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BEZIER APPROACH TO SMOOTHING ZIG-ZAG DATA FOR USE IN SHIP MANEUVERING MODEL IDENTIFICATION

ABSTRACT The Bezier curve is used as a basis of zig-zag data approximation. It improves an estimate of external excitations acting during such a type of maneuver. In addition to the previous report about a turning test modeling, this makes available most of the existing full-scale trial results for the force-based identification of ship maneuvering mathematical model.

INTRODUCTION

A ship maneuvering motion is governed by a set of differential equations. Derivatives of velocities are directly involved there. a best method of tuning formulas of external excitations and parameters thereof in those equations utilizes force (moment) level as opposed to another alternative, namely - a motion related identification (validation). The latter is sometimes called an input-output method.

The force approach requires differentiation of originally acquired (during trials recorded) motion data. This includes computing:

- derivative of course angle (if e.g. a rate of turn indicator is absent onboard) to get the yaw velocity, and a subsequent differentiation to arrive at the yaw acceleration,
- derivative of track (contributing to the ship drift angle and thus sway velocity, if. e.g. a two-axes doppler log is not installed), the final attempt is an additional differentiation to reach the sway acceleration,
- derivative of surge velocity (the velocity normally directly measured by the water-related logs) termed the surge acceleration.

In all kinds of above data some noise exists (which constrains significantly a further analysis), whether they are taken directly, i.e. numerically measured (random errors), or indirectly, i.e. from the published plots (extra digitization random errors).

When producing any derivative from noisy discrete data (spaced at relatively short time interval), it is well recognized that such noise will be 'amplified', the plot of the derivative will be unreadable and hard to investigate. On the other hand, a larger time step implies somehow an average value rather than instantaneous one.

A method of numerical differentiation is also very crucial, the most suitable (confirmed in initial investigations) is so called the centered differentiation according to the following scheme:

$$\frac{\partial f}{\partial x} = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} + o\left((\Delta x)^2\right) \tag{1}$$

This way, a data smoothing generally takes place prior to the differentiation. In [Artyszuk, 2000] a smoothing method was given for ship turning test results. Exponential curves were used in that case. The situation is a bit different in case of e.g. zig-zag test. It is difficult to find the best approximation. Plots of zig-zag test data show in a certain manner a rather periodical nature which could suggest the use of Fourier series. But the problem lies in uneven periods (and/or semi-periods) of zig-zag curves, e.g. a ship course change diagrams. The present paper concentrates upon the Bezier curve fitting (aaproximation) to such scattered data, the 1st and 2nd derivative of which are generated automatically.

THE CONCEPT

A parametric representation of complex curves (including surfaces) has a very long story. Among them, of superior properties appear so called cubic polynomial functions, e.g. [Foley et al., 1995]. Mutual interactions between technical developments in computer technology and demands for computer aided design (CAD) revealed in a huge advance of the theory of such functions, e.g. [Barsky, 1988]. An area of application quickly extended from the 2D/3D shape graphical modeling to nearly all kinds of engineering sciences under a common term of function approximation, this particularly comprises the control theory (signal processing) and applied statistics (economy, medical science, etc.), e.g. [Silverman, 1985].

The basic definition of cubic polynomial curve is as follows:

$$\begin{cases} x = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \\ y = b_0 + b_1 t + b_2 t^2 + b_3 t^3 \end{cases} \tag{2}$$

where:

a_i, b_i - shape defining constant parameters,

t - variable parameter, responsible for point movement along the curve, normally from a specific range.

The greatest advantage is that we can construct any complex shape from smaller pieces as per eq. (2) applying some continuity conditions upon their nodes. There exist plenty algorithms (methods) in the literature describing different schemes of curve divisions and behavior control. All types of such curves (the most famous are e.g. Bezier curve, B-spline) are mutually (internally) exchangeable, e.g. [Rogers, 1977], [Foley et al., 1995].

The Bezier curve seems to fulfill better the purpose of zig-zag test data modeling. We are supplied with a very good control (flexibility) and sufficient accuracy at minimum number of nodes. However, we pay for it by the lack of 2nd derivative continuity. The latter is a benefit of the B-spline. An algorithm for the Bezier curve could be constructed very easily in an interactive, graphical, and manual control form in most common electronic spreadsheets - a numerical update of chart data point movement is necessary. An automatic and efficient adjustment of the Bezier curve parameters to come as close as possible to an arbitrary shape is moved to the nearest future. a development of Bezier approximation procedure, without loss of generality, will be based on three initial semi-periods of the zig-zag course change diagram - a typical range in performing this trial. Fig. 1 shows 10°/10° zig-zag data for a chemical tanker [Artyszuk, 2000], consisting of surge v_x and sway v_y velocities (as derived from the track and speed over ground plots) and course change angle ψ. The latter two magnitudes will be considered hereafter.

A definition of a single Bezier curve segment reads:

$$\begin{cases} x = x_A(1-t)^3 + 3x_B t(1-t)^2 + 3x_C t^2(1-t) + x_D t^3 \\ y = y_A(1-t)^3 + 3y_B t(1-t)^2 + 3y_C t^2(1-t) + y_D t^3 \end{cases}, \tag{3}$$

where:

$A(x_A, y_A)$, $D(x_D, y_D)$ - the starting and ending point of the segment,

$B(x_B, y_B)$, $C(x_C, y_C)$ - control (shape defining) points, vectors $\vec{AB}$ and $\vec{CD}$ are tangential at a and D,

t - variable parameter from a normalized range <0,1>.

Fig. 2 presents, as an example, how a change of the point B (full dot curve) into B' affects the curve shape (empty dots).

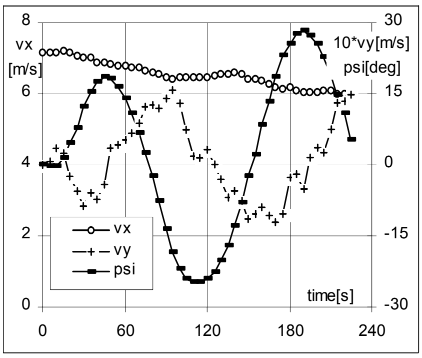

Fig.1. 10°/10° zig-zag full-scale data.

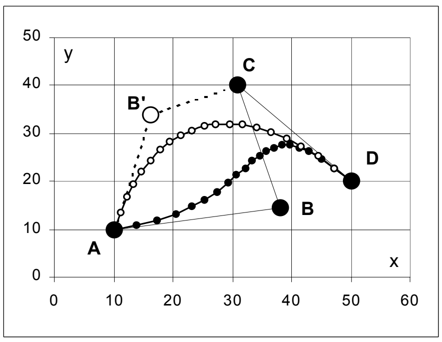

Fig.2. The Bezier curve behavior.

The problem lies now in dividing the assumed curve (Fig. 1 - the 'ψ' parameter) at minimum efforts. One can take e.g. 7 nodes - four 'time'-axis intersection points (zero points, though they are not necessary the curve flex points) and three extrema between them. This leads to 12 control points but 15 scalar parameters, taking into account the 1st derivative both at the axes origin and extrema as null and its continuity at all the nodes. From initial investigations it is clear that extrema could be neglected and, preserving the accuracy, our curve will consist of three Bezier segments - 6 control points and thus 9 scalar parameters.

They are marked in Fig. 3, x coordinate relates to the 'time', while y coordinate to the 'ψ' angle. There are four nodes - '1' to '4', three main active control points - '1B', '2B', '3B' and three active scalar values - 'x_{1A}', 'k_{2A}', 'k_{3A}'. The latter contribute to inactive control points '1A', '2A', '3A' as follows:

$$\begin{cases} '1A'\left(x_{1A},0\right) \\ '2A'\left(x_{1B}+\left(x_2-x_{1B}\right)k_{2A}, y_{1B}+\left(0-y_{1B}\right)k_{2A}\right) \\ '3A'\left(x_{2B}+\left(x_3-x_{2B}\right)k_{3A}, y_{2B}+\left(0-y_{2B}\right)k_{3A}\right) \end{cases} \quad (4)$$

Each of three segments in Fig. 3 has its own independent range of t parameter but still belonging to <0,1>. Discretization step of 0.05 is fairly good from practical point of view. However, the points on the curve corresponding to equally spaced t parameter do not give an equally spaced x variable which is a basis for the measured (being approximated) y data. This way, prior to applying any proximity criterion (e.g. a uniform or least squares method) between the measured and simulated values, we should perform e.g. a linear interpolation as to convert the relationship $y=f(x)|_t$ (at constant t step) into $y=f(x)|_x$ (at constant x step), conforming to the table of original measurements.

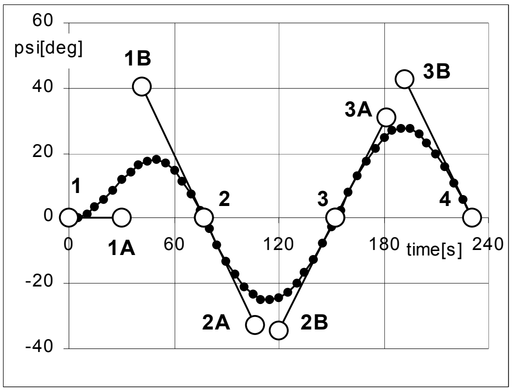

Fig.3. Selection of parameters in Bezier approximation.

The best asset we possess is that we are able to model the variation of e.g. course change mainly independently in each of three segments. Among one general approximation criterion (the whole curve), one could consider also some local criteria (for each segment).

Together with fixing the nodes at zero points, this will prohibit a possibility of wrong zig-zag data approximation as explained in [Artyszuk, 2001].

The calculation of 1st and 2nd derivatives of the Bezier segments is made by means of the conventional rules of differentiation calculus for parametrically defined functions. Introducing the following symbols:

$$x'_t = \frac{dx}{dt}, \quad y'_t = \frac{dy}{dt}, \quad x''_{t^2} = \frac{d^2x}{dt^2}, \quad y''_{t^2} = \frac{d^2y}{dt^2} \tag{5}$$

both derivatives are expressed by:

$$\frac{dy}{dx} = \frac{y'_t}{x'_t}, \quad \frac{d^2y}{dx^2} = \frac{x'_t y''_{t^2} - x''_{t^2} y'_t}{(x'_t)^3} \tag{6}$$

Due to the obvious symmetry between x and y in eq. (3) it is essential to point out only the derivatives of x:

$$x'_t = 3(1-t)^2(x_B - x_A) + 6t(1-t)(x_C - x_B) + 3t^2(x_D - x_C) \tag{7}$$

$$x''_{t^2} = 6(1-t)(x_A - 2x_B + x_C) + 6t(x_B - 2x_C + x_D) \tag{8}$$

NUMERICAL RESULTS

Let's assume a uniform approximation based upon the minimization of the following expression (see also [Artyszuk, 2000]):

$$\Delta^Y_{avg} = \frac{1}{n}\sum_{i=1}^{n}\left|Y_i^{org} - Y_i^{sm}\right| \tag{9}$$

where:

- Δ^Y_{avg} - average smoothing error for Y variable (e.g. ψ),
- n - number of data points,
- Y_i^{org} - original data,
- Y_i^{sm} - simulated data (Bezier approximation - eq.(3)).

In case of the course change angle ψ in Fig. 1, one can reach at hand a total accuracy Δ^{ψ}_{avg} of order 0.62[°] (1st segment is giving 0.90[°], while the latter two are supplying 0.44[°] and 0.52[°] respectively). Tab. 1 gathers detailed values of the Bezier approximation parameters in this situation.

Tab.1. Best fit parameters.

$(x_1=0)$ $x_2=$	77.4	$x_{1A}=$	30	$x_{1B}=$	41.7	$y_{1B}=$	40.6
$x_3=$	152.7	$k_{2A}=$	1.8	$x_{2B}=$	120.5	$y_{2B}=$	-34.6
$x_4=$	230.5	$k_{3A}=$	1.9	$x_{3B}=$	192.5	$y_{3B}=$	42.6

Fig. 4 presents a comparison between the original and just smoothed curve. Fig. 5 demonstrates the smoothing effectiveness in calculating derivatives from noisy data as per eqs. (1) and (6) - ω_z stands for yaw velocity (1st derivative of ψ) and ε_z means yaw acceleration (2nd derivative). The algorithm described in this paper could be used of course for other kinds of data in the zig-zag test, specially the sway velocity v_y - Fig. 3. The results are summarized in Fig. 6 (v_y) and 7 (a_y as sway acceleration).

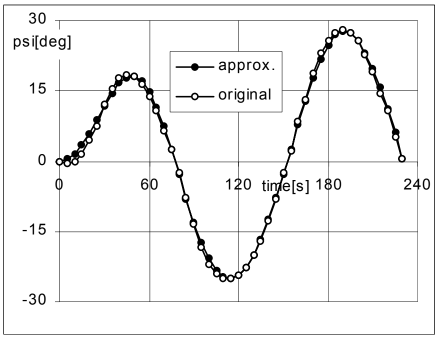

Fig.4. Course (yaw) angle approximation

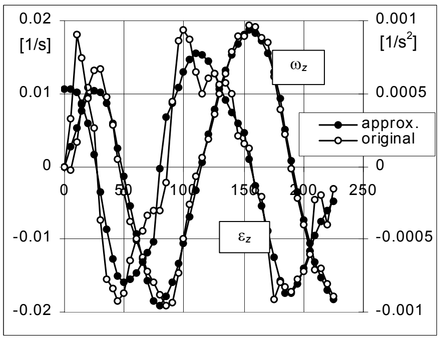

Fig.5. Calculated yaw velocity and acceleration

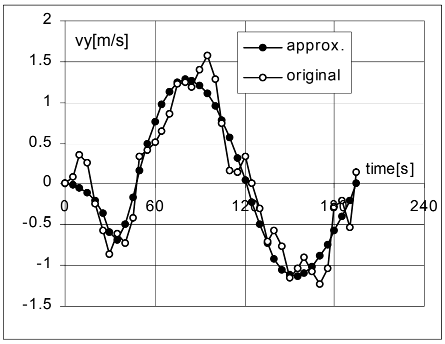

Fig.6. Sway velocity approximation.

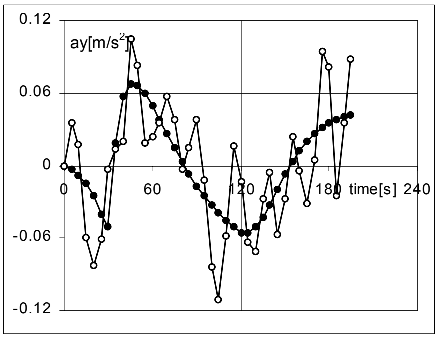

Fig.7. Calculated sway acceleration.

FINAL REMARKS

The present study is of the application type. The aim was to search among known solutions for similar problems as the investigated one, test them in new circumstances, modify them to suit exactly our demands and finally validate with a real data.

The main interest has been limited to the yaw (course) angle (and/or velocity) and sway velocity during the zig-zag test. Both magnitudes are of a clear and easy to analyze image. This way, the next challenge should be a research upon the surge component of zig-zag ship maneuvering motion, more complex, and to a greater extent subject to a correlation with sway and yaw velocities (see the inertia terms in ship-fixed differential equations of maneuvering motion).

There is of course a relationship between the latter two variables, but hard to be detected.

The accuracy of the considered sway velocity has allowed using the same procedure as for the yaw angle. However, as shown in [Abkowitz, 1980], the chart of v_y could be a bit different and thus requiring a little refined conduct, including e.g. the mentioned before extreme of the function.

The second derivative problem (concerning exclusively the transformation of the yaw angle into the yaw acceleration) has not been experienced severely in the supplied example (the data and method of curve division - nodes are close to the flex points). When such problem occurs, it is possible to skip the yaw angle modeling and try to smooth directly a derived yaw velocity, or to introduce more nodes (more control parameters per whole curve).

A conclusion is due also that the Bezier curve is very sensitive upon its parameters i.e. the control points coordinates. This and the fact that the latter are contained within the order of the function (to be modeled) values will simplify an expected automatic approximation. The increment (or decrement) of control points should be interconnected with the ordered accuracy of approximation and the accuracy of original data. The use of least squares criterion is necessary if someone is going to attempt an analytical determination of Bezier regression parameters as opposed to a simulation method.

The power of the Bezier approximation suggests its deployment in the turning test modeling, because the exponential functions proposed in [Artyszuk, 2000] are relatively stiff.

It is worthwhile finally to note that smoothing process is not the last word in forces/moment computing from the ship maneuvering differential equations. Of the same importance appears the problem of accounting for usually unknown current effects while producing the water-related sway velocity. When the current is against a ship initial course at a medium rate of e.g. half a knot, then a general shape and amplitude of sway velocity is kept constant but a large phase shift takes place. This has a reflection in analysis results of e.g. the surge motion component.

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