Regular paper

A new method of frequency offset correction using coherent averaging

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Abstract— This paper describes a new method of frequency offset correction to improve clock stability in communication systems where the temporary drift or jitter of phase angle is not accepted. This method bases on coherent spectral averaging with a special phase scanning algorithm. Achieved results show that proposed method is effective for strongly degraded signals. Method is useful for precise phase angle reconstruction in these systems where clock stability of the transceiver and receiver is insufficient.

Keywords— coherent averaging, jitter, drift, frequency offset correction.

1. Introduction

The phase noise or fluctuation is defined as a phase shift of the output signal in relation to declared value of the input signal [1–5]. A value of the shift gives us information, how frequency of the received signal differs from the declared value. Proposed method bases on coherent averaging of signal spectrum. To fulfil the coherence conditions during averaging process, a new phase angle scanning method was elaborated and examined.

2. Coherent averaging

Coherent averaging method is used to increase the signal-to-noise ratio (SNR). It is based on multiple calculation of the signal complex spectrum determined in the time domain, and then computation of average spectrum.

Let's assume that the signal is degraded by additive noise:

$$c(n) = y(n) + noise(n), \tag{1}$$

where: $c(n)$ – set of samples degraded by noise; $y(n)$ – periodically repeated signal, with fixed amplitude distribution; $noise(n)$ – n-samples set of noise realization.

The averaged signal can be expressed as:

$$c_{mean}(k) = \frac{1}{M}\sum_{n=1}^{M} c\big(k + L(n-1)\big). \tag{2}$$

Here, it is assumed, that M separated, repeatable sets of samples are averaged, and the signal $y(n)$ possesses repetition period L.

Output deviation δ_{mean} for set $c_{mean}(k)$ depends on signal deviation δ_{org} before averaging and is expressed as:

$$\delta_{mean} = \frac{\delta_{org}}{\sqrt{M}}. \tag{3}$$

Basing on expression (3), it is possible to decrease a measurement uncertainty by decreasing the signal deviation during averaging.

Let us consider a signal with a given amplitude y and deviation δ_{org}. The SNR is here defined as:

$$SNR_{org} = \frac{y}{\delta_{org}}. \tag{4}$$

For the averaged signal, we have:

$$SNR_{mean} = \frac{y}{\delta_{mean}}. \tag{5}$$

Thus, calculated SNR gain coefficient equals:

$$SNR_{coh} = \frac{\delta_{org}}{\delta_{org}/\sqrt{M}} = \sqrt{M}. \tag{6}$$

The SNR gain coefficient can be expressed as logarithm value for fixed number of averaging M:

$$SNR_{coh}[\mathrm{dB}] = 20\lg\left(SNR_{coh}\right) = 20\lg\left(\sqrt{M}\right) = 10\lg(M). \tag{7}$$

In frequency domain, coherent averaging is realised as separate averaging of real and imaginary parts of particular components of power spectrum. We decided to use fast Fourier transform (FFT) as an effective method of power spectrum density (PSD) computation.

The composition of complex values of FFT bins formed in this way gives as result the coherent averaged spectrum. For given FFT bin, it can be expressed as:

$$F_{coh}(N\text{–}1) = \frac{F_1(M\text{–}1)_{\mathrm{Re}} + F_2(M\text{–}1)_{\mathrm{Re}} + \ldots + F_k(M\text{–}1)_{\mathrm{Re}}}{k} + \mathrm{j}\frac{F_1(M-1)_{\mathrm{Im}} + F_2(M-1)_{\mathrm{Im}} + \ldots + F_k(M-1)_{\mathrm{Im}}}{k}. \tag{8}$$

When frequency offset takes place, it introduces a phase shift between successive spectrum repetitions and the coherence condition is not fulfilled, so we cannot significantly increase the *SNR*:

$$SNR_{coh} = 20\lg\left(\frac{A_{bin}}{noise_{bin}}\right), \quad (9)$$

where: A_{bin} – signal amplitude in given FFT bin; $noise_{bin}$ – noise envelope in given FFT bin.

3. Phase angle scanning method

To fulfil coherence condition during averaging process, a new phase angle scanning method is proposed. This method assumes that accuracy of phase restoration should be lower than:

$$\eta_{scan} = 1 \cdot 10^{-5}\,\mathrm{rad}/dl_{seg}\,, \quad (10)$$

where: dl_{seg} – length of averaged segment (frame).
Assuming that: length of averaged frames equals 512 samples, fixed sampling frequency is 44100 Hz and fixed signal duration is 1 second, the method resolution η_{scan} is:

$$\eta_{scan} = 0.00086\,\mathrm{rad/s} = 0.05\,\mathrm{deg/s}\,. \quad (11)$$

Algorithm for phase scanning method is shown in Fig. 1.

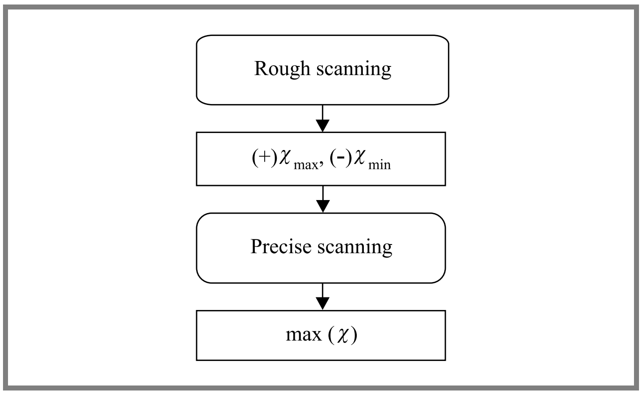

Fig. 1. Algorithm for phase scanning method.

Proposed method bases on computation of maximum value of the virtual spectral line F_v that is formed as a sum of absolute values of N spectral lines (pilots) with fixed amplitudes in given FFT bins:

$$F_v = \sum_{i=1}^{N}\left(\mathrm{abs}(F_{i\chi})\right), \quad (12)$$

where: F_v – virtual spectral line; $F_{i\chi}$– complex value of *i*th pilot spectral line.
We notice that:

$$F_{i\chi} = \sum_{k=1}^{M}\mathrm{Re}(F_{ik}) + \sum_{k=1}^{M}\mathrm{Im}(F_{ik})\,. \quad (13)$$

The $F_{i\chi}$ is computed by averaging (in M-iterations) values of the spectral lines possessing the same index i. Here the real and imaginary parts of the complex values are separately averaged for each phase angle correction coefficient χ and for all M iterations.
The iterative procedure for rough and precise blocks in scanning algorithm is used (Fig. 2). Value of the virtual spectral line is computed for the selected value of correction

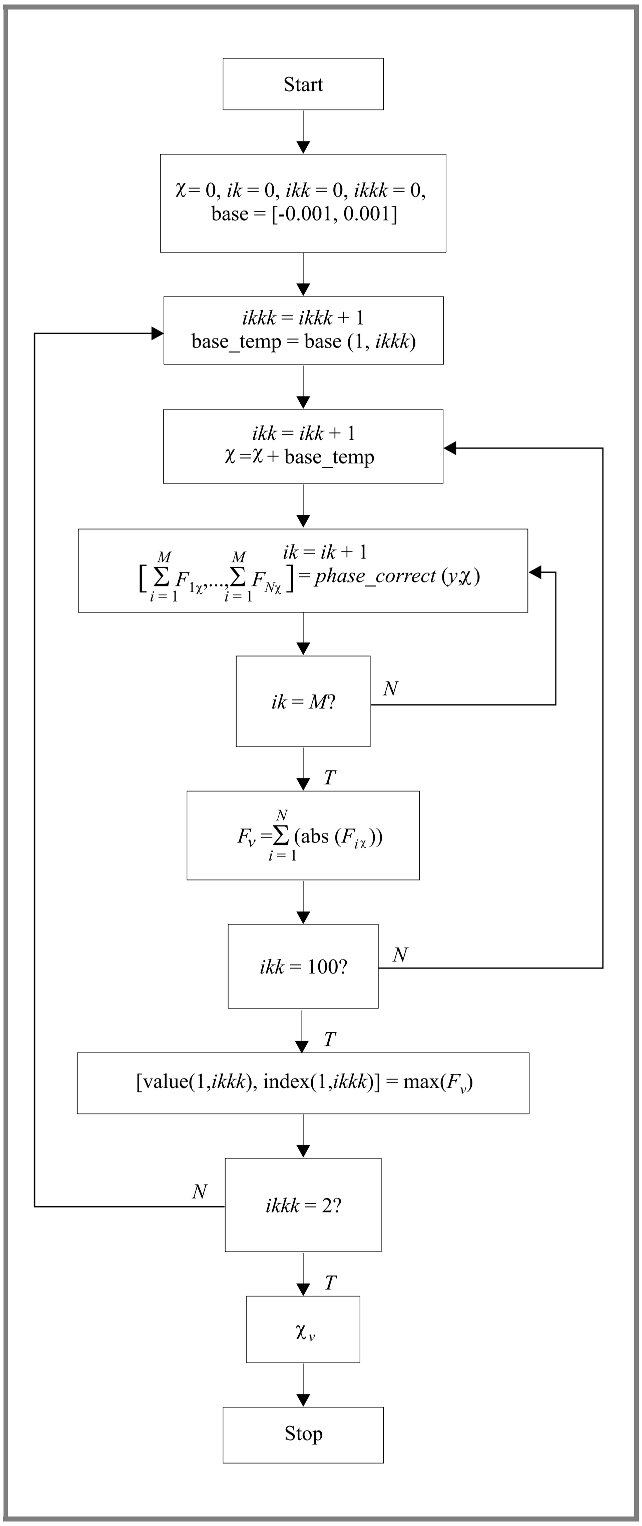

Fig. 2. Iterative algorithm used in rough and precise scanning stages.

coefficient χ [rad]. This value is constant for given iteration but it is changing between successive iterations according to the scanning range $\chi \in \langle \chi_1, \ldots, \chi_D \rangle$.

The virtual spectral line value F_v is buffered for each correction coefficient and the maximum value is found after all iterations. This maximum value represents the searched value of the phase correction. Index D is dependent on method resolution. For our laboratory experiment, it was given as: $D = D_{\mathrm{I}} + D_{\mathrm{II}} = 400$, and a scanning range was fixed as:

$$\chi \in \langle -1 \cdot 10^{-5}\,\mathrm{rad},\ 1 \cdot 10^{-5}\,\mathrm{rad} \rangle . \tag{14}$$

The two stage algorithm for searching of rough and precise values χ is proposed to decrease the computation time for declared scanning range with given resolution.

The following procedure *phase_correct* is used for phase angle correction for selected spectral line value:

$$
\begin{aligned}
&[F_1, \ldots, F_N] = phase\_correct(y, \chi)\\
&\mathrm{all}\_fft = fft\big(y(:, ik)\big)\\
&F_{1\mathrm{temp}} = \mathrm{abs}(\mathrm{all}\_fft(1, ik)).^{*}\exp(\mathrm{j}^{*}(\mathrm{angle} - \chi))\\
&F_{N\mathrm{temp}} = \mathrm{abs}(\mathrm{all}\_fft(n, ik)).^{*}\exp(\mathrm{j}^{*}(\mathrm{angle} - \chi))\\
&F_1 = F_1 + F_{1\mathrm{temp}}\\
&F_N = F_N + F_{N\mathrm{temp}}
\end{aligned}
\tag{15}
$$

As a result, the pair of values $[(-)\chi_{\max}, (+)\chi_{\max}]$, for positive and negative phase correction coefficients is computed in a rough stage. The same procedure is carried out in precise stage where range of scanning is specified between $(-\chi_{\max})$ and $(+\chi_{\max})$.

After two stages the maximum value χ_v is obtained.

4. Experimental results

The method described above has been implemented to correct the frequency drift (jitter), which was result of clocks frequencies offset in D/A and A/D converters used in transmitter and receiver of telecommunication system.

The system transmits the same periodically repeated sequences of signal with uncorrelated noise. At the receiver, noise component is reduced by using the coherent averaging process.

Pilot signal consists of six harmonics in dedicated frequency bins, was interleaved with information spectral bins, modulated with transmitted data.

The results of the signal processing are shown in Figs. 3–9. The module of the automatic drift correction computed coefficient equals: $\chi = (-0.00040)$ rad/frame and it was used during coherent spectrum averaging of the signal. The *SNR* gain for 1148 frames was: $SNR_{coh} = 30$ dB.

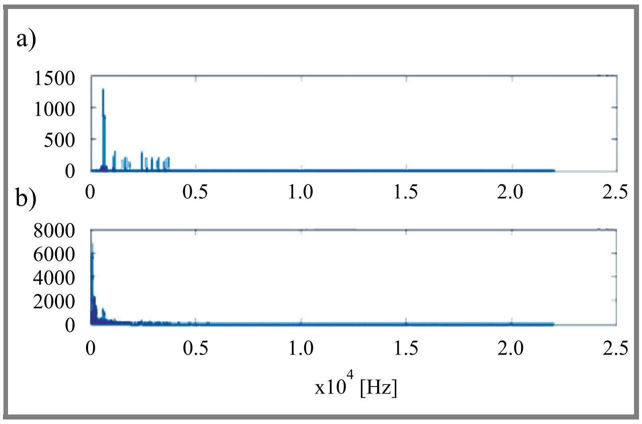

Fig. 3. Frequency domain structures of the signal: (a) original signal; (b) the same signal degraded by another signal.

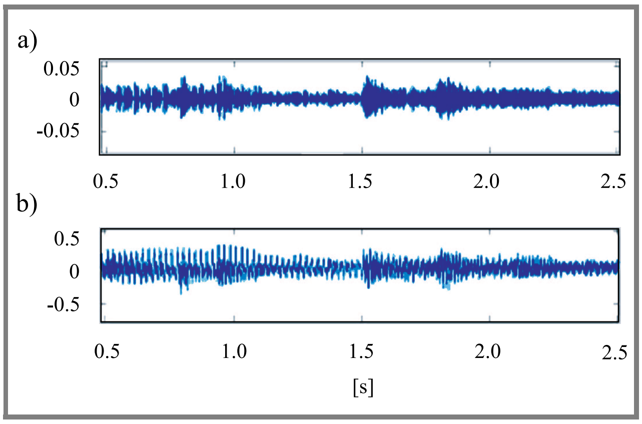

Fig. 4. Time domain structure: (a) of the original signal; (b) the same signal degraded by another signal.

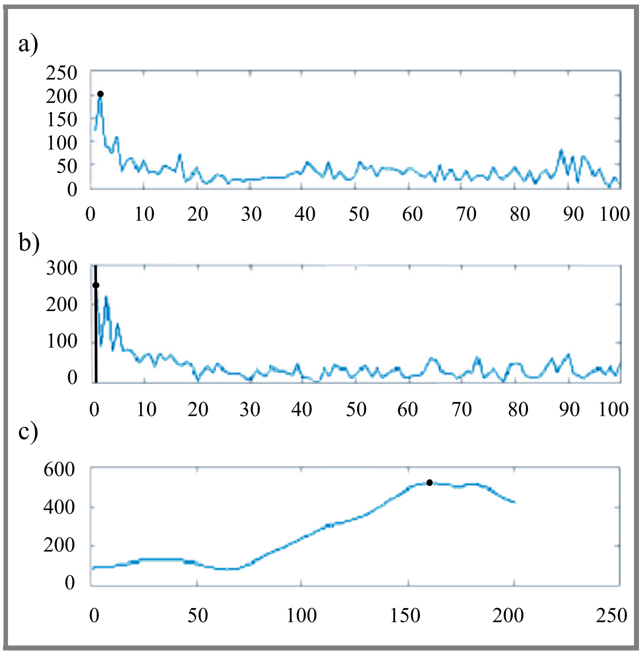

Fig. 5. Two stages of scanning method: (a),(b) rough scanning for positive and negative offsets; (c) precise scanning. Scanning results: $\chi = -0.00040$ rad/frame.

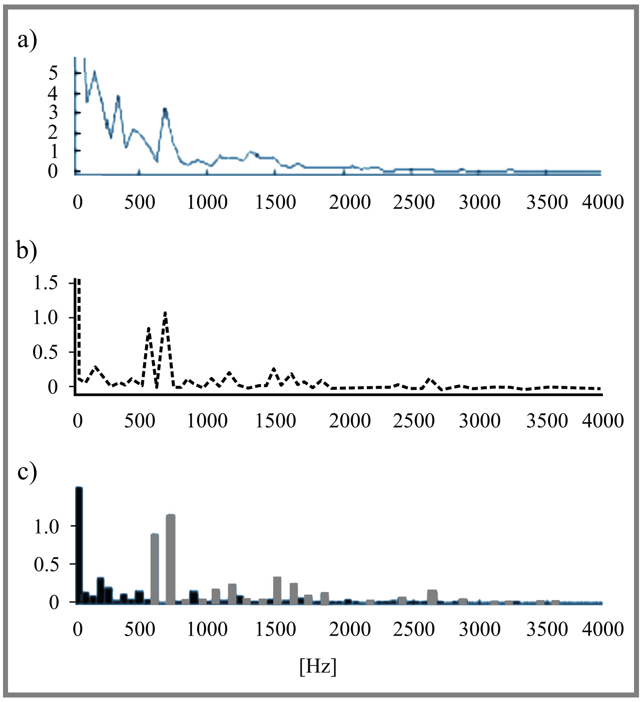

Fig. 6. Degraded spectrum signal: (a) before and (b),(c) after coherent averaging with phase correction coefficient $\chi = -0.00040$ rad/frame.

An example of the coherent averaging procedure for system with drift value: $\chi = 0.00003$ rad/frame is shown in Fig. 7.

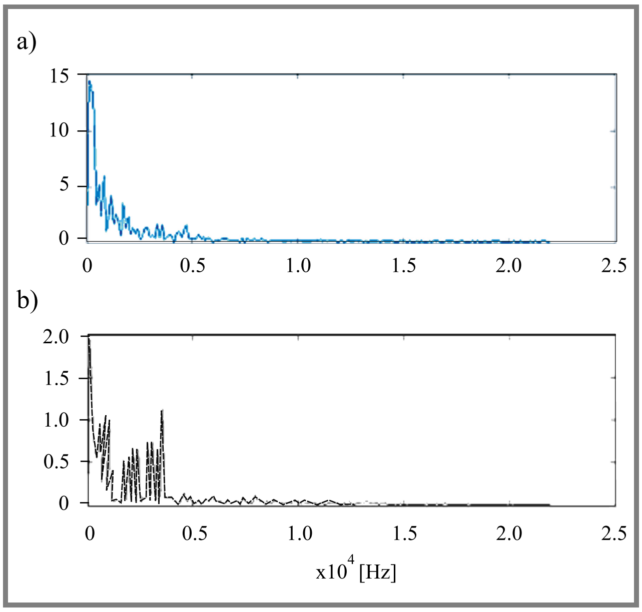

Fig. 7. Spectrum of the degraded signal before (a) and after (b) coherent averaging.

First four pilots in (Re, Im) planes before and after correction with value $\chi = -0.00040$ rad/frame are shown in Figs. 8 and 9. The signal duration was 10 seconds. Correction coefficient was found using scanning phase method.

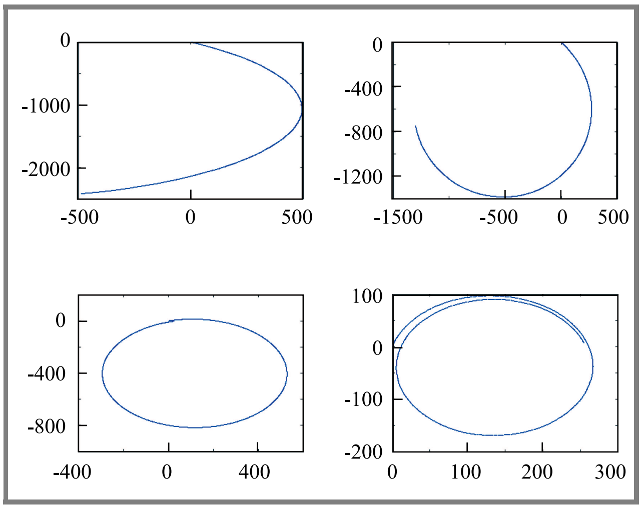

Fig. 8. First four pilots in (Re, Im) planes before phase correction.

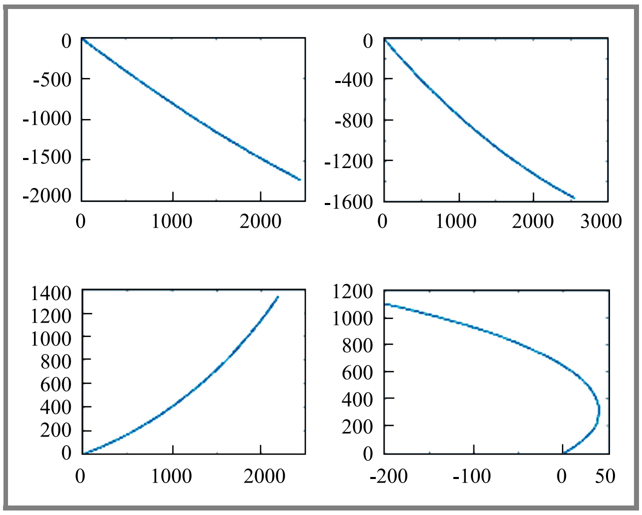

Fig. 9. First four pilots in (Re, Im) planes after phase correction with value $\chi = -0.00040$ rad/frame.

5. Conclusions

The proposed method of coherent averaging is very effective. As experimental results shown that acceptable results were obtained even if the original signal was degraded by another signal stronger by 30–40 dB.

It is possible to obtain further extension of this method to improve the calculation accuracy of drift χ. This method can be also optimised to minimise its time consumption, when is used as a component of standard digital signal processing (DSP) library.

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