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VARIANTS OF STRUCTURAL AND MEASUREMENT MODELS OF AN INTEGRATED NAVIGATIONAL SYSTEM

ABSTRACT The paper presents two structural and measurement variants of the Kalman filter in an integrated navigational system. The shape of a particular model is determined by the measuring capacity of navigational parameters and the assumed form of the state vector.

INTRODUCTION

The last years of the twentieth century started the integration and globalisation process of navigation. The large family of satellite navigational systems users currently embraces not only navigators – be it maritime, terrestrial, aerial or space – but also geodesists, geologists, foresters and all those who want to know their geographic location with high precision. This would not be possible without the suitable technological level of contemporary navigational and computer techniques.

The high precision of satellite and autonomous navigational systems (dead reckoning) frequently sets high demands for the methods of navigational data processing. It is often assumed that the handling of measurements is aimed exclusively at their optimal processing with respect to random disturbances; this is why estimation of navigational data is usually used, or the parameters of their disturbance distribution. It is attempted to eliminate surplus and systematic errors at the stage of measurement qualification (primary processing). Real measurement conditions do not always support such assumptions, reflected in the past by so-called measurement tests, and contemporaneously as an integrity test of the navigational appliance of system.

The paper presents the subject of building various navigational models integrated by a suitable Kalman filter construction – a model of state and a model of measurements.

KALMAN FILTER

Methods of Kalman filtration can be applied at various levels of navigational information treatment, starting from primary processing – the estimation of measurement errors (of the order of physical values like phase, time, amplitude) and ending at position coordinates and other navigational parameters (geometrical values). In all of these cases the same calculation algorithm is used. Seeing that contemporaneously digital systems are used for measurement and calculation, the essence of this algorithm can be presented on the example of a discrete random dynamic system. A discrete random dynamic system is described by two equations:

- state equation (structural model)

$$\mathrm{x}_{i+1} = A_{i+1,i}\, x_i + \mathrm{w}_i, \tag{1}$$

- measurement equation (measurement model)

$$\mathbf{z}_{i+1} = \mathbf{C}_{i+1}\mathbf{x}_{i+1} + \mathbf{v}_{i+1}, \tag{2}$$

where: **x** – n[th] dimension state vector,
w – r[th] dimension state disturbance vector,
z - m[th] dimension measurement vector,
v – p[th] –dimension state disturbance vector (measurement noise),
A - n×n-dimension transition matrix,
C - m×n-dimension measurement matrix,
$r \le n$, $p \le m$.

Besides, it is assumed for disturbance vectors **w** and **v** that they are Gaussian noise of normal distribution, of zero mean vector and are mutually not correlated. The state equation describes the trend of the vector we are concerned with, and the measurement model gives the functional dependence of the measurement on this vector. The solution to the equation system (1), (2), taking into consideration the limitations imposed on disturbance vectors, is in the Kalman filter. The estimation of the state vector in the filter can be presented by the diagram below:

- state vector forecast

$$\tilde{\mathbf{x}}_{i+1,i} = \mathbf{A}_{i+1,i}\hat{\mathbf{x}}_i, \tag{3}$$

where $\tilde{\mathbf{x}}$ – forecast value of the state vector,
$\hat{\mathbf{x}}$ –estimated value of the state vector,

- covariance value of the forecast state vector

$$\mathbf{P}_{i+1,i} = \mathbf{A}_{i+1,i}\mathbf{P}_i\mathbf{A}^{\mathrm{T}}_{i+1,i} + \mathbf{Q}_i, \tag{4}$$

where **Q** is the matrix of the covariance state disturbance (of vector **w**),

- innovation process

$$\varepsilon_{i+1} = \mathbf{z}_{i+1} - \mathbf{C}_{i+1}\tilde{\mathbf{x}}_{i+1,i}, \tag{5}$$

- covariance matrix of the innovation process

$$\mathbf{S}_{i+1} = \mathbf{R}_{i+1} + \mathbf{C}_{i+1}\mathbf{P}_{i+1,i}\mathbf{C}_{i+1}^{\mathrm{T}}, \tag{6}$$

where **R** is the covariance matrix of measurement disturbance (of vector **v)**

- filter amplification matrix

$$\mathbf{K}_{i+1} = \mathbf{P}_{i+1,i}\mathbf{C}_{i+1}^{\mathrm{T}}\mathbf{S}_{i+1}^{-1}, \tag{7}$$

- estimate of the state vector from filtration after making measurement $\mathbf{z}_{k+1}$

$$\hat{\mathbf{x}}_{i+1} = \tilde{\mathbf{x}}_{i+1,i} + \mathbf{K}_{i+1}\varepsilon_{i+1}, \tag{8}$$

- covariance matrix of the estimated state vector

$$\mathbf{P}_{i+1} = \left(\mathbf{I} - \mathbf{K}_{i+1}\mathbf{C}_{i+1}\right)\mathbf{P}_{i+1,i}. \tag{9}$$

As mentioned before, the calculation algorithm remains the same, but in particular applications we shall have various forms and dimensions of vectors and matrices. What follows below are solution variants of integrated navigation based on various structural and measurement models.

When accepting a particular model of integrated navigation we have to formulate two equations: the structural model and the measurement model. The structural model is determined by the navigational process model accepted. This process is defined by the state vector components and its evolution (matrix **A**). The state vector to be selected depends on the parameters to be estimated, that is, the final navigational parameters or their errors (systematic components in the forms of corrections). Apart from this, we have to know in advance if we have the possibility to make measurements of physical values that are functionally bound with the parameters estimated. It follows from this that we must have at least an approximate picture of the measurement model, and this is what we actually do in practice. We assume an initial concept that defines which values are to be estimated and we check if there are necessary measurement capacities.

The measurement model (equation 2) describes the dependence of measurements on the state vector. In the case of deterministic calculation of position coordinates (without taking into account random disturbances of state and measurement) or estimation by the least square method, the dependence is expressed by the Jacobie matrix (matrix of position surface gradients) [Banachowicz, 1991]. Let us illustrate this by the example of two navigational models. In the first, DR measurements are used, a satellite navigational system, and a terrestrial navigational system. In the other model, only one positional system is used (GPS or DGPS) and two DR navigational systems – a log-gyrocompass and inertial navigation (INS).

INTEGRATED DR/GPS/RADIONAVIGATION SYSTEM

In this case what we have at our disposal are measurements by a satellite system (GPS, GLONASS, DGPS, DGLONASS), a terrestrial radionavigation system (LORAN or a close-range system) and measurements from the log and the gyrocompass. In marine navigation first of all position coordinates (φ, λ) are accepted as elements of the state vector and their derivatives, e.g. components of the velocity vector, acceleration vector etc. Let us assume that the following parameters will be the estimated values: position coordinates (φ, λ), projections of the speed vector in relation to the bottom onto the meridian and the parallel (V_N, V_E), the systematic error of the track angle in relation to the bottom (COG – Course Over Ground -ΔCOG) and the systematic speed error in relation to the bottom (SOG – Speed Over Ground-ΔSOG). The state vector will assume the following shape for this situation:

$$\mathbf{x} = \left[\varphi, \lambda, V_N, V_E, \Delta COG, \Delta SOG\right]^{\mathrm{T}}. \tag{10}$$

As we remember, the structural model creates a state equation (formula 1). This is why we also have to determine the structure of transition matrix **A**. Let us assume it in the following form:

$$\mathbf{A}_{i+1,i} = \begin{bmatrix} 1 & 0 & k_\varphi \cdot \Delta t_i & 0 & 0 & 0 \\ 0 & 1 & 0 & k_\lambda \cdot \Delta t_i & 0 & 0 \\ 0 & 0 & 1+\Delta V_N & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+\Delta V_E & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{11}$$

where:

$$\Delta V_N = \frac{\overline{V}_N - V_N}{V_N}, \tag{12}$$

$$\Delta V_E = \frac{\overline{V}_E - V_E}{V_E},$$

$$k_\varphi = \frac{\sqrt{\left(1 - e^2 \sin^2 \varphi\right)^3}}{a\left(1 - e^2\right)},$$

$$k_{\lambda} = \frac{\sqrt{1 - e^2 \sin^2 \varphi}}{a \cos\varphi}, \tag{13}$$

φ - geographic latitude,
λ - geographic longitude,
a – semi-major axis of the earth's ellipsoid,
e – first eccentricity of the earth's ellipsoid.

Components of the average speed $\overline{V}_N$ and $\overline{V}_E$ can be calculated as the resultant speed from a continuum of the radionavigation position system. It is also frequently assumed in a simplified form for synchronic measurements that Δt_i = 1 sec – this is usually applied for synchronic measurements, obtained from a GPS receiver.

A supplementary element of the structural model is the covariance matrix of state disturbance vector **Q**. The particular elements of this matrix determine a *priori* the disturbance distribution of the estimated values. The interpretation of this matrix from the point of view of navigational practice is as follows – its elements determine the confidence intervals, where the estimated navigational parameters may be inherent. Elements (1,1) and (2,2) of matrix **Q** define the yaw interval of the vessel, speaking more strictly, they define the movement disturbance on the geographic latitude and longitude. For the state vector defined by formula (10) matrix **Q** can assume the following shape:

$$\mathbf{Q}_i = \begin{bmatrix} \sigma_{\varphi}^2 k_{\varphi}^2 + \Delta t_i^{\,2} \sigma_{V_{\varphi}}^2 & \Delta t_i^{\,2} \sigma_{V_{\varphi} V_{\lambda}} & 0 & 0 & 0 & 0 \\ \Delta t_i^{\,2} \sigma_{V_{\varphi} V_{\lambda}} & \sigma_{\lambda}^2 k_{\lambda}^2 + \Delta t_i^{\,2} \sigma_{V_{\lambda}}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{V_N}^2 & \sigma_{V_N V_E} & 0 & 0 \\ 0 & 0 & \sigma_{V_N V_E} & \sigma_{V_N}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\Delta COG}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\Delta SOG}^2 \end{bmatrix}, \tag{14}$$

where: σ_{φ} – disturbance of the vessel's movement along geographic latitude,

σ_{λ} – disturbance of the vessel's movement along geographic longitude,

$$\sigma_{V_N}^2 = \left[(\sigma_{SOG} \cos COG)^2 + (SOG \sigma_{COG} \sin COG)^2 \right], \tag{15}$$

$$\sigma_{V_E}^2 = \left[(\sigma_{SOG} \sin COG)^2 + (SOG \sigma_{COG} \cos COG)^2 \right], \tag{16}$$

$$\sigma_{V_N V_E} = \frac{1}{2} \left(\sigma_{SOG}^2 - SOG^2 \sigma_{COGd}^2 \right) \sin 2COG, \tag{17}$$

COG – Course Over Ground,
SOG – Speed Over Ground,
σ_{cog} – COG measurement error,
σ_{SOG} – SOG measurement error,
$\sigma_{\Delta COG}$ – correction determination error ΔCOG,
$\sigma_{\Delta SOG}$ – correction determination error ΔSOG.

Equations (10) – (17) determine the structural model of the navigation process, where the estimated values are position coordinates components of the speed vector in relation to the bottom and the corrections – of the track angle in relation to the bottom and the speed in relation to the bottom.

Let us assume the following parameters as values measured in the measurement system: coordinates of system positions DGPS (φ_{DGPS}, λ_{DGPS}), the terrestrial radionavigational system (φ_L, λ_L), track angle in relation to the bottom (COG) and speed in relation to the bottom (SOG). Thus, the following will be the elements of the measurement vector:

$$\mathbf{z} = \left[\varphi_{DGPS}, \lambda_{DGPS}, \varphi_L, \lambda_L, COG, SOG\right]^{\mathrm{T}}. \tag{18}$$

The measurement matrix will be a Jacobie matrix:

$$\mathbf{C} = \begin{bmatrix} \frac{\partial\varphi_{DGPS}}{\partial\varphi} & \frac{\partial\varphi_{DGPS}}{\partial\lambda} & \frac{\partial\varphi_{DGPS}}{\partial V_\varphi} & \frac{\partial\varphi_{DGPS}}{\partial V_\lambda} & \frac{\partial\varphi_{DGPS}}{\partial\Delta COG} & \frac{\partial\varphi_{DGPS}}{\partial\Delta SOG} \\ \frac{\partial\lambda_{DGPS}}{\partial\varphi} & \frac{\partial\lambda_{DGPS}}{\partial\lambda} & \frac{\partial\lambda_{DGPS}}{\partial V_\varphi} & \frac{\partial\lambda_{DGPS}}{\partial V_\lambda} & \frac{\partial\lambda_{DGPS}}{\partial\Delta COG} & \frac{\partial\lambda_{DGPS}}{\partial\Delta SOG} \\ \frac{\partial\varphi_L}{\partial\varphi} & \frac{\partial\varphi_L}{\partial\lambda} & \frac{\partial\varphi_L}{\partial V_\varphi} & \frac{\partial\varphi_L}{\partial V_\lambda} & \frac{\partial\varphi_L}{\partial\Delta COG} & \frac{\partial\varphi_L}{\partial\Delta SOG} \\ \frac{\partial\lambda_L}{\partial\varphi} & \frac{\partial\lambda_L}{\partial\lambda} & \frac{\partial\lambda_L}{\partial V_\varphi} & \frac{\partial\lambda_L}{\partial V_\lambda} & \frac{\partial\lambda_L}{\partial\Delta COG} & \frac{\partial\lambda_L}{\partial\Delta SOG} \\ \frac{\partial COG}{\partial\varphi} & \frac{\partial COG}{\partial\lambda} & \frac{\partial COG}{\partial V_\varphi} & \frac{\partial COG}{\partial V_\lambda} & \frac{\partial COG}{\partial\Delta COG} & \frac{COGd}{\partial\Delta SOG} \\ \frac{\partial VSOG}{\partial\varphi} & \frac{\partial SOG}{\partial\lambda} & \frac{\partial SOG}{\partial V_\varphi} & \frac{\partial SOG}{\partial V_\lambda} & \frac{\partial SOG}{\partial\Delta COG} & \frac{\partial VSOG}{\partial\Delta SOG} \end{bmatrix}. \tag{19}$$

After calculating the particular partial derivatives and their suitable ordering we will obtain the following shape of matrix **C**:

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & B_1 & B_2 & -1 & 0 \\ 0 & 0 & E_1 & E_2 & E_3 & -1 \end{bmatrix},$$

where:

$$B_1 = \frac{COG}{2V_N}, \quad B_1 = \frac{COG}{2} \text{ dla } V_N = 0,$$

$$B_2 = \frac{COG}{2V_E}, \quad B_2 = \frac{COG}{2} \text{ dla } V_E = 0,$$

$$E_1 = \cos COG + \frac{V_E}{V_N^2 + V_E^2} V_N \sin COG - \frac{V_E}{V_N^2 + V_E^2} V_E \cos COG,$$

$$E_2 = \sin COG + \frac{V_N}{V_N^2 + V_E^2} V_E \cos COG - \frac{V_N}{V_N^2 + V_E^2} V_N \sin COG,$$

$$E_3 = V_N \sin COG - V_E \cos COG,$$

$$\frac{V_E}{V_N^2 + V_E^2} = 0 \text{ for } V_N = 0 \;\&\; V_E = 0,$$

$$\frac{V_N}{V_N^2 + V_E^2} = 0 \text{ for } V_N = 0 \;\&\; V_E = 0.$$

The measuring model is supplemented by the covariance matrix of measurement disturbance (measurement vector):

$$\mathbf{R} = \begin{bmatrix} \sigma^2_{\varphi_{DGPS}} & \sigma_{\varphi\lambda_{DGPS}} & \sigma_{\varphi_{DGPS}\varphi_L} & \sigma_{\varphi_{DGPS}\lambda_L} & \sigma_{\varphi_{DGPS}COG} & \sigma_{\varphi_{DGPS}SOG} \\ \sigma_{\varphi\lambda_{DGPS}} & \sigma^2_{\lambda_{DGPS}} & \sigma_{\lambda_{DGPS}\varphi_L} & \sigma_{\lambda_{DGPS}\lambda_L} & \sigma_{\lambda_{DGPS}COG} & \sigma_{\lambda_{DGPS}SOG} \\ \sigma_{\varphi_{DGPS}\varphi_L} & \sigma_{\lambda_{DGPS}\varphi_L} & \sigma^2_{\varphi_L} & \sigma_{\varphi\lambda_L} & \sigma_{\varphi_L COG} & \sigma_{\varphi_L SOG} \\ \sigma_{\varphi_{DGPS}\lambda_L} & \sigma_{\lambda_{DGPS}\lambda_L} & \sigma_{\varphi\lambda_L} & \sigma^2_{\lambda_L} & \sigma_{\lambda_L COG} & \sigma_{\lambda_L SOG} \\ \sigma_{\varphi_{DGPS}COG} & \sigma_{\lambda_{DGPS}COG} & \sigma_{\varphi_L COG} & \sigma_{\lambda_L COG} & \sigma^2_{COG} & \sigma_{COG,SOG} \\ \sigma_{\varphi_{DGPS}SOG} & \sigma_{\lambda_{DGPS}SOG} & \sigma_{\varphi_L SOG} & \sigma_{\lambda_L SOG} & \sigma_{COG,SOG} & \sigma^2_{SOG} \end{bmatrix}. \quad (20)$$

As some measured values are not mutually correlated – e.g. DGPS measurements and the radionavigation terrestrial system, the matrix will be reduced to the following shape:

$$\mathbf{R} = \begin{bmatrix} k_\varphi^2\sigma^2_{\varphi_{DGPS}} & k_\varphi k_\lambda \sigma_{\varphi\lambda_{DGPS}} & 0 & 0 & 0 & 0 \\ k_\varphi k_\lambda \sigma_{\varphi\lambda_{DGPS}} & k_\lambda^2\sigma^2_{\lambda_{DGPS}} & 0 & 0 & 0 & 0 \\ 0 & 0 & k_\varphi^2\sigma^2_{\varphi_L} & k_\varphi k_\lambda \sigma_{\varphi\lambda_L} & 0 & 0 \\ 0 & 0 & k_\varphi k_\lambda \sigma_{\varphi\lambda_L} & k_\lambda^2\sigma^2_{\lambda_L} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{COG} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{SOG} \end{bmatrix}.$$

If we assume concrete values of particular variances and covariances occurring in this matrix, we will get:

$$\mathbf{R} = \begin{bmatrix} 4k_\varphi^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.3k_\lambda^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3.6k_\varphi^2 & 0.6k_\varphi k_\lambda & 0 & 0 \\ 0 & 0 & 0.6k_\varphi k_\lambda & 6.8k_\lambda^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0007 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.25 \end{bmatrix}.$$

This model was applied in a navigational system of stabilising the position of a rescue vessel. The following measurement parameters were assumed in the algorithm and the programming:

- DGPS system – $\sigma_\varphi = 2{,}0$ m, $\sigma_\lambda = 1{,}5$ m, coordinates not correlated; research performed in the Szczeciński Lagoon and the Pomorska Gulf;
- radionavigation terrestrial system AD-2 – $\sigma_\varphi = 1{,}9$ m, $\sigma_\lambda = 2{,}6$ m, $\sigma_{\varphi\lambda} = 0{,}6$ m^2 (covariance); research performed in the Gdańska Bay;
- track angle in relation to the bottom – $\sigma_{COG} = 1.5^0$;
- speed in relation to the bottom – $\sigma_{SOG} = 0.5$ knots.

INTEGRATED INS/GPS

Another solution is provided by a situation when there are following values to be estimated: position coordinates (φ, λ), projections of the speed vector in relation to the bottom onto the meridian and the parallel (V_N, V_E), acceleration vector projections onto the meridian and the parallel (a_N, a_E) and the projections of acceleration vector derivatives in relation to the bottom onto the meridian and the parallel (a'_N, a'_E). In this case the state vector will have the following elements:

$$\mathbf{x} = \left[\varphi, \lambda, V_N, V_E, a_N, a_E, a_N^{'}, a_E^{'}\right]^{\mathrm{T}}. \tag{21}$$

Transition matrix a will be defined as follows:

$$A_{i+1,i} = \begin{bmatrix} 1 & 0 & k_\varphi \Delta t_i & 0 & k_\varphi \Delta t_i^{\,2}/2 & 0 & k_\varphi \Delta t_i^{\,3}/6 & 0 \\ 0 & 1 & 0 & k_\lambda \Delta t_i & 0 & k_\lambda \Delta t_i^{\,2}/2 & 0 & k_\lambda \Delta t_i^{\,3}/6 \\ 0 & 0 & 1 & 0 & k_\varphi \Delta t_i & 0 & k_\varphi \Delta t_i^{\,2}/2 & 0 \\ 0 & 0 & 0 & 1 & 0 & k_\lambda \Delta t_i & 0 & k_\lambda \Delta t_i^{\,2}/2 \\ 0 & 0 & 0 & 0 & 1 & 0 & k_\varphi \Delta t_i & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & k_\lambda \Delta t_i \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{22}$$

Now the state evolution is defined by higher-order derivatives of particular estimated navigational parameters. The state disturbance covariance matrix will also obtain a form adapted to the new state vector elements. Thus, we will get:

$$\mathbf{Q}_i = \begin{bmatrix} \sigma_\varphi^2 k_\varphi^2 + \Delta t_i^{\,2} \sigma_{V_\varphi}^2 & \Delta t_i^{\,2} \sigma_{V_\varphi V_\lambda} & 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta t_i^{\,2} \sigma_{V_\varphi V_\lambda} & \sigma_\lambda^2 k_\lambda^2 + \Delta t_i^{\,2} \sigma_{V_\lambda}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{V_N}^2 & \sigma_{V_N V_E} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{V_N V_E} & \sigma_{V_N}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{a_N}^2 & \sigma_{a_a a_E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{a_a a_E} & \sigma_{a_E}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{a_N^{'}}^2 & \sigma_{a_N^{'} a_E^{'}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{a_N^{'} a_E^{'}} & \sigma_{a_E^{'}}^2 \end{bmatrix}. \tag{23}$$

The state disturbance covariance matrices (14) and (23) differ only as to the elements which correspond to various elements of the state vectors.

In this model let us assume the following as the measured values: position coordinates of the DGPS system (φ_{DGPS}, λ_{DGPS}), speed components in relation to the meridian and the parallel from DR navigation (V_N, V_E), acceleration components in relation to the meridian and the parallel from an inertial transformer (a_N, a_E).

With these assumptions the measurement vector will be as follows:

$$\mathbf{z} = [\varphi_{DGPS}, \lambda_{DGPS}, V_N, V_E, a_N, a_E]^{\mathrm{T}}. \tag{24}$$

The measurement matrix, as above, will be a Jacobie matrix

$$\mathbf{C} = \begin{bmatrix} \frac{\partial\varphi_{DGPS}}{\partial\varphi} & \frac{\partial\varphi_{DGPS}}{\partial\lambda} & \frac{\partial\varphi_{DGPS}}{\partial V_N} & \frac{\partial\varphi_{DGPS}}{\partial V_E} & \frac{\partial\varphi_{DGPS}}{\partial a_N} & \frac{\partial\varphi_{DGPS}}{\partial a_E} & \frac{\partial\varphi_{DGPS}}{\partial a'_N} & \frac{\partial\varphi_{DGPS}}{\partial a'_E} \\ \frac{\partial\lambda_{DGPS}}{\partial\varphi} & \frac{\partial\lambda_{DGPS}}{\partial\lambda} & \frac{\partial\lambda_{DGPS}}{\partial V_N} & \frac{\partial\lambda_{DGPS}}{\partial V_E} & \frac{\partial\lambda_{DGPS}}{\partial a_N} & \frac{\partial\lambda_{DGPS}}{\partial a_E} & \frac{\partial\lambda_{DGPS}}{\partial a'_N} & \frac{\partial\lambda_{DGPS}}{\partial a'_E} \\ \frac{\partial V_N}{\partial\varphi} & \frac{\partial V_N}{\partial\lambda} & \frac{\partial V_N}{\partial V_N} & \frac{\partial V_N}{\partial V_E} & \frac{\partial V_N}{\partial a_N} & \frac{\partial V_N}{\partial a_E} & \frac{\partial V_N}{\partial a'_N} & \frac{\partial V_N}{\partial a'_E} \\ \frac{\partial V_E}{\partial\varphi} & \frac{\partial V_E}{\partial\lambda} & \frac{\partial V_E}{\partial V_N} & \frac{\partial V_E}{\partial V_E} & \frac{\partial V_E}{\partial a_N} & \frac{\partial V_E}{\partial a_E} & \frac{\partial V_E}{\partial a'_N} & \frac{\partial V_E}{\partial a'_E} \\ \frac{\partial a_N}{\partial\varphi} & \frac{\partial a_N}{\partial\lambda} & \frac{\partial a_N}{\partial V_N} & \frac{\partial a_N}{\partial V_E} & \frac{\partial a_N}{\partial a_N} & \frac{\partial a_N}{\partial a_E} & \frac{\partial a_N}{\partial a'_N} & \frac{\partial a_N}{\partial a'_E} \\ \frac{\partial a_E}{\partial\varphi} & \frac{\partial a_E}{\partial\lambda} & \frac{\partial a_E}{\partial V_N} & \frac{\partial a_E}{\partial V_E} & \frac{\partial a_E}{\partial a_N} & \frac{\partial a_E}{\partial a_E} & \frac{\partial a_E}{\partial a'_N} & \frac{\partial a_E}{\partial a'_E} \end{bmatrix}. \tag{25}$$

Let us calculate the particular partial derivatives and put them in order. We will obtain a very simple form of matrix **C** then:

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

Now the measurement matrix is a block matrix, which makes calculations very simple and significantly reduces numerical errors.

The matrix of measurement disturbance covariance (measurement vector) will look as follows In this case:

$$\mathbf{R} = \begin{bmatrix} \sigma^2_{\varphi_{DGPS}} & \sigma_{\varphi\lambda_{DGPS}} & \sigma_{\varphi_{DGPS}V_N} & \sigma_{\varphi_{DGPS}V_E} & \sigma_{\varphi_{DGPS}a_N} & \sigma_{\varphi_{DGPS}a_E} \\ \sigma_{\varphi\lambda_{DGPS}} & \sigma^2_{\lambda_{DGPS}} & \sigma_{\lambda_{DGPS}V_N} & \sigma_{\lambda_{DGPS}V_E} & \sigma_{\lambda_{DGPS}a_N} & \sigma_{\lambda_{DGPS}a_E} \\ \sigma_{\varphi_{DGPS}V_N} & \sigma_{\lambda_{DGPS}V_N} & \sigma^2_{V_N} & \sigma_{V_NV_E} & \sigma_{V_Na_N} & \sigma_{V_Na_E} \\ \sigma_{\varphi_{DGPS}V_E} & \sigma_{\lambda_{DGPS}V_E} & \sigma_{V_NV_E} & \sigma^2_{V_E} & \sigma_{V_Ea_N} & \sigma_{V_Ea_E} \\ \sigma_{\varphi_{DGPS}a_E} & \sigma_{\lambda_{DGPS}a_N} & \sigma_{V_Na_N} & \sigma_{V_Ea_N} & \sigma^2_{a_N} & \sigma_{a_Na_E} \\ \sigma_{\varphi_{DGPS}a_E} & \sigma_{\lambda_{DGPS}a_E} & \sigma_{V_Na_E} & \sigma_{V_Ea_E} & \sigma_{a_Na_E} & \sigma^2_{a_E} \end{bmatrix}. \tag{26}$$

As some values are not mutually correlated, as in the previous model, the matrix will assume the following shape:

$$\mathbf{R} = \begin{bmatrix} k_\varphi^2 \sigma_{\varphi_{DGPS}}^2 & k_\varphi k_\lambda \sigma_{\varphi\lambda_{DGPS}} & 0 & 0 & 0 & 0 \\ k_\varphi k_\lambda \sigma_{\varphi\lambda_{DGPS}} & k_\lambda^2 \sigma_{\lambda_{DGPS}}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{V_N}^2 & \sigma_{V_N V_E} & 0 & 0 \\ 0 & 0 & \sigma_{V_N V_E} & \sigma_{V_E}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{a_N}^2 & \sigma_{a_N a_E} \\ 0 & 0 & 0 & 0 & \sigma_{a_N a_E} & \sigma_{a_E}^2 \end{bmatrix}.$$

Assuming concrete values of particular variances and covariances we can reduce the matrix to a matrix with constant elements:

$$\mathbf{R} = \begin{bmatrix} 4k_\varphi^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.3k_\lambda^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.01 & 0.1 \\ 0 & 0 & 0 & 0 & 0.1 & 0.01 \end{bmatrix}.$$

In this case we assumed the following values of variance and covariance for particular measurements:

- DGPS system – $\sigma_\varphi = 2.0$ m,
- $\sigma_\lambda = 1.5$ m,
- coordinates not correlated,
- research performed in the Szczeciński Lagoon and the Pomorska Gulf, as in the previous model:
  - speed components – $\sigma_V = 0.1$ m/s;
  - acceleration components – $\sigma_a = 0.01$ m/s^2.

Because of better consideration of the vessel's dynamics, this model has an essential advantage over the former. It turns out that for speeds close to zero and active rudders working the Doppler log is characterized by large measurement errors. Then it becomes necessary to apply the INS/GPS model, in spite of the high cost of the inertial transformer. [Banachowicz, Bober, 1999].

CONCLUSIONS

The navigational process models presented above do not specified all the possible solutions. Basically as many examples can be given as there are requirements put for the set of navigational parameters and the measurement capacities. It should be clearly stressed that the shape of these models is highly decisive for the success or failure of the filter worked out. This concerns first of all a correct, adequate for the real state of things determination of elements for particular matrices – transition **A**, measurements **C** and covariance matrices **Q** and **R**. In the case of covariance matrix the mutual relation of particular elements is most essential.

If too large errors of state disturbance are accepted, the filter becomes 'rigid' and lags behind the measurements too much [Banachowicz, 1995]. This results in the measurement results being not filtered. If the assumed state disturbance values are too small, the filter will reject measurement values deviating too much from the forecast. This is particularly important in real measurement situations when there is lack of concordance between measurements made by various navigational appliances and systems and with low-reliability measuring apparatus and measurement results. [Banachowicz, Bober, 1997], [Banachowicz, Bober, 1999]. Neither should we forget the problem of synchronizing the time scales of particular measuring instruments, the length of measurement cycles and discretization intervals. As measurement precision is very high nowadays, and so is the speed of navigating objects, the assumption about measurement simultaneity is often fictitious. It may occur that the measurement value will differ considerably from the prognosticated value. It is a systematic error of the time scale; there simply takes place a parallel shift of a series of measurements in relation to a series of forecasts. Another problem that vexes the programmers is the numeric stability of calculations. The matrix-vector recording is very convenient and well interpretable. The creation of applications is very easy, due to the existence of ready procedure libraries and sub-programs. This brings about over-extension and over-development of algorithms which results in numerical error accumulation and a slow-down of calculations performed.

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