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THEORETICAL BASIS OF THE PROBABILISTIC-FUZZY METHOD FOR ASSESSMENT OF DANGEROUS SITUATION OF A SHIP MANOEUVRING IN A RESTRICTED AREA

ABSTRACT The paper presents theoretical basis of the probabilistic-fuzzy method for assessing the safety of a ship manoeuvring on fairways. The method allows to account for such different states / conditions as the danger to the safety and the intuition that the safety of a ship manoeuvre is threatened, based on the probabilistic analysis of risk and elements of fuzzy logic. A dangerous navigational situation is defined as a fuzzy event. Two methods of determining the probability of a fuzzy event occurrence are presented. The results are compared with those obtained from a quantitative method of safety assessment based on probabilistic methods.

INTRODUCTION

In order to assess navigational risk in restricted areas (fairways, harbour entrances, harbour basins, turning basins, etc.) the most frequently used are methods of quantitative analysis based on probabilistic methods. One approach in such cases utilises data from simulated passages conducted by navigators. From the data one can estimate parameters of the density distribution function of the random variable "distance from danger". The use of the density function enables to determine the probability that a ship will move out of the safe boundaries of the fairway, in which case a ship may run aground or strike a port structure. Such an event results in the change of the system state to safety failure (navigational accident).

One shortcoming of the probabilistic method is that it offers no possibility to assess a threatened safety situation, that is a situation when a ship is manoeuvring close to the fairway boundary. This state, if no or inadequate measures are taken, can result in safety system failure.

The presented approach allows a full description and quantitative definition of a dangerous situation of a ship, thus opening new ways to analyses of ship manoeuvring safety.

THE PROBABILISTIC METHOD OF ASSESSING THE SAFETY OF SHIP MANOEUVRE IN A RESTRICTED AREA

The probabilistic method of assessing the safety of a manoeuvring ship consists in determining the probability of a collision of a ship in motion, understood as ship's deviation outside a designated manoeuvring area. Depending on the area shape, a collision may result in hitting a wharf, port structure or grounding.

The determination of collision probability is based on simulated or field studies consisting of a series of trials in which navigators handle a ship through a fairway. In the trials, the fairway in question is divided into sections defined by lines perpendicular to the fairway centre line[Iribarren 1999].

The trials of reliable size, guaranteeing a preset level of confidence, are followed by the estimation of parameters of distributions of the function of density of random variable probability: maximum distance of ship's points to the starboard side and port side from the fairway centre line, separately for particular sections.

Two random variables are determined in each of the sections: maximum distances of ship's extreme points to the starboard and port side from the fairway centre line. When the phenomenon model (type of distribution) is determined, its parameters are estimated.

In order to calculate the probability of grounding, the following equations are used:

$$p_{air} = \int_{x=d\max}^{+\infty} g_{ir}(x)d(x) \tag{1}$$

$$p_{ail} = \int_{x=-\infty}^{-d\max} g_{il}(x)d(x) \tag{2}$$

where: $g_{ir}(x)$ - density function of the random variable x of the maximum distance of ship's extreme points to the starboard side from the fairway centre line in the i-th section,

$g_{il}(x)$ - density function of the random variable x of the maximum distance of ship's extreme points to the port side from the fairway centre line in the i-th section,

d_{max} - distance from the fairway centre to the fairway border (half of the fairway width).

When we consider separately the starboard and port border of the fairway, vectors of safety failure for the starboard (P_{ar}) and port side (P_{al}) of the fairway can be written as follows:

$$P_{ar} = [p_{a1r}, p_{a2r} \ldots p_{anr}] \tag{3}$$

$$P_{al} = [p_{a1l}, p_{a2l} \ldots p_{anl}] \tag{4}$$

where:

p_{air} - navigational safety failure in the i-th section of the starboard side of the fairway,

p_{ail} - navigational safety failure in the i-th section of the port side of the fairway,

n -number of sections

Thus obtained vectors, with the order equal to the number of the sections represent a probability of a ship's deviation from the fairway (accident occurrence) for particular fairway sections (see Fig. 1).

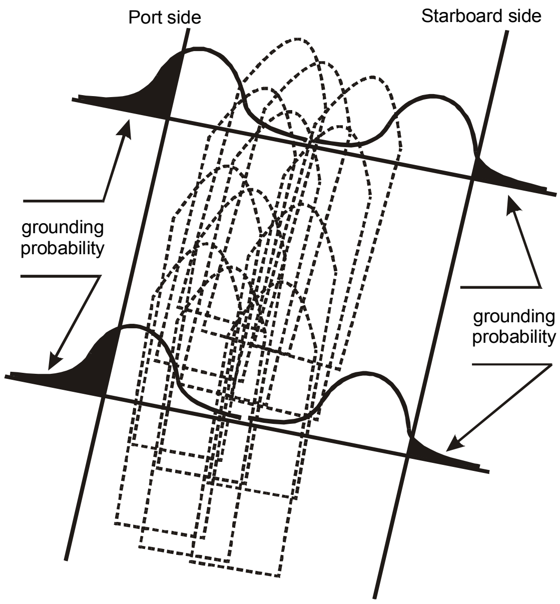

Fig. 1. Probability of an accident in a fairway

PROBABILITY OF A FUZZY EVENT

The approach presented above does not account for dangerous situations, in which a ship comes too close to the fairway boundary, as such situations may lead to situations of safety failure. This, for instance, refers to areas difficult for navigation in which, although not many collisions have occurred, dangerous situations are observed [Gucma & Pietrzykowski 2001a, 2001b].

The probability of a fuzzy event refers to a situation when there co-exist uncertainties of fuzzy and probabilistic type. Such a case takes place when an attempt is made to determine a probability of a dangerous situation for a ship moving in a restricted area such as a fairway.

A dangerous situation is meant to be a system state which, if no or inadequate countermeasures are taken, may change to the state of safety failure.

In the connection with the above, the term “dangerous navigational situation” denotes the state of the ship-area system that may but does not have to lead to a navigational accident. Such situations are referred to as incidents. As in most cases incidents do not end in a collision, i.e. accident, they are not registered or included in any statistics.

The term “dangerous situation” is an example of a linguistic variable described by the membership function of the fuzzy set “dangerous situation”, which roughly corresponds to the fuzzy set “dangerous distance”. An event understood as an occurrence of such situation is an example of a fuzzy event. Hence the determination of the probability that a dangerous situation will occur comes down to the determination of a probability of a fuzzy event.

There are two basic approaches to the determination of fuzzy event probability: Zadeh’s approach and Yager’s approach [Kasprzyk 1986, Rommelfanger 1994]. In the former the probability of a fuzzy event occurrence is a real number in the [0, 1] interval. The latter uses a fuzzy set.

ZADEH’S APPROACH

This is the most frequent approach found in the literature on the subject describing the probability of a fuzzy event [Driankov et al. 1993, Kasprzyk 1986].

Definition 1. A fuzzy event **A** is called a fuzzy subset of a space of elementary events **E**, i.e. a $\subseteq$ E with the membership function $\mu_A(x)$ measurable according to Borel:

$$A = \{x, \mu_A(x) \mid x \in E\} \tag{5}$$

The membership function μ_A attributes to each element x its degree of membership to a fuzzy set A, where $\mu_A(x) \in [0, 1]$.

Definition 2. Let there be a given fuzzy event **A** $\subseteq$ E $=\{x_1, x_2,..., x_n\}$. Each space element E is attributed its probability of occurrence: $p(x_1)$, $p(x_2)$,..., $p(x_n)$. The probability of a fuzzy event **A** is denoted as p(A) and defined as

$$p(A) = \sum_{i=1}^{n} \mu_A(x_i) * p(x_i) \tag{6}$$

Definition 3. A fuzzy event **A** in the space R^n is called a fuzzy subset A

$$A = \{x, \mu_A(x) \mid x \in R^n\} \tag{7}$$

with the membership function $\mu_A(x)$ measurable according to Borel.

Definition 4. The probability of a fuzzy event **A** according to definition 3 is then defined as Lebesques-Stieljes integer:

$$\mathrm{p(A)} = \int_{R^n} \mu_A(x)dp \tag{8}$$

For n=1 and when the probability p can be described by the density function g(x) [Rommelfanger 1994]:

$$\mathrm{p}(-\infty, \mathrm{x}) = \int_{x=-\infty}^{x} g(x)dx \tag{9}$$

the probability of a fuzzy event **A** can be written in this form:

$$\mathrm{p(A)} = \int_{x=-\infty}^{\infty} \mu_A(x)g(x)d(x) \tag{10}$$

where:

$\mu_A(x)$ - membership function to a fuzzy set,

$g(x)$ - function of density of the random variable x

The presented method can be used for defining other notions applied in the theory of probability, e.g. conditional probability.

It should be noted that in the approach presented above a real number is attributed to a fuzzy event. The number describes the probability of event occurrence.

YAGER'S APPROACH

A slightly different approach to the description and determination of fuzzy event probability is presented in [Kasprzyk 1986, Rommelfanger 1994]. It has been assumed that if a fuzzy event occurs, then the probability of the event occurrence should be described with a fuzzy set defined in the [0, 1] interval. In order to describe an event the notion of a α-cut of a fuzzy set is used.

Definition 5. α-cut of a fuzzy set $A \subseteq E$ with the membership function $\mu_A(x)$ is called a non-fuzzy set:

$$A_\alpha = \{x, \mu_A(x) \geq \alpha \mid x \in E\} \tag{11}$$

According to the decomposition theorem a fuzzy set A can be presented as the sum of its α-cuts

$$A = \sum_{\alpha \in [0,1]} \alpha A_\alpha \tag{12}$$

Then the probability of a fuzzy event A is defined as follows:

Definition 6. If A is a fuzzy event defined in the space E, and p: $B \rightarrow [0, 1]$ is the function of probability assigning the number p(.) to Borel's subsets of the set E, then the fuzzy probability of a fuzzy event a will be denoted p(A) and defined as

$$p(A) = \sum_{\alpha \in [0,1]} \alpha / p(A_\alpha) \tag{13}$$

where: $p(A_\alpha)$ – probability of a (non-fuzzy) event A_α, expressed as a real number in the [0, 1] interval.

Due to the fact that $p(A_\alpha) \in [0, 1]$ the fuzzy set p(A) is defined in the [0, 1] interval.

Definition 6 for a fuzzy event defined by the definition 3 applies also to uncountable spaces R^n.

Similarly to the definition of probability in Zadeh's approach, the presented method can be used for defining other notions used in the theory of probability.

This approach allows determining the probability of a fuzzy event in the form of a fuzzy set, which intuitively seems more correct. From an adequate number of α-cuts one can reproduce with a preset accuracy the function of membership to a fuzzy set *p(A)*. The process of defuzzification makes it possible to obtain a non-fuzzy value of this probability, which has been done in this work.

THE PROBABILISTIC-FUZZY METHOD OF ASSESSING A NAVIGATIONAL SITUATION IN A RESTRICTED AREA

The presented approaches make it possible to assess navigational safety taking into account dangerous situations, i.e. situations when a ship comes to close to a fairway border which may result in safety failure situation. Such assessment is necessary in areas where navigation is difficult, although not many collisions have occurred.

Two functions are essential in this task: the function of density of random variable probability and the function of membership of a fuzzy event "dangerous situation". Similarly to the probabilistic method, dangerous situations on the port and starboard sides of the fairway are considered separately. Therefore, there is a need to determine an appropriate function of density of the random variable x of the distance from port and starboard sides of the fairway as well as a function of membership of fuzzy sets, also for the port and starboard fairway sides (Fig. 2, 3).

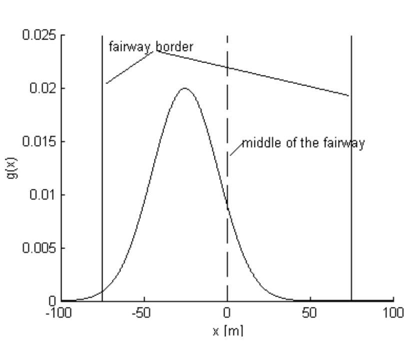

Fig. 2. Density function of random variable x of the distance of the extreme point on the port side of the ship water plane to the fairway border: normal distribution, m=-25m, σ=20m

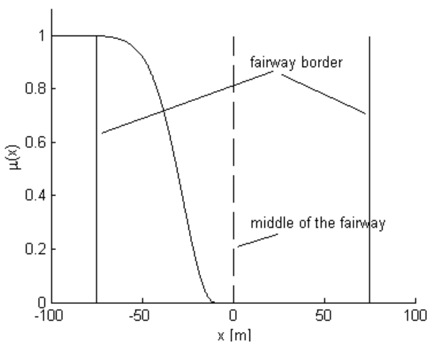

Fig. 3. Function of membership to a fuzzy set "dangerous situation" for the port side of the fairway

Using the two functions we can determine the probability of a fuzzy event "dangerous situation". Two approaches discussed above are herein used for the determination of fuzzy probability (Zadeh's and Yager's approaches).

ZADEH'S APPOACH

With the assumption that we know the density distributions of random variable x for, respectively, extreme starboard ($g_r(x)$) and port ($g_l(x)$) positions of ship water plane points as well as their corresponding functions of membership to a set "dangerous situation" $\mu_{Ar}(x)$ and $\mu_{Al}(x)$, the probability that a dangerous situation will occur can be expressed by the equations:

$$\mathrm{p_z(A_r)} = \int_{x=-\infty}^{\infty} \mu_{Ar}(x) g_r(x) d(x) \tag{14}$$

$$\mathrm{p_z(A_l)} = \int_{x=-\infty}^{\infty} \mu_{Al}(x) g_l(x) d(x) \tag{15}$$

Accepting the previously presented division of the fairway into sections we can determine the probability of a dangerous situation for the starboard and port sides of the fairway in each section. As the starboard and port sides are considered separately, by analogy to relation (3) and (4), vectors of probability of a dangerous situation can be written in this form:

$$P_z(A_r) = [p_z(A_{1r}), p_z(A_{2r})\dots p_z(A_{nr})] \tag{16}$$

$$P_z(A_l) = [p_z(A_{1l}), p_z(A_{2l})\dots p_z(A_{nl})] \tag{17}$$

where:

$p_z(A_{ir})$ - probability of a dangerous situation in i-th section of the starboard side of the fairway,

$p_z(A_{il})$ - probability of a dangerous situation in i-th section of the port side of the fairway,

n - number of sections

While analysing the equations (14) and (15) one can notice that when the function of membership to a fuzzy set "dangerous situation" equals 0, the function for the determination of dangerous situation probability also equals 0. This means that there is no danger. On the other hand, when the membership function equals 1, the function for the determination of dangerous situation probability assumes values the same as the function of density of the random variable of ship's extreme points occurring within the fairway.

The following functions have been taken for interpretation of the function for the determination of dangerous situation probability:

- Function of density of the random variable of ship's extreme points occurring within the fairway - port side (normal distribution):

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}} \tag{18}$$

where:

m - mean distance of ship extreme points from the fairway centre line,

σ- standard deviation of distances of ship's extreme points from the fairway centre line.

- Function of membership to a fuzzy set "dangerous situation" for the port side of the fairway in the following form [James 1986]:

$$\begin{cases} \mu_{Ail}(\boldsymbol{x}) = 1 - \exp(-((\boldsymbol{x} - \boldsymbol{x_o}) / \boldsymbol{x}1)^2) & \textbf{\textit{for}} \quad \boldsymbol{x} < \boldsymbol{x}_0 \\ \mu_{Ail}(\boldsymbol{x}) = 0 & \textbf{\textit{otherwise}} \end{cases} \tag{19}$$

Three examples of the density functions of random variable of ship's extreme points occurring in the fairway were assumed; their constant standard deviation σ=8m and the mean values were as follows:

1. $m_1 = -13$m
2. $m_2 = -25$m
3. $m_3 = -35$m

For the membership function to a fuzzy set "dangerous situation" the respective parameter values were assumed to be x_0 = -10 [m] and x_1 = 25 [m].

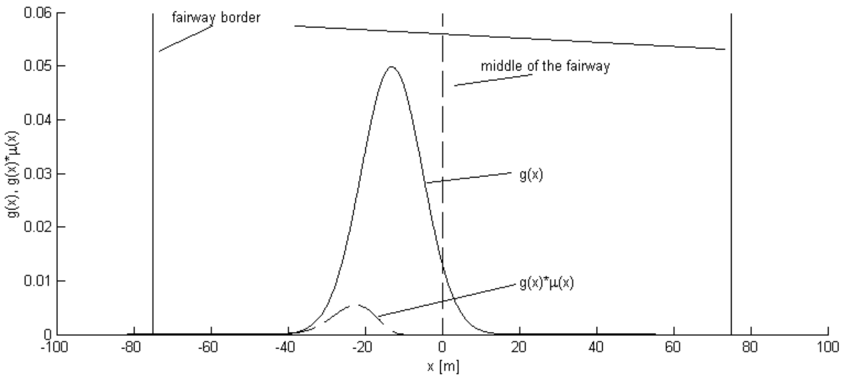

Fig. 4. A function for determining the probability of a dangerous situation for σ=8m and m_1= -13m

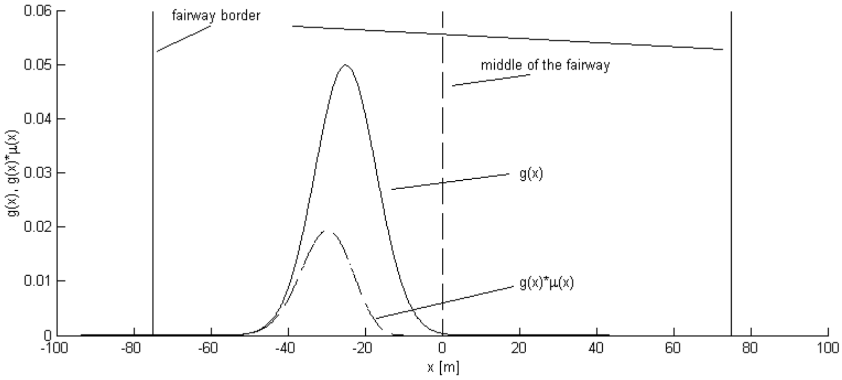

Fig. 5. A function for determining the probability of a dangerous situation for σ=8m and m_2= -25m

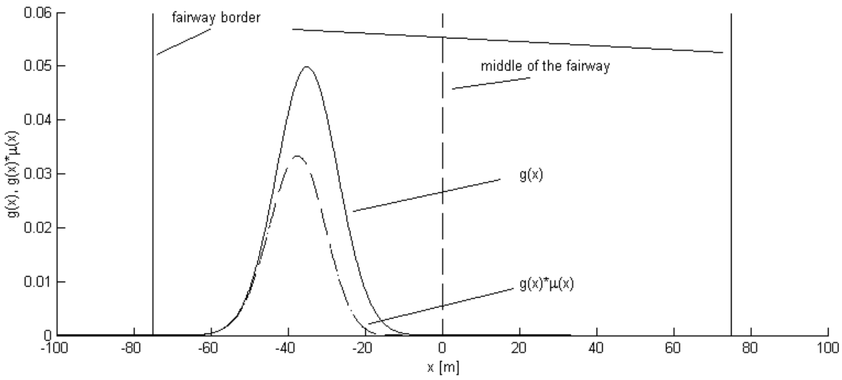

Fig. 6. A function for determining the probability of a dangerous situation for σ=8m and m_3= -35m

The corresponding functions for the determination of dangerous situation probability for the constant membership function (Figure 3) are shown in Figures 4, 5 and 6.

The probability of dangerous situation occurrence and the probability that ship's deviates outsides fairway borders are shown in Table 1.

Table 1. Probability of dangerous situation occurrence and the probability of an accident for constant standard deviation σ=8m.

Situation	Probability of dangerous situation occurrence	Probability of ship's deviating outside the fairway borders (accident)
1. (m_1=–13m)	0.07571	$4.7101 \cdot 10^{-15}$
2. (m_2=–25m)	0.32347	$2.0677 \cdot 10^{-10}$
3. (m_3=–35m)	0.60273	$2.8709 \cdot 10^{-7}$

One may notice that as the mean value m shifts towards danger (borders of the fairway), the probability that a dangerous situation will occur clearly increases with the constant membership function.

It is more difficult the point in which the function for the determination of a dangerous situation reaches the maximum value. However, one should note that the point is shifted away from the mean of ship's extreme points towards danger, depending on the inclination angle of the membership function. The point indicates that in the given location of the fairway the most dangerous situations have took place, with the degree of danger being accounted for.

How the standard deviation affects the density function of the random variable of ship's extreme points occurrence in the fairway is shown for two examples in which the functions with the same mean value m= -13m and the following standard deviations:

1. σ_1=8m
2. σ_2=20m

The resultant function for the determination of dangerous situation probability is shown in Figures 3 and 7.

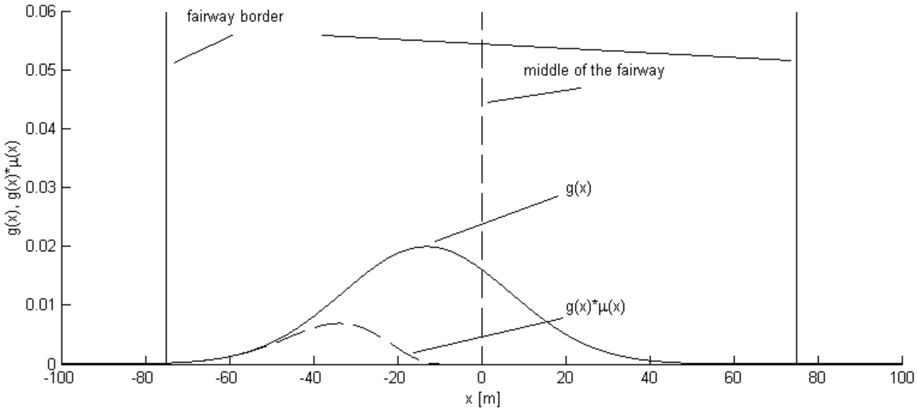

Fig. 7. Function for the determination of dangerous situation probability for σ_2=20m and m=-13m

The probability that a dangerous situation occurs and that a ship moves outside the fairway borders for the analysed cases are presented in Table 2.

Table 2. The probabilities of a dangerous situation and an accident for the constant mean m=-13 [m].

Situation	Probability of a dangerous situation	Probability of ship movement outside the fairway borders (accident)
1 (σ_1=8m)	0.07571	$4.7101 \cdot 10^{-15}$
2 (σ_2=20m)	0.20452	$9.6080 \cdot 10^{-4}$

It can be noted that as the standard deviation increases, the probability of a dangerous situation clearly increases as well, and the point at which the function reaches its maximum shifts towards danger.

In summary, the function for the determination of dangerous situation probability quantitatively defines a probable occurrence of a dangerous situation in a fairway. Its maximum indicates a location in the fairway where the most dangerous situations have occurred, with the degree of danger being accounted for.

The use of the above approach makes it possible to describe more completely the navigational safety in a restricted area, including the probability that a ship moves outside the fairway and situations considered as dangerous by navigators.

YAGER'S APPROACH

With the assumption that we know the density distributions of random variable x for, respectively, extreme starboard ($g_r(x)$) and port ($g_l(x)$) positions of ship water plane points as well as their corresponding functions of membership to a set "dangerous situation" $\mu_{Ar}(x)$ and $\mu_{Al}(x)$, the probability that a dangerous situation will occur can be expressed by the equations:

$$p_y(A_r) = \sum_{\alpha \in [0,1]} \alpha / p(A_{\alpha r}) \tag{20}$$

$$p_y(A_l) = \sum_{\alpha \in [0,1]} \alpha / p(A_{\alpha l}) \tag{21}$$

where: $p(A_{\alpha r})$ – probability of a (non-fuzzy) event $A_{\alpha r}$

$p(A_{\alpha l})$ – probability of a (non-fuzzy) event $A_{\alpha l}$

Accepting the previously presented division of the fairway into sections we can determine the probability of a dangerous situation for the starboard and port sides of the fairway in each section. As the starboard and port sides are considered separately, by analogy to relations (3) and (4), vectors of probability of a dangerous situation can be written in this form:

$$P_y(A_r) = [p_y(A_{1r}), p_y(A_{2r}) \ldots p_y(A_{nr})] \tag{22}$$

$$P_y(A_l) = [p_y(A_{1l}), p_y(A_{2l}) \ldots p_y(A_{nl})] \tag{23}$$

where:

$p_y(A_{ir})$ - probability that a dangerous situation will occur in the i-th section of the starboard side of the fairway,

$p_y(A_{il})$ - probability that a dangerous situation will occur in the i-th section of the port side of the fairway,

n - number of sections

For the interpretation of the presented method of determining the probability of a dangerous situation the same density function of random variable and the same membership function to a fuzzy set "dangerous situation" have been accepted as in relations (18) and (19).

As in Zadeh's approach, three examples were taken of density functions of random variable of ship's extreme points occurrence in the fairway.

The values of membership function $\mu_A(x)$ were determined for the accepted discretization step dx. The obtained values were taken as values α for α-cuts of the fuzzy set a 'dangerous situation'. Thus the set a was described as the sum of its α cuts.

$$A_l = \sum_{\alpha \in [0,1]} \alpha A_{\alpha l} \tag{24}$$

The values of probability p of events $A_{\alpha i}$ were calculated for the determined values of α_i cuts ($\alpha_i=\mu_A(x_i)$), using following equation:

$$p_y(A_{\alpha il}) = \int_{x=-\infty}^{x_i} g(x)d(x) \tag{25}$$

In this manner the probability of a fuzzy event "dangerous situation" was determined - in the form of a fuzzy set – described through its α-cuts. This example is illustrated by Figure 8.

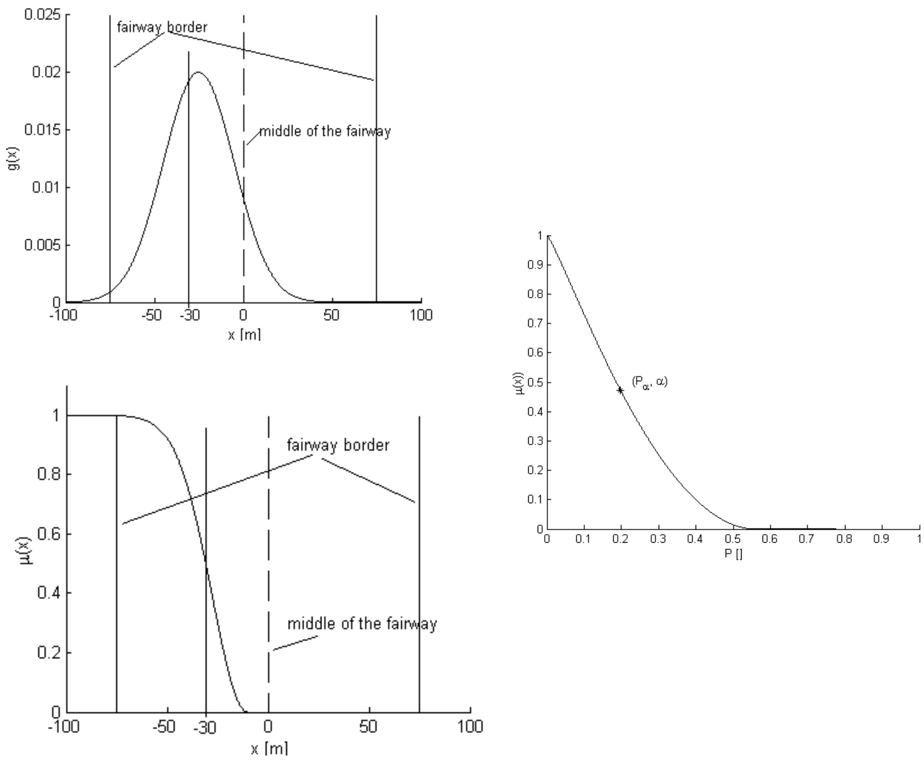

Fig. 8. Probability of a fuzzy event "dangerous situation" – in the form of a fuzzy set, described through its α-cuts for σ=20m and m=-25m, x_i=-30, $P_{\alpha i}$=1.9766e-001 and $\mu(x_i)=\alpha_i$=4.7271e-001

With a small discretization step dx we can reproduce from determined α-cuts the membership function describing a fuzzy set of the probability of fuzzy event "dangerous situation" for the three above mentioned cases of functions of density of random variable x (Fig. 9.)

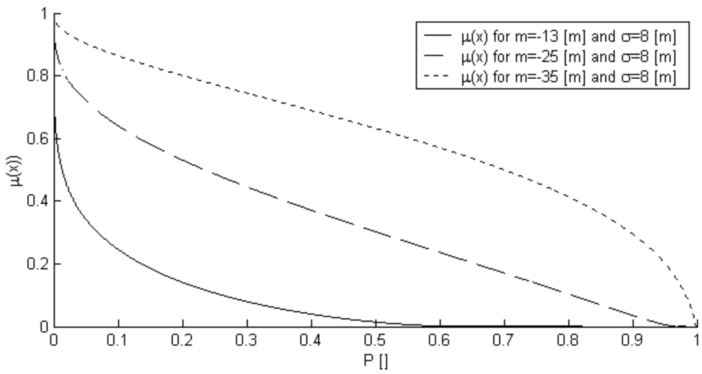

Fig. 9. Membership functions of fuzzy sets of probability that a fuzzy event "dangerous situation" will take place for three examples of density function of the random variable x: σ=8m and m_1=-13m, m_2=-25m, m_3=—35m.

One should note that the centre of gravity of the membership function (Fig.9) moves as expected towards higher probability with an increase in the value of mean density function of the random variable of distances of ship's points from the fairway centre line.

In practical applications as well as for the comparison of results with those obtained by using Zadeh's approach, we can obtain a real value of the probability through the process of defuzzification.

Table 3 presents the values of probability of a fuzzy event 'dangerous situation' afetr defuzzification by the centre of gravity method for the three cases of density function of the random variable x.

Table 3. Probability of a dangerous situation and the probability of an accident for constant standard deviation σ=8m.

Situation	Probability of a dangerous situation	Probability that a ship moves outside the fairway borders (accident)
1. (m_1=—13m)	0.13816	$4.7101 \cdot 10^{-15}$
2. (m_2=—25m)	0.30422	$2.0677 \cdot 10^{-10}$
3. (m_3=—35m)	0.39933	$2.8709 \cdot 10^{-7}$

If we compare the results in Tables 1 and 3 presenting the probability of a dangerous situation occurrence for three examples, determined by the two methods (Zadeh's and Yager's approaches) we can find out that:

- values of the probability of a dangerous situation are similar in both methods,
- differences between the probabilities calculated by the two methods are not linearly dependent on the mean random variable of the distance of ship's points from the fairway centre line.

It should be noted that in Yager's approach the method of defuzzification, the discretization step for variable x and the integration method influence the final value of the probability of a dangerous situation.

CONCLUSIONS

The article presents two methods which make it possible to take into account, apart from the reliability of safety used in traditional approaches, uncertainty caused when a dangerous situation takes place, which may lead to a state referred to as safety failure. The probability of a fuzzy event "dangerous situation" may serve as a new indicator in assessing the safety of manoeuvring.

The calculations have shown substantial convergence of results obtained by the two presented methods, although they are based on a completely different approach from each other (Zadeh's and Yager's aproaches).

The discussed methods have considerable practical importance as they allow to obtain quantitative identification of the probability of a dangerous event, which so far has not been possible.

The new approach described in this article consists in combining the classical probabilistic method (probability of an accident) and notions and methods of fuzzy logic used in order to reflect navigator's judgement (state of threatened safety).

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