 3. The energy absorbable in both ranges of the anchor cable operation, namely B and C (see before), up to the anchor breaking load is presented in Tab. 4.

Table 3. Ship kinetic energy [kJ]

DWT	6k	87k	135k
v_x [m/s]= 0.25	281	3250	4844
0.50	1125	13000	19375

Table 4. Cable energy absorption [kJ] within anchor maximum loads

DWT	6k	87k	135k	6k	87k	135k
$h/l= 1.5$	70	190	274	173	468	675
3.0	409	1105	1595	708	1873	2686
6.0	1143	3086	4454	1526	3869	5363
	Range <i>B</i>			Range <i>B+C</i>		

It could be concluded that 6kDWT ship is able to come safely to rest from the initial 0.5[m/s] velocity already in the range *B* providing the cable scope equal 6. The lower velocity, e.g. 0.25[m/s], a complete stoppage is achieved at lower cable scopes, here of order 3. The larger ships can be decelerated only from velocity 0.25[m/s] and if the cable is long enough i.e. 6 times the water depth. In other conditions, not enumerated above, the anchor will be damaged.

As mentioned before, this analysis is rather static and excludes the usual anchor dragging during a cable deployment phase. So this is an extreme situation and does not mean at all that the anchor would fail actually. Though the cable break load is 2-3 times higher than those corresponding to the anchor itself, the safe anchoring velocity for larger ships seems to remain however nearly the same to some extent if the cable maximum load instead of the anchor related one is incorporated into (28). Such an increase in the cable tension is without significant effect on the cable potential energy.

The obtained restrictions in velocity for larger ships are more serious than those quoted e.g. in [Nowicki, 1999]. However, it is difficult to make a deeper reference to the latter data as detailed computational assumptions are not published at all. It should be also pointed out that safe anchoring velocities cited elsewhere in literature are sometimes referred to quite different criteria- like e.g. a prohibition of anchor dragging or those pertaining to specific rough weather conditions ([Brook and Byrne, 1984]). Such data when applied for comparison purposes should be considered with care.

FINAL REMARKS

In the present study the sea current profile and wave effects upon the anchor cable performance have been omitted as they are relatively small, see e.g. [Polderdijk, 1985], and the actual current/wave estimations are generally unreliable (uncertain). Further research should go towards more adequate modelling of all anchor equipment failures and the associated windlass technical operation, since many wrong conclusions can be drawn with regard both to the anchor performance limits and anchoring procedure itself. This concerns the anchor application not only for a ship stay but for her manoeuvring ability improvement as well. Particular aspects of modelling could refer e.g. to a cable deployment technique (i.e. the windlass operation dynamics) during the anchoring initial phase when the whole length of anchor cable has not been yet paid out- see e.g. [Brook and Byrne, 1984]. This is required among others for an appropriate simulation of anchor early dragging due to still large cable bottom slope angles α_D at that moment.

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