POLISH ACADEMY OF SCIENCES
POLISH NAVIGATION FORUM

ANNUAL of NAVIGATION

Cezary Specht

Availability, Reliability and Continuity Model of Differential GPS Transmission

Address for correspondence

Instytut Nawigacji i Hydrografii Morskiej
Akademia Marynarki Wojennej
ul. Śmidowicza 69
81 - 103 Gdynia
afel@amw.gdynia.pl

Published
in affiliation
with the Gdańsk Branch
of the Polish Academy of Sciences

Citation
When citing papers from this volume the following reference should be used:
Author(s), Title, Annual of Navigation, N^{o} 5, 2003, Pages 85

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INTRODUCTION, AIM AND SCOPE OF WORK

The problem of fixing position coordinates for navigational needs considered only in terms of measurement error seems to have already been solved in a global scale. Its realization with higher or lower precision is only a function of the technical solution adopted. Therefore, other, equally important, although often omitted, exploitation parameters of navigation systems become crucial. These are: reliability, availability and continuity. We may even state that reliability [Farrell J.; Graas F., 1992] as well as characteristics of reliability origin, such as: availability [Ghashghai E., 2000] and continuity [Nayak R. et al, 2000] considered with regard to different levels of radionavigational positioning systems structures, seem to be one of the major directions to be investigated in the field of navigation. In general, the quality status of a modern navigation system is described by a set of characteristics derived from uniformly evaluated parameters, which should enable the unambiguous system benchmark.

System providers anticipate the needs of maritime users and publish the results of investigations concerning the individual functional characteristics of certain navigation systems [SPS, 2001]. Their forecasts are mainly based on statistical analysis and long term observations. Statistics [IALA, 1989] and parametric evaluation of empirical distributions are fundamental methods used for investigation in this field. Numerous rules and principles have been formulated this way. It is worthwhile underlining the fact that the structures validated this way are rather complicated (system, signal, space segment, etc.) ones, for which total credible probabilistic models, which would take into account all additive components, have not yet been worked out.

As a result of users' demand for information, the empirical approach to multi-criteria evaluation of navigation system performance have become common in defining the criteria mentioned above with regard to different reliability structures (system, radio-link, space segment). This approach does not support the search for mutual inter-element relations, either logical or numerical, often leading to doubts related to interpretation of fundamental notions. The most essential example to notice is a terminological evolution in "SPS Signal Specification", versions dated 1993 [SPS, 1993] and 2001 [SPS, 2001], which is the main standard description of GPS system for civilian users.

The complexity of processes and lack of faithful mathematical models justify the experimental methods used in navigational systems investigations, which obviously influences their characteristics. Already existing general and detailed mathematical theory must not be omitted here, as it combines such parameters as: availability, reliability and continuity of navigational system into common relations based on reliability. As a result, we get a limited range of interpretation with regard to the values presented and absence of possibility of integration all individual statistical models into more complex structures. It is often necessary to link the results over freely chosen time periods (reliability, continuity) to obtain estimations related to various types and kinds of navigation systems. We may conclude that statistical methods, not the probabilistic ones, constitute the essential tools for parametrical evaluation of modern systems in navigation.

Undoubtedly, the probabilistic model to describe performance and malfunction states has not been applied in this field up till now, due to the unsatisfactory truthfulness of the existing models and their complexity. The problems of reliability, availability and continuity of system performance, including DGPS, are nowadays the key parameters for service quality. They constitute reliable information related to usefulness of the system (structure) in certain local conditions, which include the characteristics of equipment used at DGPS reference station and the solution of telemetric radio link.

In the case of differential GPS systems constituting a serial reliability structure on general level (GPS and radio-link), their reliability, availability and continuity are dependent on both components. Typical reliability characteristics of GPS have been broadly described in world-wide literature [SPS, 1993; SPS, 2001] by means of long-term statistics. However, up to this day no analytic relations have been presented with regard to the telemetric differential system and direct connections with signals of space segment [RNAV6, 1996]. Then we may pose a question, being a genesis of the problem to investigate (hypothesis): **Is it possible, and to what extent, to predict (calculate, foresee) the reliability, availability and continuity of GPS differential transmission at the stage of planning the system or before a specific measurement campaign?**

This is a key problem for any group using differential techniques dependent on both signals but it may also be useful as an analytical tool before establishing reference stations for water basins or land areas. The circumstances above persuaded the author to formulate the main scientific aim of this dissertation as follows: **to develop a mathematical model of reliability, continuity and availability of transmission for differential real time GPS system to be used in navigation.** The essence of the work will be to develop a mathematical model of the process of receiving differential corrections broadcast according to RTCM-104 standard, taking into account a probabilistic nature of error in differential telemetric transmission used in selected DGPS radio-links.

Although the problem formulated above defines very precisely the scope of scientific work, it would be impossible to solve without elaboration of a general and particular mathematical theory of reliability, continuity and availability of navigation systems. It will provide for unambiguous definitions of concepts as well as mathematical relations between individual criteria. The general theory constitutes an introductory problem of the dissertation, and its solution will enable clear definition of the main scientific problem.

To solve the problem stated above, it is necessary to solve a series of fundamental research problems. The most significant ones of the scientific-research nature are as follows:

1. Comparative analysis of terminology related to the criteria considered - based on study of literature. Preliminary discussion of the terminology concerned [Specht, 2002c] has shown significant differences in interpretation of terms discussed, as well as some shortcomings in contemporary methods of their evaluation.
2. Elaboration of general and particular (for exponential distributions of lifetimes and times of failures) mathematical models of navigation system their reliability, continuity and availability of service. This issue is the initial research aim. Mathematical modeling of reliability processes with restoration will be the essential investigation method.
3. Description of the reliability structure for DGPS, components of the system, determination of functional substructures, including the process of differential transmission, the description of existing limitations for conducted theoretical considerations. These are problems of structural analysis and system modeling.
4. Elaboration of mathematical models of reliability, continuity and availability of differential GPS transmission together with chosen indexes of reliability. This will constitute the main scientific nucleus of the dissertation. Its realization is based on the model of differential GPS data transmission coded with the use of RTCM 104 standard.
5. Validation of simultaneous use of two or more DGPS reference stations (redundancy) as well as its effect on the characteristics considered. The issue in question will be considered with the application of reliability theory of systems built with redundancy structures.
6. Development of author’s theoretical concept of DGPS system with synchronous, multi-channel telemetric link. This concept was presented [Specht 2002a] and found by International Association of Lighthouse and Aids to Navigation Authorities (IALA) Committee of Radionavigation to be one of possible future directions for DGNSS development [RNAV17, 2002].

Utility of the considerations above with regard to the subject, indicate the existence of tasks displaying features of typical implemental nature. These are:

1. Software tools designed for navigators, to provide for numerical validation of certain reliability features.
2. Evaluation of coverage areas for DGPS signals from reference stations in the South Baltic with regard to availability, reliability and continuity. Calculated ranges should be based on measured signal strengths and developed models.
3. The proposal for possible modification of maritime differential emissions with the application of synchronized multichannel access, to improve parameters of the DGNSS system.

In order to solve the problems formulated above, the dissertation is divided into four chapters:

Chapter I. „Terminological synthesis of concepts: availability, reliability and continuity in literature on navigation" – covers the study of literature in the field of navigation with regard to the three concepts discussed. They will be characterized with respect to both their nature, and methods of determination navigational purposes. As a result of the analysis, drawn will be conclusions related to the existing differences in perception of the terms, narrowing down of their meaning in navigation, as well as constraints of the models recommended.

Chapter II. „General model of the availability, reliability and continuity" - here the definition of navigational structure will be constructed. This definition is indispensable for the process of system modeling. The mathematical model of alternative process with restoration will be worked out, based on the mathematical description of navigational systems' states. As a result, mathematical relations of availability, reliability and continuity will be formulated, describing them on a general level. The same model will enable the transposition to time-related formulas, typical of a number navigation processes, i.e. processes - where distributions of lifetimes and times of failures are expressed by exponential equations.

Chapter III. „Classical differential GPS systems"- the main thesis will be considered here and include structural models of differential DGPS transmission, providing, in this way, for making a separate individual navigational structure - differential GPS transmission. Consequently, models and measures of reliability, availability and continuity will be proposed for transmission of the RTCM messages (type 1 or 9-3) commonly used in navigation.

Chapter IV. „Differential GPS system network" – presents the validation method of DGPS systems in the areas where multiple coverage of several radiobeacons and reference stations exists. An analogous method of modelling as in the classic systems is proposed. A general validation of transmission methods for differential GPS will also be proposed in this chapter.

The issues considered in this dissertation, will exemplify a group of single element models - GPS telemetric transmission - within the complex structure of differential GPS. We can assume that a new scientific area will be opened for future deliberations in the fields related to:

1. Optimisation of decision making process when deploying maritime reference stations (DGPS) or land based (permanent GPS/RTK), taking into account: user reliability features, minimum requirements for the basin (area), maximization of task effectiveness and many others.
2. Advantages of multistation system nets in the aspect of position accuracy and operation reliability, also other typical features influencing exploitation.

The mathematical model proposed in this dissertation, fills the gap concerning methodology used to determine the magnitude of reliability, continuity and transmission availability in differential GPS systems on surveying level. Thus, more effective way of use in navigation has been provided. General and particular theory introduced to describe navigation systems by means of reliability characteristics enables theoretical considerations over individual elements and easy integration into more complex structures.

Chapter 1

TERMINOLOGICAL SYNTHESIS OF CONCEPTS: AVAILABILITY, RELIABILITY AND CONTINUITY IN LITERATURE ON NAVIGATION

1.1. The comparable criteria of navigation systems

The parametric assessment of navigation systems during the last decade has been the most common way of their classification with regard to their quality. Within the scope of this evaluation critical space is provided. This space is very closely related to the navigation requirements set for its various forms. The comparable criteria of navigation systems are often discussed in world literature [ERP, 1996; FRP, 1999] as well as in Polish publications [Kopacz Z. et al, 1996]. These criteria have been re-estimated as a result of technological developments and the needs of navigation process, but are accompanied by the change in their hierarchy. This re-estimation reflects the technical developments in the form the ways chosen and increased number of research projects conducted by various research institutions in investigation directions that are aimed at highlighting selected exploitation features.

The analysis of the criteria allows distinguishing three main groups, which are identical with particular phases of positioning systems development over the years (Fig. 1.1), and they are as follows:

- Positioning criteria - system characteristics in quality of position fixing. They have in their scope 3 types of accuracy (predictable, repeatable, relative) as well as fix rate and dimension, system: capacity, ambiguity, and coverage.
- Reliability criteria – they form a separate group of indicators with reference to characteristics of systems exploitation. Reliability, availability and continuity are among them.
- The safety of exploitation criteria – their function is to give the user current information about the quality (status) of operating system allowing for the proper level of their utility. So far, integrity, the only criterion belonging to this group, has been characterised by a wide range of variables such as: time to alarm, the probability of false alarm etc. [Ober P. B., 1999].

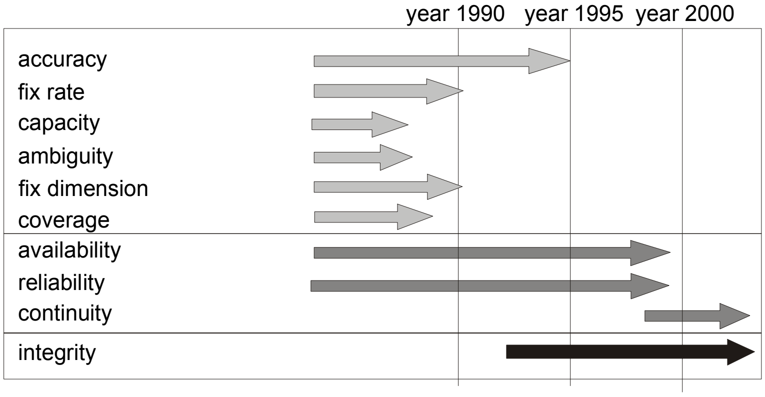

Fig. 1.1. The importance of the navigation systems comparable criteria during the last years

By observing the changeable nature of the comparison criteria, it is difficult not to notice their direct relationship with each of the phases of the evolution of satellite positioning systems (GPS, Glonass), which are dominant in the contemporary navigation. The first of these groups – positioning criteria – were the main exploitation characteristics till the 1990s [IALA, 1990]. The previous systems were usually of poor precision as well as fix rate (Omega, Loran, Decca, Transit) and they lacked the fully ambiguity of measurement (Loran, Decca). Consequently, these positional measurements were the main, if not the only, characteristics of such systems. When, in the mid-1990s, the GPS system became a fully operational fulfilling almost all navigation requirements of the users looking for precision of positioning, the research connected with positioning criteria was combined into the group of criteria of reliability theorem origin. At first the availability and reliability [IALA, 1989] and then continuity [FRP, 1999] described with reference to various systems, structures or functional blocks, allowed evaluating the capacity of the systems to non-failure performance. By doing so, they also made its characteristics capable of being compared with efficiency and economic factors.

The third group to do with safe of exploitation is regarded as a very quickly developing part of research of the beginning of the 21st century in the navigation. No doubt the integrity be can compared with current functional diagnosis which seems to be synonymous with modernity in all of the contemporary navigation systems.

1.2. Reliability and availability

The analysis of the terms: reliability and availability in navigation literature shows that there are two clearly distinguishable periods. The first one dates back the end of 80-ties when the positioning characteristics were dominated. The second one starts from the beginning of 90-ties where multicriterial analyses of the navigational systems were developed.

1.2.1. Literature till the year 1990

The term reliability till the end of the 1980s was present in the navigation usually as the indicator of the evaluation of the operating condition of appliances both in Eastern [Zarudnyj B. N., 1973] as well as in Western literature. In the most extensive elaboration [IALA, 1989], being in effect also in all country, and concerning reliability and availability of the system of aids, the following information can be read:

Reliability [IALA, 1989] – is the ability of an aid, or system of aids, to perform a required function under stated conditions for a stated period of time. A wide range of other standard documents also used the recommendation given here. The technical approach to the reliability criterion made the MTBF (Mean Time Between Failures) the unique parameter used hereafter to characterise the reliability, which is also the parameter used in data collecting by most Lighthouse Authorities. The suggested reliability calculating method was based on a range of examples of evaluating MTBF of navigation appliances combined in series and parallel structures, for which the reliability indicators of combined structure MTBFS were respectively [IALA, 1989]:

In the case of system structures connected in series (independent failures of components):

$$\frac{1}{MTBFS} = \frac{1}{MTBF_1} + \frac{1}{MTBF_2} + \ldots + \frac{1}{MTBF_i} \quad , \qquad (1.1)$$

where:

$MTBF_i$ - Mean Time Between Failures of the i -th component,

$MTBFS$ - Mean Time Between Failures of the System,

as well as for two-element passive redundancy, without repair system:

$$MTBFS = MTBF_1 + MTBF_2 \ . \qquad (1.2)$$

Similar analyses for active redundancy, with or without systems were done there.

It should be noted that, despite limiting the term reliability to single indicator – $MTBFS$, in this document there are also formulas related to reliability function and the failure rate indicator for p components with independent failures, connected in series:

$$R(t) = e^{-\sum_{i=1}^{p} \lambda_i t} , \tag{1.3}$$

$$\lambda_s = \sum_{i=1}^{p} \lambda_i , \tag{1.4}$$

where:

$R(t)$ - reliability function of the system,

λ_s - failure rate of the system,

λ_i - failure rate of the i-th component.

In the case of active redundancy parallel structure without repair reliability function was analyzed

$$R(t) = 2e^{-\lambda t} - e^{-2\lambda t} . \tag{1.5}$$

The whole document refers to appliances of optical aids to navigation, including technical elements of known characteristics ($MTBF$ or λ).

The second of the terms discussed here – availability [IALA, 1989] is seen as the probability that an aid or system of aids performing a required function under stated conditions at any randomly chosen instant in time. It is shown by the following formula:

$$A = \frac{MTBF}{MTBF + MTTR} , \tag{1.6}$$

where:

$MTTR$ - Mean Time To Repair,

A - system availability.

The availability is the system evaluation criterion, allowing for stating how well its functions are performed. Thanks to such an understanding of definition, system categories related to the availability were established. Their evaluation is carried out by taking into account the formula (1.6) based on numerous enough measurement test. The method of determining this indicator is obvious as far as single technical appliances or groups of such appliances are concerned. But with reference to navigation systems, which are strongly influenced by the environment factors (propagation circumstances, weather impact etc), this methodology can not be accepted.

1.2.2. Literature after the year 1990

The beginning of the nineties in navigation is the time of the satellite navigation systems domination in positioning. Together with their implementation many documents describing their exploitation appeared. The demand was caused by the necessity to provide the users of the systems with precise information about the properties of the systems. The comparison criterions suggested in [IALA, 1990] including both positional (accuracy, fix rate, ambiguity, fix dimension) as well as exploitation characteristics (coverage, reliability, availability and integrity) became an introduction to multi-criteria assessment of the radionavigation systems.

Taking into account the analysis carried out, the document called 'Global Positioning System (GPS) Standard Positioning Service, Signal Specification' [SPS, 1993] seems to be the most interesting comparable material. This document presents the exploitation characteristics of GPS system made accessible for civilian users. It was revised over the years to appear in 2001 in its final form [SPS, 2001] with selective availability excluded. When comparing the definitions of reliability and availability of GPS service taken from these two references mentioned above, one can read that:

- Service availability [SPS, 1993] – given coverage, the percentage of time over a specified time interval that a sufficient number of satellites are transmitting a usable ranging signal within view of any point on or near the Earth.
- Service availability [SPS, 2001] – defined to be the percentage of time over any 24-hour interval that the predicted 95 % positioning error is less than its threshold for any given point within the service volume.
- Service reliability [SPS, 1993] given coverage and service availability, the percentage of time over a specified time interval that the instantaneous predictable horizontal error is maintained within a specified reliability threshold at any point on or near the Earth.
- Service reliability [SPS, 2001] – the percentage of time over a specified time interval that the instantaneous Signal-in-Space SPS User Range Error is maintained within a specified reliability threshold at any given point within the service volume, for all healthy GPS satellites.

From such a comparison of definitions, the conclusion may be drawn that during almost seven years the definitions have significantly changed. In the case of reliability the lack of system usability was at first described as positioning error [SPS, 1993] being geometrical (DOP's) and precision of the pseudorange accuracy measurement function, but later it became the term referring only to one of its two components - pseudorange measurement error. Consequently, this new definition is not influenced by satellite configuration represented by DOP factors. The term: availability was also affected by a similar change. In the definition of 1993, availability is only related to usability of radio signals reaching the user, whereas the new meaning of this term refers to position solution.

Trying to establish the cause of such a change in meaning of these two terms, it should be noted that the essence of both definitions – the probability or the ratio of functional times to total time being in fact the measurement of probability, have not changed. But the definition of the conditions which are regarded as fulfilled to name the system a correctly functioning one, has changed considerably. Similar conclusions can be drawn by analyzing many standard documents referring to satellite navigation where different forms of availability and reliability are defined. These forms include reliability and availability of: transmission, broadcast, reference station, signal and the user [USCG, 1993] and also for availability we can name: PDOP factor, horizontal or vertical of the service [SPS, 2001].

1.3. Continuity

At the end of the 90s a new criterion appeared - namely the service continuity, which is connected directly with the navigation task carried out and the system used to support it. Continuity is the probability that the specified system performance will be maintained for the duration of a phase of operation, presuming that the system was available at the beginning of that phase of operation [FRP, 1999]. Seeing it as a navigation task, it seems to be a very essential criterion because the scope of its usage refers to specified period when a navigation object is to use the navigation system to perform a set task provided that at the beginning of it (t - time) the system was available. It should be noted that as it happens in the navigation (maritime or air), the task starts when the system is available.

As for the practice, the navigator starts the process (of survey, landing, docking etc.) when the appropriate navigation system is available after a short no operating interval. The definition presented here might seem non-ambiguous as far as its meaning is concerned, but in European literature it can be found as: "continuity is the ability of a system to function within specified performance limits without interruption during a specified period (normally short term). There is no need to include the availability at the beginning of the time period of the operation because if there is no service then the operation will be not commence" [IALA 2001]. Consequently:

$$C = e^{-\frac{CTI}{MTBF}} . \tag{1.7}$$

If $MTBF >> CTI$ then

$$C \cong 1 - \frac{CTI}{MTBF} , \tag{1.8}$$

where:

C - service continuity,

CTI - Continuity Time Interval. For maritime applications CTI equal 3 hours [IALA, 2001].

The definitions presented here are diverse. The main difference is connected with determining the operating condition (availability or lack of it) at the beginning. Then it is questionable the view presented in [IALA, 2001] - as it states that continuity refers to short term reliability.

The synthesis of the meaning of notions discussed above: reliability, availability and continuity was carried out by taking into account the most extensive and formal (standards and recommendations) pieces of navigation literature. The following conclusions are the result of the procedure:

(literature till the year 1990)

- the reliability term was discussed only in their technical aspects of the appliances with the result being the acceptance of the numerical value of MTBF and MTBFS as reliability factors,
- the reliability function and the failure rate are determined by including identical technical elements,
- the literature lacks analysis of other reliability factors,
- the analysis was carried out with the appliances of zero time of restoration,
- the usage of simplified models of operating systems (exponential distributions of lifetimes and times of failures),
- the suggested calculating methodology lacks modification into more complex processes (e.g. alternative with restoration).

(literature after the year 1990)

- reliability and availability refers to different functional structures,
- the definition of continuity is ambiguous,
- the lack of mathematical connection between availability, reliability and continuity,
- vague procedures and methods of determining each of the criteria,
- the measurement of the criteria is based on statistic analysis of empirical measurement data.

The conclusions presented indicate the necessity to develop a unified theory of the navigational criteria under consideration and to determine the relations between them. The next chapter will present a general model, which will be later used for determining: availability reliability and continuity of differential GPS transmission.

<u>Chapter II</u>

GENERAL MODEL OF THE AVAILABILITY, RELIABILITY AND CONTINUITY

2.1. The components and structures of the navigational system

The analysis of the subject literature shows that the characteristics considered – originating the general theory of the navigation systems and devices reliability – are often referred to various functional structures (like a system, signal, radio link etc.) Their numerical representation is mainly obtained in an empirical way and is based on the measurement procedures with a diversified representative sample. The structures difference and the accuracy of the requisite sample length connected with the variable credibility of the determinations result in the lack of any possibility of the direct components adaptation of the reliability theory. Due to this fact, some mathematical correlations connecting various reliability indexes do not have the direct application in assessment of the work process of the navigational system or its components.

To consider the navigational systems on the general level on the basis of this theory it is necessary to precisely consider its reliability structure, which means the process of the series-parallel modelling the relations between components. The estimation of the characteristics of each component states the basis of the further inference related to more complex forms. Taking the remarks mentioned above into consideration, we need to define the notion of the structure components of the navigational system as it enables applying identical measures and reliability indexes.

Let an ordered set:

$$\left(S_1, S_2,..., S_n, S, \psi\right) \tag{2.1}$$

be a mathematical model of a complex navigational system (object), where ψ denotes the function representing the navigational structure of the system defined as follows

$$\psi : S_1 \times S_2 \times ... \times S_n \to S \,, \tag{2.2}$$

where

$$S_1 \times S_2 \times ... \times S_n = \{(x_1, x_2, \ldots, x_n) : x_i \in S_i, i = 1,2,\ldots n\}. \tag{2.3}$$

The structure ψ assigns the system state to the components states. $S_1, S_2, ..., S_n$ mean the components states while S means the system state. The state of component i in the time t is random variable $\xi_i(t)$ taking its values from the set S_i.

In navigational considerations we assign two states to the components and the systems, which are connected with their functioning: '0' if the component or the system is failed and '1' if the component or system is functioning properly:

$$S_1 = ... = S_n = S = B = \{0,1\}. \tag{2.4}$$

Then it is stated that the components and the systems are binary. Due to such formalized notations each of the systems, groups or the single components can be independently modeled or estimated in the availability sense. We emphasize that the physical objects (devices) can be the system components. Different structures can include additional components, whose states are influenced by factors not directly related to technical equipment. It means the possibility of joining, within the framework of the structure, additional components dependent on local weather conditions (optical or radar systems etc.), propagation characteristics of the medium (radionavigation and acoustic systems) or hydrological properties.

It is also worthwhile pointing out that within the framework of modeling structures or systems it is acceptable to define any subsystems incorporating the components subsets of the more complex structures. It carries out to consider the reliability criteria (reliability, availability, continuity) in any subset of S. Fig. 2.1. presents three examples of the navigational structures refer to the DGPS system or to its substructures. First one (a) is a general, binary, series structure of the logical form

$$\psi(x_1, x_2) \equiv x_1 \wedge x_2 = \min(x_1, x_2) \ , \tag{2.5}$$

where x_1, x_2 are binary indicator variables describing the states of components e_1, e_2.

DGPS system has a complex structure, so its characteristics like availability can be obtained only by means of statistical analysis, comprising the representative measurement sample. Due to the complexity of the processes influencing the functioning states (or failure states) existing in both of the components the probabilistic description of the working process is up till now unsolved.

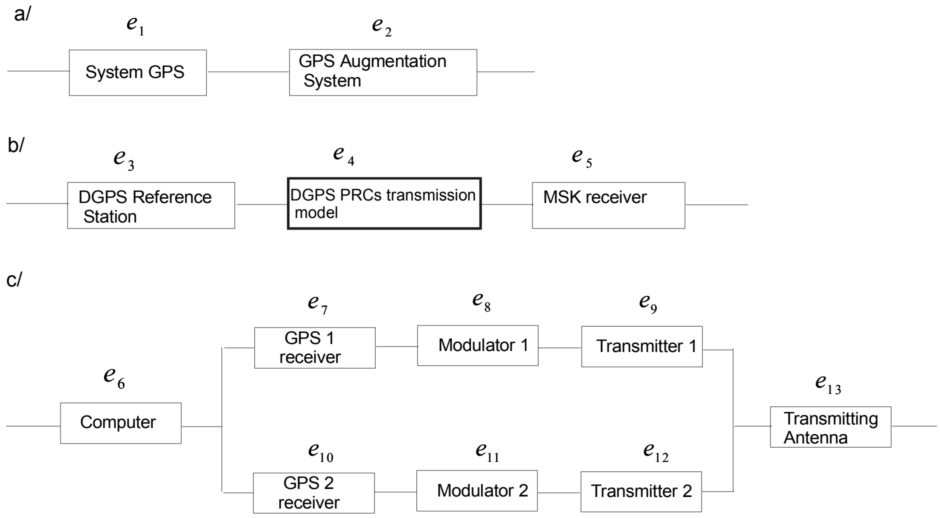

Fig. 2.1. The chosen navigational structures of the DGPS system: a/ the general structure of the system, b/ the structure of pseudorange corrections transmission, c/ the structure of the reference station

The second of the above structures - (b) describes the process of the pseudorange corrections transmission between the reference station and the user receiver. It is a substructure of (a) as it is included in element e_2. It is worthwhile pointing out that components e_3, e_5 of the structure (b) have a character of the technical devices whose reliability characteristics enable determination of various reliability indexes. That structure contains also component e_4 – the model of the pseudorange corrections transmission, which is not a technical equipment. If the pseudorange corrections transmission could be described in an analytical (probabilistic) way it would be possible to obtain analogous characteristics with regard to it as with the rest of the components. As a consequence it enables a mathematical description of the whole structure (b).

The third of the structures - (c) is a typical DGPS reference station model with series – parallel form of the elements:

$$\psi(x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}) \equiv x_6 \wedge [(x_7 \wedge x_8 \wedge x_9) \vee (x_{10} \wedge x_{11} \wedge x_{12})] \wedge x_{13} , \quad (2.6)$$

where:

$x_6, x_7, ..., x_{13}$ - states of the components: $e_6, e_7, ..., e_{13}$.

It constitutes a system of the technical devices with the determined functioning characteristics where modeling of the stages can be based on typical technical qualities [IALA, 1989].

The above analysis leads to tree fundamental conclusions:

1. The structures can contain technical devices and other factors influencing the functioning state.
2. It is admissible to model the availability, reliability or continuity of any structures and any of their components. For the navigational systems it is, for example, possible to consider the availability of the DGPS reference station, telemetric transmission, user's receiver or all of these factors together.
3. For the navigation systems it is possible determine the characteristics such as reliability, availability or continuity on the basis of the probabilistic model of the functioning or failure times for each of the components $e_1, e_2, ..., e_n$.

2.2. The failure process of the navigational system

Now we consider the navigational system operating in time. Let $X_1, X_2, ...$ denote independent, non-negative, with the same distributions random variables representing the lifetimes of the renewal system while random variables $Y_1, Y_2, ...$ correspond to their failure times. Hence, random variables $Z_n^{'} = X_1 + Y_1 + X_2 + Y_2 + ... + Y_{n-1} + X_n$, $n = 1,2,...$ mean the moments of failures while $Z_n^{"} = Z_n^{'} + Y_n$, $n = 1,2,...$ mean the moments of renewal.

We denote the distribution of the system lifetime by

$$P(X_i \le x) = F(x), \quad i = 1,2,... \tag{2.7}$$

and the distribution of the system failure time by

$$P(Y_i \le y) = G(y), \quad i = 1,2,... \ . \tag{2.8}$$

Moreover, we assume that the random variables X_i and Y_i have the finite expected values

$$E(X_i) = E(X), \tag{2.9}$$

$$E(Y_i) = E(Y) \ , \qquad i = 1,2,... \tag{2.10}$$

and variances

$$V(X_i) = \sigma_1^2 \ , \quad V(Y_i) = \sigma_2^2 . \qquad (2.11), (2.12)$$

To exclude the degenerated (deterministic) cases we need to assume the condition:

$$\sigma_1^2 + \sigma_2^2 > 0 . \tag{2.13}$$

2.3. Availability of the navigational system

Supporting the assumptions mentioned above we define the process of operation of a system as

$$\alpha(t) = \begin{cases} 1, & Z_n^{"} \le t < Z_{n+1}^{'} \\ 0, & Z_{n+1}^{'} \le t < Z_{n+1}^{"} \end{cases} \quad \text{for} \quad n = 0,1,... \tag{2.14}$$

and $\alpha(t) = 1$ means that a discussed system is functioning and $\alpha(t) = 0$ represents failure state at time t (Fig. 2.2).

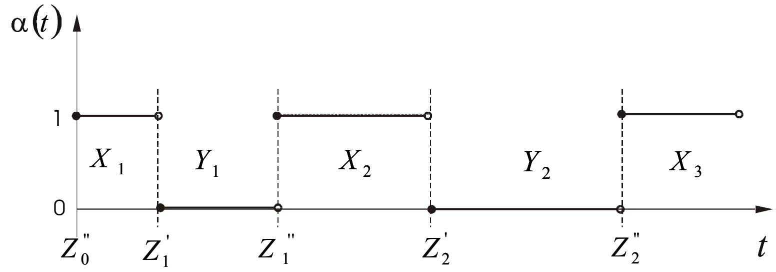

Fig. 2.2. The functioning and failure states of the navigational system

The availability $A(t)$ of a system at the t time can be defined by

$$A(t) = P[\alpha(t) = 1]. \tag{2.15}$$

To find its estimation we consider the following sequence of events: $V_0, V_1,...V_n$ such that

$$V_n = \{Z_n^{"} \le t < Z_{n+1}^{'}\}, \quad n = 0,1,2.... \tag{2.16}$$

The event V_n means that at the time t the system is available (functioning) and up to the time t exactly n - renewals (the changes of the operating state) have occurred. As the events $V_0, V_1,..., V_n$ are exclusive in pairs then

$$P[\alpha(t) = 1] = P\left(\bigcup_{n=0}^{\infty} V_n\right) = \sum_{n=0}^{\infty} P(V_n). \tag{2.17}$$

The graphic interpretation of the availability notion is shown in Fig. 2.3.

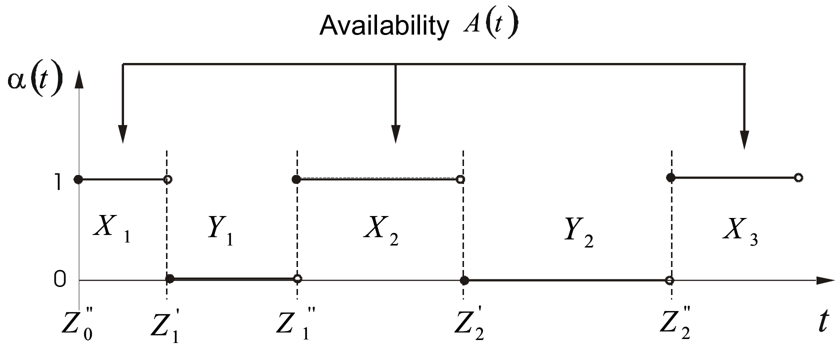

Fig. 2.3. The graphic interpretation of the availability

To find the values $P(V_n)$ we need the following notations

$$S_n' = X_1 + X_2 + ... + X_n \ , \ \ P(S_n' \leq x) = F_n(x) \tag{2.18}$$

and

$$S_n'' = Y_1 + Y_2 + ... + Y_n \ , \ \ P(S_n'' \leq y) = G_n(y), \ \text{for} \ \ n = 1,2.... \tag{2.19}$$

The variables S_n' and S_n'' represent system cumulative times of functioning and failure respectively. The interpretation of random variables S_n' and S_n'' is shown in Fig. 2.4.

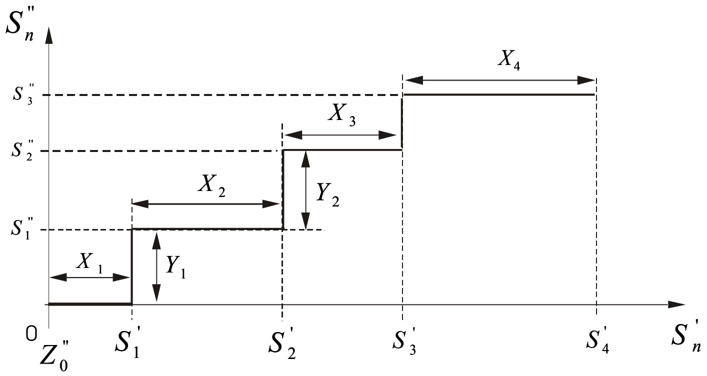

Fig. 2.4. The cumulative lifetime and failure time of the system

The distributions of the lifetime $F_n(x)$ and failure time $G_n(y)$ can be found by n-times convolution operation.

For $n = 2$ we get

$$F_2(t) = \int_0^t F(t - x)\, dF(x), \tag{2.20}$$

$$G_2(t) = \int_0^t G(t - y)\, dG(y). \tag{2.21}$$

For $n = 3$ we have

$$F_3(t) = P(X_1 + X_2 + X_3 \le t), \tag{2.22}$$

$$G_3(t) = P(Y_1 + Y_2 + Y_3 \le t). \tag{2.23}$$

Since from formulae (2.18) and (2.19) it follows that

$$S_2^{'} = X_1 + X_2, \qquad S_2^{''} = Y_1 + Y_2, \tag{2.24}$$

then

$$F_3(t) = P(S_2^{'} + X_3 \le t) = \int_0^t F_2(t - x)\, dF(x), \tag{2.25}$$

$$G_3(t) = P(S_2^{'} + Y_3 \le t) = \int_0^t G_2(t - y)\, dG(y). \tag{2.26}$$

Proceeding in a similar way, for any *n,* we get the final form as

$$F_n(t) = \int_0^t F_{n-1}(t - x)\, dF(x), \tag{2.27}$$

$$G_n(t) = \int_0^t G_{n-1}(t - y)\, dG(y), \qquad n = 2,3\ldots\ , \tag{2.28}$$

where $F_1(x) = F(x)$ and $G_1(y) = G(y)$. Moreover, since $Z_n^{''} = S_n^{'} + S_n^{''}$, then

$$\Phi_n(t) = P(Z_n^{''} \le t) = \int_0^t F_n(t - u)\, dG_n(u), \ \ n = 1,2\ldots, \tag{2.29}$$

where $\Phi_n(t)$ denotes the distribution of the random variable $Z_n^{''}$.

To determine $A(t)$ we separately compute the probabilities of $P(V_0)$ and $\sum_{n=1}^{\infty} P(V_n)$.

As for $t \geq 0$ it is

$$V_0 = \{Z_0^{''} \leq t < Z_{n+1}^{'}\} = \{0 \leq t < X_1\}, \tag{2.30}$$

so

$$P(V_0) = P(X_1 > t) = 1 - P(X_1 \leq t) = 1 - F(t). \tag{2.31}$$

While we find $P(V_n)$, for $n = 1,2...$, using the formula of total probability

$$\begin{aligned} P(V_n) &= P(Z_n^{''} \leq t < Z_{n+1}^{'}) = \\ &= P(Z_n^{''} \leq t < Z_n^{''} + X_{n+1}) = \\ &= \int_0^t P[Z_n^{''} \in [x, x + dx)] \cdot P(X_{n+1} > t - x), \end{aligned} \tag{2.32}$$

then

$$P(V_n) = \int_0^t [1 - F(t - x)] d\Phi_n(x). \tag{2.33}$$

The availability is computed as the sum of probabilities of exclusive events

$$\begin{aligned} A(t) &= P[\alpha(t) = 1] = \\ &= \sum_{n=0}^{\infty} P(V_n) = P(V_0) + \sum_{n=1}^{\infty} P(V_n). \end{aligned} \tag{2.34}$$

Replacing (2.31) and (2.33) to (2.34) we obtain

$$A(t) = 1 - F(t) + \sum_{n=1}^{\infty} \int_0^t [1 - F(t - x)] d\Phi_n(x), \tag{2.35}$$

then

$$A(t) = 1 - F(t) + \sum_{n=1}^{\infty} \int_0^t [1 - F(t - x)] d\left[\sum_{n=1}^{\infty} \Phi_n(x)\right]. \tag{2.36}$$

Finally the availability of the system takes the following form

$$A(t) = 1 - F(t) + \int_0^t [1 - F(t - x)] dH_{\Phi}(x), \tag{2.37}$$

where

$$H_{\Phi}(x) = \sum_{n=1}^{\infty} \Phi_n(x) \tag{2.38}$$

is a function of a renewal stream made of the renewal moments of the navigational system.

The colloquially understood availability of the navigational system can be referred to any time interval in which it is estimated (mostly in an empirical way). However, the most competent representation of the availability is its limiting value, which is defined as an availability coefficient - A. Therefore A can be written as follows

$$A = \lim_{t\to\infty} P[\alpha(t) = 1] = \lim_{t\to\infty} A(t) = \lim_{t\to\infty}\left\{1 - F(t) + \int_0^t [1 - F(t-x)]\, dH_\Phi(x)\right\}. \tag{2.39}$$

Since $\lim_{t\to\infty}[1 - F(t)] = 0$ then we get

$$A = \lim_{t\to\infty}\left\{\int_0^t [1 - F(t-x)]\, dH_\Phi(x)\right\}. \tag{2.40}$$

To estimate the value of A we use the fundamental theorem of the renewal theory [Kopociński B., 1973]:

Theorem 1. (Smith)
If the time between failures has an aperiodic distribution then the renewal function in a simple renewal stream satisfies the condition:

$$\lim_{t\to\infty}\int_0^t g(t-u)\, dH(u) = \frac{1}{\theta}\int_0^\infty g(u)\, du, \tag{2.41}$$

where θ is the finite expected value of distribution $F(x)$ and $g: R \to R$ is an integrable function in the interval $[0,\infty)$.

Using Theorem 1 we find the availability coefficient of the navigational system replacing $1 - F(t)$ by $g(t)$ and $\frac{1}{\lambda} + \frac{1}{\mu}$ by θ obtaining

$$\begin{aligned} A &= \lim_{t\to\infty} A(t) = \lim_{t\to\infty}\frac{1}{E(X)+E(Y)}\int_0^\infty [1 - F(u)]\, du = \\ &= \lim_{t\to\infty}\frac{1}{E(X)+E(Y)}\int_0^\infty R(u)\, du, \end{aligned} \tag{2.42}$$

where $R(u)$ is a reliability function.

From

$$\int_0^\infty R(u)\, du = E(X), \tag{2.43}$$

it follows that

$$A = \frac{E(X)}{E(X) + E(Y)}. \tag{2.44}$$

Analogous relation can be found in [IALA, 1989], namely a statistical estimator of A. It is given there by the formula

$$\hat{A} = availability = \frac{MTBF}{MTBF + MTTR}, \tag{2.45}$$

where $MTBF$ denotes Mean Time Between Failures and $MTTR$ denotes Mean Time To Repair.

Evolving in (2.45)

$$MTBF = \frac{X_1 + X_2 + X_3 + \ldots + X_n}{n}, \tag{2.46}$$

$$MTTR = \frac{Y_1 + Y_2 + Y_3 + \ldots + Y_n}{n} \tag{2.47}$$

we get

$$\hat{A}_n = \frac{\dfrac{X_1 + X_2 + X_3 + \ldots + X_n}{n}}{\dfrac{X_1 + X_2 + X_3 + \ldots + X_n}{n} + \dfrac{Y_1 + Y_2 + Y_3 + \ldots + Y_n}{n}} . \tag{2.48}$$

Due to the strong law of large numbers, with $n \to \infty$

$$\frac{X_1 + X_2 + X_3 + \ldots + X_n}{n} \to E(X) \text{ with the probability equal to 1} \tag{2.49}$$

and

$$\frac{Y_1 + Y_2 + Y_3 + \ldots + Y_n}{n} \to E(Y) \text{ with the probability equal to 1.} \tag{2.50}$$

Hence, for $n \to \infty$

$$\hat{A}_n \to A = \frac{E(X)}{E(X) + E(Y)} \text{ with the probability equal to 1.} \tag{2.51}$$

As we see $\hat{A}_n \approx A$ for the large n.

The well known, from literature, dependence (2.51) is very useful for computing the component availability of the navigational system based on statistical analysis of run of the process: the registered data or the measure campaign. For the navigation process understood generally this statistical estimation [IALA, 1989; SPS, 1993; SPS, 2001] corresponds to an observation in the interval $[0, T]$, which can be rewritten as

$$A_{av} = \frac{1}{T}\int_{0}^{T} A(t)\,dt\,, \tag{2.52}$$

where A_{av} means the average availability of the navigational system in the interval $[0, T]$.

For navigational systems a single-day statistics [SPS, 2001] or thirty-day statistics [SPS, 1993] are typical periods of time.

The division, offered above, of both of the availability measures (2.37) and the availability coefficient (2.44) of the navigational system is a general model referred to the probabilistic characteristics described by means of the distributions and the expecting values (2.9), (2.10), (2.11), (2.12). This model can be applied in mathematical modeling of the navigational structure components, their groups, subsystems or navigational systems.

2.4. Reliability of the navigational system

Analogous to the availability we determine the reliability of the navigational system in the interval of time $[t, t+\tau)$ defined as the survival probability of a system. The graphic interpretation of the reliability notion is Fig. 2.5.

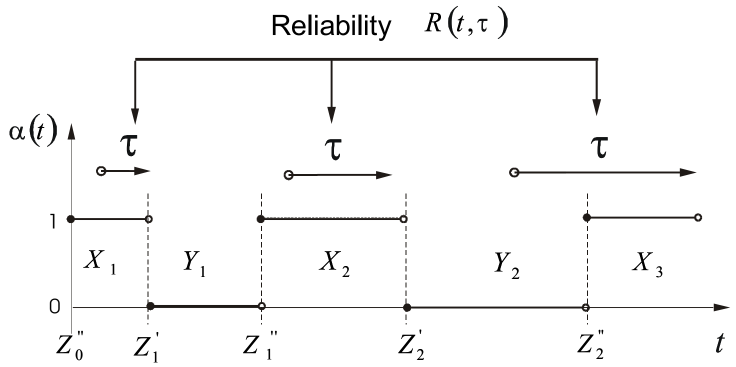

Fig. 2.5. The graphic interpretation of the reliability

To find the reliability value we consider the sequence of events A_n consisting in occurrence exactly n renewals of the availability in $[t, t+\tau)$. Moreover, in that time interval a loss of the availability did not occur. Notice that the events $A_0, A_1, ..., A_n$ are pairwise mutually exclusive. Let us define an event B consisting in fact that in the interval $[t, t+\tau)$ the component is available. Then

$$B = \bigcup_{n=0}^{\infty} A_n \tag{2.53}$$

and

$$\begin{aligned} R(t,\tau) &= P(B) = P\left(\bigcup_{n=0}^{\infty} A_n\right) = \\ &= \sum_{n=0}^{\infty} P(A_n) = P(A_0) + \sum_{n=1}^{\infty} P(A_n). \end{aligned} \tag{2.54}$$

From (2.16), for $n = 1,2...$, we can write

$$A_n = \left\{Z_n'' \le t < t+\tau < Z_n'' + X_{n+1}\right\} \tag{2.55}$$

and hence

$$P(A_n) = P\left(Z_n'' \le t < t+\tau < Z_n'' + X_{n+1}\right). \tag{2.56}$$

From independence of events Z_n'' and X_{n+1} it follows that

$$\begin{aligned} P(A_n) &= \int_0^t P\left(Z_n'' \in [x, x+dx)\right) \cdot P\left(X_{n+1} > t+\tau - x\right) = \\ &= \int_0^t P\left(Z_n'' \in [x, x+dx)\right) \cdot \left[1 - P\left(X_{n+1} \le t+\tau - x\right)\right], \end{aligned} \tag{2.57}$$

then

$$P(A_n) = \int_0^t \left[1 - F(t+\tau-x)\right] d\Phi_n(x). \tag{2.58}$$

Hence

$$\begin{aligned} R(t,\tau) &= P(A_0) + \sum_{n=1}^{\infty} P(A_n) = \\ &= P(X_{n+1} > t+\tau) + \sum_{n=1}^{\infty} \int_0^t \left[1 - F(t+\tau-x)\right] d\Phi_n(x) = \\ &= 1 - P(X_{n+1} \le t+\tau) + \int_0^t \left[1 - F(t+\tau-x)\right] d\left[\sum_{n=1}^{\infty} \Phi_n(x)\right]. \end{aligned} \tag{2.59}$$

Taking into consideration (2.38) we obtain

$$R(t,\tau)=1-F(t+\tau)+\int_0^t[1-F(t+\tau-x)]\,dH_\Phi(x)\,. \tag{2.60}$$

Similarly to the availability we estimate the limit

$$\lim_{t\to\infty}R(t,\tau)=\lim_{t\to\infty}\left[1-F(t+\tau)+\int_0^t[1-F(t+\tau-x)]\,dH_\Phi(x)\right]. \tag{2.61}$$

It is obvious that

$$\lim_{t\to\infty}[F(t+\tau)]=1\,, \tag{2.62}$$

so

$$\lim_{t\to\infty}[1-F(t+\tau)]=0\,. \tag{2.63}$$

Hence and from Theorem 1 it follows that

$$\lim_{t\to\infty}R(t,\tau)=\frac{1}{E(X)+E(Y)}\int_0^\infty[1-F(\tau+x)]\,dx\,. \tag{2.64}$$

Substituting $x+\tau=u$; $0\le x<\infty$ and $\tau\le u<\infty$ it is

$$(x=0)\Rightarrow(u=\tau), \tag{2.65}$$

$$(x\to\infty)\Rightarrow(u\to\infty). \tag{2.66}$$

The final form of the limiting reliability is as follows:

$$\lim_{t\to\infty}R(t,\tau)=\frac{1}{E(X)+E(Y)}\int_\tau^\infty[1-F(u)]\,du\,. \tag{2.67}$$

2.5. The continuity of the navigational system

Let us define the continuity of the navigational system as a probability that the system functions properly throughout the interval $[t,t+\tau)$ under the condition that the system is available at the moment t.

The graphic way of presenting the functioning continuity of a system is shown in Fig. 2.6.

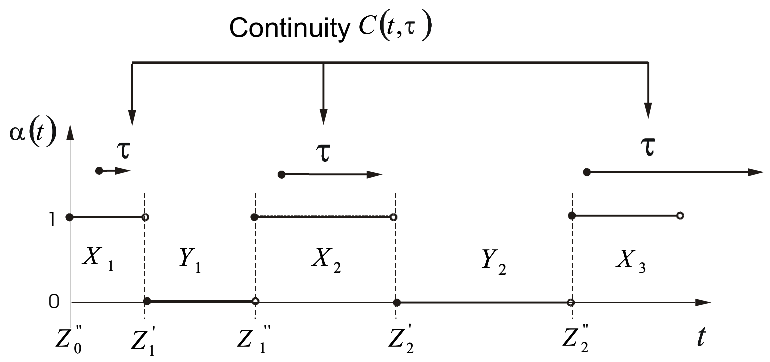

Fig. 2.6. The graphic interpretation of the continuity

To determine the continuity let us consider two events:

1. An event $D = \{\alpha(t) = 1\}$ consisting in fact that a system is available at the moment t.
2. An event $E = \{\alpha(t) = 1,\ \ \alpha(x) = 1 \text{ for } x \in [t, t+\tau)\}$ consisting in fact that a navigational system was available in the interval $[t, t+\tau)$ and in that period of time a change in state (loss of the availability) did not occur.

Define the functioning continuity as a conditional probability

$$C(t,\tau) = P(E/D). \tag{2.68}$$

Notice that $E \subset D$. Hence and from the conditional probability formula we get

$$C(t,\tau) = P(E/D) = \frac{P(D \cap E)}{P(D)} = \frac{P(E)}{P(D)}. \tag{2.69}$$

From (2.40) and (2.60)

$$P(E) = R(t,\tau) \quad \text{and} \quad P(D) = A(t), \tag{2.70}$$

then the formula of the functioning continuity of the navigational system takes the following form:

$$C(t,\tau) = \frac{1 - F(t+\tau) + \int_0^t [1 - F(t+\tau-x)]\, dH_\Phi(x)}{1 - F(t) + \int_0^t [1 - F(t-x)]\, dH_\Phi(x)}. \tag{2.71}$$

Similarly to reliability and availability we estimate the limiting continuity for $t \to \infty$:

$$\lim_{t\to\infty} C(t,\tau) = \frac{\lim\limits_{t\to\infty}\left\{1 - F(t+\tau) + \int\limits_0^t [1 - F(t+\tau-x)]\, dH_\Phi(x)\right\}}{\lim\limits_{t\to\infty}\left\{1 - F(t) + \int\limits_0^t [1 - F(t-x)] dH_\Phi(x)\right\}} \ . \tag{2.72}$$

Using (2.40) and (2.67) we have

$$\lim_{t\to\infty} C(t,\tau) = \frac{\frac{1}{E(X)+E(Y)}\int\limits_\tau^\infty [1 - F(u)]\, du}{\lim\limits_{t\to\infty}\left\{\int\limits_0^t [1 - F(t-x)] dH_\Phi(x)\right\}} \ . \tag{2.73}$$

As (2.44) takes place then

$$\lim_{t\to\infty} C(t,\tau) = \frac{\frac{1}{E(X)+E(Y)}\int\limits_\tau^\infty [1 - F(u)] du}{\frac{E(X)}{E(X)+E(Y)}} \tag{2.74}$$

and finally

$$\lim_{t\to\infty} C(t,\tau) = \frac{1}{E(X)}\int\limits_\tau^\infty [1 - F(u)] du \ . \tag{2.75}$$

The relationship between reliability, availability and continuity of the navigational system follows from (2.71) and (2.74):

$$R(t,\tau) = A(t)\cdot C(t,\tau), \tag{2.76}$$

$$\lim_{t\to\infty} R(t,\tau) = \lim_{t\to\infty} A(t)\cdot \lim_{t\to\infty} C(t,\tau). \tag{2.77}$$

The dependence presented is meaningful because it enables recalculations between the criteria discussed, which leads to combining them to a uniform model.

2.6. Availability, reliability and continuity of the navigational system with the exponential distributions of life and failure times

Typical realizations of the operating time of the navigational systems are characterized by the exponential distributions of the lifetime and the time of failures due to the property called the ”memoryless” property [Grabski F., 1981]:

Theorem 2.

If T is the life length of a component having exponential life distribution with the function density:

$$f(t) = \begin{cases} \lambda e^{-\lambda t} & , \text{ for } \quad t > 0 , \\ 0 & , \text{ for } \quad t \le 0 \end{cases} \tag{2.78}$$

and

$$F(t) = \begin{cases} 1 - e^{-\lambda t} & , \text{ for } \quad t > 0 , \\ 0 & , \text{ for } \quad t \le 0 , \end{cases} \tag{2.79}$$

where λ is a strictly fixed positive parameter called failure rate, then

$$P[T > t + x \,|\, T > t] = P[T > t] = e^{-\lambda x} \quad , \tag{2.80}$$

for all $x \geq 0$, independent of t.

This property tells us that a used exponential component is essentially “as good as a new one”. This property has important practical and theoretical consequences. Assuming the exponential life distribution it follows that:

1. since a used component is as good as a new one (stochastically), there is no advantage in following the policy of planned replacement of used components known to be still functioning,
2. in statistical estimation of mean life, percentiles, reliability, and so on, data may be collected consisting only of the number of periods of time of observed life and of the numbers of observed failures; the ages of components under observation are irrelevant.

Denote

$$G(t)=\begin{cases}1-e^{-\mu t} & , \text{ for } \quad t>0\,, \\ 0 & , \text{ for } \quad t\le 0\,,\end{cases} \tag{2.81}$$

where μ denotes renewal rate.

Below we determine each of the criteria considered putting the dependences (2.79) and (2.81) in the places of distributions of the life and failure times.

2.6.1. Availability

From (2.37) and (2.79) we get

$$\begin{aligned} A_{\exp}(t) &= 1-F(t)+\int_0^t \left[1-F(t-x)\right] dH_{\Phi}(x) = \\ &= e^{-\lambda t}+\int_0^t \left[1-\left(1-e^{-\lambda(t-x)}\right)\right] dH_{\Phi}(x)\,, \end{aligned} \tag{2.82}$$

where $A_{\exp}(t)$ denotes the availability of the navigational system in the case of the exponential life and failure times distributions.

To estimate the value of $H_{\Phi}(x)$ it is necessary to use Laplace transform. Notice that the transforms of the life and failures density for the exponential distributions are as follows [Bobrowski D., 1985]:

$$\widetilde{f}(s)=\frac{\lambda}{s+\lambda}, \tag{2.83}$$

$$\widetilde{g}(s)=\frac{\mu}{s+\mu}, \tag{2.84}$$

where $\widetilde{f}(s)$ is a density transform of lifetime, and $\widetilde{g}(s)$ is a density transform of failure time.

Determine the renewal density on the basis of its Laplace transform [Kopociński B., 1973]:

$$\tilde{h}(s) = \frac{\tilde{f}(s)\tilde{g}(s)}{1-\tilde{f}(s)\tilde{g}(s)} = \frac{\frac{\lambda}{s+\lambda}\cdot\frac{\mu}{s+\mu}}{1-\frac{\lambda}{s+\lambda}\cdot\frac{\mu}{s+\mu}} = \\ = \frac{\lambda\mu}{s(s+\lambda+\mu)} = \frac{\lambda\mu}{\lambda+\mu}\left(\frac{1}{s}-\frac{1}{s+\lambda+\mu}\right), \tag{2.85}$$

then

$$H(x) = \frac{\lambda\mu}{\lambda+\mu}\left[1-e^{-(\lambda+\mu)x}\right]. \tag{2.86}$$

Replacing (2.86) to (2.82) we have

$$\begin{aligned} A_{\exp}(t) &= e^{-\lambda t} + \int_0^t \left[1-\left(1-e^{-\lambda(t-x)}\right)\right]\frac{\lambda\mu}{\lambda+\mu}\left[1-e^{-(\lambda+\mu)x}\right]dx = \\ &= e^{-\lambda t} + \frac{\lambda\mu}{\lambda+\mu}\int_0^t e^{-\lambda t}e^{\lambda x}\left[1-e^{-\lambda x}e^{-\mu x}\right]dx = e^{-\lambda t} + \frac{\lambda\mu}{\lambda+\mu}e^{-\lambda t}\int_0^t \left[e^{\lambda x}-e^{-\mu x}\right]dx = \\ &= e^{-\lambda t} + \frac{\lambda\mu}{\lambda+\mu}e^{-\lambda t}\left(\int_0^t e^{\lambda x}dx - \int_0^t e^{-\mu x}dx\right) = \\ &= e^{-\lambda t} + \frac{\lambda\mu}{\lambda+\mu}e^{-\lambda t}\left[\left.\left(\frac{1}{\lambda}e^{\lambda x}\right)\right|_0^t - \left.\left(-\frac{1}{\mu}e^{-\mu x}\right)\right|_0^t\right] = \\ &= e^{-\lambda t} + \frac{\lambda\mu}{\lambda+\mu}e^{-\lambda t}\left[\frac{1}{\lambda}e^{\lambda t} + \frac{1}{\mu}e^{-\mu t} - \frac{1}{\lambda} - \frac{1}{\mu}\right] = \\ &= e^{-\lambda t} + \frac{\lambda\mu}{\lambda+\mu}e^{-\lambda t}\frac{1}{\lambda}e^{\lambda t} + \frac{\lambda\mu}{\lambda+\mu}e^{-\lambda t}\frac{1}{\mu}e^{-\mu t} - \frac{\lambda\mu}{\lambda+\mu}e^{-\lambda t}\frac{1}{\lambda} - \frac{\lambda\mu}{\lambda+\mu}e^{-\lambda t}\frac{1}{\mu}. \end{aligned} \tag{2.87}$$

And finally we obtain

$$A(t)_{\exp} = e^{-\lambda t} + \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t} - \frac{\mu}{\lambda+\mu}e^{-\lambda t} - \frac{\lambda}{\lambda+\mu}e^{-\lambda t}, \tag{2.88}$$

then

$$A(t)_{\exp} = e^{-\lambda t} + \frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t} - \left(\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}\right)e^{-\lambda t}. \tag{2.89}$$

Hence

$$A_{\exp}(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} . \qquad (2.90)$$

2.6.2. The availability coefficient

From the properties of the exponential distribution

$$E_{\exp}(X_i) = \frac{1}{\lambda}, \qquad (2.91)$$

$$E_{\exp}(Y_i) = \frac{1}{\mu} \text{ for } i = 1,2,\ldots, \qquad (2.92)$$

where $E_{\exp}(X_i)$ is an expected value of the exponential life distribution and $E_{\exp}(Y_i)$ is an expected value of the exponential failure distribution.

Hence and from (2.44) it follows that

$$A_{\exp} = \frac{\frac{1}{\lambda}}{\frac{1}{\lambda} + \frac{1}{\mu}} = \frac{\mu}{\mu + \lambda}, \qquad (2.93)$$

where $A_{\exp}$ denotes the availability factor of the navigational system with the exponential life and failure times distributions.

2.6.3. Reliability

As

$$R_{\exp}(t, \tau) = 1 - F(u) + \int_0^t [1 - F(u - x)] dH_{\Phi}(x), \qquad (2.94)$$

where $R_{\exp}(t, \tau)$ denotes the reliability of the navigational system with the exponential life and failure distributions. Taking into account (2.79) we get

$$
\begin{aligned}
R_{\exp}(t,\tau) &= e^{-\lambda(t+\tau)} + \int_0^t e^{-\lambda(t+\tau-x)} \frac{\lambda\mu}{\lambda+\mu}\left[1-e^{-(\lambda+\mu)x}\right]dx = \\
&= e^{-\lambda(t+\tau)} + \frac{\lambda\mu}{\lambda+\mu}\int_0^t \left[e^{-\lambda(t+\tau)}e^{\lambda x} - e^{\lambda x-\lambda x-\mu x}\right]dx = \\
&= e^{-\lambda(t+\tau)} + \frac{\lambda\mu}{\lambda+\mu}\cdot e^{-\lambda(t+\tau)}\left(\int_0^t e^{\lambda x}dx - \int_0^t e^{-\mu x}dx\right) = \\
&= e^{-\lambda(t+\tau)} + \frac{\lambda\mu}{\lambda+\mu}\cdot e^{-\lambda(t+\tau)}\left[\left(\frac{1}{\lambda}e^{\lambda x}\right)\Bigg|_0^t - \left(-\frac{1}{\mu}e^{-\mu x}\right)\Bigg|_0^t\right] = \\
&= e^{-\lambda(t+\tau)} + \frac{\lambda\mu}{\lambda+\mu}\cdot e^{-\lambda(t+\tau)}\left[\frac{1}{\lambda}e^{\lambda t} - \frac{1}{\lambda} + \frac{1}{\mu}e^{-\mu t} - \frac{1}{\mu}\right] = \qquad (2.95) \\
&= e^{-\lambda(t+\tau)} + \frac{\mu}{\lambda+\mu}e^{-\lambda\tau} - \frac{\mu}{\lambda+\mu}e^{-\lambda(t+\tau)} + \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}\cdot e^{-\mu t} - \frac{\lambda}{\lambda+\mu}e^{-\lambda(t+\tau)} = \\
&= e^{-\lambda(t+\tau)} + \frac{\mu}{\lambda+\mu}e^{-\lambda\tau} - \frac{\mu}{\lambda+\mu}e^{-\lambda(t+\tau)} + \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}\cdot e^{-\mu t} - \frac{\lambda}{\lambda+\mu}e^{-\lambda(t+\tau)}.
\end{aligned}
$$

Hence we have

$$
\begin{aligned}
R_{\exp}(t,\tau) &= e^{-\lambda(t+\tau)}\frac{\lambda+\mu}{\lambda+\mu} - e^{-\lambda(t+\tau)}\frac{\mu}{\lambda+\mu} - e^{-\lambda(t+\tau)}\frac{\lambda}{\lambda+\mu} + \\
&+ \left[\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}\right]\cdot e^{-\lambda\tau}.
\end{aligned} \qquad (2.96)
$$

As

$$
e^{-\lambda(t+\tau)}\frac{\lambda+\mu}{\lambda+\mu} - e^{-\lambda(t+\tau)}\frac{\mu}{\lambda+\mu} - e^{-\lambda(t+\tau)}\frac{\lambda}{\lambda+\mu} = 0, \qquad (2.97)
$$

finally we obtain

$$
R_{\exp}(t,\tau) = \left[\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}\right]\cdot e^{-\lambda\tau}. \qquad (2.98)
$$

2.6.4. The limiting reliability value

Compute the limit:

$$
\lim_{t\to\infty} R_{\exp}(t,\tau) = \lim_{t\to\infty}\left[\left(\frac{\mu}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}\right)\cdot e^{-\lambda\tau}\right]. \qquad (2.99)
$$

As

$$\lim_{t\to\infty}\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}=0, \tag{2.100}$$

then

$$\lim_{t\to\infty}R_{\exp}(t,\tau)=\frac{\mu}{\lambda+\mu}\cdot e^{-\lambda\tau}. \tag{2.101}$$

2.6.5. Continuity

Because (2.71) takes place then

$$C_{\exp}(t,\tau)=\frac{1-F(t+\tau)+\int_0^t[1-F(t+\tau-x)]\,dH_\Phi(x)}{1-F(t)+\int_0^t[1-F(t-x)]\,dH_\Phi(x)}, \tag{2.102}$$

where $C_{\exp}(t,\tau)$ denotes the continuity of the navigational system with the exponential life and failure distributions and hence

$$C_{\exp}(t,\tau)=\frac{\left[\frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}\right]\cdot e^{-\lambda\tau}}{\frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t}} \tag{103}$$

and finally

$$C_{\exp}(t,\tau)=e^{-\lambda\tau}. \tag{2.104}$$

2.6.6. The limiting continuity value

Compute the limit:

$$\begin{aligned}\lim_{t\to\infty}C_{\exp}(t,\tau)&=\frac{1}{E(X)}\int_\tau^\infty[1-F(u)]\,du=\frac{1}{E(X)}\int_\tau^\infty\left(1-1+e^{-\lambda u}\right)du=\\&=\frac{1}{E(X)}\int_\tau^\infty e^{-\lambda u}du=\frac{1}{E(X)}\lim_{a\to\infty}\int_\tau^a e^{-\lambda u}du=\lambda\lim_{a\to\infty}\left[\left(-\frac{1}{\lambda}e^{-\lambda u}\right)\Big|_\tau^a\right]=\\&=\lambda\lim_{a\to\infty}\left(-\frac{1}{\lambda}e^{-\lambda a}+\frac{1}{\lambda}e^{-\lambda\tau}\right).\end{aligned} \tag{2.105}$$

Since $\lim_{a\to\infty}\left(-\frac{1}{\lambda}e^{-\lambda a}\right)=0$ then the final form of the limit is equal to

$$\lim_{t\to\infty} C_{\exp}(t,\tau)=e^{-\lambda\tau}. \tag{2.106}$$

It is obvious that the dependences (2.76) and (2.77) between availability, reliability and continuity for a particular model (with the exponential life and failure times distributions) are also satisfied:

$$R_{\exp}(t,\tau)=A_{\exp}\cdot C_{\exp}(t,\tau), \tag{2.107}$$

$$\lim_{t\to\infty} R_{\exp}(t,\tau)=\lim_{t\to\infty} A_{\exp}(t)\cdot\lim_{t\to\infty} C_{\exp}(t,\tau). \tag{2.108}$$

Chapter III

CLASSICAL DIFFERENTIAL GPS SYSTEMS

The general and particular mathematical model (with the exponential life and failure times) of the availability, reliability and continuity of the navigation systems presented in Chapter II states the theoretical basis of considering these criteria due to the system of the differential GPS transmission. Section III is a particular solution referred to a certain problem – modeling of the differential GPS transmission.

3.1. General reliability structure of the differential GPS systems – determination the object of study

To uniquely determine the modeled navigational structure it is necessary to consider the differential GPS system from the point of view of modeling the reliability structures. The differential GPS system, fundamentally, is the series structure of two subsystems: space – in the aspect of pseudorange measurements and land whose task is to send the differential corrections. Hence it follows that the unreliability state of any component causes the unreliability state of the whole structure. Let us define that bicomponent structure, the elements of which are: the GPS system and **L**ocal **A**rea **A**ugmented **S**ervice - LAAS. Let $\mathbf{X}_{\mathbf{diff}}$ be the state vector of the differential GPS system

$$\mathbf{X}_{\mathbf{diff}} = \left[x_{GPS}, x_{LAAS}\right], \tag{3.1}$$

where

$$x_{GPS} = \begin{cases} 1 & \text{- working state of the GPS system,} \\ 0 & \text{- failure state of the GPS system} \end{cases} \tag{3.2}$$

and

$$x_{LAAS} = \begin{cases} 1 & \text{- working state of the LAAS system,} \\ 0 & \text{- failure state of the LAAS system.} \end{cases} \tag{3.3}$$

The structure function for the form (3.1) may be written as

$$\psi(\mathbf{X_{diff}}) \equiv x_{GPS} \wedge x_{LAAS} = \min(x_{GPS}, x_{LAAS}), \tag{3.4}$$

where:

$\psi(\mathbf{X_{diff}})$ - structure function of the differential GPS system,

x_{GPS} - state of the GPS system,

x_{LAAS} - state of the Local Area Augmented Service - LAAS.

As the state of the system and its components can be described with the binary random variables

$$\psi(\mathbf{X_{diff}}), x_{GPS} x_{LAAS} \in \mathbf{B}. \tag{3.5}$$

The empirical statistics of the GPS system are elaborated in detail [SPS 1993; SPS, 2000] so in further considerations we are restricted to analyzing the second of the elements - LAAS. It can be stated by any forms of the telemetric distribution of the pseudorange corrections like: marine DGPS reference stations (283.5-325 kHz) [Poppe D.; Last J. D., 1994], permanent GPS/RTK stations [Baran L. W.; Oszczak S., 1999], based on the standards: NMT (450 MHz), GSM (900 MHz), DCS (1800 MHz) mobile systems and also systems where PRCs are transmitted by the geostationary satellites: WAAS, EGNOS [Cydejko J.; Oszczak S., 2002].

A typical Differential GPS system consists of at least two and at most of four basic components. In more complicated systems (marine or air) the reference station, monitoring station, control station, user's segment are those components. The elements like the reference station, and the rover representing GPS RTK systems are the minimal configuration. Therefore for both of the differential solutions the system state vectors: DGPS - $\mathbf{X_{DGPS}}$ and GPS/RTK - $\mathbf{X_{RTK}}$ take the forms

$$\mathbf{X_{DGPS}} = [x_{RS}, x_{IM}, x_{CS}, x_{US}] \tag{3.6}$$

and

$$\mathbf{X_{RTK}} = [x_{RS}, x_{US}], \tag{3.7}$$

for which

$$\psi(\mathbf{X_{DGPS}}) \equiv x_{RS} \wedge x_{IM} \wedge x_{CS} \wedge x_{US} = \min(x_{RS}, x_{IM}, x_{CS}, x_{US}), \tag{3.8}$$

$$\psi(\mathbf{X_{RTK}}) \equiv x_{RS} \wedge x_{US} = \min(x_{RS}, x_{US}), \tag{3.9}$$

where:
$\psi(\mathbf{X_{DGPS}})$- structure function of DGPS system,
$\psi(\mathbf{X_{RTK}})$- structure function of GPS/RTK system,
x_{RS} – state of the DGPS or GPS/RTK reference station,
x_{IM} – state of the monitoring station,
x_{CS} – state of the control station,
x_{US} – state of the user's segment.

Similarly to the structure $\psi(\mathbf{X_{diff}})$ we can write that

$$\psi(\mathbf{X_{DGPS}}), \psi(\mathbf{X_{RTK}}), x_{RS}, x_{IM}, x_{CS}, x_{US} \in \mathbf{B}, \tag{3.10}$$

where the interpretation of the binary states is the same as in (3.2) and (3.3).

Consider the DGPS (GPS/RTK) reference station as a separate structure with the following components: technical architecture of the reference station, transmitting antenna system and the differential GPS pseudorange correction transmission system. The state vector of that structure takes the form

$$\mathbf{X_{RS}} = [x_{TS}, x_{TN}, x_S], \tag{3.11}$$

and the structure function corresponding to it can be rewritten as

$$\psi(\mathbf{X_{RS}}) \equiv x_{TS} \wedge x_{TN} \wedge x_S = \min(x_{TS}, x_{TN}, x_S), \tag{3.12}$$

for $\psi(\mathbf{X_{RS}}), x_{TS}, x_{TN}, x_S \in B$, where:
$\psi(\mathbf{X_{RS}})$ - structure function of the DGPS or GPS/RTK reference station.
x_{TS} – state of the technical architecture of the reference station,
x_{TN} – state of the transmitting antenna system,
x_S – state of the differential GPS pseudorange correction transmission system.

Due to the possibility of various setting the components of the DGPS system, it is justified to define them for purposes of the work. Hence:

- The equipment of the reference station (computer, GPS antenna, reference receivers) destined to determine the PRCs are the technical architecture of the reference station.
- The transmitting antenna system is a set of technical devices destined to broadcast the differential signals. It is composed of modulators, amplifiers and the aerial subsystem.
- The differential GPS pseudorange correction transmission system is the structure including the format and the methods used for relaying the current values of PRCs between the DGPS reference station and the user.

Notice that the structures presented here essentially concern the components with three types of features;

1. those related to the empirical reliability statistics of GPS system included in standard documents [SPS, 1993; SPS, 2001], elaborated on the basis of the long observations,
2. reliability features defined by producers,
3. the differential GPS pseudorange correction transmission system with unknown reliability characteristics.

The first two of the factors presented here were for years under extremely broad investigations. The relevant literature concerning the subject is fundamentally based on empirical studies, which in the case of the GPS system cover almost 30 years of statistics. It enables the reliability evaluation of any mixed structures. However, for the last of the elements a probabilistic model has not up till now been developed, which is directly confirmed by the publications of IALA [RNAV6, 1996]. Elaborating such a model will constitute the main investigative element of that dissertation.

3.2. Working and failure states

The fundamental problem which constitutes the starting point for further studies concerning the reliability characteristics of the process of transmission the differential corrections is to precisely determine working and failure states. In contradistinction to the classical reliability structures mostly related to typical technical devices, the working state (availability state) of the pseudorange corrections transmission is defined very atypically. For the technical devices the working state is associated with the situation when that device is in working condition and satisfies its pre-set functions [Kopociński B., 1973]. Translating this way of reasoning into navigational categories, those states correspond to the notions of availability and inaccessibility. If, by analogy, we referred the way of reasoning presented to the system of the pseudorange corrections transmission then we should find out that: if the particular pseudorange corrections (PRC) reach the differential receiver (they are correctly decoded) the system ought to be regarded as 'available'. However, the specification of RTCM format (messages composition) prevent the statement mentioned from satisfying all the conditions concerned with the availability state. The additional factor which decides about the state of DGPS system is the age of corrections. Using it bring about the necessity of taking into account the availability of the system during a definite time interval, despite the fact that the pseudorange corrections are not correctly decoded. That time is referred to as the maximum age of corrections.

The differential data are transmitted in the form of binary sequences. However, the particular bits create higher structures – RTCM messages, in which a failure of even one component (bit) causes neglecting the data of the whole message.

Then, while considering the higher structure such as the RTCM message, it can be noticed that though the DGPS receiver decodes correctly a single RTCM message (i.e. 9-3) containing corrections to only 3 satellites, this will not prevent solving the navigational problem in the sense of determining the coordinates of position (latitude, longitude, height: φ, λ, h), for which corrections from at least 4 satellites are necessary. Continuing that reasoning we conclude that the differential GPS transmission system is available under the following conditions (conjunction of events):

1. The user receiver received a sufficient number of pseudorange corrections for position determination.
2. The age of pseudorange corrections is lower than the arbitrarily accepted.

The first condition refers to a faultless reception of such a number of RTCM messages that would include corrections to a listed – minimal number of satellites. In the case of transmitting the RTCM message type 1 (including the corrections to all satellites being over the minimal elevation of the reference station) that criterion is satisfied for a single message of that type. For RTCM message type 9-3 including corrections to a subset of satellites (typically 3) it is necessary for the DGPS receiver to receive at least 2 messages with the pseudorange corrections to various satellites.

The second criterion causes the system to be available for as long as the time which passed from the defining the quantities of corrections on the DGPS reference station to the moment they are accounted for in navigational solution do not exceed the fixed limit. Let us notice that the problem being considered, due to the special defining of the availability (working) state is extremely rare, even from the point of view of the reliability theory.

Logical dissertations quoted here permit us to adopt a strategy aimed at deriving a mathematical model of the particular reliability criteria. It will consist in modeling 3 types of structures in turn – from the simplest: binary transmission, through the messages transmission, to the availability of the whole system inclusive. Hence, it is obvious that it is necessary to determine three states of availability related to the mentioned structures:

1. availability of the binary transmission – it is the probability of correct reception the basic information units - bits by the DGPS receiver,
2. availability of the RTCM message transmission – it is the probability of correct decoding the RTCM messages by the DGPS receiver,
3. availability of the differential GPS transmission – it is the probability of correct decoding, by the DGPS receiver, at least the minimal number of pseudorange corrections, indispensable for the navigational determining the position, under assumption that their age is lower then the set up one.

3.2.1. Binary transmission

The radio signal on its propagation way between the DGPS reference station and the user is subject to natural and artificial interferences. When these influences are strong enough, the deformation of the received signal may be so large that the decision made in the detector concerning the impulse level (0 or 1) will turn to be wrong. The statistical measure of that event is Bite Error Ratio - BER corresponding to the average number of incorrectly received bits in their sequences with a specific length. The above measure is correlated directly with the signal (S) to Noise (Z) ratio. For a special case of the interferences with Gaussian white noise (Z ≡ N) the measure can be approximated as [Rasiukiewicz, Leśnicki, 1983]

$$BER = a \cdot erfc\left(b\sqrt{SNR}\right) = 10^{-y}\,, \tag{3.11}$$

where:
a,b – constants depend on the kind of modulation used to overcode the input binary sequence and also on assumed definitions of the signal/noise ratio,
SNR - signal/noise ratio,
$erfc(.)$ - complementary error function of the form

$$erfc(x) = \frac{2}{\sqrt{\pi}}\int_x^{\infty} e^{-t^2}\,dt\,. \tag{3.12}$$

The useful approximation of (3.11) is the following dependence [Rasiukiewicz, Leśnicki, 1983]

$$SNR(dB) = 11.42\,\lg\left[y + \lg(2a)\right] + 1.65 - 20\lg b\,, \tag{3.13}$$

notice that in $4 \le y + \lg(2a) \le 15$ range of the estimation do not exceed ± 0.05 dB.

For basic types of modulation used in the differential GPS the values of constants are given in Tab. 3.1.

Table 3.1. The coefficients of the equation (3.11) as a function of the modulation type [Rasiukiewicz, Leśnicki, 1983].

Modulation type	*a*	*b*
2-PSK, MSK, SFSK	$\frac{1}{2}$	1
2-ASK, 2FSK	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
TFM	$\frac{1}{8}$	0.83
M-PSK$(M \ge 4)$ O-PSK$(M=4)$	1	$\sin\frac{\pi}{M}$

Formula (3.11) and its relation with the SNR value can be used to elaborate the analytic relation connecting BER and the availability of transmission for the recommended by the standard RTCM methods of the transmission. Hence, let us consider the binary transmission of the pseudorange corrections in differential GPS systems as a process in which the binary random variables X_i, $i = 1,...,n$, (where n denotes the number of a consecutive bit) are such that

$$X_i = \begin{cases} 1 & \text{- when the single bit of a RTCM message was correctly received,} \\ 0 & \text{- when the single bit of a RTCM message was not correctly received.} \end{cases} \quad (3.14)$$

The process of failures of the binary transmission is presented in Fig. 3.1.

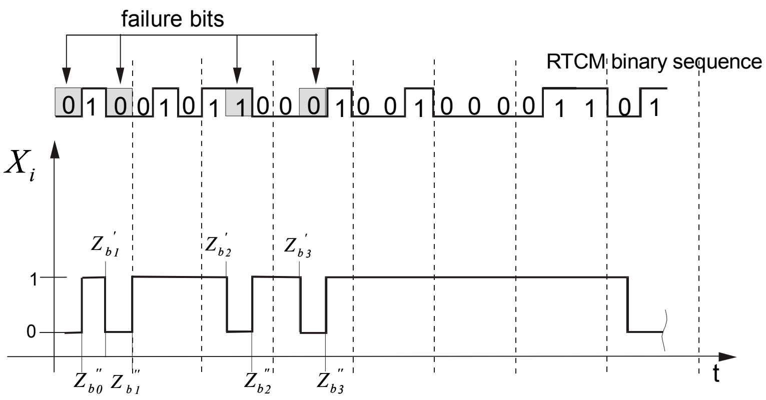

Fig. 3.1. Working and failure states of the RTCM binary transmission. Notations Z'_{b_i} and Z''_{b_i} as in Fig. 2.2.

For the process being considered

$$P(X_i = 0) = BER = q_b, \quad (3.15)$$

$$P(X_i = 1) = 1 - BER = p_b, \quad (3.16)$$

where:
p_b - probability of correctly received single binary element of RTCM message,
q_b - probability of not correctly received single binary element of RTCM message.

3.2.2. Messages transmission

Define the process of a single RTCM message transmission by means of the binary vector of state $\mathbf{X_d}$ with components X_i

$$\mathbf{X_d} = [X_1, X_2, ..., X_n]. \quad (3.17)$$

As the structure of the RTCM message transmission is related to the number of satellites for which the corrections are transmitted and the type of the message so the structure function of the message takes the form

$$\psi_j(\mathbf{X_d}) = \psi_j(X_1, X_2, ..., X_n), \qquad \text{for } j = \{1,2,3,4\}, \tag{3.18}$$

where:

$\psi_j(\mathbf{X_d})$ - structure function of the messages transmission,

j - index defining the number of RTCM messages needed for sending the corrections to all satellites located over the minimal elevation of the DGPS reference station.

The failure process of the binary and messages transmission is shown in Fig.3.2.

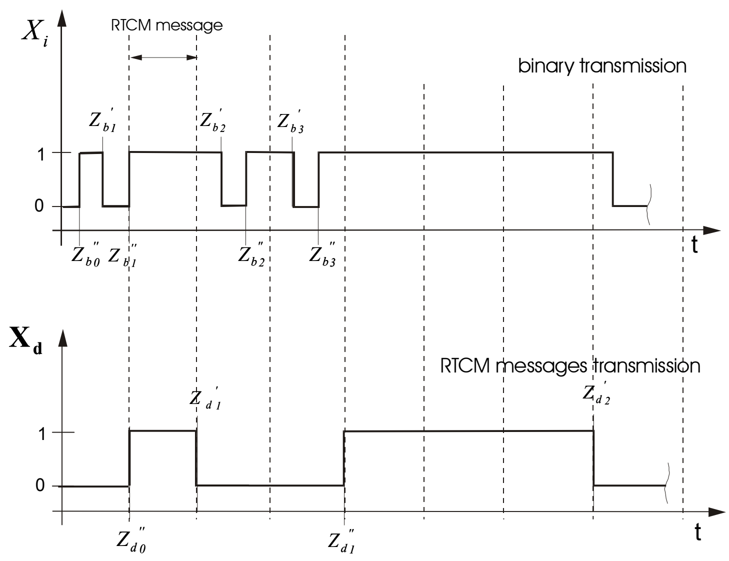

Fig. 3.2. Working and failure states of the RTCM messages transmission. For the presentation purpose the messages have the length which do not correspond to the real conditions (3 bits)

The introduced index j, according to the RTCM standard for the message type 1, can takes only the value 1, because all the pseudorange corrections are transmitted within the framework of single message, whereas for the RTCM message type 9-3 it can take the values: 2, 3 or 4 correspondingly.

For the transmission methods being considered

$$\begin{aligned} &P[\psi_j(\mathbf{X_d}) = 1] == P[\psi_j(X_1, X_2, ..., X_n) = 1] = 0 \cdot P[\psi_j(\mathbf{X_d}) = 1] + \\ &+ 1 \cdot P[\psi_j(\mathbf{X_d}) = 1] = E[\psi_j(\mathbf{X_d}) = 1] \end{aligned} \tag{3.19}$$

and as $X_1, X_2, ..., X_n$ are independent so

$$P[\psi_j(\mathbf{X_d}) = 1] = p_d = 1 - (q_b)^n \ . \tag{3.20}$$

Then the probability of the opposite event takes the following form

$$P[\psi_j(\mathbf{X_d}) = 0] = 1 - P[\psi_{k,j}(\mathbf{X_d}) = 1] = q_d = 1 - p_d \ , \tag{3.64}$$

where:

p_d - probability of correctly received the RTCM message consisted of the n-th number of bits,

q_d - probability of not correctly received the RTCM message consisted of the n-th number of bits.

3.2.3. Differential GPS transmission

Define the process of the differential GPS transmission by means of the binary vector $\mathbf{X_s}$ of state of the pseudorange corrections transmission system:

$$\mathbf{X_s} = [X_{j,k}] \ , \tag{3.22}$$

where k - corresponds to the next number of the same RTCM message type.

As the RTCM standard admits two types of the messages for differential Global Positioning System code measurements, hence:

1. the DGPS reference station transmitting RTCM message type 1:

$$\mathbf{X_s} = [X_{1,1}, X_{1,2}, ..., X_{1,k}] \ , \tag{3.23}$$

2. the DGPS reference station transmitting RTCM message type 9-3:

- for $j = 2$ (the station transmits pseudorange corrections for: 4, 5 or 6 satellites)

$$\mathbf{X_s} = [X_{1,1}, X_{1,2}, ..., X_{1,k}, X_{2,1}, X_{2,2}, ..., X_{2,k}] \ , \tag{3.24}$$

- for $j = 3$ (the station transmits pseudorange corrections for: 7, 8 or 9 satellites)

$$\mathbf{X_s} = [X_{1,1}, X_{1,2}, ..., X_{1,k}, X_{2,1}, X_{2,2}, ..., X_{2,k}, X_{3,1}, X_{3,2}, ..., Y_{3,k}] \ , \tag{3.25}$$

- for $j = 4$ (the station transmits pseudorange corrections for: 10, 11 or 12 satellites)

$$\mathbf{X_s} = [X_{1,1}, X_{1,2}, ..., X_{1,k}, X_{2,1}, X_{2,2}, ..., X_{2,k}, X_{3,1}, X_{3,2}, ..., X_{3,k}, X_{4,1}, X_{4,2}, ..., X_{4,k}] \ . \tag{3.26}$$

Figure 3.3 shows an example of the process of failure of the structure described by (3.25), which corresponds to the RTCM message type 9-3 transmitting the pseudorange corrections for 9 satellites. Figure *(a)* presents the process of messages transmission $\mathbf{X_d}$, while the diagrams: *(b),(c),(d)* refers to the states of $X_{1,k}$, $X_{2,k}$, $X_{3,k}$ correspondingly. The state of the system *(e)* represented by the binary vector $\mathbf{X_s}$, which takes into account the determined in subsection 3.2, definition of availability of the differential GPS transmission. The vector $\mathbf{X_d}$ reaches the working state ($\mathbf{X_d} = 1$) after correctly decoding the RTCM message and stays in that state through the time interval equal to a maximum age of corrections, even in the case of incorrectly decoding the next messages with the corrections to the same set of satellites.

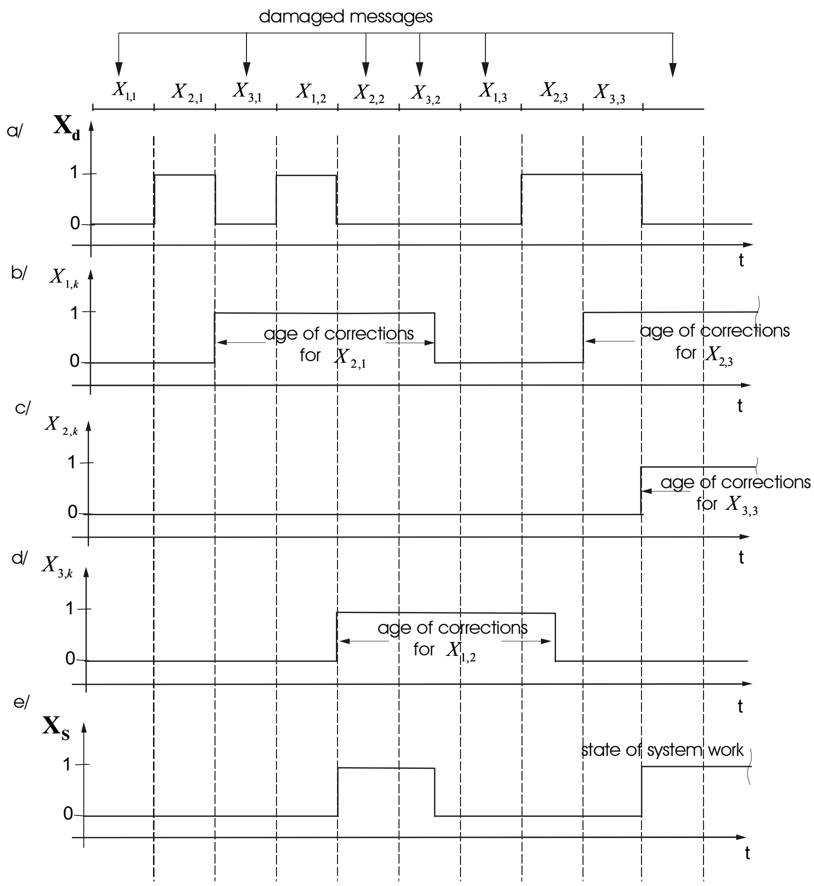

Fig. 3.3. States of: RTCM messages transmission (a), messages transmission related to the same set of satellites (b,c,d), differential GPS transmission (e), for pseudorange corrections for 9 satellites (three messages type 9-3).

From the presented analysis of the binary, messages and differential GPS transmission we arrive at three important rules indispensable in further modeling process:

1. Change of the system state from not available to available (from '0' to '1') may occur in moments of time when the messages are correctly decoded.
2. Change of the system state from available to not available (from '1' to '0') due to, the determined by the user's requirements, the value of maximum age of corrections, may occur in any moments of time.
3. Change of the system state from not available to available (from '0' to '1') may occur in such moments of time which are integer multiples of the quotient of the message length and the baud rate.

3.3. The availability model

3.3.1. RTCM message type 1

To determine the notion of availability of the differential GPS transmission for DGPS systems supporting the RTCM message type 1 it is necessary to define the condition deciding about the state of the system. It is as follows: „the system DGPS transmitting the pseudorange corrections using RTCM message type 1 is available if the age of the corrections do not exceed the maximum value". In the case of messages of that type all corrections are transmitted within the framework of the same message. That is why their age is the same. The correct transmission of the message corresponds to lack of damage any of its bits. The criterion of the maximum age of corrections follows from the, fundamental in navigation literature, opinion that the DGPS system is available after the loss of the pseudorange corrections does not exceed 30 s.

Unlike the RTCM message 9-3 message type 1 does not have constant length (L_1), which can be presented as a function of the number of tracked satellites N_s in the following way:

$$L_1 = \begin{cases} 60 + 50N_s & \text{, for } N_s = \{3,6,9,12\}, \\ 70 + 50N_s & \text{, for } N_s = \{4,7,10\}\ , \\ 80 + 50N_S & \text{, for } N_s = \{2,5,8,11\}, \end{cases} \tag{3.27}$$

where
N_s – the number of satellites for which the DGPS reference station transmits the pseudorange corrections.

Due to the variable length of the message being the function of the number of satellites for which the reference station transmits PRCs, in availability analysis, consideration should be given to the time needed to transmit the RTCM message type 1 between the reference station and the receiver T_{L_1} defined as latency:

$$T_{L_1} = \frac{L_1}{R}, \tag{3.28}$$

where:
R - baud rate [bps],
L_1 - length of RTCM message type 1 [bits].

The time T_{L_1} is a part of the maximum age of corrections T_{max} . The number of transmitted messages type 1 - $n(L_1)$ in the time $(T_{\max} - T_{L_1})$ can be determined from the relation

$$n(L_1) = \frac{(T_{\max} - T_{L_1}) \cdot R}{L_1}, \tag{3.29}$$

where
T_{max} – maximum age of the corrections.

The quantity $(T_{\max} - T_{L_1})$ being the age of corrections, counted from the moment they have been decoded on the user's side (5-th and 6-th column in tab. 3.2.), is not a constant multiple of a message transmission time - T_{L_1} (included in 3-rd and 4-th column). Hence it follows that the corrections cease to be taken into account in moments different from the moments in which next messages are received.

Table 3.2. The connection between: the number of satellites, message length, baud rate and the number of transmitted messages for the maximum age of corrections = 30 s.

N_s	L_1 [bits]	T_{L_1} [s]		$(T_{\max} - T_{L_1})$ [s]		$n(L_1)$	
		100 bps	200 bps	100 bps	200 bps	100 bps	200 bps
1	2	3	4	5	6	7	8
3	210	2.1	1.55	27.9	28.95	13.28	27.57
4	270	2.7	1.35	27.3	28.65	10.11	21.22
5	330	3.3	1.65	26.7	28.35	8.09	17.18
6	360	3.6	1.8	26.4	28.2	7.33	15.67
7	420	4.2	2.1	25.8	27.9	6.14	13.29
8	480	4.8	2.4	25.2	27.6	5.25	11.5
9	510	5.1	2.55	24.9	27.45	4.88	10.76
10	570	5.7	2.85	24.3	27.15	4.26	9.53
11	630	6.3	3.15	23.7	26.85	3.76	8.52
12	660	6.6	3.3	23.4	26.7	3.55	8.09

As a result of that observation, in considering the process of PRCs transmission there is a necessity of taking into account two periodical time windows Δ_1^1, Δ_2^1 with different lengths (Fig. 3.4) divided the transmission time of each message to 2 parts. Figure 3.4. presents the process of RTCM message type 1. The pseudorange corrections are determined at the moments t_{k-n}, in order to be decoded at t_{k-n+1} by the user's receiver.

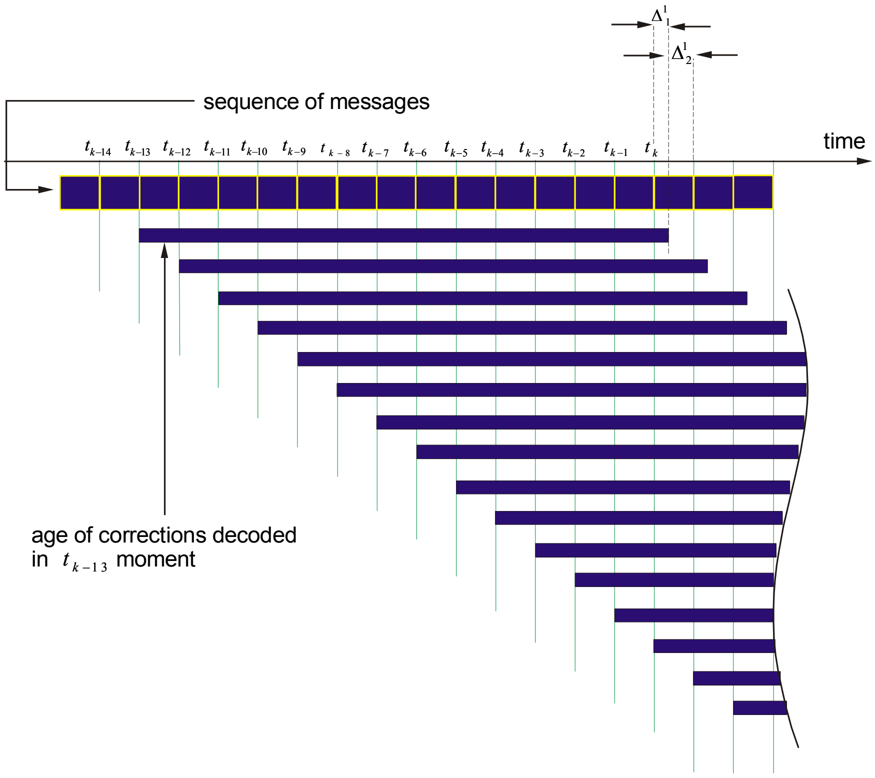

Fig. 3.4. The process of the PRCs transmission using the message RTCM type 1

The length of the particular time intervals: Δ_1^1, Δ_2^1 can be determined from the relations

$$\Delta_1^1 = \frac{\{n(L_1) - \text{int}[n(L_1)]\} \cdot L_1}{R}, \tag{3.30}$$

$$\Delta_2^1 = \frac{L_1}{R} - \Delta_1^1, \tag{3.31}$$

where:

$\text{int}(.)$ - entier $(.)$.

The dependences (3.30) and (3.31) are presented in graphical form in Fig. 3.5. As the intervals Δ_1^1, Δ_2^1 are not equal to each other then also the availability values corresponding to them are not the same.

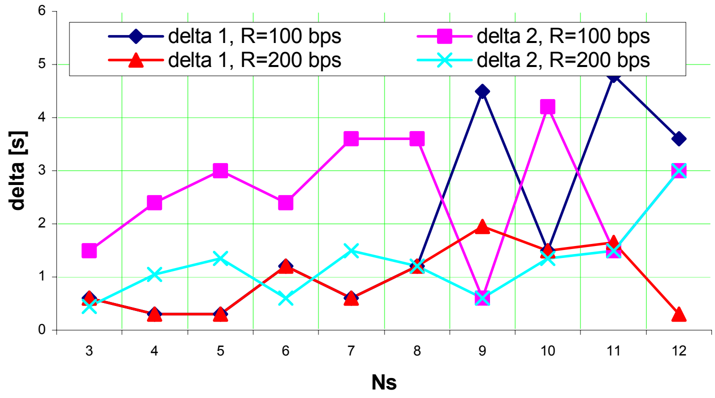

Fig. 3.5. The values Δ_1^1 and Δ_2^1 as a function of the number of satellites for which the PRCs are transmitted and baud rate.

The availability - $A(t)$ of the differential GPS transmission can be computed as

$$A(t) = \begin{cases} P[\psi(\Lambda_1) = 1], & \text{for} \quad t \in [t_k, \Delta_1^1), \\ P[\psi(\Lambda_2) = 1], & \text{for} \quad t \in [t_k + \Delta_1^1, \Delta_2^1), \; k \in Z, \end{cases} \tag{3.32}$$

where the binary state vectors for the time intervals Δ_1^1 and Δ_2^1 are written as follows

$$\mathbf{\Lambda_1} = [x_{1,k}, x_{1,k-1}, \ldots, x_{1,k-\xi}] \;, \tag{3.33}$$

$$\mathbf{\Lambda_2} = [x_{1,k}, x_{1,k-1}, \ldots, x_{1,k-(\xi-1)}] \tag{3.34}$$

and their structure functions

$$\psi(\mathbf{\Lambda_1}) = [x_{1,k} \amalg x_{1,k-1} \amalg \ldots \amalg x_{1,k-\xi}] = \coprod_{i=0}^{\xi} x_{1,k-i} \;, \tag{3.35}$$

$$\psi(\mathbf{\Lambda_2}) = [x_{1,k} \amalg x_{1,k-1} \amalg \ldots \amalg x_{1,k-(\xi-1)}] = \coprod_{i=0}^{\xi-1} x_{1,k-i} \;, \tag{3.36}$$

where

$$\xi = \text{int}[n(L_1)] \tag{3.37}$$

and also

$x_{1,k}$ - state of k-th message,

$\mathbf{Z}$ - set of integers.

In the notation offered used is a symbol $\amalg$ (introduced by Barlow & Proshan, 1983), which denotes the following operations

$$\coprod_{i=1}^{n} u_i \equiv 1 - \prod_{i=1}^{n} (1 - u_i), \tag{3.38}$$

$$u_1 \amalg u_2 = 1 - (1 - u_1)(1 - u_2) \, . \tag{3.39}$$

Submitting (3.35) and (3.36) with the structure decomposition we obtain

$$A(t) = \begin{cases} P[\psi(\Lambda_1) = 1] = 1 - (q_d)^{\xi+1}, & \text{for} \quad t \in [t_k, \Delta_1^1), \\ P[\psi(\Lambda_2) = 1] = 1 - (q_d)^{\xi}, & \text{for} \quad t \in [t_k + \Delta_1^1, \Delta_2^1). \end{cases} \tag{3.40}$$

3.3.2. RTCM message type 9-3

Transmission availability modeling of that type, due to the format, requires considering three separate cases, depending on the number of satellites situated over the minimal elevation of the DGPS reference station. Analyzing the global visibility of the space segment GPS [SPS, 1993] for a nominal constellation of 24 satellites, the cases considered can attributed some specific probabilities of the occurrence.

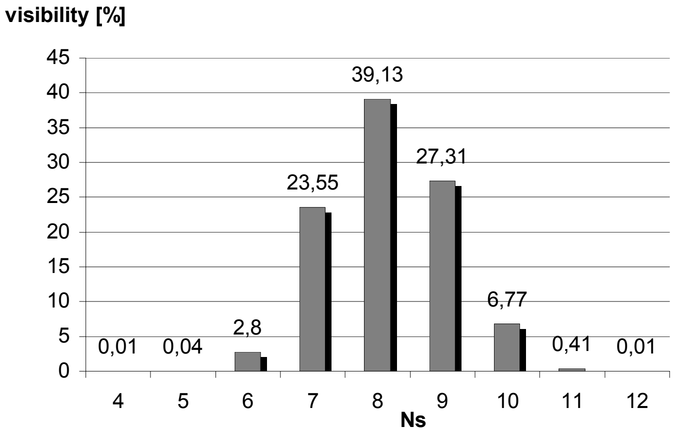

Fig. 3.6. The global GPS visibility (%). Total constellation of satellites - 24 SV [SPS, 1993]

From the values presented on Fig. 3.6 it follows that

$$P[N_s = \{4,5,6\}] = 0.0285\,, \tag{3.41}$$

$$P[N_s = \{7,8,9\}] = 0.8999\,, \tag{3.42}$$

$$P[N_s = \{10,11,12\}] = 0.0719\,. \tag{3.43}$$

We came into conclusion that almost 90 % of time the DGPS reference stations transmit the pseudorange corrections using three RTCM messages type 9-3.

3.3.2.1. Transmission of PRCs for: 4, 5 or 6 satellites

For the differential GPS transmission using the message RTCM type 9-3 the availability is the state in which the age of at least 4 pseudorange corrections related to various satellites is lower than the maximum one. It corresponds to correct reception of two different messages RTCM type 9-3 for which the age of corrections is less than the age defined as maximum. This type of transmission has a fixed length of the message ($L_{9\text{-}3}$) equaled to 210 bits. Hence, the probability of correct reception each is as follows

$$p_d = 1 - (BER)^{L_{9-3}}\,. \tag{3.44}$$

In order to define the navigational structure function (by means of the minimal path or minimal cut) it is well-founded to illustrate the process considered in a graphic form. Figure 3.7 shows the process of the differential GPS transmission of two type 9-3 messages. Each of them, after decoding, is available for a defined time interval equal to the age of corrections reduced by the time needed for transmission. For the analyses, the classical case was adopted: the age of corrections $T_{max} = 30\,\mathrm{s}$ and the baud rate R=100 bps are used contemporarily by almost all DGPS reference stations.

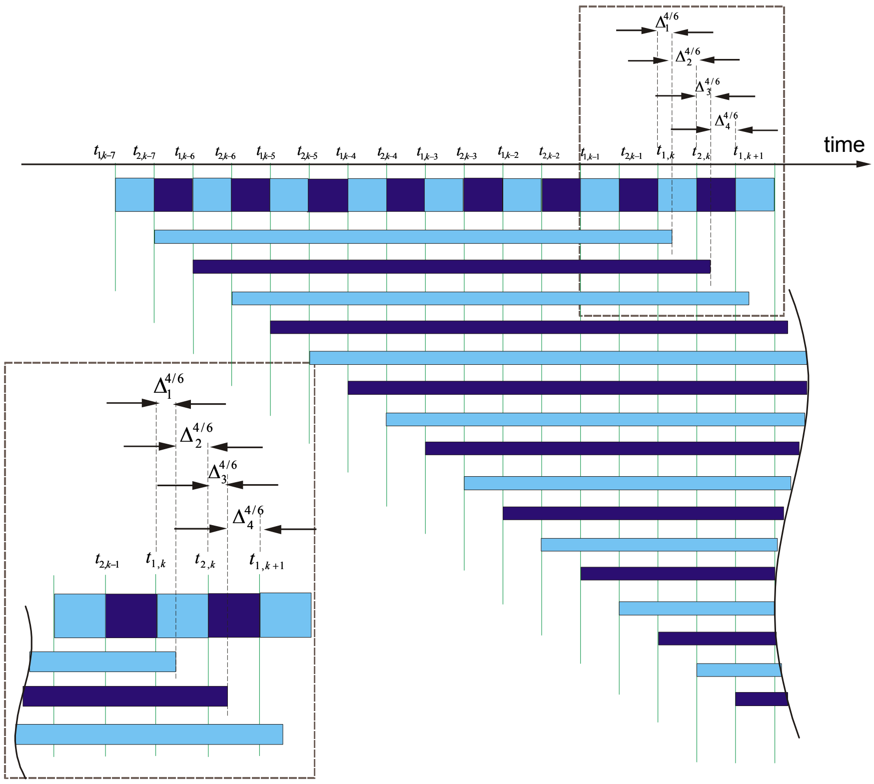

Fig. 3.7. The process of the PRCs transmission for N_s ={4,5,6}

The basic factor which makes the process of mathematical modeling complex is the variable value of the age of corrections, which is not a constant multiple of lasting time of a single message 9-3 (in graphic example-2.1 s). Hence the considerations related to the availability require taking into account the periodically repeated time intervals $\Delta_1^{4/6}, \Delta_2^{4/6}, \Delta_3^{4/6}, \Delta_4^{4/6}$, similarly to the case of the RTCM message type 1.

Let us conclude that for any $t \geq 0$

$$P\left(t \in \Delta_1^{4/6}\right) = P\left(t \in \Delta_3^{4/6}\right) \text{ and } P\left(t \in \Delta_2^{4/6}\right) = P\left(t \in \Delta_4^{4/6}\right), \tag{3.45}$$

hence the availability - $A(t)$ of the differential GPS transmission is computed as follows

$$A(t) = \begin{cases} P[\psi(\Lambda_1) = 1], & \text{for} \quad t \in \left[t_k, \Delta_1^1\right), \\ P[\psi(\Lambda_2) = 1], & \text{for} \quad t \in \left[t_k + \Delta_1^1, \Delta_2^1\right). \end{cases} \tag{3.46}$$

The binary state vectors of the system for Δ_1^1 and Δ_2^1 have the form

$$\mathbf{\Lambda_1} = \left[x_{2,k-7}, x_{1,k-6}, x_{2,k-6}, x_{1,k-5}, \ldots, x_{1,k-1}, x_{2,k-1}, x_{1,k}\right], \tag{3.47}$$

$$\mathbf{\Lambda_2} = \left[x_{1,k-6}, x_{2,k-6}, x_{1,k-5}, \ldots, x_{1,k-1}, x_{2,k-1}, x_{1,k}\right] \tag{3.48}$$

and the functions of their structures

$$\psi(\mathbf{\Lambda_1}) = \left(x_{2,k-7} \amalg x_{2,k-6} \amalg, \ldots, \amalg x_{2,k-1}\right)\left(x_{1,k-6} \amalg x_{1,k-5} \amalg, \ldots, \amalg x_{1,k}\right) = \coprod_{i=1}^{7} x_{2,k-i} \coprod_{i=0}^{6} x_{1,k-i}, \tag{3.49}$$

$$\psi(\mathbf{\Lambda_2}) = \left(x_{2,k-6} \amalg x_{2,k-5} \amalg, \ldots, \amalg x_{2,k-1}\right)\left(x_{1,k-6} \amalg x_{1,k-5} \amalg, \ldots, \amalg x_{1,k}\right) = \coprod_{i=1}^{6} x_{2,k-i} \coprod_{i=0}^{6} x_{1,k-i}. \tag{3.50}$$

Using the decomposition to (3.49), (3.50) we get

$$A(t) = \begin{cases} \left[1 - (1 - p_d)^7\right] \cdot \left[1 - (1 - p_d)^6\right] = 1 - q^6 - q^7 + q^{13}, \text{ where} & t \in \left[t_k, \mathbf{\Delta}_1^1\right), \\ \left[1 - (1 - p_d)^6\right] \cdot \left[1 - (1 - p_d)^6\right] = 1 - 2q^6 + q^{12}, \quad \text{where} & t \in \left[t_k + \mathbf{\Delta}_1^1, \mathbf{\Delta}_2^1\right). \end{cases} \tag{3.51}$$

3.3.2.2. Transmission of PRCs for: 7, 8 or 9 satellites

Let us consider the transmission of three RTCM messages of type 9-3 transmitted by the DGPS reference station related to the satellites: 1-3, 4-6, 7-9 correspondingly. The availability will be the state in which the age of at least 4 pseudorange corrections related to various satellites is lower than the maximum. It corresponds to two RTCM messages correctly received, for which the age of corrections is lower than the age defined as maximum. In contrast with the problem considered in previous subsection it is necessary to think over the case „2-out-of-3' in the sense of Bernoulli distribution.

That model, in comparison with the two remain cases, is extremely important because the probability of its occurrence is almost 90 %. The analysis of the process indicates the necessity of separating 12 intervals: $\Delta_1^{7/9}$, $\Delta_2^{7/9}$, $\Delta_3^{7/9}$, $\Delta_4^{7/9}$, $\Delta_5^{7/9}$, $\Delta_6^{7/9}$, $\Delta_7^{7/9}$, $\Delta_8^{7/9}$, $\Delta_9^{7/9}$, $\Delta_{10}^{7/9}$, $\Delta_{11}^{7/9}$, $\Delta_{12}^{7/9}$ (Fig. 3.8).

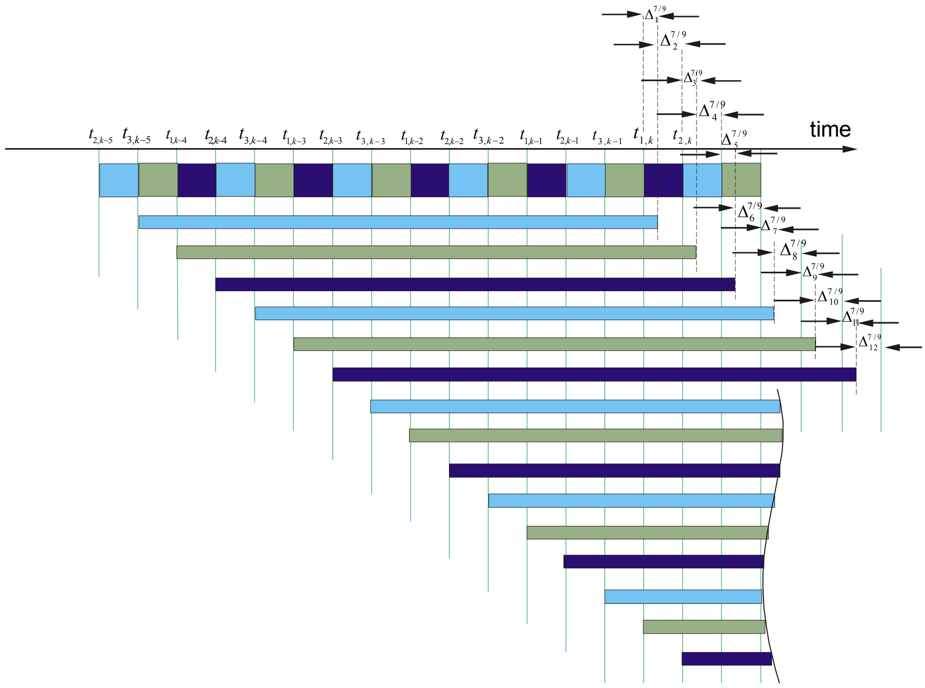

Fig. 3.8. The process of PRCs transmission for $N_s = \{7,8,9\}$

Similarly to the case of the corrections for 4-6 SV let us make an observation:

$$P\left(t \in \Delta_h^{7/9}\right) = const \quad , \quad h = 2s + 1 \, , \tag{3.52}$$

$$P\left(t \in \Delta_h^{7/9}\right) = const \quad , \quad h = 2s \, , \qquad s = 1,2,3\ldots \, . \tag{3.53}$$

Then the binary state vectors of the system for the time intervals Δ_1^1 and Δ_2^1 are of the form

$$\mathbf{\Lambda_1} = \left[\left(x_{1,k-4}, x_{1,k-3}, \ldots, x_{1,k}\right), \left(x_{2,k-4}, x_{2,k-3}, \ldots, x_{2,k-1}\right), \left(x_{3,k-5}, x_{3,k-4}, \ldots, x_{3,k-1}\right)\right] \, , \tag{3.54}$$

$$\mathbf{\Lambda_2} = \left[\left(x_{1,k-4}, x_{1,k-3}, \ldots, x_{1,k}\right), \left(x_{2,k-4}, x_{2,k-3}, \ldots, x_{2,k-1}\right), \left(x_{3,k-4}, x_{3,k-3}, \ldots, x_{3,k-1}\right)\right] \tag{3.55}$$

and their structure functions can be presented as follows

$$\begin{aligned}\psi(\mathbf{\Lambda_1}) &= (x_{3,k-5} \amalg x_{3,k-4} \amalg \ldots \amalg x_{3,k-1})(x_{1,k-4} \amalg x_{1,k-3} \amalg \ldots \amalg x_{1,k}) \amalg (x_{1,k-4} \amalg x_{1,k-3} \amalg \ldots \amalg x_{1,k}) \\ &(x_{2,k-4} \amalg x_{2,k-3} \amalg \ldots \amalg x_{2,k-1}) \amalg (x_{3,k-5} \amalg x_{3,k-4} \amalg \ldots \amalg x_{3,k-1})(x_{2,k-4} \amalg x_{2,k-3} \amalg \ldots \amalg x_{2,k-1}) \amalg \\ &\amalg (x_{1,k-4} \amalg x_{1,k-3} \amalg \ldots \amalg x_{1,k})(x_{2,k-4} \amalg x_{2,k-3} \amalg \ldots \amalg x_{2,k-1})(x_{3,k-5} \amalg x_{3,k-4} \amalg \ldots \amalg x_{3,k-1}) = \\ &\coprod_{i=1}^{5} x_{3,k-i} \coprod_{i=0}^{4} x_{1,k-i} \amalg \coprod_{i=0}^{4} x_{1,k-i} \coprod_{i=1}^{4} x_{2,k-i} \amalg \coprod_{i=1}^{5} x_{3,k-i} \coprod_{i=1}^{4} x_{2,k-i} \amalg \coprod_{i=0}^{4} x_{1,k-i} \coprod_{i=1}^{4} x_{2,k-i} \coprod_{i=1}^{5} x_{3,k-i} \ ,\end{aligned} \tag{3.56}$$

$$\begin{aligned}\psi(\mathbf{\Lambda_2}) &= (x_{3,k-4} \amalg x_{3,k-3} \amalg \ldots \amalg x_{3,k-1})(x_{1,k-4} \amalg x_{1,k-3} \amalg \ldots \amalg x_{1,k}) \amalg (x_{1,k-4} \amalg x_{1,k-3} \amalg \ldots \amalg x_{1,k}) \\ &(x_{2,k-4} \amalg x_{2,k-3} \amalg \ldots \amalg x_{2,k-1}) \amalg (x_{3,k-4} \amalg x_{3,k-3} \amalg \ldots \amalg x_{3,k-1})(x_{2,k-4} \amalg x_{2,k-3} \amalg \ldots \amalg x_{2,k-1}) \amalg \\ &\amalg (x_{1,k-4} \amalg x_{1,k-3} \amalg \ldots \amalg x_{1,k})(x_{2,k-4} \amalg x_{2,k-3} \amalg \ldots \amalg x_{2,k-1})(x_{3,k-4} \amalg x_{3,k-3} \amalg \ldots \amalg x_{3,k-1}) = \\ &\coprod_{i=1}^{4} x_{3,k-i} \coprod_{i=0}^{4} x_{1,k-i} \amalg \coprod_{i=0}^{4} x_{1,k-i} \coprod_{i=1}^{4} x_{2,k-i} \amalg \coprod_{i=1}^{4} x_{3,k-i} \coprod_{i=1}^{4} x_{2,k-i} \amalg \coprod_{i=0}^{4} x_{1,k-i} \coprod_{i=1}^{4} x_{2,k-i} \coprod_{i=1}^{4} x_{3,k-i} \ .\end{aligned} \tag{3.57}$$

The functions $\psi(\mathbf{\Lambda_1})$, $\psi(\mathbf{\Lambda_2})$ can be rewritten in an equivalent form as

$$\psi(\mathbf{\Lambda_1}) = \max[\mathbf{\Omega}_1^1, \mathbf{\Omega}_2^1, \mathbf{\Omega}_3^1, \mathbf{\Omega}_4^1], \tag{3.58}$$

$$\psi(\mathbf{\Lambda_2}) = \max[\mathbf{\Omega}_1^2, \mathbf{\Omega}_2^2, \mathbf{\Omega}_3^2, \mathbf{\Omega}_4^2], \tag{3.59}$$

where

$$\psi(\mathbf{\Omega}_1^1) = \coprod_{i=1}^{5} x_{3,k-i} \coprod_{i=0}^{4} x_{1,k-i} \ , \qquad \psi(\mathbf{\Omega}_1^2) = \coprod_{i=1}^{4} x_{3,k-i} \coprod_{i=0}^{4} x_{1,k-i} \ , \tag{3.60}$$

(3.61)

$$\psi(\mathbf{\Omega}_2^1) = \psi(\mathbf{\Omega}_2^2) = \coprod_{i=0}^{4} x_{1,k-i} \coprod_{i=1}^{4} x_{2,k-i} \ , \tag{3.62}$$

$$\psi(\mathbf{\Omega}_3^1) = \coprod_{i=1}^{5} x_{3,k-i} \coprod_{i=1}^{4} x_{2,k-i} \ , \qquad \psi(\mathbf{\Omega}_3^2) = \coprod_{i=1}^{4} x_{3,k-i} \coprod_{i=1}^{4} x_{2,k-i} \ , \qquad (3.63),(3.64)$$

$$\psi(\mathbf{\Omega}_4^1) = \coprod_{i=0}^{4} x_{1,k-i} \coprod_{i=1}^{4} x_{2,k-i} \coprod_{i=1}^{5} x_{3,k-i} \ , \tag{3.65}$$

$$\psi(\mathbf{\Omega}_4^2) = \coprod_{i=0}^{4} x_{1,k-i} \coprod_{i=1}^{4} x_{2,k-i} \coprod_{i=1}^{4} x_{3,k-i} \ , \tag{3.66}$$

for which

$$P[\psi(\mathbf{\Omega}_1^1) = 1] = (1 - q_d{}^5)(1 - q_d{}^5) = 1 - 2q_d{}^5 + q_d{}^{10} \ , \tag{3.67}$$

$$P\left[\psi\left(\mathbf{\Omega}_{\mathbf{1}}^{\mathbf{2}}\right)=1\right]=\left(1-q_d^{\ 4}\right)\left(1-q_d^{\ 5}\right)=1-q_d^{\ 4}-q_d^{\ 5}+q_d^{\ 9}, \tag{3.68}$$

$$P\left[\psi\left(\mathbf{\Omega}_{\mathbf{2}}^{\mathbf{1}}\right)=1\right]=P\left[\psi\left(\mathbf{\Omega}_{\mathbf{2}}^{\mathbf{2}}\right)=1\right]=\left(1-q_d^{\ 5}\right)\left(1-q_d^{\ 4}\right)=1-q_d^{\ 4}-q_d^{\ 5}+q_d^{\ 9}, \tag{3.69}$$

$$P\left[\psi\left(\mathbf{\Omega}_{\mathbf{3}}^{\mathbf{1}}\right)=1\right]=\left(1-q_d^{\ 5}\right)\left(1-q_d^{\ 4}\right)=1-q_d^{\ 4}-q_d^{\ 5}+q_d^{\ 9}, \tag{3.70}$$

$$P\left[\psi\left(\mathbf{\Omega}_{\mathbf{3}}^{\mathbf{2}}\right)=1\right]=\left(1-q_d^{\ 4}\right)\left(1-q_d^{\ 4}\right)=1-2q_d^{\ 4}+q_d^{\ 8}, \tag{3.71}$$

$$\begin{aligned} P\left[\psi\left(\mathbf{\Omega}_{\mathbf{4}}^{\mathbf{1}}\right)=1\right]&=\left(1-q_d^{\ 5}\right)\left(1-q_d^{\ 4}\right)\left(1-q_d^{\ 5}\right)= \\ &=1-q_d^{\ 4}-2q_d^{\ 5}+q_d^{\ 9}+q_d^{\ 10}-q_d^{\ 14}, \end{aligned} \tag{3.72}$$

$$\begin{aligned} P\left[\psi\left(\mathbf{\Omega}_{\mathbf{4}}^{\mathbf{2}}\right)=1\right]&=\left(1-q_d^{\ 5}\right)\left(1-q_d^{\ 4}\right)\left(1-q_d^{\ 4}\right)= \\ &=1-2q_d^{\ 4}+q_d^{\ 8}-q_d^{\ 5}+2q_d^{\ 9}-q_d^{\ 13}. \end{aligned} \tag{3.73}$$

Introducing the additional variable z_u, where $u=\{1,2,3,4\}$, such that

$$z_u=\begin{cases} P\left[\psi\left(\mathbf{\Omega}_{\mathbf{u}}^{\mathbf{1}}\right)=1\right] & \text{, for } \ t\in\left[t_k,\Delta_1^1\right), \\ P\left[\psi\left(\mathbf{\Omega}_{\mathbf{u}}^{\mathbf{2}}\right)=1\right] & \text{, for } \ t\in\left[t_k+\Delta_1^1,\Delta_2^1\right), \end{cases} \tag{3.74}$$

we determine the availability of the transmission using the formula of the sum of events [Borowkow, 1975]

$$A(t)=\sum_{k=1}^{4}z_k-\sum_{k<l}z_k z_l+\sum_{k<l<m}z_k z_l z_m-\ldots(-1)^3 z_1\ldots z_4, \tag{3.75}$$

obtaining after some operations the final form

$$\begin{aligned} A(t)&=z_1+z_2+z_3+z_4-z_1z_2-z_2z_4-z_2z_3-z_3z_4-z_1z_3-z_1z_4+z_1z_3z_4 \\ &+z_2z_3z_4+z_1z_2z_3+z_1z_2z_4-z_1z_2z_3z_4. \end{aligned} \tag{3.76}$$

3.3.2.3. Transmission of PRCs for: 10, 11 or 12 satellites

It seems to be unimportant to consider that event in spite of a very small probability of its occurrence. However, the model will enable answering the following questions:

1. What influence does the increasing the number of satellites for which DGPS reference station transmits the PRCs have on the availability, reliability and continuity ?

2. Is it justifiable to limit the number of PRCs transmitted by the DGPS reference station due to the still increasing number of satellites in the GPS system ?

Let us restrict the analysis of availability to the cases of even and odd time intervals Δ presented on Fig. 3.9.

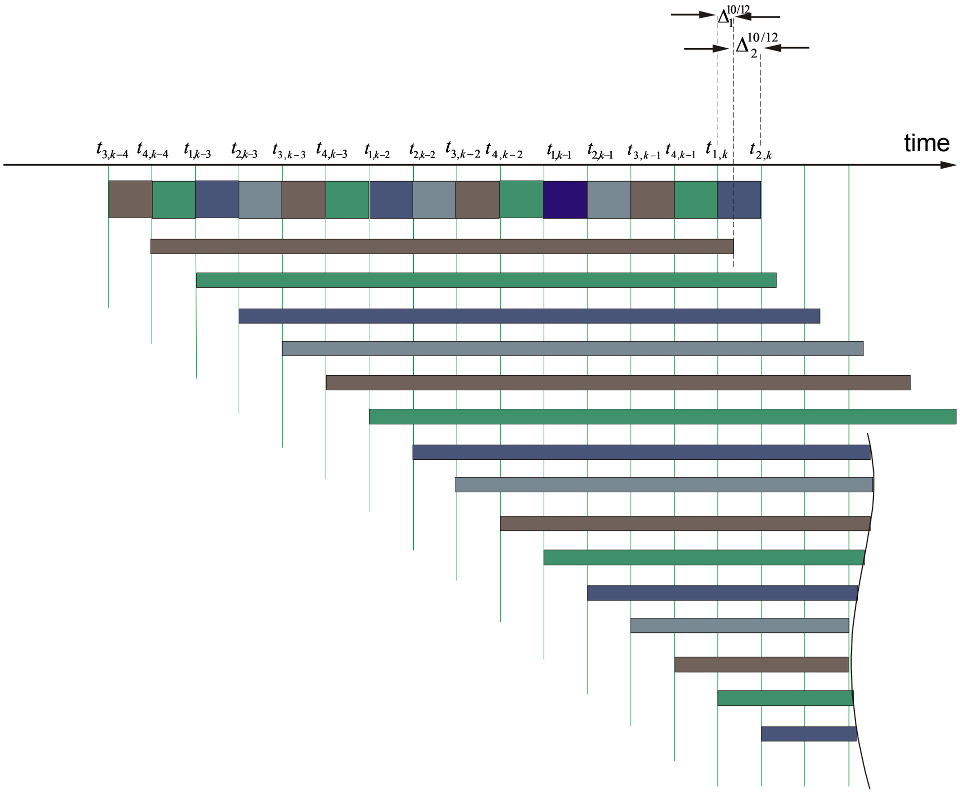

Fig. 3.9. The process of PRCs transmission for $N_s = \{10,11,12\}$

Similarly to the former cases

$$P\left(t \in \Delta_h^{10/12}\right) = const\ , \quad h = 2s + 1\ , \tag{3.77}$$

$$P\left(t \in \Delta_h^{10/12}\right) = const\ , \quad h = 2s\ , \quad s = 1,2,3...\ , \tag{3.78}$$

then

$$\mathbf{\Lambda_1} = \Big[\left(x_{1,k-3}, x_{1,k-2}, \ldots, x_{1,k}\right), \left(x_{2,k-3}, x_{2,k-2}, x_{2,k-1}\right), \left(x_{3,k-3}, x_{3,k-2}, x_{3,k-1}\right), \\ , \left(x_{4,k-4}, x_{4,k-2}, x_{4,k-1}\right)\Big], \tag{3.79}$$

$$\mathbf{\Lambda_2} = \Big[\left(x_{1,k-3}, x_{1,k-2}, \ldots, x_{1,k}\right), \left(x_{2,k-3}, x_{2,k-2}, x_{2,k-1}\right), \left(x_{3,k-3}, x_{3,k-2}, x_{3,k-1}\right), \\ , \left(x_{4,k-3}, x_{4,k-2}, x_{4,k-1}\right)\Big] \tag{3.80}$$

and their structure functions take the following forms

$$\begin{aligned}
\psi(\mathbf{\Lambda_1}) = {} & \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{3} x_{2,k-i} \amalg \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{3} x_{3,k-i} \amalg \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{4} x_{4,k-i} \amalg \\
& \amalg \coprod_{i=1}^{3} x_{2,k-i} \coprod_{i=1}^{3} x_{3,k-i} \amalg \coprod_{i=1}^{3} x_{2,k-i} \coprod_{i=1}^{4} x_{4,k-i} \amalg \coprod_{i=1}^{3} x_{3,k-i} \coprod_{i=1}^{4} x_{4,k-i} \amalg \\
& \amalg \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{3} x_{2,k-i} \coprod_{i=1}^{3} x_{3,k-i} \amalg \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{3} x_{2,k-i} \coprod_{i=1}^{4} x_{4,k-i} \amalg \\
& \amalg \coprod_{i=1}^{3} x_{2,k-i} \coprod_{i=1}^{3} x_{3,k-i} \coprod_{i=1}^{4} x_{4,k-i} \amalg \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{3} x_{3,k-i} \coprod_{i=1}^{4} x_{4,k-i} \amalg \\
& \amalg \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{3} x_{2,k-i} \coprod_{i=1}^{3} x_{3,k-i} \coprod_{i=1}^{4} x_{4,k-i} ,
\end{aligned} \tag{3.81}$$

$$\begin{aligned}
\psi(\mathbf{\Lambda_2}) = {} & \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{3} x_{2,k-i} \amalg \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{3} x_{3,k-i} \amalg \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{3} x_{4,k-i} \amalg \\
& \amalg \coprod_{i=1}^{3} x_{2,k-i} \coprod_{i=1}^{3} x_{3,k-i} \amalg \coprod_{i=1}^{3} x_{2,k-i} \coprod_{i=1}^{3} x_{4,k-i} \amalg \coprod_{i=1}^{3} x_{3,k-i} \coprod_{i=1}^{3} x_{4,k-i} \amalg \\
& \amalg \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{3} x_{2,k-i} \coprod_{i=1}^{3} x_{3,k-i} \amalg \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{3} x_{2,k-i} \coprod_{i=1}^{3} x_{4,k-i} \amalg \\
& \amalg \coprod_{i=1}^{3} x_{2,k-i} \coprod_{i=1}^{3} x_{3,k-i} \coprod_{i=1}^{3} x_{4,k-i} \amalg \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{3} x_{3,k-i} \coprod_{i=1}^{3} x_{4,k-i} \amalg \\
& \amalg \coprod_{i=0}^{3} x_{1,k-i} \coprod_{i=1}^{3} x_{2,k-i} \coprod_{i=1}^{3} x_{3,k-i} \coprod_{i=1}^{3} x_{4,k-i}
\end{aligned} \tag{3.82}$$

The structures $\psi(\mathbf{\Lambda}_1)$, $\psi(\mathbf{\Lambda}_2)$ can be rewritten in equivalent form

$$\psi(\mathbf{\Lambda_1}) = \max\left[\mathbf{\Omega}_1^1, \mathbf{\Omega}_2^1, \mathbf{\Omega}_3^1, \mathbf{\Omega}_4^1, \mathbf{\Omega}_5^1, \mathbf{\Omega}_6^1, \mathbf{\Omega}_7^1, \mathbf{\Omega}_8^1, \mathbf{\Omega}_9^1, \mathbf{\Omega}_{10}^1, \mathbf{\Omega}_{11}^1\right] , \tag{3.83}$$

$$\psi(\mathbf{\Lambda_2}) = \max\left[\mathbf{\Omega}_1^2, \mathbf{\Omega}_2^2, \mathbf{\Omega}_3^2, \mathbf{\Omega}_4^2, \mathbf{\Omega}_5^2, \mathbf{\Omega}_6^2, \mathbf{\Omega}_7^2, \mathbf{\Omega}_8^2, \mathbf{\Omega}_9^2, \mathbf{\Omega}_{10}^2, \mathbf{\Omega}_{11}^2\right], \tag{3.84}$$

where

$$\psi\left(\mathbf{\Omega}_1^1\right)=\psi\left(\mathbf{\Omega}_1^2\right)=\prod_{i=0}^{3} x_{1,k-i} \prod_{i=1}^{3} x_{2,k-i}\ ,\quad \psi\left(\mathbf{\Omega}_2^1\right)=\psi\left(\mathbf{\Omega}_2^2\right)=\prod_{i=0}^{3} x_{1,k-i} \prod_{i=1}^{3} x_{3,k-i}\ , \quad (3.85), (3.86)$$

$$\psi\left(\mathbf{\Omega}_3^1\right)=\prod_{i=0}^{3} x_{1,k-i} \prod_{i=1}^{4} x_{4,k-i}\ ,\qquad \psi\left(\mathbf{\Omega}_3^2\right)=\prod_{i=0}^{3} x_{1,k-i} \prod_{i=1}^{3} x_{4,k-i}\ , \quad (3.87), (3.88)$$

$$\psi\left(\mathbf{\Omega}_4^1\right)=\psi\left(\mathbf{\Omega}_4^2\right)=\prod_{i=1}^{3} x_{2,k-i} \prod_{i=1}^{3} x_{3,k-i}\ , \quad (3.89), (3.90)$$

$$\psi\left(\mathbf{\Omega}_5^1\right)=\prod_{i=1}^{3} x_{2,k-i} \prod_{i=1}^{4} x_{4,k-i}\ ,\qquad \psi\left(\mathbf{\Omega}_5^2\right)=\prod_{i=1}^{3} x_{2,k-i} \prod_{i=1}^{3} x_{4,k-i}\ , \quad (3.91), (3.92)$$

$$\psi\left(\mathbf{\Omega}_6^1\right)=\prod_{i=1}^{3} x_{3,k-i} \prod_{i=1}^{4} x_{4,k-i}\ ,\qquad \psi\left(\mathbf{\Omega}_6^2\right)=\prod_{i=1}^{3} x_{3,k-i} \prod_{i=1}^{3} x_{4,k-i}\ , \quad (3.93), (3.94)$$

$$\psi\left(\mathbf{\Omega}_7^1\right)=\psi\left(\mathbf{\Omega}_7^2\right)=\prod_{i=0}^{3} x_{1,k-i} \prod_{i=1}^{3} x_{2,k-i} \prod_{i=1}^{3} x_{3,k-i}\ , \quad (3.95)$$

$$\psi\left(\mathbf{\Omega}_8^1\right)=\prod_{i=0}^{3} x_{1,k-i} \prod_{i=1}^{3} x_{2,k-i} \prod_{i=1}^{4} x_{4,k-i}\ , \quad (3.96)$$

$$\psi\left(\mathbf{\Omega}_8^2\right)=\prod_{i=0}^{3} x_{1,k-i} \prod_{i=1}^{3} x_{2,k-i} \prod_{i=1}^{3} x_{4,k-i}\ , \quad (3.97)$$

$$\psi\left(\mathbf{\Omega}_9^1\right)=\prod_{i=1}^{3} x_{2,k-i} \prod_{i=1}^{3} x_{3,k-i} \prod_{i=1}^{4} x_{4,k-i}\ , \quad (3.98)$$

$$\psi\left(\mathbf{\Omega}_9^2\right)=\prod_{i=1}^{3} x_{2,k-i} \prod_{i=1}^{3} x_{3,k-i} \prod_{i=1}^{3} x_{4,k-i}\ , \quad (3.99)$$

$$\psi\left(\mathbf{\Omega}_{10}^1\right)=\prod_{i=0}^{3} x_{1,k-i} \prod_{i=1}^{3} x_{3,k-i} \prod_{i=1}^{4} x_{4,k-i}\ , \quad (3.100)$$

$$\psi\left(\mathbf{\Omega}_{10}^2\right)=\prod_{i=0}^{3} x_{1,k-i} \prod_{i=1}^{3} x_{3,k-i} \prod_{i=1}^{3} x_{4,k-i}\ , \quad (3.101)$$

$$\psi\left(\mathbf{\Omega}_{11}^1\right)=\prod_{i=0}^{3} x_{1,k-i} \prod_{i=1}^{3} x_{2,k-i} \prod_{i=1}^{3} x_{3,k-i} \prod_{i=1}^{4} x_{4,k-i}\ , \quad (3.102)$$

$$\psi\left(\mathbf{\Omega}_{11}^2\right)=\prod_{i=0}^{3} x_{1,k-i} \prod_{i=1}^{3} x_{2,k-i} \prod_{i=1}^{3} x_{3,k-i} \prod_{i=1}^{3} x_{4,k-i}\ , \quad (3.103)$$

for which

$$P\left[\psi\left(\mathbf{\Omega}_1^1\right)=1\right]=P\left[\psi\left(\mathbf{\Omega}_1^2\right)=1\right]=P\left[\psi\left(\mathbf{\Omega}_2^1\right)=1\right]=P\left[\psi\left(\mathbf{\Omega}_2^2\right)=1\right]= \\ P\left[\psi\left(\mathbf{\Omega}_3^2\right)=1\right]=\left(1-q_d{}^4\right)\left(1-q_d{}^3\right)=1-q_d{}^3-q_d{}^4+q_d{}^7, \tag{3.104}$$

$$P\left[\psi\left(\mathbf{\Omega}_4^1\right)=1\right]=P\left[\psi\left(\mathbf{\Omega}_4^2\right)=1\right]=P\left[\psi\left(\mathbf{\Omega}_5^2\right)=1\right]=P\left[\psi\left(\mathbf{\Omega}_6^2\right)=1\right]= \\ =\left(1-q_d{}^3\right)\left(1-q_d{}^3\right)=1-2q_d{}^3+q_d{}^6, \tag{3.105}$$

$$P\left[\psi\left(\mathbf{\Omega}_5^1\right)=1\right]=P\left[\psi\left(\mathbf{\Omega}_6^1\right)=1\right]=\left(1-q_d{}^3\right)\left(1-q_d{}^4\right)=1-q_d{}^3-q_d{}^4+q_d{}^7, \tag{3.106}$$

$$P\left[\psi\left(\mathbf{\Omega}_3^1\right)=1\right]=\left(1-q_d{}^4\right)\left(1-q_d{}^4\right)=1-2q_d{}^4+q_d{}^8, \tag{3.107}$$

$$P\left[\psi\left(\mathbf{\Omega}_7^1\right)=1\right]=P\left[\psi\left(\mathbf{\Omega}_7^2\right)=1\right]=P\left[\psi\left(\mathbf{\Omega}_8^2\right)=1\right]=P\left[\psi\left(\mathbf{\Omega}_{10}^2\right)=1\right]= \\ =\left(1-q_d{}^4\right)\left(1-q_d{}^3\right)\left(1-q_d{}^3\right)=1-2q_d{}^3-q_d{}^4+q_d{}^6+2q_d{}^7-q_d{}^{10}, \tag{3.108}$$

$$P\left[\psi\left(\mathbf{\Omega}_8^1\right)=1\right]=P\left[\psi\left(\mathbf{\Omega}_{10}^1\right)=1\right]=\left(1-q_d{}^4\right)\left(1-q_d{}^3\right)\left(1-q_d{}^4\right)= \\ =1-q_d{}^3-2q_d{}^4+2q_d{}^7+q_d{}^8-q_d{}^{11}, \tag{3.109}$$

$$P\left[\psi\left(\mathbf{\Omega}_9^1\right)=1\right]=\left(1-q_d{}^3\right)\left(1-q_d{}^3\right)\left(1-q_d{}^4\right)= \\ =1-2q_d{}^3-q_d{}^4+q_d{}^6+2q_d{}^7-q_d{}^{10}, \tag{3.110}$$

$$P\left[\psi\left(\mathbf{\Omega}_9^2\right)=1\right]=\left(1-q_d{}^3\right)\left(1-q_d{}^3\right)\left(1-q_d{}^3\right)=1-3q_d{}^3+3q_d{}^6-q_d{}^9, \tag{3.111}$$

$$P\left[\psi\left(\mathbf{\Omega}_{11}^1\right)=1\right]=\left(1-q_d{}^4\right)\left(1-q_d{}^3\right)\left(1-q_d{}^3\right)\left(1-q_d{}^4\right)= \\ ==1-2q_d{}^3-2q_d{}^4+q_d{}^6+4q_d{}^7+q_d{}^8-2q_d{}^{10}-2q_d{}^{11}+q_d{}^{14}, \tag{3.112}$$

$$P\left[\psi\left(\mathbf{\Omega}_{11}^2\right)=1\right]=\left(1-q_d{}^4\right)\left(1-q_d{}^3\right)\left(1-q_d{}^3\right)\left(1-q_d{}^3\right)= \\ =1-3q_d{}^3-q_d{}^4+3q_d{}^6+3q_d{}^7-q_d{}^9-3q_d{}^{10}+q_d{}^{13}. \tag{3.113}$$

Introducing the additional variable z_u, $u=1,2,...,11$, such that

$$z_u=\begin{cases} P\left[\psi\left(\Omega_u^1\right)=1\right], & \text{where} \quad t\in\left[t_k,\Delta_1^1\right), \\ P\left[\psi\left(\Omega_u^2\right)=1\right], & \text{where} \quad t\in\left[t_k+\Delta_1^1,\Delta_2^1\right). \end{cases} \tag{3.114}$$

Then the availability of the transmission is based on the formula of the sum of events [Borowkow, 1975]

$$A(t)=\sum_{k=1}^{11} z_k - \sum_{k<l} z_k z_l + \sum_{k<l<m} z_k z_l z_m - \ldots (-1)^{10} z_1 \ldots z_{11}. \tag{3.115}$$

Due to the high upper factor (11) of the first sum the final form of $A(t)$ is not presented here.

3.4. Reliability and continuity model

The reliability and continuity of the navigational systems considered in Chapter II require determining two parameters characterizing the time of work of the system. They are: failure rate - λ and renewal rate - μ. From the definition [Bobrowski, 1985] we have

$$\lambda(t)=-\frac{d}{dt}[\ln R(t)]=\frac{f(t)}{R(t)}, \tag{3.116}$$

where:

$f(t)$ - probability density function,

$\lambda(t)$- failure rate.

Notice that if $t \to 0$, then

$$P[T \in (t, \Delta t)/T \geq t]=\frac{P\{t < T \leq t+\Delta t\}}{P\{T > t\}}=\frac{R(t)-R(t+\Delta t)}{R(t)} \approx \lambda(t)\Delta t, \tag{3.117}$$

which means that the quantity $\lambda(t)\Delta t$ is approximately equal to the probability of failure in the time interval $[t, t+\Delta t)$, under the condition that up to the moment t the component has worked without any failure. Analogically, for μ we obtain

$$\mu(t)\Delta t \approx \frac{U(t)-U(t+\Delta t)}{U(t)}, \tag{3.118}$$

where $U(t)$ is the unreliability function [Bobrowski, 1985] of the following form

$$U(t)=P(T<t)=1-R(t). \tag{3.119}$$

The considered process of the differential transmission GPS indicates the existence of two values of probability of the system occurrence in the availability state. These probabilities are related to the intervals Δ_1 and Δ_2. Hence, for both of the intervals the failure rates and renewal rates can be determined as

$$\lambda_1 = \frac{A(t) - A(t)A(t+\Delta_1)}{\Delta_1 \cdot A(t)}, \text{ for } \quad t \in [t_k, \Delta_1), \tag{3.120}$$

$$\mu_1 = \frac{[1-A(t)] - [1-A(t)][1-A(t+\Delta_1)]}{\Delta_1 \cdot [1-A(t)]}, \text{ for } \quad t \in [t_k, \Delta_1), \tag{3.121}$$

$$\lambda_2 = \frac{A(t) - A(t)A(t+\Delta_2)}{\Delta_2 \cdot A(t)}, \text{ for } \quad t \in [t_k + \Delta_1, \Delta_2), \tag{3.123}$$

$$\mu_2 = \frac{[1-A(t)] - [1-A(t)][1-A(t+\Delta_2)]}{\Delta_2 \cdot [1-A(t)]}, \text{ for } \quad t \in [t_k + \Delta_1, \Delta_2), \tag{3.124}$$

where:

λ_1, μ_1 - failure and renewal rates for $t \in [t_k, \Delta_1)$,

λ_2, μ_2 - failure and renewal rates for $t \in [t_k + \Delta_1, \Delta_2)$.

As the variable t related to the beginning of the interval, for which the reliability is being considered, can belong to one of two times intervals: $[t_k, \Delta_1)$ or $[t_k + \Delta_1, \Delta_2)$, $k \in \mathbf{Z}$, and as the moment $t + \tau$ related to the end of that interval τ can also belong to one of the intervals mentioned, so it is necessary to consider 4 possible cases. First of them is illustrated in Fig. 3.10.

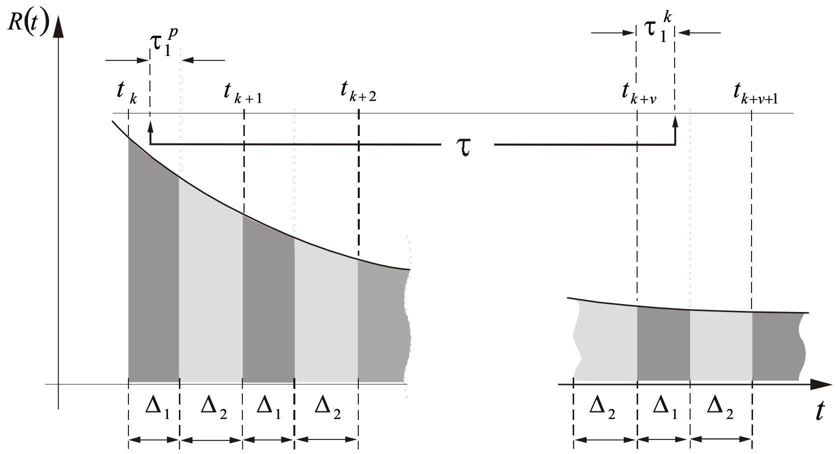

Fig. 3.10. The essence of determining the reliability of the differential GPS transmission in the interval τ, when $t \in [t_k, \Delta_1)$ and $t + \tau \in [t_{k+v}, \Delta_1)$, $k, v \in \mathbf{Z}$.

variant 1.

For: $t \in [t_k, \Delta_1)$, $t + \tau \in [t_{k+v}, \Delta_1)$, where $k, v \in \mathbf{Z}$ we have

$$R[t,\tau) = R[t,\tau_1^p) R[t_k + \Delta_1, \Delta_2) R[t_{k+1}, \Delta_1) \cdot \ldots \cdot R[t_{k+v-1}, \Delta_1) \cdot \\ \cdot R[t_{k+v-1} + \Delta_1, \Delta_2) R[t_{k+v}, \tau_1^k), \tag{3.125}$$

where:

τ_1^p - time interval between the moments: t, $t_k + \Delta_1$,

τ_1^k - time interval between the moments: t_{k+v}, $t+\tau$.

The variables τ and v are related to each other in the following way

$$\tau = \tau_1^p + (v-1)\Delta_1 + v\Delta_2 + \tau_1^k \tag{3.126}$$

and as

$$R[t_k, \Delta_1) = R[t_{k+1}, \Delta_1) = \ldots = R[t_{k+v}, \Delta_1) \tag{3.127}$$

and also

$$R[t_k + \Delta_1, \Delta_2) = R[t_{k+1} + \Delta_1, \Delta_2) = \ldots = R[t_{k+v} + \Delta_1, \Delta_2), \tag{3.128}$$

then

$$R[t,\tau) = R[t,\tau_1^p) \, R[t_k + \Delta_1, \Delta_2)^v R[t_k, \Delta_1)^{v-1} \, R[t_{k+v}, \tau_1^k) \,. \tag{3.129}$$

Using (2.95), the particular factors of the product (3.129) are determined as follows

$$R[t,\tau_1^p) = \left\{ \frac{\mu_1}{\lambda_1+\mu_1} + \frac{\lambda_1}{\lambda_1+\mu_1} \exp[-(\lambda_1+\mu_1)t] \right\} \exp(-\lambda_1 \tau_1^p), \tag{3.130}$$

$$R[t_{k+v},\tau_1^k) = \left\{ \frac{\mu_1}{\lambda_1+\mu_1} + \frac{\lambda_1}{\lambda_1+\mu_1} \exp[-(\lambda_1+\mu_1)t_{k+v}] \right\} \exp(-\lambda_1 \tau_1^k), \tag{3.131}$$

$$R[t_k,\Delta_1) = \left\{ \frac{\mu_1}{\lambda_1+\mu_1} + \frac{\lambda_1}{\lambda_1+\mu_1} \exp[-(\lambda_1+\mu_1)t_k] \right\} \exp(-\lambda_1 \Delta_1), \tag{3.132}$$

$$R[t_k + \Delta_1,\Delta_2) = \left\{ \frac{\mu_2}{\lambda_2+\mu_2} + \frac{\lambda_2}{\lambda_2+\mu_2} \exp[-(\lambda_2+\mu_2)(t_k+\Delta_1)] \right\} \exp(-\lambda_2 \Delta_2). \tag{3.133}$$

In order to define the continuity we apply the assumption,

$$\frac{\mu_1}{\lambda_1+\mu_1}+\frac{\lambda_1}{\lambda_1+\mu_1}\exp[-(\lambda_1+\mu_1)t]=1\,, \tag{3.134}$$

then

$$C[t,\tau)=\exp(-\lambda_1\tau_1^p)\,R[t_k+\Delta_1,\Delta_2)^v R[t_k,\Delta_1)^{v-1}\,R[t_{k+v},\tau_1^k). \tag{3.135}$$

variant 2.

For $t\in[t_k,\Delta_1)$, $t+\tau\in[t_{k+v}+\Delta_1,\Delta_2)$, $k,v\in\mathbf{Z}$ we have

$$\begin{aligned}R[t,\tau)=R[t,\tau_1^p)R[t_k+\Delta_1,\Delta_2)R[t_{k+1},\Delta_1)\cdot\ldots\cdot R[t_{k+v-1}+\Delta_1,\Delta_2)\cdot\\ \cdot R[t_{k+v},\Delta_1)R[t_{k+v}+\Delta_1,\tau_2^k),\end{aligned} \tag{3.136}$$

where:

τ_2^k - time interval between the moments: $t_{k+v}+\Delta_1$, $t+\tau$.

As the variables τ and v are related in the following way

$$\tau=\tau_1^p+v\Delta_1+v\Delta_2+\tau_2^k\;, \tag{3.137}$$

then

$$R[t,\tau)=R[t,\tau_1^p)\,R[t_k+\Delta_1,\Delta_2)^v R[t_k,\Delta_1)^v\,R[t_{k+v}+\Delta_1,\tau_2^k), \tag{3.138}$$

where

$$R[t_{k+v}+\Delta_1,\tau_2^k)=\left\{\frac{\mu_2}{\lambda_2+\mu_2}+\frac{\lambda_2}{\lambda_2+\mu_2}\exp[-(\lambda_2+\mu_2)(t_{k+v}+\Delta_1)]\right\}\exp(-\lambda_2\tau_2^k). \tag{3.139}$$

Similarly to (3.135), the continuity takes form

$$C[t,\tau)=\exp(-\lambda_1\tau_1^p)\,R[t_k+\Delta_1,\Delta_2)^v R[t_k,\Delta_1)^v\,R[t_{k+v}+\Delta_1,\tau_2^k). \tag{3.140}$$

variant 3.

For $t\in[t_k+\Delta_1,\Delta_2)$, $t+\tau\in[t_{k+v},\Delta_1^1)$, $k,v\in\mathbf{Z}$, we have

$$\begin{aligned}R[t,\tau)=R[t,\tau_2^p)R[t_{k+1},\Delta_1)R[t_{k+1}+\Delta_1,\Delta_2)\cdot\ldots\cdot R[t_{k+v-1},\Delta_1)\cdot\\ \cdot R[t_{k+v-1}+\Delta_1,\Delta_2)R[t_{k+v},\tau_1^k),\end{aligned} \tag{3.141}$$

where:

τ_2^p - time interval between the moments: t, t_{k+1}.

The variables τ and v are connected with the following relation

$$\tau = \tau_2^p + (v-1)\Delta_1 + (v-1)\Delta_2 + \tau_1^k, \tag{3.142}$$

then

$$R[t,\tau) = R[t,\tau_2^p)\, R[t_k + \Delta_1, \Delta_2)^{v-1}\, R[t_k, \Delta_1)^{v-1}\, R[t_{k+v}, \tau_1^k), \tag{3.143}$$

where

$$R[t,\tau_2^p) = \left\{ \frac{\mu_2}{\lambda_2 + \mu_2} + \frac{\lambda_2}{\lambda_2 + \mu_2} \exp[-(\lambda_2 + \mu_2)t] \right\} \exp(-\lambda_2 \tau_2^p). \tag{3.144}$$

The final form of the continuity is performed as follows

$$C[t,\tau) = \exp(-\lambda_2 \tau_2^p)\, R[t_k + \Delta_1, \Delta_2)^{v-1}\, R[t_k, \Delta_1)^{v-1}\, R[t_{k+v}, \tau_1^k). \tag{3.145}$$

variant 4.

For $t \in [t_k + \Delta_1, \Delta_2)$, $t + \tau \in [t_{k+v} + \Delta_1, \Delta_2)$, $k, v \in \mathbf{Z}$, we have

$$\begin{aligned} R[t,\tau) = R[t,\tau_2^p) R[t_{k+1}, \Delta_1) R[t_{k+1} + \Delta_1, \Delta_2) \cdot \ldots \cdot R[t_{k+v-1} + \Delta_1, \Delta_2) \cdot \\ \cdot R[t_{k+v}, \Delta_1) R[t_{k+v} + \Delta_1, \tau_2^k), \end{aligned} \tag{3.146}$$

where:

τ_2^k - time interval between the moments: $t_{k+v} + \Delta_1$, $t + \tau$.

As the variables τ and v are related in the following way

$$\tau = \tau_2^p + v\Delta_1 + (v-1)\Delta_2 + \tau_2^k, \tag{3.147}$$

then

$$R[t,\tau) = R[t,\tau_2^p)\, R[t_k + \Delta_1, \Delta_2)^{v-1}\, R[t_k, \Delta_1)^{v}\, R[t_{k+v} + \Delta_1, \tau_2^k), \tag{3.148}$$

where

$$R[t_{k+v} + \Delta_1, \tau_2^k) = \left\{ \frac{\mu_2}{\lambda_2 + \mu_2} + \frac{\lambda_2}{\lambda_2 + \mu_2} \exp[-(\lambda_2 + \mu_2)(t_{k+v} + \Delta_1)] \right\} \exp(-\lambda_2 \tau_2^k). \tag{3.149}$$

The continuity is defined as follows

$$C[t,\tau) = \exp(-\lambda_2 \tau_2^p)\, R[t_k + \Delta_1, \Delta_2)^{v-1}\, R[t_k, \Delta_1)^{v}\, R[t_{k+v} + \Delta_1, \tau_2^k). \tag{3.150}$$

The relations presented in this subchapter allow for reliability and continuity analytical calculations for both RTCM message (type 1 and 9-3).

DIFFERENTIAL GPS SYTEM NETWORK

4.1 Outline of DGPS network methods

High level requirements with regard to availability and reliability related to the safety of navigation have motivated the international organizations, responsible for aids to navigation systems (IALA, IMO), to increase requirements for maritime radionavigation broadcast and also DGPS services. Consequently, the desirability of DGPS multicoverage in areas of higher maritime traffic risk has been taken into consideration, since signals broadcast from a single DGPS base station may turn out to be insufficient to satisfy high navigational requirements [IALA, 1999].

Within the internal basins, such as the Baltic Sea, there occurs a unique navigational situation with respect to operating differential GPS beacons. The area features multicoverage over the same area by broadcast zones of several neighbouring DGPS reference stations. This situation is exceptionally evident in the Southern Baltic, and is caused by the following:

1. special shape of shore line, the distance between the opposite shores is less then 300 km,
2. high number of littoral states with well-developed economies and well-developed maritime infrastructure,
3. high maritime traffic and cargo operations intensity,
4. high number of maritime ports requiring aids to navigation and hydrographical support, with regard to accurate ship positioning,
5. absence of international co-operation with regard to common strategy related to hydrographical and navigational support of navigation.

It is worthwhile mentioning here that the basin under consideration is unique in the global scale. It is not common to meet similar density of reference base stations. Research projects centred on navigational advantages of system multicoverage were initiated in Poland in mid 1990s and have mainly been directed towards 2 aspects:

1. Extrapolation of pseudorange correction based upon data received from several local reference stations [Cydejko J.; Król J., 2002],
2. Increased reliability exploitation parameters of a system composed of several DGPS reference stations working in synchronized networking mode [Specht, 2002a].

4.2. Coverage of networking mode

The starting point for network analysis of DGPS systems was to determine the basins where redundancy existence structures offered the possibility for their simultaneous use (signals from several DGPS reference stations). It was essential to evaluate the coverage zone of all available DGPS base stations by means of direct measurements combined with the theoretical model of propagation, worked out on the basis of Polish publications [Kopacz Z., Urbański J., 1981] and international literature [Enge P. et al, 1987; Kalafus R. et al, 1993].

Dedicated expert software was developed to pre-analyse all coverage zones of DGPS beacons in the Southern Baltic. The shoreline combined of 100 000 points was vectorized and a database of ground conductivity and dielectric constant was added [Dołuchanow M.P., 1965]. Millington's method [Kalafus R. et al, 1993] was used to calculate the field strength of beacons' signal. Measurement campaigns of signal strength performed during period 2000-2002 proved the simultaneous availability of up to seven reference stations (Tab. 4.1).

Table 4.1. Typical results of signal strength of DGPS beacons in the Southern Baltic, measured on fixed position: 53° 55' 15'' N, 14° 17'06'' E (summer 2002 year).

no	Reference station	SS [dBu]	SNR [dB]	f [kHz]
1	Dziwnów	45	19	283.5
2	Holmsjo	30	9	292
3	Hammerodde	34	11	289
4	Hoburg	32	11	297.5
5	Kullen	28	7	293
6	Wustrow	48	18	308
7	Rozewie	26	26	301

SS - signal strength level,

f - frequency.

Coverage zones of three examples of DGPS Baltic reference stations are shown in the picture below.

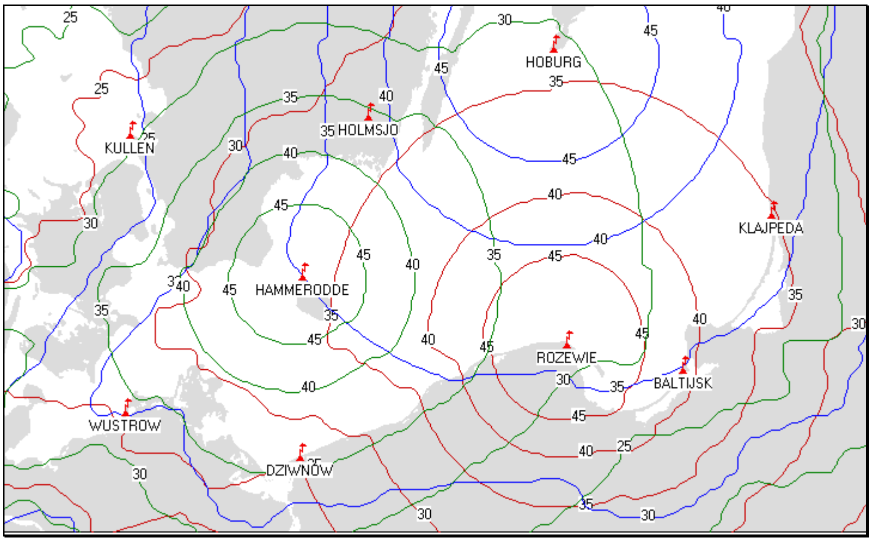

Fig. 4.1. Coverage areas of DGPS reference stations: Rozewie, Hoburg and Hammerodde

Determination of coverage areas for every single reference station enables determining common areas, simultaneously covered by broadcast signals, thus enabling networking mode (Fig. 4.2).

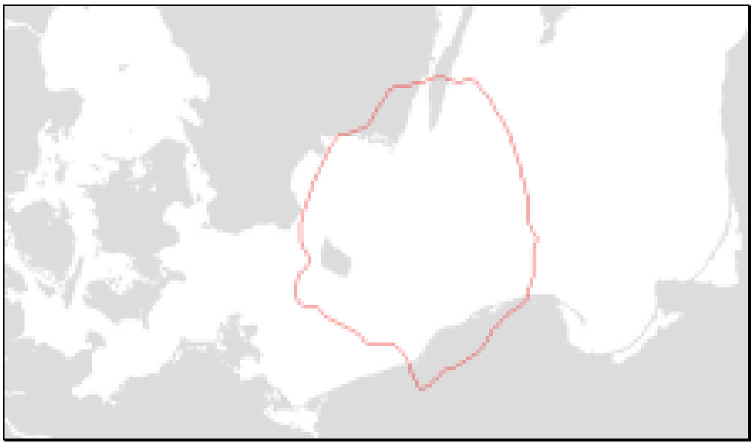

Fig. 4.2. Common coverage area within the range of DGPS: Rozewie, Hoburg and Hammerodde

For the area presented above, 34 dBuV was assumed as the minimum signal strength level (theoretical limit for coverage area border line).

4.3. Model of availability, reliability and continuity

4.3.1. Non-synchronised system

For the networking DGPS method, the probability of signal detections from every station is independent. Hence for number g – of independent broadcasting reference stations, the transmission availability of networking system can be evaluated as

$$A_s(t) = 1 - [1 - A_1(t)][1 - A_2(t)] \cdot \ldots \cdot [1 - A_g(t)] \ . \tag{4.1}$$

To determine the reliability of networking systems, the reliability function for parallel systems can be used [Grabski F., 1981], in the form

$$R_s[t,\tau) = 1 - [1 - R_1[t,\tau)][1 - R_2[t,\tau)] \cdot \ldots \cdot [1 - R_g[t,\tau)] \tag{4.2}$$

and by analogy, for the continuity, we get

$$C_s[t,\tau) = 1 - [1 - C_1[t,\tau)][1 - C_2[t,\tau)] \cdot \ldots \cdot [1 - C_g[t,\tau)] \tag{4.3}$$

where

$A_s(t)$ - availability of DGPS transmission for networking system in moment of time *t*,

$R_s[t,\tau)$ - reliability of DGPS transmission for networking system for the period τ,

$C_s[t,\tau)$ - continuity of DGPS transmission for networking system for the period τ.

Relations (4.1)-(4.3) can be expressed as a sum of independent events [Borowkow, 1975], which in the case of networking DGPS system incorporating g reference stations has the form

$$A_s(t) = \sum_{k=1}^{g} A_k(t) - \sum_{k<l} A_k(t)A_l(t) + \sum_{k<l<m} A_k(t)A_l(t)A_m(t) - \ldots (-1)^{g-1} A_1(t)\ldots A_g(t), \tag{4.4}$$

$$\begin{aligned} R_s[t,\tau) = {} & \sum_{k=1}^{g} R_k[t,\tau) - \sum_{k<l} R_k[t,\tau)R_l[t,\tau) + \sum_{k<l<m} R_k[t,\tau)R_l[t,\tau)R_m[t,\tau) + \\ & - \ldots (-1)^{g-1} R_1[t,\tau)\ldots R_g[t,\tau), \end{aligned} \tag{4.5}$$

$$C_s[t,\tau)=\sum_{k=1}^{g} C_k[t,\tau)-\sum_{k<l} C_k[t,\tau)C_l[t,\tau)+\sum_{k<l<m} C_k[t,\tau)C_l[t,\tau)C_m[t,\tau)+ \\ -\ldots(-1)^{g-1}C_1[t,\tau)\ldots C_g[t,\tau). \tag{4.6}$$

4.3.2. Synchronized systems

One of the concepts how to take advantage of a redundant number of reference DGPS signals is embodied in the synchronized networking method [Specht C., 2002c]. That idea has been considered as one of potential feature directions for DGPS systems development in the nearest future [RNAV 17, 2002].

The classical solution for maritime DGPS in the LF/MF range was based on frequency division access, and independent work of autonomous, single reference station resulted from this solution. A proposal for further investigations related to DGPS systems operating in the LF/MF range concerns the creation of reference stations chains working in the ordered mode (Fig. 4.3).

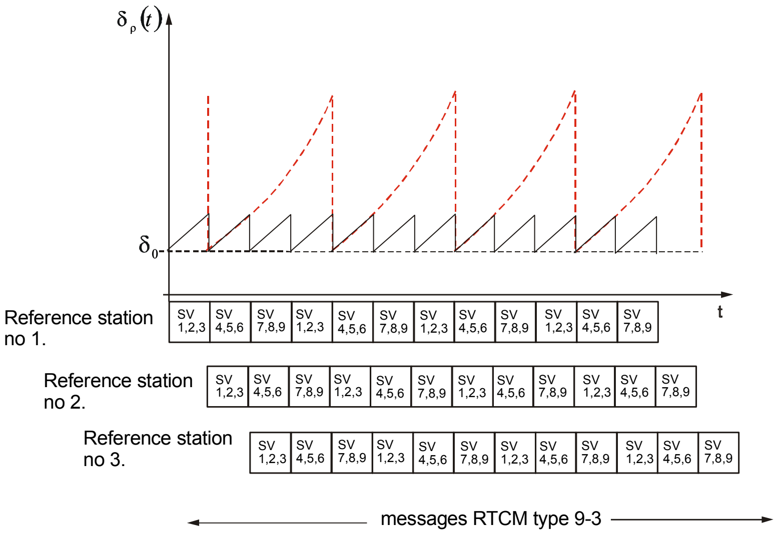

Fig. 4.3. A concept of multi-station system performance with telemetric link for chain of 3 DGPS reference stations and expected increase in the pseudorange accuracy measurement

The main problem, which would be a time synchronization of broadcast DGPS signals, might in practice be regarded as solved, due to the fundamental GPS function, which, apart from the positioning service, is time distribution (GPS time). GPS time is transferred by satellite SPS signals with error not exciding of 5 nanoseconds in relation to UTC.

It would be necessary to establish a synchronized PRC transmission method, including message type 9-3 with baud rate 100 bps. As a result, a users' receiver could detect pseudorange corrections simultaneously from several reference stations. Evident diminution of data age would influence the increase in system positioning accuracy.

The expected improvement of pseudorange accuracy measurement in the system (Fig. 4.3) would result from higher availability of message transmission and shortened time delay. The fundamental feature of the solution proposed is the fact that nothing would be changed in characteristics observed by a user, who would prefer to continue to work with only one, single reference station. The moment of switching the station into the chain (network) should only be done in a precisely described moment of time, not at any moment as it takes place now.

For the synchronized system the binary state vectors of the system for time intervals Δ_1 and Δ_2 are of the form

$$\begin{aligned}\mathbf{\Lambda_1} = \big[&\left(x^1_{1,k-4}, x^1_{1,k-3},\ldots,x^1_{1,k}\right),\left(x^1_{2,k-4}, x^1_{2,k-3},\ldots,x^1_{2,k-1}\right),\left(x^1_{3,k-5}, x^1_{3,k-4},\ldots,x^1_{3,k-1}\right),\\ &\left(x^2_{2,k-4}, x^2_{2,k-3},\ldots,x^2_{2,k}\right),\left(x^2_{3,k-4}, x^2_{3,k-3},\ldots,x^2_{3,k-1}\right),\left(x^2_{1,k-5}, x^2_{1,k-4},\ldots,x^2_{1,k-1}\right),\\ &\left(x^3_{3,k-4}, x^3_{3,k-3},\ldots,x^3_{3,k}\right),\left(x^3_{1,k-4}, x^3_{1,k-3},\ldots,x^3_{1,k-1}\right),\left(x^3_{2,k-5}, x^3_{2,k-4},\ldots,x^3_{2,k-1}\right)\big],\end{aligned} \tag{4.7}$$

$$\begin{aligned}\mathbf{\Lambda_2} = \big[&\left(x^1_{1,k-4}, x^1_{1,k-3},\ldots,x^1_{1,k}\right),\left(x^1_{2,k-4}, x^1_{2,k-3},\ldots,x^1_{2,k-1}\right),\left(x^1_{3,k-4}, x^1_{3,k-3},\ldots,x^1_{3,k-1}\right),\\ &\left(x^2_{2,k-4}, x^2_{2,k-3},\ldots,x^2_{2,k}\right),\left(x^2_{3,k-4}, x^2_{3,k-3},\ldots,x^2_{3,k-1}\right),\left(x^2_{1,k-4}, x^2_{1,k-3},\ldots,x^2_{1,k-1}\right),\\ &\left(x^3_{3,k-4}, x^3_{3,k-3},\ldots,x^3_{3,k}\right),\left(x^3_{1,k-4}, x^3_{1,k-3},\ldots,x^3_{1,k-1}\right),\left(x^3_{2,k-4}, x^3_{2,k-3},\ldots,x^3_{2,k-1}\right)\big],\end{aligned} \tag{4.8}$$

where upper index of variable x identifies one of the three reference stations working in the synchronized networking mode.

Their structure functions take the following forms

$$\begin{aligned}\psi(\mathbf{\Lambda_1}) = &\left(\coprod_{i=0}^{4} x^1_{1,k-i} \amalg \coprod_{i=1}^{5} x^2_{1,k-i} \amalg \coprod_{i=1}^{4} x^3_{1,k-i}\right)\left(\coprod_{i=1}^{4} x^1_{2,k-i} \amalg \coprod_{i=0}^{4} x^2_{2,k-i} \amalg \coprod_{i=1}^{5} x^3_{2,k-i}\right)\amalg\\ \amalg&\left(\coprod_{i=0}^{4} x^1_{1,k-i} \amalg \coprod_{i=1}^{5} x^2_{1,k-i} \amalg \coprod_{i=1}^{4} x^3_{1,k-i}\right)\left(\coprod_{i=1}^{5} x^1_{3,k-i} \amalg \coprod_{i=1}^{4} x^2_{3,k-i} \amalg \coprod_{i=0}^{4} x^3_{3,k-i}\right)\amalg\\ \amalg&\left(\coprod_{i=1}^{4} x^1_{2,k-i} \amalg \coprod_{i=0}^{4} x^2_{2,k-i} \amalg \coprod_{i=1}^{5} x^3_{2,k-i}\right)\left(\coprod_{i=1}^{5} x^1_{3,k-i} \amalg \coprod_{i=1}^{4} x^2_{3,k-i} \amalg \coprod_{i=0}^{4} x^3_{3,k-i}\right)\amalg\\ \amalg&\left(\coprod_{i=0}^{4} x^1_{1,k-i} \amalg \coprod_{i=1}^{5} x^2_{1,k-i} \amalg \coprod_{i=1}^{4} x^3_{1,k-i}\right)\left(\coprod_{i=1}^{4} x^1_{2,k-i} \amalg \coprod_{i=0}^{4} x^2_{2,k-i} \amalg \coprod_{i=1}^{5} x^3_{2,k-i}\right)\\ &\left(\coprod_{i=1}^{5} x^1_{3,k-i} \amalg \coprod_{i=1}^{4} x^2_{3,k-i} \amalg \coprod_{i=0}^{4} x^3_{3,k-i}\right),\end{aligned} \tag{4.9}$$

$$\begin{aligned}\psi(\mathbf{\Lambda_2}) = & \left(\coprod_{i=0}^{4} x^{1}_{1,k-i} \amalg \coprod_{i=1}^{4} x^{2}_{1,k-i} \amalg \coprod_{i=1}^{4} x^{3}_{1,k-i}\right)\left(\coprod_{i=1}^{4} x^{1}_{2,k-i} \amalg \coprod_{i=0}^{4} x^{2}_{2,k-i} \amalg \coprod_{i=1}^{4} x^{3}_{2,k-i}\right) \amalg \\ & \amalg \left(\coprod_{i=0}^{4} x^{1}_{1,k-i} \amalg \coprod_{i=1}^{4} x^{2}_{1,k-i} \amalg \coprod_{i=1}^{4} x^{3}_{1,k-i}\right)\left(\coprod_{i=1}^{4} x^{1}_{3,k-i} \amalg \coprod_{i=1}^{4} x^{2}_{3,k-i} \amalg \coprod_{i=0}^{4} x^{3}_{3,k-i}\right) \amalg \\ & \amalg \left(\coprod_{i=1}^{4} x^{1}_{2,k-i} \amalg \coprod_{i=0}^{4} x^{2}_{2,k-i} \amalg \coprod_{i=1}^{4} x^{3}_{2,k-i}\right)\left(\coprod_{i=1}^{4} x^{1}_{3,k-i} \amalg \coprod_{i=1}^{4} x^{2}_{3,k-i} \amalg \coprod_{i=0}^{4} x^{3}_{3,k-i}\right) \amalg \\ & \amalg \left(\coprod_{i=0}^{4} x^{1}_{1,k-i} \amalg \coprod_{i=1}^{4} x^{2}_{1,k-i} \amalg \coprod_{i=1}^{4} x^{3}_{1,k-i}\right)\left(\coprod_{i=1}^{4} x^{1}_{2,k-i} \amalg \coprod_{i=0}^{4} x^{2}_{2,k-i} \amalg \coprod_{i=1}^{4} x^{3}_{2,k-i}\right) \\ & \left(\coprod_{i=1}^{4} x^{1}_{3,k-i} \amalg \coprod_{i=1}^{4} x^{2}_{3,k-i} \amalg \coprod_{i=0}^{4} x^{3}_{3,k-i}\right). \end{aligned} \tag{4.10}$$

Structures decomposition (4.9) and (4.10) as an arithmetical issue will not be shown in this publication because of the size of the mathematical formulas. Nevertheless, it is relatively simple to realize with the use of computer. The other characteristics such as: reliability and continuity of synchronized DGPS networking system are described by the same relations as for a single station system (equations: 3.16-3.150).

4.4. Assessment of differential GPS transmission methods

Mathematical models worked out for availability, reliability and continuity of differential GPS transmission, both for classic (single-station) and for networking mode (multi-station), enabled realization of a number of comparative analyses with influencers, such as: message format, baud rate of telemetric channel, number of satellites supported by transmitted pseudorange corrections, with regard to the considered characteristics. Based on the above, we may draw a series of conclusions, of which the most important are:

The performed tests have shown that there exist two values of transmission availability, depending on time t, its appurtenance to the range: Δ_1 or Δ_2. The curves presented in Fig. 4.4 reflect the influence of minimum and maximum values of availability. Significant differences can be noted for reference stations using message RTCM type 1, particularly at the limits of coverage range (BER=10^{-3}). For the systems transmitting message type 9-3 these values are approximate.

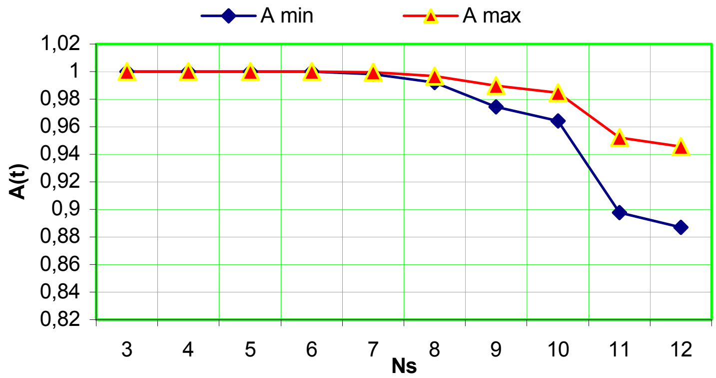

Fig. 4.4. Minimum and maximum availability of differential GPS transmission, for RTCM message type 1, BER=10^{-3}, R=100 bps, as a function of the number corrected satellites

Baud rate increase of telemetric link results in higher availability, reliability and continuity (Fig. 4.5).

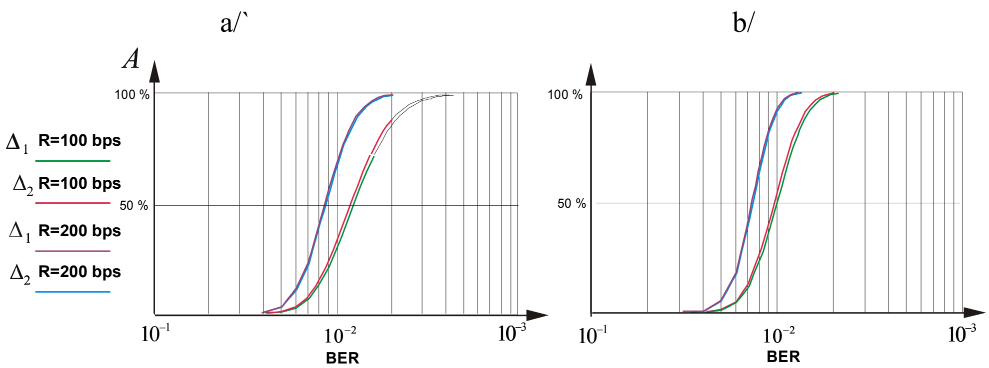

Fig. 4.5. DGPS transmission system availability as a function of bit error ratio (BER) and baud rate(R), where: Ns={4,5,6} (Fig. a) or Ns={7,8,9} (Fig. b).

The increase in the number of satellites for which the referential station transmits pseudorange corrections within the framework of message RTCM type 9-3 type influences the availability (Fig. 4.5), reliability and continuity (Fig. 4.6) of differential transmission. The diagram below presents the continuity of differential GPS transmission, for BER=10^{-3}, message RTCM type 9-3, number of satellites as a parameter, versus time.

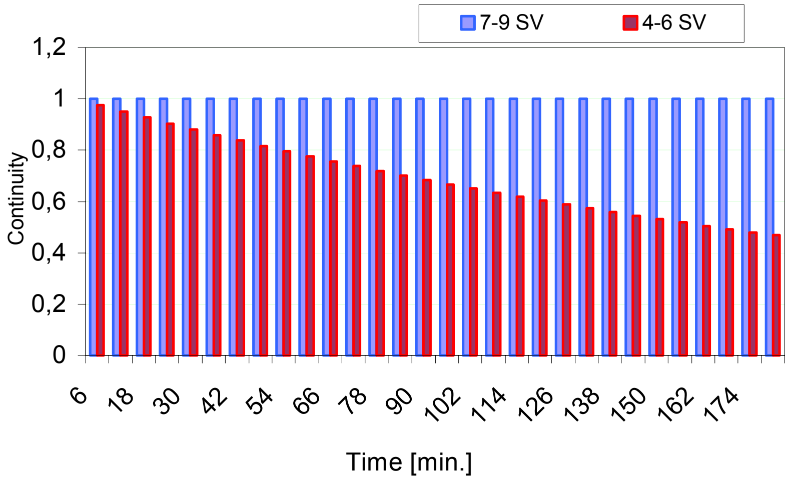

Fig. 4.6. Continuity of differential GPS transmission for: message RTCM type 9-3, BER=10^{-3}, Ns={4,5,6} and Ns={7,8,9}

Computer simulations have shown that application of DGPS networking mode of work significantly improves reliability characteristics of the system, and for BER=10^{-2} in the networking mode, of synchronized DGPS system, the availability of the transmission was 0.99992. The coefficient for a single-station mode was 0.5409 and it would be far too insufficient.

Models developed within this dissertation, together with dedicated software, can be used to forecast reliability criteria for navigation systems with regard to differential GPS transmission. The figure below, presents an example of simulation of transmission availability zones for DGPS reference station in the Baltic.

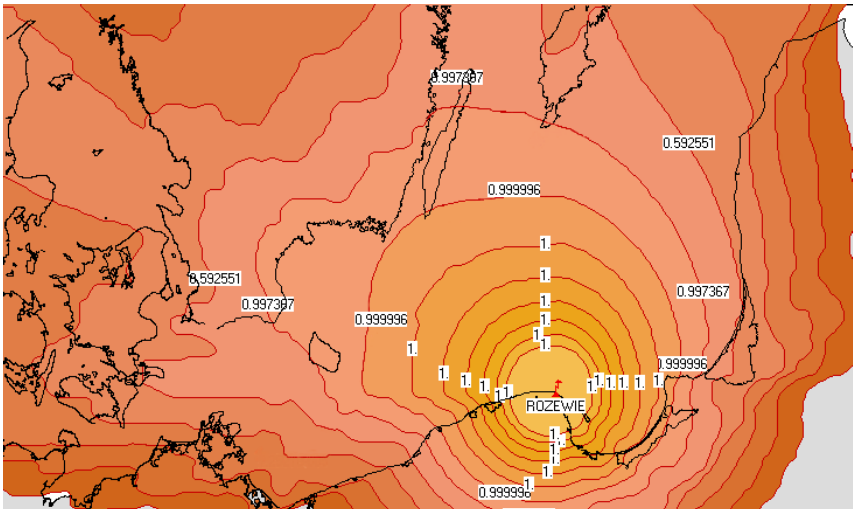

Fig. 4.7. Transmission availability of DGPS reference station Rozewie

SUMMARY AND CONCLUSIONS

The aim of this dissertation has been to develop a mathematical model of availability, reliability and continuity of differential GPS transmission based on probabilistic approach. The problem was important because of the lack of methods, which would enable the parameter prognosis with regard to the process of navigation. In this light, the problem reliability characteristics constitutes one of the fundamental issues for multi-criteria assessment of modern systems at the stage of system testing and exploitation as well. The necessity of general theory development for availability, reliability and continuity of navigation systems became obvious during the preliminary research work, which was justified by the existing terminological discrepancies in this field. The theory the author worked out has been referred to the probabilistic characteristics of navigation systems functioning such as expected value, variance, or standard deviation. For this reason it may be applied for freely chosen distribution processes, not only for the systems where healthy or unhealthy functioning periods were given by exponential equations (which is typical of the processes in navigation).

Another essential problem related to the proposed theory of general availability, reliability and continuity is the suggestion that additive components, which interfere with the working states of the systems should be taken into consideration. Their binary representation states may be smoothly evaluated by means of statistic analysis. This way, it was shown that modeling reliability indexes in navigation do not need to be limited only to equipment characteristics [IALA, 1989] but they should also encompass the process occurrence in the system environment. We may assume that in a due course of time, when the individual sub-processes occurring in the system are recognized, the full mathematical description will only be a question of knowledge development.

Conceptual unification with regard to the characteristics included herein has enabled the application of elements of reliability theory to structural analysis of DGPS systems applied in navigation, where a separated structure has been studied. On this basis, the model of differential GPS transmission standard using RTCM SC-104 applied for navigation messages has also been investigated.

In addition, the following research results have been obtained:

- Empirical results concerning RTCM transmission of message type 9-3 instead of type 1 have been confirmed by analytical results justified also by DGPS reliability characteristics (availability, reliability, continuity),

- Evidence of positive influence of increase in the number of satellites, for which a differential DGPS station transmits PRC corrections, on availability, reliability, continuity of GPS transmission,
- Computation of DGPS coverage zones for the Southern Baltic area, and determination of zones where the use of DGPS networking mode is possible,
- Concept of DGPS synchronized mode has been presented and reliability characteristics for this mode have been worked out, also some aspects of exploitation have been shown.

The new concept of specification of reliability criteria for navigation systems as presented here, is a theoretical model with reference to general and particular issues. The theory enables;

- Unambiguous determination of navigation structures and systems on the basis of reliability theory and evaluation of their availability, reliability and continuity,
- Comparable validation of differential GPS transmissions from single reference DGPS station related to pseudorange data format and transmission baud rate,
- Designing DGPS stations spatial deployment around the marine basins, depending on precise requirements of navigation service exploitation characteristics,
- Usefulness analysis of DGPS signal multicoverage over water basins, and characteristics determination in networking mode application.

The cognitive elements of this dissertation are listed below:

1. Elaboration of a continuity mathematical model for navigation systems operation, which enabled establishment of relations between availability, reliability and continuity.

2. Realization of mathematical model for the considered criteria related to DGPS transmission applied in navigation and using RTCM messages type 1 or 9-3.

3. Concept of synchronized DGPS system and validation of its availability, reliability and continuity.

It should be emphasized that the research presented here is theoretical. Thus, the fundamental limitation of the differential GPS transmission model proposed is the absence of additive interfering factors related to both natural and man made sources of electromagnetic emission. This issue concerns mainly DGPS system application in land navigation or urban environment, where the presence of various electromagnetic interferences can significantly modify the characteristics of differential transmission. It is mainly due to its broad extent that more profound considerations have not been given to the interference problem in this dissertation. However, the issues may be further developed in a separate research project.

References

- Baran L.W., Oszczak S., (1999), GPS/DGPS Permanent Stations in Poland, Reports on Geodesy, No 3 (44).
- Barlow R. E., Proshan F., (1975), Statistical Theory of Reliability and Life Testing, Holt, Rinehart and Winston, inc., USA.
- Bobrowski D., (1985), Modele i metody matematyczne teorii niezawodności w przykładach i zadaniach, Wydawnictwo Naukowo-Techniczne, Warszawa, Poland.
- Borowkow A., (1975), Rachunek prawdopodobieństwa, Państwowe Wydawnictwo Naukowe, Warszawa, Poland.
- Cydejko J., Król A., (2002), Analiza wartości poprawek różnicowych z kilku stacji referencyjnych GPS, Proceedings of XIIth International Scientific and Technical Conference – The Part of Navigation in Support of Human Activity on the Sea, Gdynia, Poland.
- Cydejko J., Oszczak S., (2002), Test of EGNOS System Test Bed (ESTB) Performance in Poland, Proceedings of The European Navigation Conference GNSS'2002, CD paper no 193, Copenhagen, Denmark.
- Dziewicki M., Felski A., Specht C., (1998), Availability Of DGPS Reference Stations Signals on South Baltic, Proceedings of the 2nd European Symposium on Global Navigation Satellite Systems – GNSS'98, Touluse, France.
- Dołuchanow M.P., (1965), Rozchodzenie się fal radiowych, Państwowe Wydawnictwo Naukowe, Warszawa, Poland.
- Enge P. K., Ruane M., Langlais D., (1988), Coverage of a Radio Beacon-Based Differential GPS Network, Navigation, vol. 34, No. 4, winter 1987-88.
- Felski A., (1999), O rozumieniu kilku podstawowych pojęć związanych z nawigacją satelitarną, VIII International Scientific and Technical Conference on Maritime Traffic Engineering, Szczecin Maritime University, Poland.
- Felski A., Grabski F., Specht C., (1999), Availability of the DGPS radiobeacon signals – analytical method, Proceedings of the 3rd European Symposium on Global Navigation Satellite Systems – GNSS'99, Genova, Italy.
- Farrell J., Graas F., (1992), Statistical Validation for GPS Integrity Test, Navigation, vol. 39, No 2, summer.
- Ghashghai E., (2000), GPS Availability of the Terrestrial Navigation, Proceedings of the IAIN World Congress, US ION 56th Annual Meeting, San Diego, CA, USA.
- Grabski F., (1981), Matematyczne podstawy bada operacyjnych, WSMW, Gdynia, Poland.

- Grabski F., (2002), Semi-markowskie modele niezawodności i eksploatacji, Polska Akademia Nauk, Instytut Badań Systemowych, tom 30, Warszawa, Poland.
- Kalafus R., Enge P., Levin P., Hansen A., (1993), Coverage of DGPS/Radiobeacons, Navigation vol. 39, No 4, winter.
- Kopacz Z. et al., (1996), Osłona radionawigacyjna obszarów morskich R.P, raport końcowy z projektu celowego KBN, Gdynia, Poland.
- Kopacz Z., Urbański J., (1981), Zasady wystawiania i kalibracji radionawigacyjnych systemów hydrograficznych, WSMW, Gdynia, Poland.
- Kopociński B., (1973), Zarys teorii odnowy i niezawodności, Biblioteka Naukowa Inżyniera, Państwowe Wydawnictwo Naukowe, Warszawa, Poland.
- Kremer G., Kalafus R., Loomis P., Reynolds J., (1990), The Effect of Selective Availability on Differential GPS Corrections, Navigation, vol. 37, No 1, spring.
- Nayak R. A., Cannon M.E., Wilson C., Zhang G., (2000), Use of Multiple GPS Antennae And A Low Coast IMU for Reliable And Continuous Urban Navigation, Proceedings of the IAIN World Congress, U.S. ION 56th Annual Meeting, San Diego, CA, USA.
- Ober P.B., (1999), Designing Integrity into Position Estimation, Proceedings of the 3rd European Symposium on Global Navigation Satellite Systems – GNSS'99, Genova, Italy.
- Oszczak S., Wasilewski A., Rzepecka Z., Baran L.W., Kapcia J., (1999), RTK/DGPS Reference Network for Three City Area: Gdansk, Gdynia, Sopot (Poland), Proceedings of the ION GPS'99, Nashville, Tennessee, USA.
- Pietraszewski D., Spalding J., Viehweg C., Luft L., (1988), U.S. Coast Guard Differential GPS Navigation Field Test Findings, Navigation, vol. 35, No 1.
- Poppe D., Last J.D., (1994), DGPS Radio - Beacon Coverage Prediction in the European Environment, the Third International Conference on Differential Satellite Navigation Systems, London, UK.
- Rasiukiewicz W., Leśnicki A., (1983), Podstawy systemów horyzontowych linii radiowych, Wydawnictwo Komunikacji i Łączności, Warszawa, Poland.
- Specht. C., (2000), Model dostępności transmisji systemów DGPS pracujących w oparciu o depesze RTCM nr 1 i 9, XII Międzynarodowa Konferencja Naukowo-Techniczna 'Rola nawigacji w zabezpieczeniu działalności ludzkiej na morzu', Gdynia, Poland.
- Specht C., (2002a), Synchronized FDMA of DGPS Reference Stations for LF/MF Systems Proceedings of The European Navigation Conference GNSS'2002, CD paper no 101, Copenhagen, Denmark.
- Specht C., (2002b), Polish Maritime DGPS Reference Stations Coverage after the Implementation of New Frequency Net – Preliminary Results,

World Congress of European Geodetic Society EGS 2002, Nice, 21-27 april, Reports on Geodesy no 01 (61).

- Specht C., (2002c), Availability, Reliability and Continuity in Navigation and Hydrography – terminology discussion, Annual of Navigation, no 4/2002.
- Specht C., Nowak A., (2002d), DGPS Reference station coverage projecting program, Proceedings of XIIth International Scientific and Technical Conference – The Part of Navigation in Support of Human Activity on the Sea, Gdynia, Poland.
- Załęska-Fornal A., (1998), Konstruowanie i badanie probabilistycznych modeli niezawodnościowych nieodnawialnych złożonych obiektów technicznych, rozprawa doktorska, AMW, Gdynia, Poland.
- Zarudnyj B.N., (1973), Nadieżnost sudowoj nawigacionnoj aparatury, Izdatielstwo Sudostrojenie, Lieningrad, Russia.

Recommendations, regulations, reports and standards

- ERP (1996), European Radionavigation Plan, First Draft For Working Group Review, Booz Allen & Hamilton, S.A., Paris.
- FRP (1999), Federal Radionavigation Plan, U.S. Department of Transportation.
- IALA (1989), Guide to the Availability and Reliability of Aids to Navigation, International Association of Lighthouse and Aids to Navigation Authorities.
- IALA (1990), Aids to Navigation Guide, International Association of Lighthouse and Aids to Navigation Authorities.
- IALA (1999), Guidelines for the Performance and Monitoring of DGNSS Service in the Band 283.5-325 kHz, International Association of Lighthouse and Aids to Navigation Authorities.
- IALA (2001), Recommendation on the Performance and Monitoring of DGNSS Services in the Frequency Band 283.5-325 KHz, R-121, June.
- ITU-R (1997), Technical Characteristics of Differential Transmitions for Global Navigation Satellite Systems (GNSS) from Maritime Radio Beacons in the frequency band 285-325 kHz in Region 1 and 285-325 kHz in regions 2 and 3, International Telecommunications Union – Radiocommunications Bureau, Geneva.
- RNAV17 (2002), DGNSS-The Way Forward, International Association of Lighthouse and Aids to Navigation Authorities, Committee of Radionavigation, 16 september.
- RNAV6 (1996), Defining the availability of radiobeacon DGNSS, International Association of Lighthouse and Aids to Navigation Authorities, Committee of Radionavigation, WG1/4, 25 september.
- RTCM (1996), Recommended Standards for Differential Navstar GPS Reference Stations and Integrity Monitors (RSIM), version 1.0 RTCM Special Committee NO.104, Radio Technical Commission for Maritime Services (RTCM Paper 88-96/SC104).
- RTCM (2001), Recommended Standards for Differential GNSS (Global Navigation Satellite Systems) Service, version 2.3 RTCM Special Committee NO.104, Radio Technical Commission for Maritime Services (RTCM Paper 136-2001/SC104).
- SPS (1993), Global Positioning System (GPS), Standard Positioning Service, Signal Specification, Department of Defense, Positioning/Navigation Executive Committee.
- SPS (2001), Global Positioning System Standard Positioning Service, Performance Standard, Assistant Secretary of Defense.
- USCG (1993), Broadcast Standard for the USCG DGPS Navigation Service, U.S. Coast Guard Commandant Instruction (COMDTINST) M16577.1, US Department of Transportation, United States Coast Guard, Washington, DC, april.