Andrzej Felski, Franciszek Grabski, Cezary Specht
Polish Naval Academy
Gdynia

AN ANALYTICAL METHOD
OF THE DGPS RADIOBEACON SIGNAL AVAILABILITY
PREDICTION

ABSTRACT

The most popular method of determining availability of DGPS as a complex radionawigation system is the ratio of Mean Time Between Failures to total (i.e. sum MTBF and Mean Time To Repair). This simple method can be used only as a post processed method. The analytical method of availability prediction of the DGPS radio signals transmitted by the Marine Radiobeacons is discussed in the paper. Authors propose to determine it on the basis of such information like: type of RTCM message, signal to noise ratio, rate of corrections etc. To model the availability zones of each DGPS Reference Station these calculations can be used. The short version of this article was presented on the GNSS'99 Conference in Genoa. Now, the full version is presented. Some results of the implementation in practice for Polish DGPS Reference Station Rozewie are added.

ESSENCE OF AVAILABILITY PROBLEM

Availability of a radionavigation system is defined as the probability with which the service provided by the system is available for a user located within its coverage area (service access probability). It is one of the most important criteria for comparison of radionavigation systems because it specifies the real status of the service provided. In principle, availability can only be referred to its coverage area. It is necessary to emphasize that such defined concept of availability applies to a complete radionavigation service provided by the system. Therefore it means the guarantee to keep the, specified by provider, parameters of system performance. The definition of availability can not be limited only to guarantee the transfer of radio signals but it also should take into consideration such features as required or assumed fix accuracy, reliability of functioning of technical equipment and signal transmission, integrity, fix rate and many more. It should also include the technical characteristics of radio signal i.e. signal level or signal to noise ratio.

Each radionavigation system is usually considered as a complex serial structure, where a failure of even one element causes major fault of the whole system. System fault can be determined as a state caused by any factor, which makes the system functioning unproperly. It means that the system fault may be considered a state caused by failure of any internal element as well as a failure of radio link (for instance excessive noise level). These causes lead to appearance of specified time period called Time to Repair (TTR). During that period, the user assumes that if system does not function properly, it is necessary to wait until the operator removes all causes of system fault. Technical repair of any mechanism, changing of weather conditions or decreasing of atmospheric interference can be regarded as the malfunction correction. So, for an average user, Time to Repair means the time from the moment of appearance of any factor, which makes the system failed until the moment of its proper functioning.

Mean value of TTR may be used for estimation of availability. Moreover, availability can be calculated as the ratio of total time during which the system was available to the total period of time under consideration. Thus, we are able to figure it out by dividing the value of Mean Time Between Failure (MTBF) by the sum of Mean Time Between Failure (MTBF) and Mean Time To Repair (MTTR) called mission time.

$$A = \frac{MTBF}{MTBF + MTTR} \tag{1}$$

The notion of availability used in coverage area can be identified with the same term derived from universal theory of dependability, which means the probability of being in particular moment $t>0$ in state of possibility for fix calculating using packet of received radio signals:

$$K(t) = \frac{\upsilon}{\lambda + \upsilon} + \frac{\lambda}{\lambda + \upsilon} exp[-(\lambda + \upsilon)t] \tag{2}$$

Where: λ - failure rate

   υ - repair rate

From the viewpoint of universal theory of dependability, formula (1) is equivalent to availability coefficient (G), which is used, in coverage area, as an alternative measure of system availability:

$$G = \lim_{t \to \infty} K(t) \frac{\upsilon}{\lambda + \upsilon} \tag{3}$$

Therefore, we can talk about availability with reference to any created functional form being both simple element of technical structure and very complex system including components, whose technical character may be debatable (i.e. duct of radio wave propagation in atmosphere).

The conditions necessary to estimate the availability are the following:
a) Defining the structure of elements or system,
b) Defining up state and down state,
c) Assigning their numerical representation.

When relating the above considerations to DGPS we can conclude that, due to the considerable number of determining factors, estimation of availability is possible only in an empirical way, using long term research and observation.

Functional structure of DGPS has both serial and parallel character. It includes two parallel channels: GPS and LDAS (**L**ocal **D**ifferential **A**ugmented GPS **S**ervice). At the same time, both of these modules have to function properly in order that DGPS might be regarded as a correctly working unit.

Considering relationship between DGPS availability, LDAS availability and space segment of GPS it can be shown as:

$$A_{DGPS} = A_{GPS} \cap A_{LDAS} \tag{4}$$

where: A_{DGPS} – Availability of DGPS
A_{GPS} – Availability of space segment of GPS
A_{LDAS} – Availability of LDAS

Reliable shown (Fig.1) structure of LDAS causes its availability to simultaneously consist of availability of four elements:
a) Reference Station,
b) Monitoring Station,
c) Control Station,
d) User's segment.

Only their simultaneous availability may ensure the complete availability of the differential system:

$$A_{LDAS} = A_{RS} \cap A_{IM} \cap A_{CS} \cap A_{US} \tag{5}$$

where: A_{RS} – Availability of a reference station,
A_{IM} – Availability of a monitoring station,
A_{CS} – Availability of a control station,
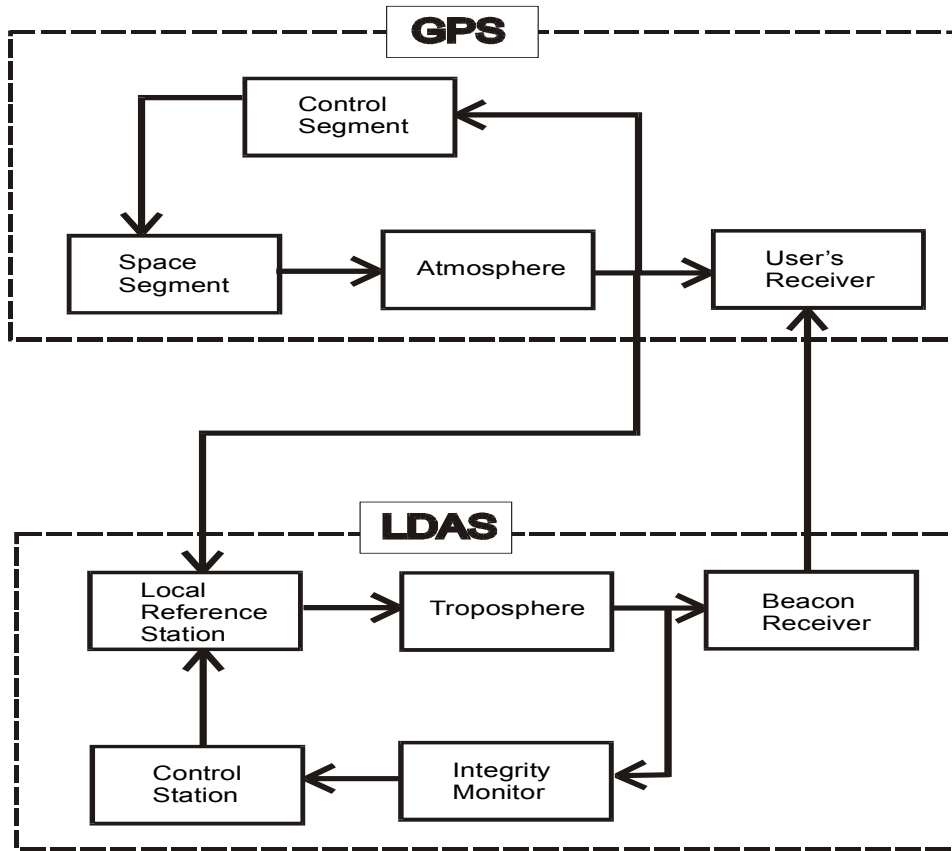A_{US} – Availability of user segment.

Fig. 1. Simplified scheme of DGPS

Within some confines of availability of a Reference Station A_{RS} we can consider availability of a technical structure concerning the reference station as well as transmitting system. Besides it we should not ignore the problem of radio signal availability connected with local radio characteristic of a given area, the type of used emission and the technology of transmission of differential corrections referred to related transfer format (i. e. RTCM SC-104).

$$A_{RS} = A_{DTA} \cap A_{DTN} \cap A_{DS} \cap A_{DT} \tag{6}$$

where: A_{DTA} – Availability of a technical structure,
  A_{IM} – Availability of a transmitting system,
  A_{CS} – Availability of a radio signal,
  A_{US} – Availability of the transmission channel.

In dependencies (4)–(6) most of variables constitute parameters of such technical devices as receiver, satellite or monitoring station.

Only one element of that very complex structure of LDAS is considered in this paper. It is *availability of transmission*, which is defined as user's availability to receive GPS differential corrections transmitted in MF/LF band according to RTCM SC-104 standard.

In the case of data transmission with MSK modulation broadcasted by DGPS Reference Station, its signal is interfered by various natural or man-made interferences. If the negative influence is strong enough, deformation of received signal may be so weighty that decision concerning the selection of impulse level (0 or 1) is worked out incorrectly. Statistical measurement of such occurrence is defined as a Bit Error Ratio (BER), i.e. mean number of errors inside their sequences of precise length. This quantity is directly correlated with the ratio of signal power (*S*) to interferences power (*N*) inside the radio link. In specific condition of Gauss white noise interferences, the above quantity can be estimated as follows [Rusin, 1990]:

$$BER = \frac{1}{2} erf \sqrt{S/N} \stackrel{def}{=} \frac{1}{\sqrt{\pi}} \int_{\sqrt{S/N}}^{\infty} e^{-x^2} dx \tag{7}$$

On the basis of the above formula and its relation with S/N (Signal to Noise Ratio) we can work out the analytical relationship [Specht, 1997] between BER and the availability of transmission for PRC transmission methods, both recommended by RTCM standard.

THE AVAILABILITY OF SIGNAL FROM DGPS REFERENCE STATION

Discussion on RTCM massage, the type 1

DGPS LF/LM capability for transmission we can define as a state in which minimum one RTCM massage, type 1 is correctly transmitted to user's receiver during time period equal to *T* seconds. Frequently the quantity of *T* is determined as 30 seconds.

By analogy the fault of DGPS LF/LM transmission can be interpreted as a state in which no RTCM massage is correctly transmitted to user's receiver during time period equal to *T* seconds. Correct transfer of RTCM message means that none of its *n* bits is damaged during transmission.

The length of RTCM message, type 1 (L_1), as a function of tracked satellites number can be presented as follows:

$$\text{For } N_S = \{3, 6, 9, 12\} \ L_1 = 60+50N_S \tag{8}$$

$$\text{For } N_S = \{4, 7, 10\} \ L_1 = 70+50N_S \tag{9}$$

$$\text{For } N_S = \{2, 5, 8, 11\} \ L_1 = 80+50N_S \tag{10}$$

where: N_S – number of satellites for which a Reference Station transmits pseudo-range corrections.

Number (n) of transmitted RTCM messages during time period equal to T_{max} is function of binary flow of telemetric link dominated by baud rate (R) and length of massage (L_1).

$$n = \frac{T_{max} \cdot R}{L_1} \qquad (11)$$

where: R – baud rate,
$\quad\quad\; T_{max}$ – age of correction limit.

Probability of correct reception of RTCM message, L_1 bits long as a function of BER value can be described by means of the following formula:

$$p_d = (1 - BER)^{L_1} \qquad (12)$$

Considering T_{max} interval (usually 30 seconds) characterizing DGPS transmission availability (p_s) we can declare that system is faulty in the case of all RTCM massages, when all transmitted during time period equal to T_{max}, were failed. Hence the probability of system availability loss (q_s), which means incorrect reception all messages during time period equal to T_{max}:

$$q_s = (1 - p_d)^n \qquad (13)$$

then:

$$p_s = 1 - q_s \qquad (14)$$

Discussion on RTCM massage, the type 9-3.

For this type of message, the DGPS LF/LM capability for transmission can be defined as a state in which minimum two different RTCM messages, type 9-3 were correctly transmitted to user's receiver during time period equal to T seconds. They should refer to two different sets of three satellites. This type of transmission has a constant message length (L_{9-3}) equal to 210 bits. Hence the probability of correct reception RTCM messages is identical for each of sets of three satellites:

$$p_d = (1 - BER)^{L_{9-3}} \qquad (15)$$

And the opposite is:

$$q_d = 1 - p_d \qquad (16)$$

Depending on a number of satellites tracked by Reference Station there are two cases:
a)   Corrections are transmitted for two sets of three satellites – $3 < N_s < 6$
b)   Corrections are transmitted for three sets of three satellites – $6 < N_s < 9$

Variant a)

Let us consider two sets of three pseudo-range corrections (PRC) related adequately to satellites: 1, 2, 3 and 4, 5, 6. Mean number (n) of RTCM massages, type 9-3 (for each of two sets of three satellites) during the time period T_{max} is defined as follows:

$$n = \frac{T_{max} \cdot R}{2 \cdot L_{9-3}} \tag{17}$$

For the conditions determined in this way the state of fault (lack of availability) will be a failure to receive correctly, during period Tmax, at least one of two sets of three PRC related to different satellites. This situation will take place if, during assigned time period, all corrections related to both or only to one of two sets of three satellites fail.

Probability of such event - qs we can describe as follows:

$$q_s = q_d{}^n \left[\sum_{k=0}^{n} \binom{n}{k} p_d{}^k q_d{}^{n-k} \right] + q_d{}^n \left[\sum_{k=1}^{n} \binom{n}{k} p_d{}^k q_d{}^{n-k} \right] \tag{18}$$

Using the next identity:

$$\left[\sum_{k=0}^{n} \binom{n}{k} p_d{}^k q_d{}^{n-k} \right] = 1 \tag{19}$$

Finally we can write:

$$q_s = q_d{}^n + q_d{}^n \left(1 - q_d{}^n \right) = q_d{}^n + q_d{}^n - 2q_d{}^{2n} = 2q_d{}^n - q_d{}^{2n} \tag{20}$$

Variant b)

Let us consider three sets of three pseudo-range corrections related adequately to satellites: 1–3, 4–6 and 7-9. Mean number (*n*) of RTCM messages, type 9-3 (for each of three sets of three satellites) during time period equal to T_{max} is defined as follows:

$$n = \frac{T_{max} \cdot R}{3 \cdot L_{9-3}} \tag{21}$$

For the conditions determined in this way the state of fault (lack of availability) will be a failure to receive correctly, during time period equal to Tmax, at least two sets of three PRC related to different satellites. This situation takes place if, during assigned time period, all PRC related to minimum two sets of three satellites are failed.

Probability of such event - qs we can describe as follows:

$$q_s = q_d{}^{2n} \left[\sum_{k=0}^{n} \binom{n}{k} p_d{}^k q_d{}^{n-k} \right] + q_d{}^{2n} \left[\sum_{k=1}^{n} \binom{n}{k} p_d{}^k q_d{}^{n-k} \right] + q_d{}^{2n} \left[\sum_{k=1}^{n} \binom{n}{k} p_d{}^k q_d{}^{n-k} \right] =$$

$$q_d{}^{2n} + q_d{}^{2n} \left(1 - q_d{}^n \right) + q_d{}^{2n} \left(1 - q_d{}^n \right) = q_n{}^{2n} \left(1 + 1 - q_d{}^n + 1 - q_d{}^n \right) = q_d{}^{2n} \left(3 - 2q_d{}^n \right) = \tag{22}$$

$$= 3q_d{}^{2n} - 2q_d{}^{3n}$$

PRACTICAL IMPLEMENTATION

DGPS LF/MF Systems are commonly used for sea-surveying applications, especially in maritime hydrography, dredging etc. The high accuracy and the high availability are preferable for this kind of measurement. The method presented can be used for availability calculations before a surveying campaign for a specified region of coverage and/or specified time.
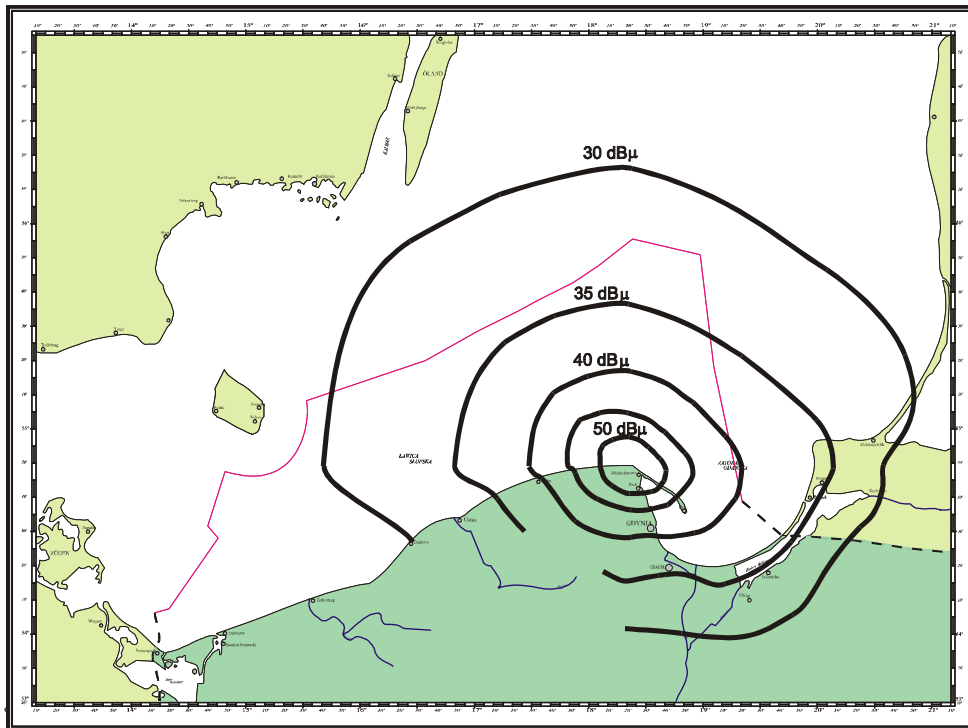

Fig. 2. Coverage zone of the RS Rozewie

Coverage of DGPS reference station needs to be determined during the test trials based on measured signal strength and signal to noise ratio for the worst time block. As an example let's consider Polish DGPS RS Rozewie, whose electric field strength is presented in fig. 2.
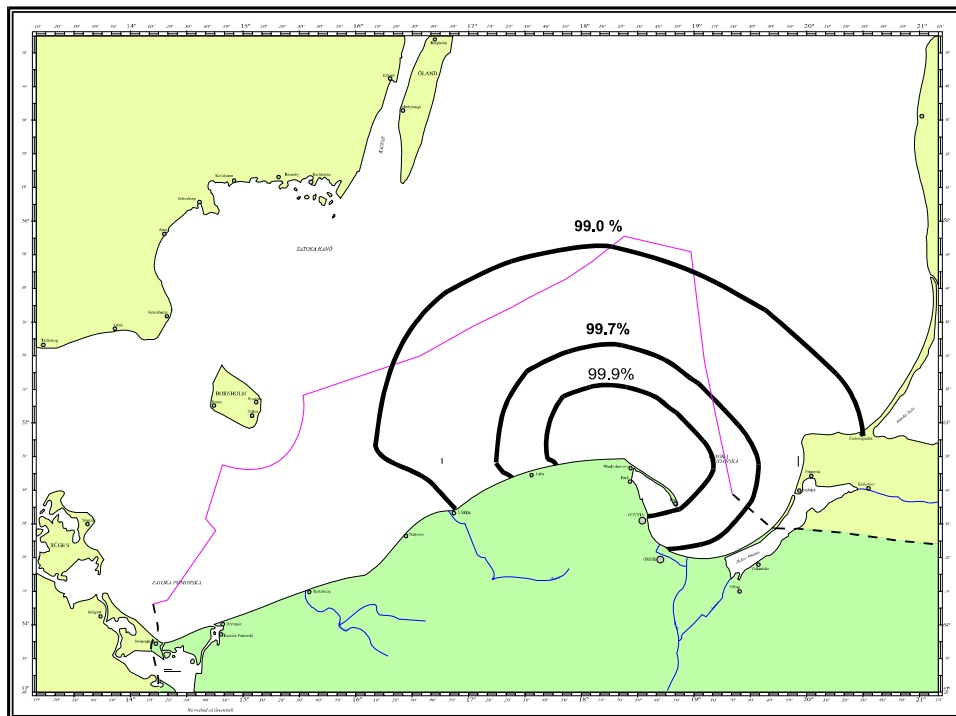
Fig. 3. Availability zone of the RS Rozewie

The SNR value was collected during a special campaign of the signal strength measurements made by authors. Next steep should be GPS space segment surveying design based on a standard software like: Trimble - Quickplan, Leica –Ski, Ashtech – Evaluate. The number of available satellites is necessary for transmission calculations during the survey. The availability zone calculated on the basis of the method proposed is presented in fig. 3. The following data: N_S =8, RTCM type 1, boud rate 100 are used for the calculation.

It is clear, that the same calculations can be done for different RTCM messages, transmission rate and number of visible satellites.

CONCLUSIONS

The availability of DGPS regarded as a probability of correctly received satellite signals and PRC emitted by a Reference Station is defined as a product of probabilities of proper functioning of system individual elements. The availability of system technical elements and probability of proper transmission from satellites to users are very high. Thus availability of DGPS system is mainly determined by PRC transmission link between the Reference Station and the user. The paper presents an analytical method of calculation of the probability of the proper PRC transmission as a function of message type, rate and condition of transmission.

On this basis we are able to predict the availability of the specific station in any area. It is also possible to carry out many analyses related to the complete system or to a particular element impact on service quality.

In fig. 4 we can see an example of such theoretical analysis. It presents the availability of PRC transmission as a function of BER, used message type (RTCM 9-3 and RTCM 1) and binary flow of telemetric link (baud rate) - 100 or 200, for 9 visible satellites. High values of BER, taking place near coverage border were assumed on purpose. It is clear from the figure that the RTCM message type 9-3 increases the system coverage because in spite of lower value of S/N (and corresponding with it BER value) enables achieving higher theoretic availability of DGPS transmission.
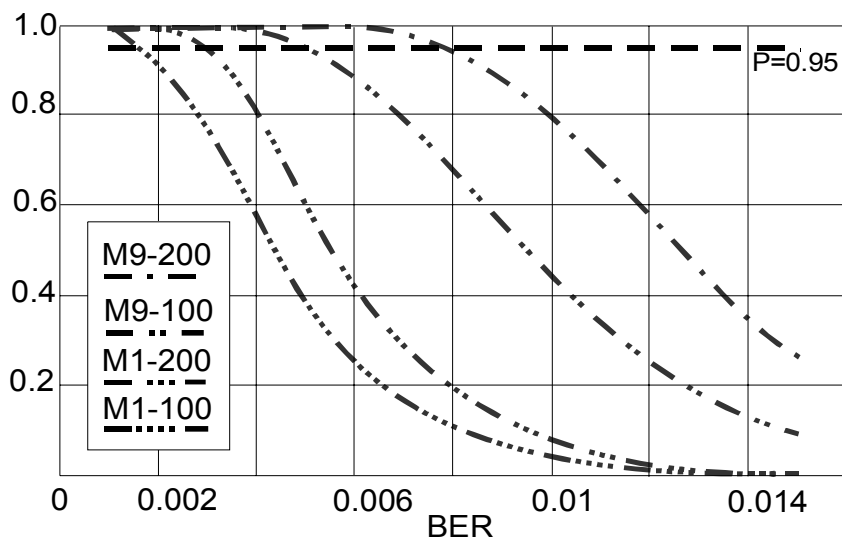

Fig. 4. Availability of DGPS transmission

BIBLIOGRAPHY

1. Ackroud N., Quality Control Considerations for Real Time Differential GPS, The International Conference of the Royal Institute of Navigation, London 1991.
2. CCIR, Report 322, World Distribution and Characteristics of Atmospheric Radio Noise, ITU, Geneve 1964.
3. Farrell J., Graas F., Statistical Validation for GPS Integrity Test. Navigation vol.39 no.2, Summer 1992.
4. Felski A., Specht C., Trials of DGPS in the Area of Northern Poland. EURNAV 96, Vehicle Navigation Location and Control, London 1996.

5. Huuhka E., Lehtoranta V., Study of Night-time Coverage of DGPS Radiobeacons in Finland, document prepared for Finnish Maritime Administration, Jokela, December 1995.
6. Pole position – Sercel Positioning News No 18, April 1995.
7. Poppe D., Last J. D., DGPS Radio-Beacon Coverage Prediction in the European Environment. Third International Conference on Differential Satellite Navigation Systems, London, 1994.
8. Rusin M., Transmission Systems – Television. Wydawnictwo Komunikacji i Łączności, Warsaw 1990 (in Polish).
9. Specht C., Multicriterial analyses of DGPS in context of radionavigation service on the South Baltic. Doctoral Thesis. Polish Naval Academy, Gdynia 1997 (in Polish).