

**Piotr Kaniewski, Stanisław Konatowski**  
**Military University of Technology**  
**Warszawa**

## POSITIONING WITH AHRS/ODOMETER/GPS SYSTEM

**ABSTRACT** The paper presents a project of an integrated positioning system composed of a dead reckoning unit (DR), and a GPS receiver. The DR subsystem includes an Attitude and Heading Reference System (AHRS) and an odometer. The data from DR and GPS are jointly processed via an algorithm of linearized complementary Kalman filter. The paper shortly introduces an idea of relative and absolute navigation. Next, it presents state-space description of the AHRS/ODOMETER/GPS integrated system. a functional structure of the system and Kalman filtering algorithm are presented. At last, the authors outline an adopted methodology of testing of AHRS/ODOMETER/GPS integrated system. Chosen results of the tests, conducted with use of real navigation data, are included in the paper. The presented positioning system may find its application in land vehicles.

### INTRODUCTION

In land navigation systems, Global Positioning System (GPS) receivers are often used as radiotechnical devices and dead-reckoning (DR) modules as autonomous systems [Forsell, 1991, Kayton & Fried, 1997]. Their output data can be processed jointly to improve accuracy of estimation of the vehicle position, increase reliability of the navigation system and provide for continuity of positioning. An additional benefit of integration of navigation data is a possibility of comfortable presentation of processing results to the user of the system.

The DR units are used as reference navigation systems because of their good short-term accuracy, good reliability and continuity, immunity to jamming and swiftness of response to rapid manoeuvres of the vehicle. Their main drawback is poor long-term accuracy. Contrary to DR systems, GPS receivers provide for a medium or high, but importantly, time-independent accuracy of positioning. Their drawbacks include vulnerability to jamming and possibility of signal outages (poor continuity).

The paper presents an example of integrated AHRS/ODOMETER/GPS system. The AHRS and odometer constitute a DR unit and the GPS receiver is used as a correcting device. The system employs a linearized complementary Kalman filter as a data processing algorithm. To reduce complexity and computational burden of this algorithm, a suboptimal Kalman filter has been applied. The design of the filter is based on simplified model of errors of navigation devices.

## PRINCIPLES OF POSITIONING

The integrated AHRS/ODOMETER/GPS system employs two entirely different methods of positioning. They are often referred to as relative and absolute positioning. The former method, also referred to as dead reckoning (DR), consists in measuring and counting distance increments, traveled by a land vehicle in short time spans  $T_{DR}$ , with respect to a known initial position [Kayton & Fried, 1997]. This type of positioning is realized by a subsystem composed of AHRS and odometer. The latter method consists in estimation of position of the vehicle by referencing it to external objects of known locations. Absolute positioning is used in GPS receivers [Spilker & Parkinson, 1996].

In the designed system, AHRS provides for precise magnetic heading  $\alpha_M$ , pitch  $\theta_y$  and roll  $\theta_x$ , whereas the odometer is used as a source of distance increments  $r$ . Following correction of AHRS magnetic heading  $\alpha_M$  with local magnetic variance  $\Delta\alpha_M$ :

$$\alpha = \alpha_M + \Delta\alpha_M \quad (1)$$

one obtains the true heading  $\alpha$ .

The relative positioning of the vehicle can be realised in various frames of reference. The most often-used navigation frames are geodetic and Cartesian Earth-centred Earth-fixed (ECEF) co-ordinates [Kayton & Fried, 1997, Salychev, 1998].

The geodetic co-ordinates of the vehicle can be calculated as follows:

$$\varphi(k+1) = \varphi(k) + \frac{w_N(k)T_{DR}}{R_N(k) + h(k)} = \varphi(k) + \frac{\cos[\theta_y(k)]\cos[\alpha(k)]r(k)}{R_N(k) + h(k)} \quad (2)$$

$$\lambda(k+1) = \lambda(k) + \frac{w_E(k)T_{DR}}{[R_E(k) + h(k)]\cos[\varphi(k)]} = \lambda(k) + \frac{\cos[\theta_y(k)]\sin[\alpha(k)]r(k)}{[R_E(k) + h(k)]\cos[\varphi(k)]} \quad (3)$$

where:  $\varphi, \lambda, h$  - latitude, longitude and altitude of the vehicle,  
 $\alpha_M$  - magnetic heading of the vehicle,  
 $\alpha$  - true heading of the vehicle,  
 $w_E, w_N$  - East and North velocity components,  
 $\theta_y$  - pitch angle of the vehicle,  
 $r$  - distance increment in an interval between  $kT_{DR}$  and  $(k+1)T_{DR}$ ,  
 $T_{DR}$  - time span between two successive DR positions,  
 $R_N$  - meridian radius of Earth's curvature,  
 $R_E$  - prime radius of Earth's curvature.

The meridian and prime radii are given as follows:

$$R_N(k) = \frac{a}{\sqrt{[1 - e^2 \sin^2 \varphi(k)]^3}} \quad (4)$$

$$R_E(k) = \frac{a(1-e^2)}{\sqrt{1-e^2 \sin^2 \varphi(k)}} \quad (5)$$

$$e^2 = 1 - b^2/a^2 \quad (6)$$

where:  $a, b$  - semi-major and semi-minor axis of Earth's reference ellipsoid,  
 $e$  - first eccentricity.

In many land navigation applications, a simplifying assumption can be made, that if a terrain relief is not diversified too much, pitch and roll angles are zeros:

$$\begin{aligned} \theta_x &\approx 0 \\ \theta_y &\approx 0 \end{aligned} \quad (7)$$

In such a case, the horizontal distance increments  $r_H$  are assumed to be equal with the distance increments  $r$  counted by an odometer:

$$r_H = r \cos \theta_y \approx r \quad (8)$$

thus, equations (2) and (3) simplify. If the assumption (7) cannot be made, the pitch measurements from AHRS should be used to calculate horizontal distance increments  $r_H$  in a locally level frame of reference Earth Navigation Unit. The ENU frame axes coincide with East, North and upward (local vertical) directions.

The GPS positioning is realised in Cartesian ECEF (WGS-84) co-ordinate system. Therefore, if a DR subsystem is to be integrated with a GPS receiver, it is comfortable to formulate relative positioning equations in Cartesian ECEF frame of reference. The following formula can be applied:

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ z(k) \end{bmatrix} + C_n^e(k) \begin{bmatrix} \sin \alpha(k) \cos \theta_y(k) \\ \cos \alpha(k) \cos \theta_y(k) \\ \sin \theta_y(k) \end{bmatrix} r(k) \quad (9)$$

$$C_n^e = \begin{bmatrix} -\sin \lambda & -\sin \varphi \cos \lambda & \cos \varphi \cos \lambda \\ \cos \lambda & -\sin \varphi \sin \lambda & \cos \varphi \sin \lambda \\ 0 & \cos \varphi & \sin \varphi \end{bmatrix} \quad (10)$$

where:  $x, y, z$  - Cartesian ECEF position of the vehicle,

$C_n^e$  - transformation matrix from ENU to ECEF frame of reference.

Under assumption of zero pitch angle (7), the following DR equation can be formulated:

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ z(k+1) \end{bmatrix} \approx \begin{bmatrix} x(k) \\ y(k) \\ z(k) \end{bmatrix} + C_n^e(k) \begin{bmatrix} \sin \alpha(k) \\ \cos \alpha(k) \\ 0 \end{bmatrix} r(k) \quad (11)$$

Taking into account the relationships:

$$w_N = \frac{r \cos(\alpha)}{T_{DR}} \quad (12)$$

$$w_E = \frac{r \sin(\alpha)}{T_{DR}} \quad (13)$$

the final relative navigation equation in Cartesian ECEF co-ordinates is as follows:

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ z(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ z(k) \end{bmatrix} + \begin{bmatrix} -\sin \lambda & -\sin \varphi \cos \lambda \\ \cos \lambda & -\sin \varphi \sin \lambda \\ 0 & \cos \varphi \end{bmatrix} \cdot \begin{bmatrix} T_{DR} w_E(k) \\ T_{DR} w_N(k) \end{bmatrix} \quad (14)$$

Contemporary GPS receivers usually form observables of pseudoranges and delta ranges (also referred to as range rates or Doppler), and sometimes also accumulated delta ranges (also referred to as carrier phase or integrated Doppler). These observables, along with positions and velocities of GPS visible satellites, calculated with data extracted from GPS navigation messages, are used in the receiver to solve for the user position and velocity [Spilker & Parkinson, 1996].

The pseudorange from the vehicle to the  $i$ -th GPS satellite is given with the following non-linear equation [Brown & Chwang, 1992, Spilker & Parkinson, 1996]:

$$\Psi_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} + b + v_i \quad (15)$$

where:  $\Psi_i$  - pseudorange between vehicle and  $i$ -th satellite,  
 $X_i, Y_i, Z_i$  -  $i$ -th satellite position,  
 $x, y, z$  - unknown vehicle location,  
 $b$  - GPS receiver's clock bias,  
 $v_i$  - pseudorange measurement error.

The receiver's clock bias  $b$  is a result of lack of synchronisation of a relatively inaccurate receiver's clock and precise atomic clocks of GPS satellites. To properly estimate user position, the receiver's clock bias must be estimated along with the co-ordinates of the vehicle.

The drawbacks and benefits of relative and absolute positioning are to large extent complementary. Thus, proper integration of the both methods allows reduction of their disadvantages and putting their advantages to good use.

### DESIGN OF AHRS/ODOMETER/GPS SYSTEM

#### Functional scheme of AHRS/ODOMETER/GPS system

The structure of the designed AHRS/ODOMETER/GPS integrated system is shown in Fig.2. The system consists of a DR subsystem, a GPS receiver, a complementary linearized Kalman filter (LCKF), and user-satellite range reckoning algorithm  $h^*$ .

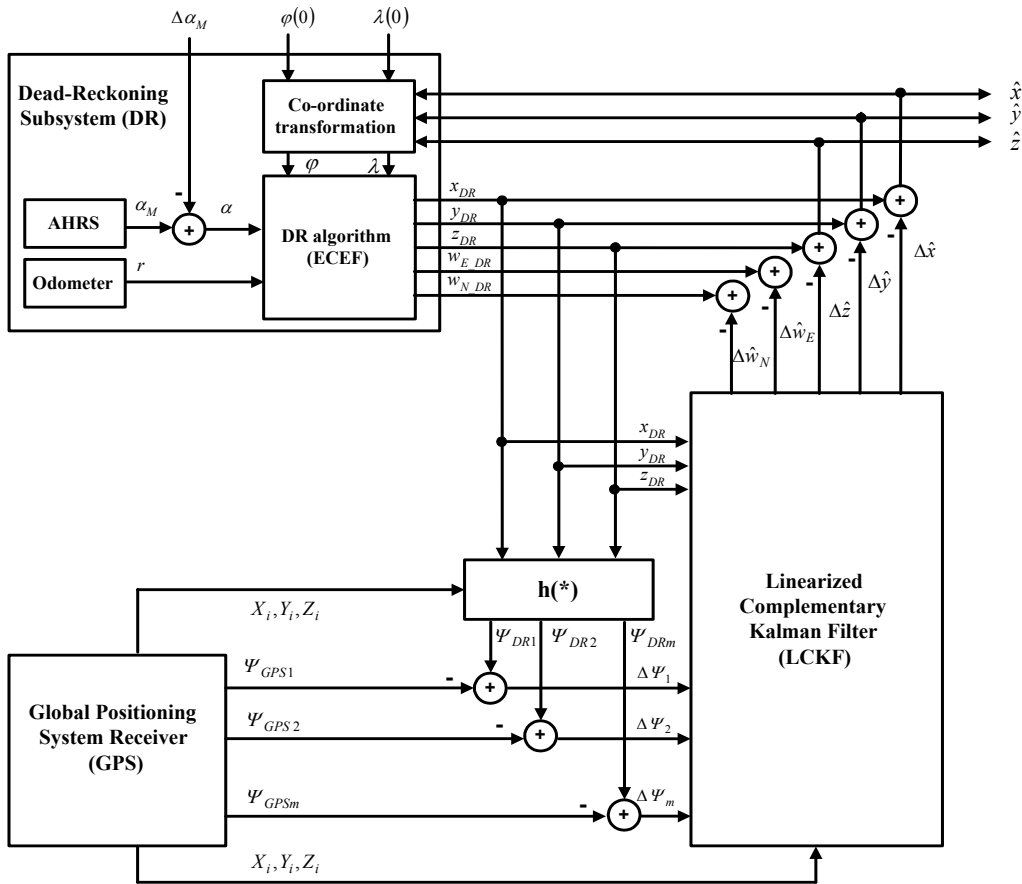


Fig.1. Functional scheme of AHRS/ODOMETER/GPS system

The DR subsystem is composed of AHRS, odometer, co-ordinate transformation algorithm, and relative positioning algorithm. The AHRS magnetic heading  $\alpha_M$  is corrected with local magnetic variance  $\Delta\alpha_M$  which can be calculated for a known geographical location.

The obtained true heading  $\alpha$  and distance increments  $r$  from the odometer are used, along with the known initial position of the vehicle, in relative positioning of the user. The DR algorithm is realised in Cartesian co-ordinates of ECEF frame of reference according to formula (14).

The position of vehicle and positions of all visible GPS satellites, are transformed through  $\mathbf{h}(\cdot)$  function, to reckon values of geometric ranges from the vehicle to the satellites. The non-linear vector function  $\mathbf{h}(\cdot)$  calculates norms of distance vectors between the GPS receiver and the GPS satellites.

The linearized complementary Kalman filter processes differences between DR calculated user-satellite ranges and GPS measured pseudoranges. The processed measurement vector  $\mathbf{z}$  is of variable size, depending on the amount of tracked GPS satellites  $m$ . Positioning and velocity errors estimated by the Kalman filter are used as DR corrections in a feed-forward correction loop. In turn, DR positions  $(x_{DR}, y_{DR}, z_{DR})$  are used as a reference trajectory in linearization of LCKF.

The co-ordinate transformation algorithm is a set of equations converting estimated Cartesian co-ordinates  $(\hat{x}, \hat{y}, \hat{z})$  of the vehicle into geodetic co-ordinates  $(\varphi, \lambda, h)$  [Salychev, 1998]. It provides for geodetic latitude  $\varphi$  and longitude  $\lambda$ , which are necessary in DR algorithm of positioning (14).

### Mathematical description of AHRS/ODOMETER/GPS system

The measurement vector  $\mathbf{z}$  presented to the Kalman filter is composed of differences between reckoned ranges user-satellite and measured pseudoranges:

$$\mathbf{z} = [\Delta\Psi_1 \quad \Delta\Psi_2 \quad \dots \quad \Delta\Psi_m]^T \quad (16)$$

where:

$$\Delta\Psi_i = \Psi_{DR_i} - \Psi_{GPS_i} \quad (17)$$

$$\begin{aligned} \Psi_{DR_i} &= \sqrt{(X_i - x_{DR})^2 + (Y_i - y_{DR})^2 + (Z_i - z_{DR})^2} = \\ &= \sqrt{[X_i - (x + \Delta x)]^2 + [Y_i - (y + \Delta y)]^2 + [Z_i - (z + \Delta z)]^2} \end{aligned} \quad (18)$$

$$\Psi_{GPS_i} = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} + b + v_i \quad (19)$$

As can be seen, the components of  $\mathbf{z}$  vector contain errors resulting from errors of relative positioning in DR subsystem  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ , GPS receiver clock bias  $b$  and pseudorange measurement errors  $v_i$ . Slowly changing and correlated DR positioning errors and GPS receiver clock bias have to be estimated in LCKF.

As the Kalman filter is a model-based estimation algorithm, it is necessary to formulate a design model of the AHRS/ODOMETER/GPS system. For the presented complementary structure of the system, the design model should include a dynamics model of errors  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$ ,  $b$  and an observation model, describing relationship between the estimated quantities and the measurement vector  $\mathbf{z}$ .

The dynamics model of relative positioning errors can be formulated as follows:

$$\begin{bmatrix} \Delta x(k+1) \\ \Delta y(k+1) \\ \Delta z(k+1) \end{bmatrix} = \begin{bmatrix} \Delta x(k) \\ \Delta y(k) \\ \Delta z(k) \end{bmatrix} + \begin{bmatrix} -\sin \lambda & -\sin \varphi \cos \lambda \\ \cos \lambda & -\sin \varphi \sin \lambda \\ 0 & \cos \varphi \end{bmatrix} \cdot \begin{bmatrix} T_{DR} \Delta w_E(k) \\ T_{DR} \Delta w_N(k) \end{bmatrix} \quad (20)$$

The equation (20) has been derived from the dead-reckoning algorithm equation (14). The positioning errors  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$  at a time  $(k+1)T_{DR}$  represent sums of their previous values from a time step  $kT_{DR}$  and new errors, caused by imperfect reckoning of East and North velocity components in DR. The DR velocity errors  $\Delta w_E$ ,  $\Delta w_N$  result from errors of counting distance increments in odometer  $\Delta r$ , errors of AHRS headings  $\Delta \alpha$  and non-zero pitch angle of the vehicle  $\theta_y$ .

In the design of the AHRS/ODOMETER/GPS system, a simple dynamics model of the system was formulated, in which only combined effects of all AHRS and odometer errors have been taken into account. The East and North velocity errors  $\Delta w_E$ ,  $\Delta w_N$  have been modelled as random-walk stochastic processes. For simplicity of the model, the above errors have been assumed independent. The equations of velocity errors are given below:

$$\Delta w_E(k+1) = \Delta w_E(k) + w_{\Delta w_E}(k) \quad (21)$$

$$\Delta w_N(k+1) = \Delta w_N(k) + w_{\Delta w_N}(k) \quad (22)$$

where:  $\Delta w_E$ ,  $\Delta w_N$  - East and North velocity errors of DR subsystem,  
 $w_{\Delta w_E}$ ,  $w_{\Delta w_N}$  - Gaussian white noises of process disturbances.

The design model of the Kalman filter requires formulation of its dynamics model in a form of the following vector equation [Candy, 1987, Minkler & Minkler, 1993]:

$$\mathbf{x}(k+1) = \mathbf{\Phi}(k+1, k) \cdot \mathbf{x}(k) + \mathbf{w}(k) \quad (23)$$

where:  $\mathbf{x}$  - state vector to be estimated by LCKF,  
 $\mathbf{w}$  - vector of process disturbances,  
 $\mathbf{\Phi}$  - state transition matrix.

Combining equations (20), (21) and (22) into one vector equation of the form (23), one obtains:

$$\begin{bmatrix} \Delta x(k+1) \\ \Delta y(k+1) \\ \Delta z(k+1) \\ \Delta w_E(k+1) \\ \Delta w_N(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -T_{DR} \sin \lambda & -T_{DR} \sin \varphi \cos \lambda \\ 0 & 1 & 0 & T_{DR} \cos \lambda & -T_{DR} \sin \varphi \sin \lambda \\ 0 & 0 & 1 & 0 & T_{DR} \cos \varphi \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \Delta x(k) \\ \Delta y(k) \\ \Delta z(k) \\ \Delta w_E(k) \\ \Delta w_N(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ w_{\Delta w_E}(k) \\ w_{\Delta w_N}(k) \end{bmatrix} \quad (24)$$

The following model should be additionally augmented with a model of GPS receiver's clock errors. A 2-state model, including clock bias  $b$  and clock drift  $d$ , has been adopted [Brovn & Hvang, 1992]. A discrete model of GPS receiver's clock errors can be presented as follows:

$$\begin{bmatrix} b(k+1) \\ d(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T_{GPS} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} b(k) \\ d(k) \end{bmatrix} + \begin{bmatrix} w_b(k) \\ w_d(k) \end{bmatrix} \quad (25)$$

Assuming equal discretization times  $T_{DR}=T_{GPS}=T$ , as a result of merging dynamics models (24) and (25), the state vector to be estimated by LCKF contains 7 states:

$$\mathbf{x} = [\Delta x \quad \Delta y \quad \Delta z \quad \Delta w_E \quad \Delta w_N \quad b \quad d]^T \quad (26)$$

and the transition matrix (is equal:

$$\Phi = \begin{bmatrix} \Phi_{DR} & \mathbf{0} \\ \mathbf{0} & \Phi_{GPSCLK} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -T \sin \lambda & -T \sin \varphi \cos \lambda & 0 & 0 \\ 0 & 1 & 0 & T \cos \lambda & -T \sin \varphi \sin \lambda & 0 & 0 \\ 0 & 0 & 1 & 0 & T \cos \varphi & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (27)$$

The combined vector of process disturbances  $\mathbf{w}$  is as follows:

$$\mathbf{w} = [0 \quad 0 \quad 0 \quad w_{\Delta w_E} \quad w_{\Delta w_N} \quad w_b \quad w_d]^T \quad (28)$$

and its covariance matrix  $\mathbf{Q}$ :



$$\mathbf{Q} = E[\mathbf{w} \mathbf{w}^T] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\Delta wE}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\Delta wN}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & TS_f + \frac{T^3 S_g}{3} & \frac{T^2 S_g}{2} \\ 0 & 0 & 0 & 0 & 0 & \frac{T^2 S_g}{2} & TS_g \end{bmatrix} \quad (29)$$

where:  $S_f, S_g$  - spectral amplitudes of white noises  $w_b$  and  $w_d$ ,  
 $\sigma_{\Delta wE}^2, \sigma_{\Delta wN}^2$  - variances of velocity errors  $w_{\Delta wE}, w_{\Delta wN}$ .

Having formulated the above dynamics model of the AHRS/ODOMETER/GPS system, it is necessary to describe the observation model. The observation model defines relationship between the measurement vector  $\mathbf{z}$  and the state vector  $\mathbf{x}$  as the following vector equation:

$$\mathbf{z}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{v}(k) \quad (30)$$

where:  $\mathbf{z}$  - measurement vector,  
 $\mathbf{x}$  - state vector,  
 $\mathbf{v}$  - vector of measurement noises,  
 $\mathbf{H}$  - observation matrix.

Because of a non-linear relationship between the GPS measurements and the ECEF position of the vehicle, the observation model is non-linear. In the presented design, the measurement equation has been linearized around the trajectory of the vehicle from DR subsystem. The linearization consists in calculating the observation matrix  $\mathbf{H}$  as a matrix of derivatives of non-linear  $\mathbf{h}(\cdot)$  function with respect to the components of the state vector  $\mathbf{x}$ . The reference points, in which derivatives are calculated, are reckoned positions of the vehicle from DR. Thus, the observation model for AHRS/ODOMETER/GPS has the following form:

$$\begin{bmatrix} \Psi_{DR_1} - \Psi_{GPS_1} \\ \Psi_{DR_2} - \Psi_{GPS_2} \\ \vdots \\ \Psi_{DR_m} - \Psi_{GPS_m} \end{bmatrix} = \begin{bmatrix} \frac{\partial \Psi_1}{\partial x} & \frac{\partial \Psi_1}{\partial y} & \frac{\partial \Psi_1}{\partial z} & 0 & 0 & -\frac{\partial \Psi_1}{\partial b} & 0 \\ \frac{\partial \Psi_2}{\partial x} & \frac{\partial \Psi_2}{\partial y} & \frac{\partial \Psi_2}{\partial z} & 0 & 0 & -\frac{\partial \Psi_2}{\partial b} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \Psi_m}{\partial x} & \frac{\partial \Psi_m}{\partial y} & \frac{\partial \Psi_m}{\partial z} & 0 & 0 & -\frac{\partial \Psi_m}{\partial b} & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta w_E \\ \vdots \\ \Delta w_N \\ b \\ d \end{bmatrix} + \begin{bmatrix} -v_1 \\ -v_2 \\ \vdots \\ -v_m \end{bmatrix} \quad (31)$$

The derivatives in the observation matrix  $\mathbf{H}$  are given as follows:

$$\frac{\partial \Psi_i}{\partial z} = \frac{-(X_i - x)}{\sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2}} \quad (32)$$

$$\frac{\partial \Psi_i}{\partial z} = \frac{-(Y_i - y)}{\sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2}} \quad (33)$$

$$\frac{\partial \Psi_i}{\partial z} = \frac{-(Z_i - z)}{\sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2}} \quad (34)$$

$$\frac{\partial \Psi_i}{\partial b} = 1 \quad (35)$$

The GPS pseudorange measurements contain errors  $\mathbf{v}$  of diverse origin and type [Spilker & Parkinson, 1996]. Some of them can be considered uncorrelated white noises whereas the others are correlated. For simplicity, errors of each measured pseudorange have been modelled as zero-mean white noise processes with variances  $\sigma_\psi^2$ . Thus, the covariance errors matrix  $\mathbf{R}$  is diagonal and contains variances  $\sigma_\psi^2$  of all pseudorange measurements:

$$\mathbf{R} = \mathbb{E}[\mathbf{v} \mathbf{v}^T] = \begin{bmatrix} \sigma_\psi^2 & 0 & \dots & 0 \\ 0 & \sigma_\psi^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_\psi^2 \end{bmatrix} \quad (36)$$

### Linearized complementary Kalman filtering algorithm

As an algorithm of estimation of positioning, velocity and GPS receiver's clock errors, a linearized complementary Kalman filter (LCKF) has been employed. The filter processes the measurement vector  $\mathbf{z}$ , composed of differences between DR and GPS measurements. The vector  $\mathbf{z}$  contains a combination of DR and GPS errors. The LCKF exploits complementary statistical properties of these errors, which is reflected in the name of the filter. The role of the LCKF consists in separation of DR and GPS errors. The estimated DR errors can be further used to correct the DR/GPS system. As the relationship between the measurement vector  $\mathbf{z}$  and the state vector  $\mathbf{x}$  is non-linear, a linearized Kalman filter has been applied.

The LCKF algorithm is composed of initialisation and a series of prediction-correction steps. The idea of data processing in LCKF is presented in the Fig. 2:

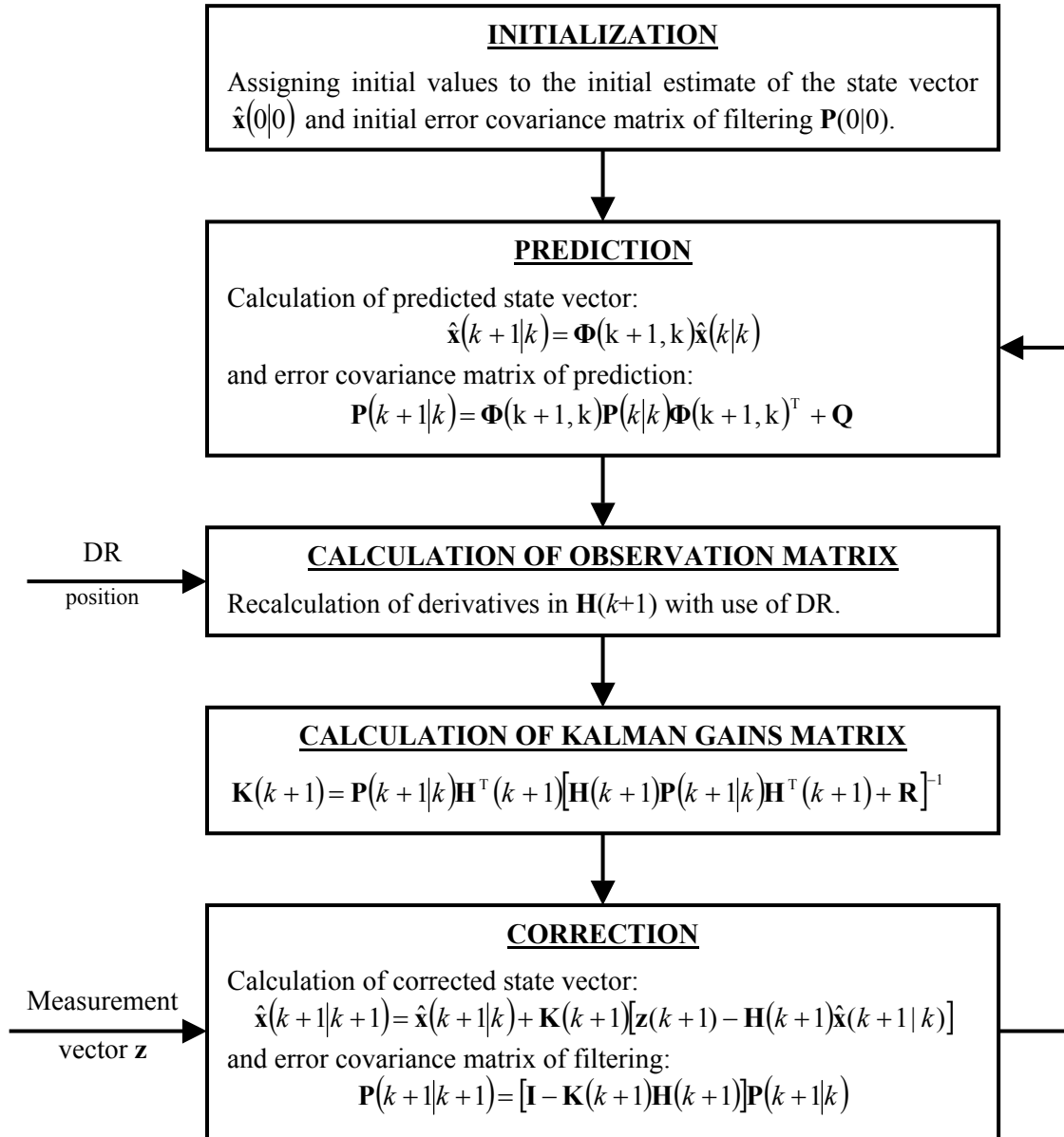


Fig.2 Algorithm of linearized complementary Kalman filter

### METHODOLOGY OF TESTING

The tests of the AHRS/ODOMETER/GPS system consisted of off-line processing of real and simulated navigation data. The route of the vehicle and trajectories of all visible GPS satellites have been generated with use of MATLAB<sup>®</sup> scientific software. Errors of distance increments from the odometer and pseudoranges from the GPS receiver have been simulated. The AHRS errors have been obtained from a real device. The tested attitude and heading reference system is a 9-axis solid-state inertial measurement system that utilizes MEMS micro-machined sensors. A starting location of the vehicle was assumed at latitude 52°N and longitude 22°E. The simulations lasted for 600 seconds. The adopted methodology of testing is presented in the Fig. 3.

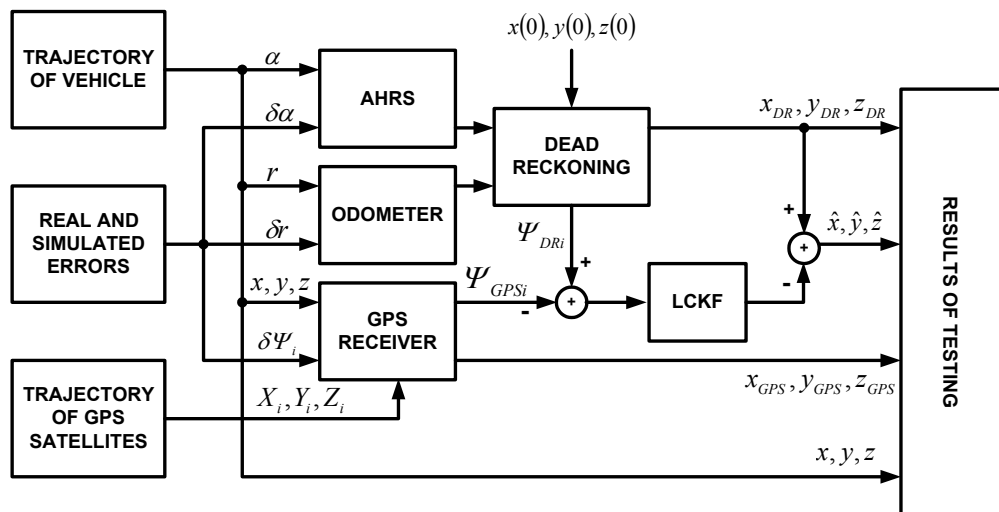


Fig.3. Methodology of testing

### RESULTS OF TESTING

To demonstrate the quality of the designed Kalman filtering algorithm, a comparison of positioning errors of AHRS/ODOMETER, GPS receiver and integrated navigation system AHRS/ODOMETER/GPS is presented. Chosen realizations of the errors are shown in Fig. 4 and 5. The positioning errors are expressed in ENU frame of reference.

The Fig. 4 demonstrates the accuracy of AHRS/ODOMETER/GPS and its subsystems in case of lack of GPS outages. An influence of GPS outages onto the quality of the system is demonstrated in the Fig. 5. As can be seen, breaks in GPS correcting data result in temporary increase of positioning errors. As soon as GPS data become available, positioning errors of the integrated system are quickly attenuated. The periods of GPS outages lasted for 100 s and 70 s respectively. They are marked with circles.

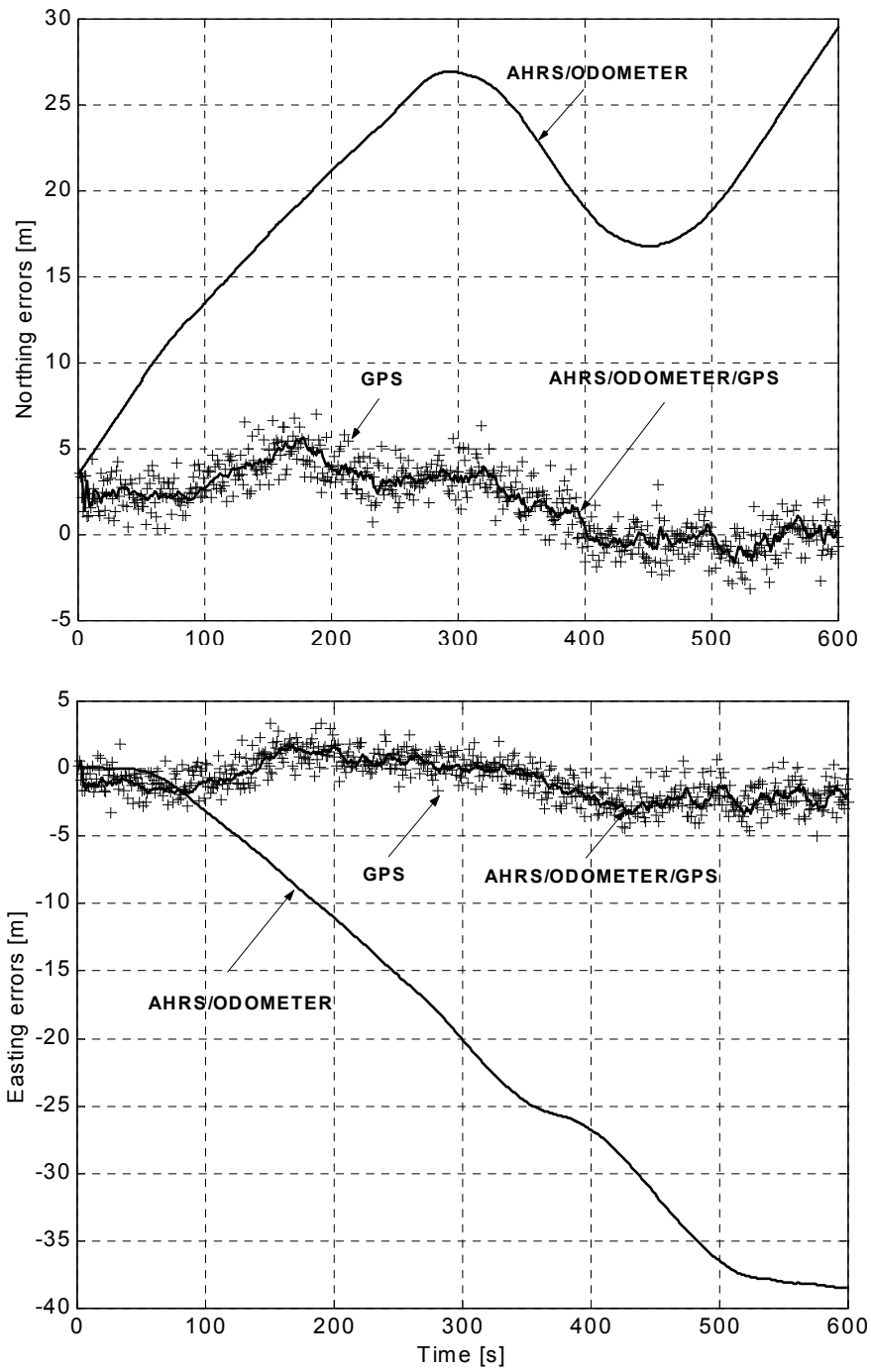


Fig.4 Positioning errors without GPS outages

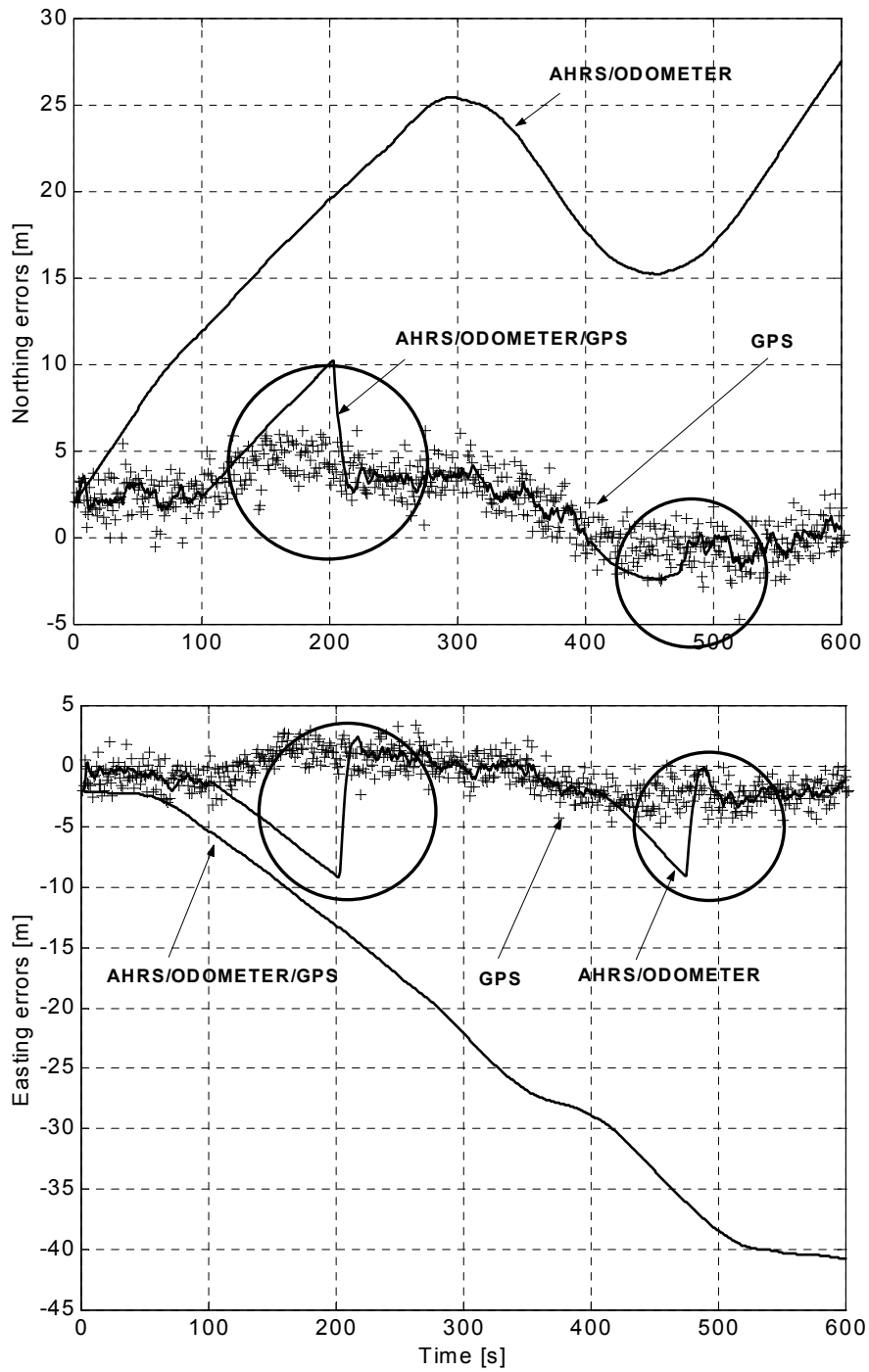


Fig.5 Positioning errors with GPS outages

## CONCLUSIONS

On the basis of the above results several conclusions can be drawn:

- The AHRS/ODOMETER/GPS system is more accurate than any of its subsystems alone. The positioning errors of AHRS/ODOMETER, increasing with a time of operation and a distance travelled, have been eliminated and the GPS receiver uncorrelated positioning errors have been significantly reduced.
- An improvement of AHRS/ODOMETER/GPS positioning accuracy in comparison to the accuracy of GPS alone depends on the character of GPS errors. Best results are achieved when the correlated GPS errors are not large in comparison to the uncorrelated ones.
- The continuity of the system has been significantly improved in comparison to the continuity of GPS alone. The GPS outages result only in temporary deterioration of positioning accuracy of AHRS/ODOMETER/GPS. Provided the periods of outages are short, the accuracy of the integrated system remains satisfactory for many applications.

## REFERENCES

1. R.G. BROWN, P.Y.C. HWANG, Introduction to random signals and applied Kalman filtering, Wiley, UK, 1992.
2. J.V. CANDY, Signal Processing the Model-Based Approach, McGraw-Hill, 1987.
3. B. FORSELL, Radionavigations systems, Prentice Hall, UK, 1991.
4. M. KAYTON, W.R. FRIED, Avionics Navigation Systems - Second Edition, John Wiley & Sons, Inc., New York, USA, 1997.
5. G. MINKLER, J. MINKLER, Theory and Application of Kalman Filtering, Magellan Book Company, USA, 1993.
6. O. SALYCHEV, Inertial Systems in Navigation and Geophysics, Bauman MSTU Press, Moscow, Russia, 1998.
7. J. SPILKER & B. PARKINSON, EDS., Global Positioning System: Theory and Applications, Vol. i and Vol. II, American Institute of Aeronautics and Astronautics, 1996.

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