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DESIGN AND SIMULATIVE TESTING OF DR/GPS SYSTEM

ABSTRACT The article presents a project and simulation results of an integrated positioning system DR/GPS. Firstly, a choice of devices for the positioning system is discussed. Secondly, a concept of integration of a dead reckoning (DR) module with a GPS receiver is described. The system processes data from an odometer, a gyro, an electronic compass, which constitute the DR unit, and from a GPS receiver with use of two complementary Kalman filters. Next, state-space model of the system and Kalman filters are shortly presented. After that, the authors outline an adopted methodology of simulative testing of DR/GPS integrated system. Chosen simulation results are included in the paper. The presented integrated positioning system may find its application as a component of an in-car navigation system or, following several alterations, also as an onboard system of other types of land vehicles.

INTRODUCTION

Nowadays, land positioning systems gain more and more interest and find new areas of application. They can be met in military vehicles, in special purpose civilian vehicles or even in higher-class or middle-class cars. The importance of integrated positioning systems for land users is strictly connected with latest developments in Intelligent Transportation Systems (ITS) [Drane, 1998]. ITS systems include navigation and tracking systems, fleet management systems, traffic information systems, etc. Their spreading will have significant impact on various aspects of our life. Potential benefits include increased safety and comfort of travelling, higher efficiency of transportation, reduction of fuel consumption, traffic congestion and pollution, to name just a few. As integrated positioning systems are key elements of ITS, any progress in them is of great significance to the progress in ITS.

Integrated positioning systems are expected to be accurate, reliable and yet not too expensive. To meet these contradictory requirements, several devices or subsystems are usually integrated into a compound system [Forsell, 1991 & Kayton, 1997].

A choice of navigation devices and their scheme of integration have significant impact on the ultimate performance of the system. The article describes a design and simulation results of an integrated positioning system DR/GPS.

PRINCIPLES OF OPERATION OF DR/GPS

The DR/GPS system is composed of a dead reckoning unit (DR) and a GPS receiver. These two subsystems are based on entirely different principles of operation. The DR subsystem realises relative positioning, whereas GPS offers an absolute method of positioning. Their drawbacks and benefits are to large extent complementary. Thus, proper integration of the both positioning methods allows reduction of their disadvantages and putting their advantages to good use.

Assuming that a vehicle positioning is realised in geodetic co-ordinates, the relative positioning consists in calculating latitude and longitude by counting north and east distance increments, travelled by the vehicle in short time spans, with respect to a known initial position. a geodetic position of the vehicle is described by its latitude φ, longitude *(*and altitude h. In most land navigation applications only latitude and longitude are of interest. They can be calculated as follows [Kayton, 1997]:

$$\varphi_N = \varphi_0 + \sum_{k=1}^{N} \Delta\varphi_k = \varphi_0 + \sum_{k=1}^{N} \frac{r_k \cdot \cos\alpha_k}{R_n + h_k} \tag{1}$$

$$\lambda_N = \lambda_0 + \sum_{k=1}^{N} \Delta\lambda_k = \lambda_0 + \sum_{k=1}^{N} \frac{r_k \cdot \sin\alpha_k}{(R_e + h_k)\cdot\cos\varphi_k} \tag{2}$$

where: φ_k, λ_k, h_k - latitude, longitude and altitude of the vehicle at a time kT_{DR},
α_k - heading of the vehicle at a time kT_{DR},
r_k - distance increment in a period between $(k\text{-}1)T_{DR}$ and kT_{DR},
T_{DR} - time span between two successive DR positions,
R_n - meridian radius of Earth's curvature,
R_e - prime radius of Earth's curvature.

The principle of this method is illustrated in Fig. 1.

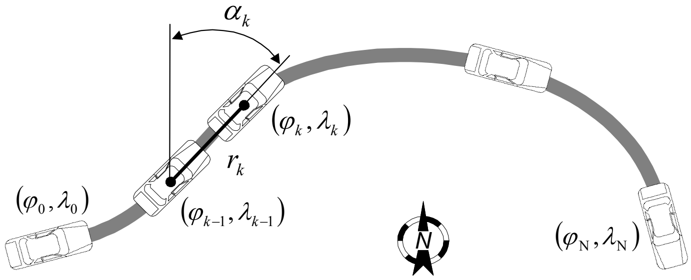


Fig. 1. Relative positioning principle

The principle of operation of the DR subsystem ensures its capability of worldwide, all-day and all-weather use. Independence on any facilities or devices external to the vehicle, as satellite or ground radio transmitters guarantees autonomous operation and consequently immunity to jamming. Further advantages of the DR subsystem include high short-term accuracy of positioning, good continuity and reliability, as well as immediate response to manoeuvres.

There is, however, a serious drawback of the dead reckoning method, i.e. increasing positioning errors along with a time of operation of the system and a distance travelled. For this reason, a dead reckoning system cannot be used alone in most applications. It has to be augmented with a device or system capable of absolute positioning, like NAVSTAR Global Positioning System (GPS) [Spilker, 1996].

Nowadays, GPS is most often-used global navigation satellite system. Contemporary GPS receivers usually form observables of pseudoranges and delta ranges (also referred to as range rates or Doppler), and sometimes also accumulated delta ranges (also referred to as carrier phase or integrated Doppler). These observables, along with positions and velocities of GPS visible satellites, calculated with data extracted from the GPS navigation message, are used in the receiver to solve for the user position and velocity. The idea of GPS absolute positioning is shown in Fig. 2.

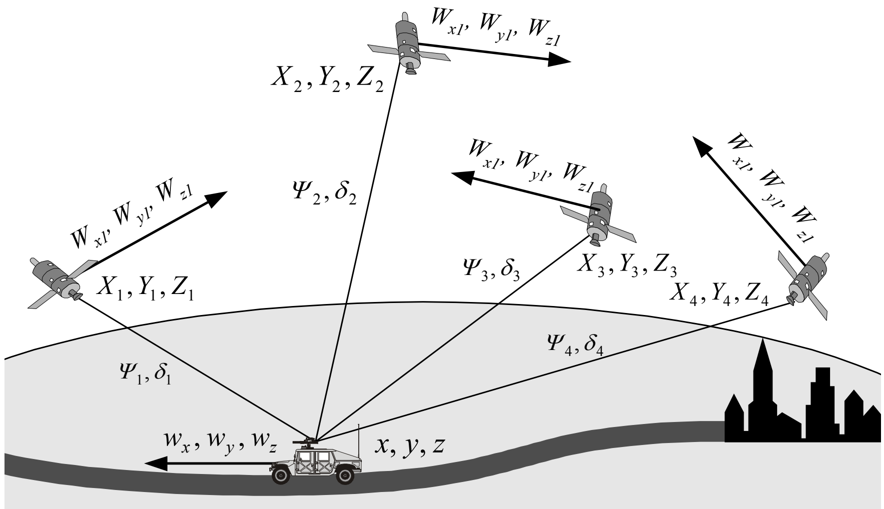


Fig. 2. Principle of absolute positioning with GPS

The GPS observables are related to the vehicle position and velocity by the following non-linear equations [Spilker, 1996]:

$$\Psi_i = \sqrt{(X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2} + c\Delta t \tag{3}$$

$$\delta_i = \frac{(x - X_i)(w_x - W_{xi}) + (y - Y_i)(w_y - W_{yi}) + (z - Z_i)(w_z - W_{zi})}{\rho_i} + d \qquad (4)$$

where: Ψ_i - pseudorange between vehicle and i-th satellite,
δ_i - delta range between vehicle and i-th satellite,
X_i, Y_i, Z_i - i-th satellite position,
W_{Xi}, W_{Yi}, W_{Zi} - i-th satellite velocity,
x, y, z - unknown vehicle location,
w_x, w_y, w_z - unknown vehicle velocity,
c - speed of light,
Δt - GPS receiver clock bias,
d - GPS receiver clock drift,
ρ_i - true range between vehicle and i-th satellite.

A simultaneous tracking of four or more satellites enables receiver to estimate user position, velocity, clock bias and drift. Positioning is realised in WGS-84 frame of reference, but its results (x, y, z) are usually converted into a more comfortable set of coordinates, e.g. geodetic coordinates (φ, λ, h) [Forsell, 1991]. Further in the article, it is assumed that the GPS receiver provides geodetic latitude and longitude.

The absolute positioning in GPS can be realised worldwide, 24 hours a day and independently on weather conditions. The system guarantees an accuracy of positioning independent on a time of operation or a distance travelled. a stand-alone GPS accuracy is to large extent dependent on the user equipment, but in the post-SA era it is sufficient for many navigation applications. Nevertheless it can be further significantly improved by use of differential techniques (DGPS) [Spilker, 1996]. Drawbacks of absolute navigation with use of GPS include possibility of signal outages, especially in urban canyons, tunnels and under dense foliage and short-term positioning accuracy worse than that in a relative navigation. As it is not an autonomous system, it is also prone to jamming.

From the foregoing one can see, that relative and absolute positioning (DR and GPS) complement one another and their integration may result in reduction of disadvantages and putting the advantages to good use.

DESIGN OF DR/GPS

Components of the system

The absolute positioning may be realised with any off-shelf GPS receiver. Depending on the system requirements with respect to accuracy, continuity, reliability and integrity, as well as a budget of the project, use of differential corrections or GPS/GLONASS integrated receiver may be considered. The presented DR/GPS integrated positioning system contains a standard GPS receiver.

Choice of navigation devices for DR is more complicated. The system must contain devices measuring a distance travelled and a heading of the vehicle or quantities related to them. Contemporary land navigation systems usually contain a combination of the following devices: odometers, accelerometers, difference odometers, gyros and electronic compasses [Drane, 1998].

Odometers or accelerometers can be used as sensors of linear motion of the vehicle. Use of accelerometers would require parallel use of precise orientation sensors, therefore an odometer has been applied. Odometers measure total distance travelled by the vehicle or distance increments in short time spans. The measurements are obtained by counting signals from wheel or drive-shaft sensors. The sensors contain a stationary and a rotary part. The stationary part generates signals when the rotary element passes it. Odometers are simple and relatively accurate.

Apart from the distance increments, a heading of the vehicle is necessary to realize the dead reckoning positioning. As the angular motion sensors, difference odometers, gyros and electronic compasses can be applied. The difference odometer is constructed as a pair of odometers located at the opposite wheels of the vehicle. On turns, the distances travelled by the outer and the inner wheels of the vehicle are different, and enable calculation of heading changes. Difference odometers are cumbersome in installation and their errors tend to increase along with the distance travelled. For these reasons they have not been employed in the designed DR subsystem.

Another candidate for the angular motion sensor in DR is a gyro. Depending on its type, it outputs continuous angular velocity or discrete angular rates around the measurement axis. The heading can be calculated by integration of the angular velocity or counting the angular rates from the gyro with its measurement axis parallel to the vertical axis of the vehicle. The following formulae can be applied:

$$\alpha(\mathrm{N}) = \alpha(0) + \sum_{k=1}^{\mathrm{N}} \Delta\alpha(k) = \alpha(0) + \sum_{k=1}^{\mathrm{N}} \left[\int_{(k-1)\mathrm{T_g}}^{k\mathrm{T_g}} \omega(t)dt \right] \tag{5}$$

where: ω - angular velocity from gyro,

α - vehicle heading,

$\Delta\alpha$ - heading increments,

$\alpha(0)$ - initial heading,

$\mathrm{T_g}$ - time span between two successive samples of gyro measurements.

A disadvantage of gyro utilisation is increasing errors of calculated heading. On the other hand, it is simple in installation and relatively cheap.

As a source of information about the heading of the vehicle, an electronic compass may also be used. Its main advantage consists in a fact that it measures magnetic heading directly, and consequently does not suffer from increasing errors, typical for difference odometers or gyros. Its use in a positioning system, along with a gyro, may correct gyro errors providing good long-term accuracy of heading.

In turn, the gyro ensures that the system is capable of immediate response to rapid manoeuvres of the vehicle. An electronic compass may be a valuable navigation device but requires cautious installation and calibration. Magnetic fields from metal parts of the vehicle and its load, as well as from nearby objects passed during the travel, may seriously affect accuracy of measurements.

For the above reasons, a gyro, an electronic compass and an odometer have been employed in the presented DR/GPS system.

Structure of the system

A scheme of the DR/GPS positioning system is shown in Fig. 3. It is composed of a DR subsystem and a GPS receiver. The DR subsystem processes distance increments from an odometer, angular rates from a vertically oriented gyro and compass headings. The compass and gyro data are jointly processed via a complementary Kalman filter (DRKF) embedded in a closed-loop configuration. Such a configuration provides estimation and removal of small heading errors at a time $k\mathrm{T}_{DR}$, remained after the last correction at $(k\text{-}1)\mathrm{T}_{DR}$. As DRKF estimates only residual gyro errors rather that whole gyro errors, a design model for the Kalman filter can be less detailed, i.e. fewer states in the state vector suffice.

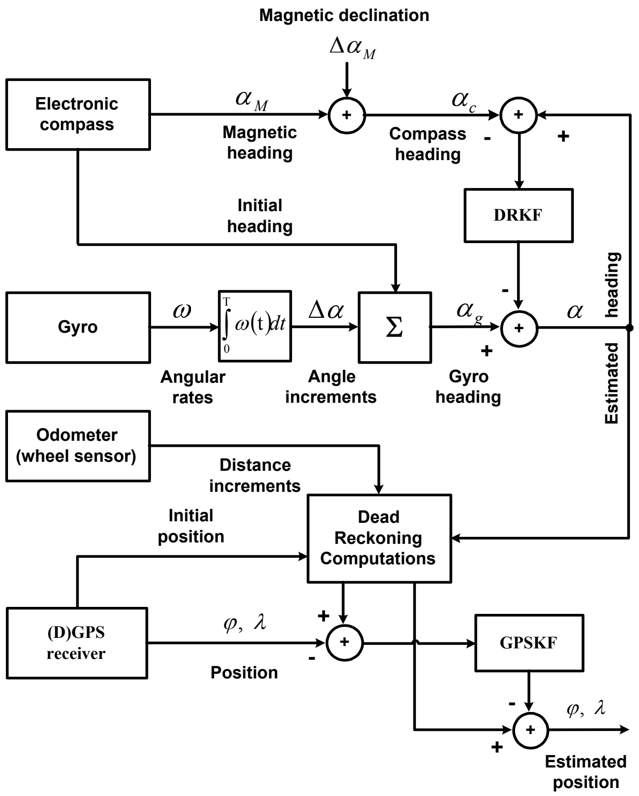


Fig. 3. Integrated positioning system DR/GPS

Moreover, as the gyro errors change slowly, DRKF corrections remain up-to-date for a relatively long period, and DRKF may work with extended periods between successive measurements.

The DR heading and distance increments are used in DR to determine geodetic latitude, longitude and horizontal velocity of the vehicle. These data can be compared with the GPS position fixes and velocity. In the presented systems, their differences are processed by the second Kalman filter (GPSKF) and estimated DR position errors are subsequently used as feed-forward corrections to the DR data.

State-space model of the system and filters' design

State-space models of respective parts of the DR/GPS positioning system have to be defined to formulate Kalman filters' equations [Brown, 1992 & Candy, 1987]. These models include vector difference equations describing dynamics and observation models.

The first Kalman filter DRKF processes data of gyro and electronic compass. As it has been assumed that compass errors are uncorrelated, the dynamics model of this part of the system describes time propagation of correlated gyro errors.

In continuous form, the dynamics model can be formulated as the following vector differential equation:

$$\dot{\mathbf{x}}(t) = \mathbf{F}(t) \cdot \mathbf{x}(t) + \mathbf{u}(t) \tag{6}$$

where: $\mathbf{x}$ - state vector,
$\mathbf{F}$ - state matrix,
$\mathbf{u}$ - vector of random inputs to the system.

A model of gyro errors [Kim, 1996] adopted in the project, includes 5 states:

$$\frac{d}{dt}\begin{bmatrix} \delta\alpha \\ \delta k_g \\ \delta k_m \\ b_g \\ b_m \end{bmatrix} = \begin{bmatrix} 0 & \omega & \omega & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\beta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\beta \end{bmatrix} \cdot \begin{bmatrix} \delta\alpha \\ \delta k_g \\ \delta k_m \\ b_g \\ b_m \end{bmatrix} + \begin{bmatrix} u_\alpha \\ u_{kg} \\ u_{km} \\ u_{bg} \\ u_{bm} \end{bmatrix} \tag{7}$$

where: $\delta\alpha$ - gyro heading error,
δk_g, δk_m - constant and Gauss-Markov scale factor errors,
b_g, b_m - constant and Gauss-Markov gyro biases,
ω - angular velocity of vehicle around gyro measurement axis,
β - inverse of time constant of Gauss-Markov process,
u_α, u_{kg}, u_{km}, u_{bg}, u_{bm} –gaussian white noises.

The above continuous dynamics model has to be converted into its discrete version:

$$\mathbf{x}(k+1) = \mathbf{\Phi}(k)\mathbf{x}(k) + \mathbf{w}(k) \tag{8}$$

The operation is laborious, but straightforward [Brown, 1992], and consists in calculation of transition matrix (and covariance matrix of discrete random inputs to the system $E[\mathbf{w}(k)\mathbf{w}^T(l)] = \mathbf{Q}(k)\delta_{kl}$. The obtained transition matrix is as follows:

$$\mathbf{\Phi} = \begin{bmatrix} 1 & \Delta\alpha & \frac{\Delta\alpha}{\beta \mathrm{T_g}}\left(1-e^{-\beta \mathrm{T_g}}\right) & \mathrm{T_g} & \frac{1}{\beta}\left(1-e^{-\beta \mathrm{T_g}}\right) \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & e^{-\beta \mathrm{T_g}} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & e^{-\beta \mathrm{T_g}} \end{bmatrix} \tag{9}$$

An observation model underlying DRKF design defines relationship between the state vector $\mathbf{x}$ and the measurement vector $\mathbf{z}$ and takes into account measurement errors $\mathbf{v}$.

$$\mathbf{z}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{v}(k) \tag{10}$$

In case of the DR/GPS system, the measurement vector processed by DRKF is in fact a scalar value, representing a difference between the estimated heading α and the compass heading α_c. An observation matrix H is as follows:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{11}$$

A design of GPSKF filter is based on a simple model, where DR north and east velocity errors have been assumed to be Wiener processes, thus latitude and longitude errors are modelled as integrated Wiener processes. The Wiener stochastic process is non-stationary and its variance increases infinitely with time. Such a model is adequate to description of infinitely increasing errors of relative positioning realised in the DR subsystem. a state vector of the model is as follows:

$$\mathbf{x} = \begin{bmatrix} \Delta\varphi & \Delta w_N & \Delta\lambda & \Delta w_E \end{bmatrix}^T \tag{12}$$

where: $\Delta\varphi$, $\Delta\lambda$ - DR latitude and longitude error,
Δw_N, Δw_E - DR north and east velocity error.

Taking into account the assumed statistical properties of the state vector elements, as well as Eq. 1 and 2, one obtains the following transition matrix of the dynamics model:

$$\mathbf{\Phi} = \begin{bmatrix} 1 & \frac{\mathrm{T_{DR}}}{\mathrm{R_n}} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{\mathrm{T_{DR}}}{\mathrm{R_e}\cdot\cos(\varphi)} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

The Kalman filter GPSKF processes a measurement vector composed of differences between DR and GPS latitude and longitude:

$$\mathbf{z} = \begin{bmatrix} \varphi_{DR} - \varphi_{GPS} \\ \lambda_{DR} - \lambda_{GPS} \end{bmatrix} \tag{14}$$

An observation matrix H in the measurement model is as follows:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{15}$$

Both filters, DRKF and GPSKF, have been designed according to the above state-space models and are slightly modified covariance Kalman filter equations. The DRKF algorithm takes into account deterministic controls introduced to its estimates. These controls exist because the filter works in a closed correction loop. Thus, its own estimates from a previous step of processing are subtracted from the present measurement vector **z**.

SIMULATION METHODOLOGY

In order to assess errors of the integrated system DR/GPS, it was tested with a series of simulations realised with use of MATLAB® scientific software. During the simulations a realistic trajectory of the vehicle and trajectories of all visible GPS satellites were generated. a starting location of the vehicle was assumed at latitude 52(N and longitude 21(E. The simulations started at 1000 second of a GPS week and lasted for 600 seconds. Fig. 4 presents the assumed route of the vehicle in a local frame of reference East-North-Up (ENU) [Kayton, 1997], with origin at the vehicle starting location.

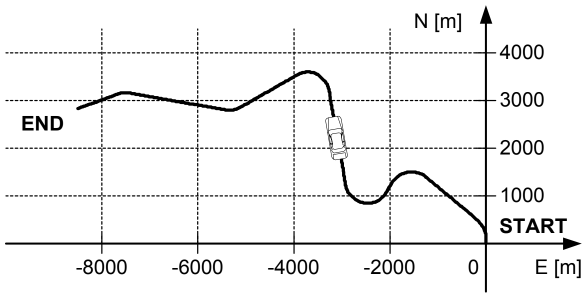


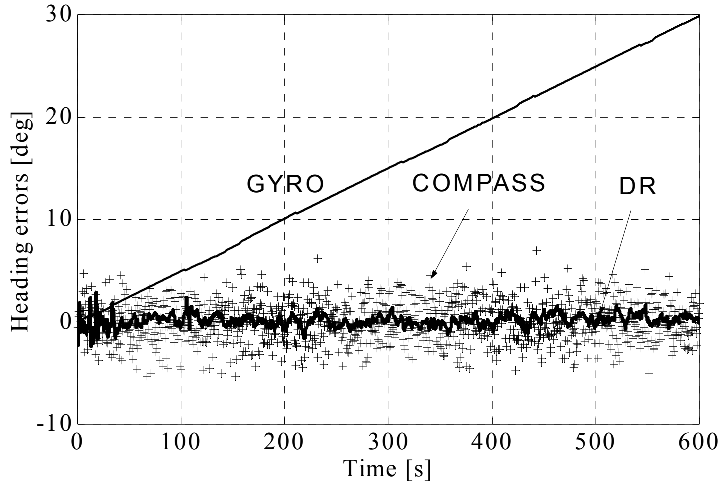
Fig. 4. Route of the vehicle assumed in simulations

Next, error-free measurements for all navigation devices, i.e. the odometer, the gyro and the electronic compass, as well as error-free pseudoranges of the GPS receiver were generated. In parallel, errors of the above devices were generated and imposed on the error-free measurements. Sampling rates of generated data were set with accordance to capabilities of typical low-cost navigation devices and were 20 Hz for gyro and odometer, 2 Hz for compass and 1 Hz for GPS receiver.

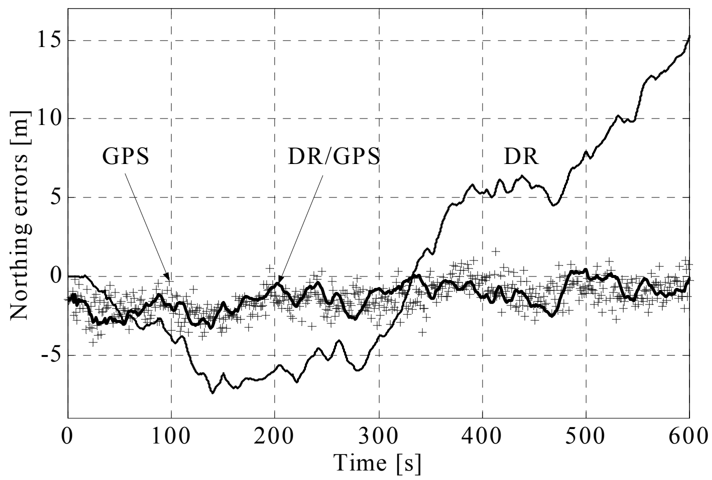
SIMULATION RESULTS

The obtained erroneous navigation data subsequently underwent processing by the designed DRKF and GPSKF procedures and the obtained results were visualised and compared. a comparison of headings' errors obtained on the basis of uncorrected gyro data, stand-alone compass measurements and heading estimated in the DR subsystem, is presented in Fig. 5(a). Fig. 5(b) and 5(c) compare north and east positioning errors of the DR subsystem, GPS receiver and DR/GPS integrated system. The positioning errors are expressed in ENU frame of reference.

(a)



(b)



(c)

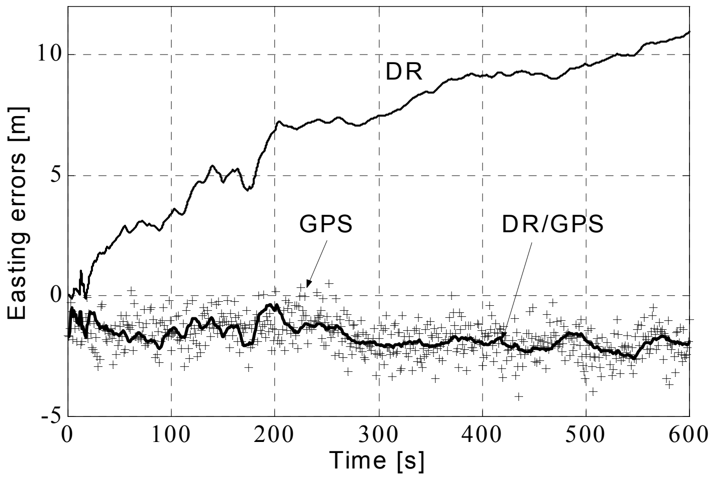


Fig. 5 Heading and positioning errors

CONCLUSIONS

On the basis of the above results several conclusions can be drawn:

- Use of DRKF for joint processing of gyro and compass measurements eliminates increasing errors of gyro heading and smoothes random errors of the electronic compass. The accuracy of estimated heading is conspicuously better than that of the gyro or compass alone.
- The DR/GPS system is more accurate than any of its subsystems alone. DR positioning errors, increasing with a time of operation and a distance travelled, have been eliminated and the GPS receiver uncorrelated positioning errors have been efficiently reduced.
- An improvement of DR/GPS positioning accuracy in comparison to the GPS alone depends on a character of GPS errors. Best results are achieved when the correlated GPS errors are not large in comparison to the uncorrelated ones.
- Further improvement of accuracy of the system could be achieved with use of differential GPS corrections. This would significantly reduce most of correlated GPS errors and enable GPSKF to more efficiently estimate DR errors.

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