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Hydraulic Conditions of Steady Motion in an Estuarial River Section under Wind Influence

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Abstract

The effect of wind influence upon swellings in river estuaries has been presented in this elaboration. The wind, which causes shear stresses at the water surface, changes the vertical profile of water flow velocity and therefore is a factor causing wind swellings. This problem is very important in the estuarial river sections, especially in river inflows to reservoirs (sea), when natural slopes of flowing water are small and components of gravity force are not able to oppose the wind effect. Then wind backwater occurs.

Notations

a – parameter in equation of river water turbulent viscosity coefficient,
B – river width,
C – velocity constant in Chezy's formula,
D_b, D_w – functions of parameter a,
e – base of natural logarithms ($e = 2.71\ldots$),
g – acceleration of gravity,
h_{meas} – backwater curve measured in field works
H – depth of river water,
I – river slope,
I_d – bottom slope,
K – coefficient of turbulent viscosity,
m, m_1, m_2 – constants in the formula describing the coefficient of turbulent viscosity,
n – roughness coefficient in Manning's formula,
p – pressure,
Rz – ordinate of water level,

W	–	wind velocity,
x, y	–	axes of co-ordinates' system,
V	–	water flow velocity,
$zh0$	–	depth in river under uniform flow,
zw	–	curve of backwater surface,
zd	–	bottom line,
$zw.w(-)$	–	backwater curve in the case when wind is blowing along the flow,
$zw.w(+)$	–	backwater curve in the case of wind blowing opposite to the river flow,
$zw.(0)$	–	backwater curve in the at no wind,
α	–	Saint-Venant's coefficient,
θ	–	turbulent viscosity coefficient ratio,
κ_w	–	constant in the formula defining wind stresses,
$\kappa_0, \kappa, \kappa_3$	–	coefficients in formulae describing turbulent viscosity coefficients,
τ	–	shear stresses,
τ_w	–	wind shear stresses,
τ_b	–	bottom shear stresses,
ρ	–	water density.

1. Introduction

River estuaries are places, where there are very complicated flow conditions due to mutual penetration of river levels caused by catchment run off and the lower boundary condition, which can bring about water swellings. Penetration of these both factors' influence is the reason why there are small river water slopes. Small slope in this area is the reason why the wind disturb considerably the flow. The wind, which is blowing above the water surface, causes the shear stresses – wind friction. These stresses causes changes in the vertical distribution of river water flow velocities – changes of tachoida. Change of the vertical velocity distribution together with the equation of flow continuity and energetic changes transforms the location of water level. If the natural river slopes are small, components of the gravity forces are not able to counteract the wind stresses and the wind blowing opposite to the water flow direction causes wind swellings. These swellings can form a backwater curve in windy conditions and wind backwater currents, which have an opposite direction to the main river flow. The analysis of the hydraulic flow condition in the river-bed, when the wind is blowing at the surface and wind shear stresses appear is the subject of the present elaboration.

2. Analysis of the Phenomenon

2.1. Introduction

The analysis of the phenomenon is based upon a hydrodynamic equation of water flow. The following simplifications have been made for the analysis:

- the case analysed is a two-dimensional flow (vertical-plane). We assume that the vertical axle y and the horizontal axle x make up the plane of a co-ordinates' system,
- mass forces have a potential,
- water density is constant (there is no density stratification),
- motion is steady,
- water motion is caused by free flow under the influence of forces of gravity and wind action.

At the water surface shear stresses (wind friction) appear, which have a positive sign, if directed opposite to direction of the river water flow. They cause shear stresses counteracting the water motion in the river. These stresses are determined by water motion and wind stresses. However, the atmospheric pressure under the water surface is assumed constant, and thus the hydrodynamic equations have the following form:

$$\frac{dV_x}{dt} = X - \frac{1}{\rho} \cdot \frac{\partial p}{\partial x} + \frac{\partial}{\partial x}(\tau_{xx}) + \frac{\partial}{\partial y}(\tau_{xy}), \tag{1}$$

$$\frac{dV_y}{dt} = Y - \frac{1}{\rho} \cdot \frac{\partial p}{\partial y} + \frac{\partial}{\partial x}(\tau_{yx}) + \frac{\partial}{\partial y}(\tau_{yy}) \tag{2}$$

the continuity equation having the following form:

$$\text{div } V = 0. \tag{3}$$

These relationships will be used in the following way. First will be defined the relationships, which determine the vertical distribution of the component of eddy viscosity tensor in order to specify friction of the water motion at the bottom and relevant vertical distribution of velocity (tachoida). Then applying equations (1) and (3), the equation of backwater curve in windy conditions will be derived in the classical manner by introducing additional wind friction at the water surface and eddy viscosity along the river, which can represent sudden changes in the riverbed cross-section.

2.2. Analysis of the Vertical Distribution of the Reynolds Stresses' Tensor Component

Equations (1) and (2) have been used for the analysis. First was differentiated $\partial/\partial y$ and then $\partial/\partial x$ then these equations were subtracted. I gave:

$$\frac{d}{dt}\left(\frac{\partial Vx}{\partial y}-\frac{\partial Vy}{\partial x}\right)=\frac{\partial^2\tau_{xy}}{\partial x\,\partial y}+\frac{\partial^2(\tau_{xy})}{\partial y^2}-\frac{\partial^2\tau_{yy}}{\partial x^2}-\frac{\partial^2\tau_{xx}}{\partial x\,\partial y}. \tag{4}$$

We analyse the case of uniform steady motion. Therefore all the terms except one are equal to zero. We have:

$$\frac{\partial^2\tau_{xy}}{\partial y^2}=0. \tag{5}$$

After taking the boundary conditions into consideration,

$$\tau_{xy}(0)=\tau_b \text{ and } \tau_{xy}(H)=-\tau_w \tag{6}$$

the solution if this equation has the following form:

$$\tau_{xy}(y)=\tau_b-(\tau_w+\tau_b)\,\frac{y}{H}. \tag{7}$$

Moreover, it can be noticed that the maximum velocity $V_{\max}$ corresponds to the ordinate y_0, which fulfils the condition:

$$\tau_{xy}(y_0)=0, \text{ hence } y_0=\frac{\tau_b}{\tau_w+\tau_b}\cdot H. \tag{8}$$

The equation (7) and Boussinesq's hypothesis specifying the components of Reynolds stresses' tensor will be the basis for defining the vertical change of velocity and the friction at the bottom in the further part of the elaboration.

The elements of water motion have been presented schematically in Fig. 1.

2.3. Energy Changes of the Water Stream along the River

Energy changes of the water stream along the river are determined on the basis of the equation (1) with the help of so-called vertical integration. We introduce the following definition of average values:

$$\frac{1}{H}\int_0^H V dy=\overline{V} \text{ and } \frac{1}{H}\int_0^H V(y)dy=V_0. \tag{9}$$

It can, moreover, be proved that in the flow conditions with a free surface, when the atmospheric pressure does not change, the terms appearing on the right side of Eqs. 1 and 2 can be described as:

$$X-\frac{1}{\rho}\frac{\partial\rho}{\partial x}=-\frac{\partial Rz}{\partial x} \text{ (derivative from the water level ordinate),} \tag{10}$$

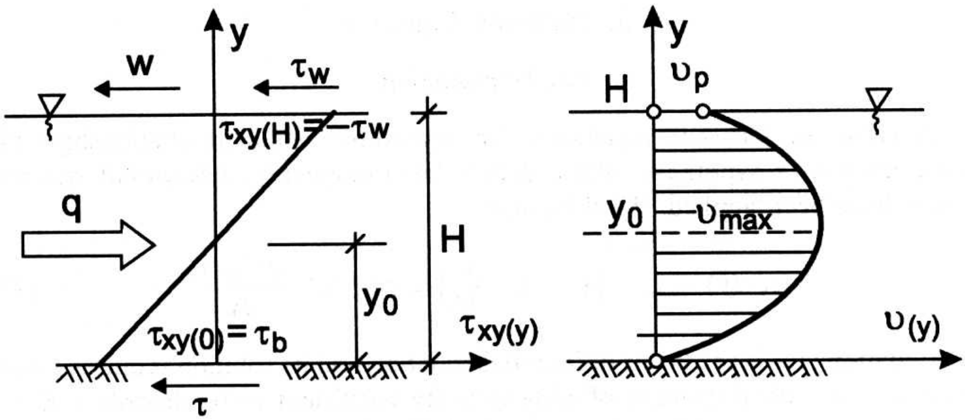

Fig. 1. Vertical distribution of shear stress and velocity

$$\frac{1}{H} \cdot \int_0^H \frac{dVx}{dt} \cdot dy = \frac{\partial}{\partial x}\left(\frac{\alpha \cdot V_0^2}{2}\right) \quad \text{where } \alpha \text{ is Saint–Venant's coefficient.} \tag{11}$$

The component of Reynolds stresses' tensor τ_{xx} can be assumed according to Meyer, Pacewicz (1984)

$$\tau_{xx} = \kappa_3 \cdot \rho \cdot HV_0 \frac{\partial V_0}{\partial x} \quad \text{hence} \tag{12}$$

$$\frac{\partial \tau_{xx}}{\partial x} = \kappa_3 \cdot \rho \cdot H \frac{\partial^2}{\partial x^2} \cdot \left(\frac{V_0^2}{2}\right). \tag{13}$$

This term represents practically additional energy losses caused by the rapid change of riverbed cross-section. Introduction of this term enables the obtaining a smooth curve of the river even in cases of very sudden narrowing.

The final energy changes of the water stream when wind stresses acting on the surface and rapid changes of riverbed cross-section can be described by the below-given equation:

$$\frac{\partial}{\partial x}\left(\frac{\alpha V_0^2}{2}\right) = -g\frac{\partial Rz}{\partial x} + \kappa_3 H \frac{\partial^2}{\partial x^2}\left(\frac{V_0^2}{2}\right) - \frac{\tau_w + \tau_b\,(\tau_w, V_0)}{\rho H}. \tag{14}$$

Equations (14) and (7), which represent the basic set of hydrodynamic equations (1), (2) will be used for further calculations.

3. Tachoida Equation

3.1. Introduction

To describe the tachoida equations, the pewviously obtained relationships (7) and Boussinesq's hypothesis, which define the components of Reynolds stresses' tensor, have been applied. We then have:

$$\tau_{xy}(y) = \tau_b - \left(\tau_b + \tau_w \cdot \frac{y}{H}\right) = \rho Ky(y) \cdot \frac{dV(y)}{dy}. \tag{15}$$

Equation (15) is the linear differential equation and its solution requires knowledge of the vertical changes of eddy viscosity coefficient to be described $K_y(y)$ and the boundary conditions to be specified. The boundary condition is assumed as follows:

$$V(0) = 0. \tag{16}$$

The solution of equation (15) can then be presented as:

$$V(y) = \frac{\tau_b}{\rho} \cdot \int_0^y \frac{dy}{Ky(y)} - \frac{\tau_w + \tau_b}{\rho H} \cdot \int_0^y \frac{y \cdot dy}{Ky(y)}. \tag{17}$$

3.2. Classical Tachoidas

The classical forms of tachoidas have been analysed further in the elaboration relating them to the previously obtained relationships.

1) Bazin's tachoida (Puzyrewski, Sawicki 1998) can be defined as:

$$Ky(y) = \kappa_0 \cdot H \cdot V_0 \quad \text{and} \quad \tau_w = 0 \tag{18}$$

after calculating, we obtain:

$$\frac{V(y)}{V_0} = 3 \cdot \frac{y}{H}\left(1 - \frac{1}{2} \cdot \frac{y}{H}\right) \tag{19}$$

and furthermore

$$\tau_b = 3\kappa_0 \rho \cdot V_0^2.$$

After comparing to Chezy's formula, we obtain:

$$\kappa_0 = \frac{g}{3c^2}. \tag{20}$$

Then the average flow velocity, which is calculated from Bazin's tachoida, is the same as the average value calculated from Chezy's formula.

2) Prandtl's tachoida (Prandtl 1956) can be obtained from the general formulae under the assumptions below:

$$Ky(y) = 0.4 \cdot \sqrt{\frac{\tau_b}{\rho}} \cdot y \quad \text{and} \quad \tau_{xy} = \tau_b = \text{const.} \tag{21}$$

As Prandtl has assumed $\tau_{xy} = \text{const}$; to satisfy this we must have $\tau_w = -\tau_b$.

This means that in fact we have a case when the wind is blowing in direction of water flow causing the stresses at the water surface $|\tau_w| = |\tau_b|$. After calculation in this case, we have:

$$V(y) = 2.5\sqrt{\frac{\tau_b}{\rho}} \cdot \ln\left(\frac{y}{y_*}\right) \tag{22}$$

and

$$V_0 = 2.5\sqrt{\frac{\tau_b}{\rho}} \cdot \ln\left(\frac{y}{e \cdot y_*}\right). \tag{23}$$

Equation (23) does not fulfil the condition that $V(0) = 0$. The zero velocity is now at point $y = y_*$. Comparison of Prandtl's and Chezy's formulae leads to the conclusion (Prandtl 1956) that the Chezy constant C is equal to:

$$C = 2.5\sqrt{g} \cdot \ln\left(\frac{H}{y_* e}\right). \tag{24}$$

Now, as according to Chezy the constant C is known, it is possible to calculate the thickness of so-called laminar sublayer y_*. Thus we have:

$$y_* = \frac{H}{e} \exp\left(-\frac{0.4C}{\sqrt{g}}\right) \tag{25}$$

in practice this thickness amounts to:

$$y_* = 0.004H \tag{26}$$

which denotes values of from 0.5 to 3 cm.

It is possible to evaluate the eddy viscosity coefficient at the point $y = y_*$. We obtain:

$$Ky(y_*) = 0.4\sqrt{g \cdot H \cdot J} \cdot \frac{H}{e} \exp\left(-\frac{0.4C}{\sqrt{g}}\right). \tag{27}$$

Under average conditions, the coefficient in the river amounts to $1.3 \cdot 10^{-4}$ [m^2/s] which means that it is about 100 times bigger than so-called kinematic water viscosity. Furthermore, it means that there is turbulent motion in the sublayer.

3) Modified Prandtl's tachoida

Modified Prandtl's tachoida can be obtained by adjusting Prandtl's concept to those, satisfying general conditions at flow:

$$\tau_{xy}(H) = -\tau_w = 0, \quad \text{and} \quad V(0) = 0 \quad \text{(tachoida without wind)}$$

moreover, the distribution of the eddy viscosity should be completed so that this parameter has no smaller values than the kinematic viscosity.

$$Ky(y) = m_1 \cdot \sqrt{\frac{\tau_b}{\rho}} \cdot y + K_0. \tag{28}$$

After calculating, we obtain:

$$\tau_{xy}(y) = \tau_b \left(1 - \frac{y}{H}\right) \tag{29}$$

and afterwards:

$$V(y) = \frac{1}{m} \cdot \sqrt{\frac{\tau_b}{\rho}} \cdot \left[(1+0)\ln\left(1 + \frac{1}{\theta} \cdot \frac{y}{H}\right) - \frac{y}{H}\right] \tag{30}$$

where:

$$\theta = \frac{K_0}{m_1 H \sqrt{\frac{\tau_b}{\rho}}}. \tag{31}$$

It is also possible to calculate the average velocity. Then we have:

$$V_0 = \frac{1}{m_1} \cdot \sqrt{\frac{\tau_b}{\rho}} \cdot [(1+\theta)]^2 \cdot \ln\left(\frac{1+\theta}{\theta}\right) - \theta - \frac{3}{2}. \tag{32}$$

Comparison of the average velocities, which are calculated according to Prandtl's and Chezy's modified method, results in the additional relationship:

$$C = \frac{\sqrt{g}}{m_1} \cdot \left[(1+\theta)^2 lu \left(\frac{1+\theta}{\theta}\right) - \theta - \frac{3}{2}\right]. \tag{33}$$

Assuming after Prandtl's $m_1 = 0.4$, it is possible to evaluate the eddy viscosity at the bottom. Thus we have:

$$K_0 = 0.4\sqrt{gHI} \cdot \exp\left[-\frac{3}{2} - \frac{0.4C}{\sqrt{g}}\right]. \tag{34}$$

For conditions of the Central Odra River, we obtain $K_0 \cong 30 \cdot \nu$; in the Lower – $K_0 \cong 100 \cdot \nu$; in the estuarial section – $K_0 \cong 30 \cdot \nu$.

The example diagrams of tachoida assumed by Prandtl and Bazin have been presented in Fig. 2.

3.3. Tachoida under Conditions of Blowing Wind

3.3.1. Introduction

In order to describe the tachoida under windy conditions, the general relationship (7) was applied, which gives the vertical distribution of the τ_{xy} component

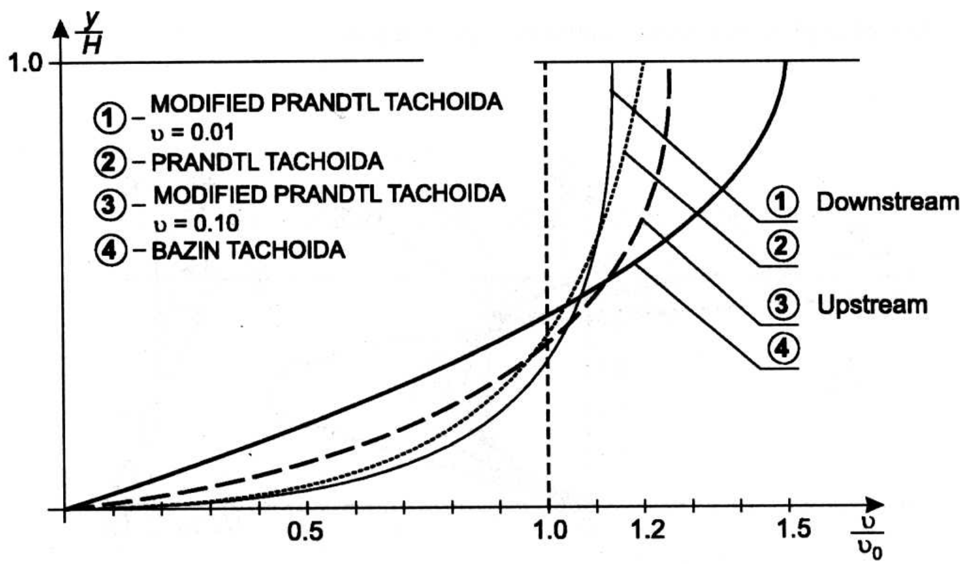

Fig. 2. Plots of various tachoidas

of turbulent shear stress tensor and the known (assumed) distribution of eddy viscosity. This results from Boissinesq's assumed hypothesis. The selection of the formula describing the eddy viscosity should reflect the nature of the phenomenon on the one hand, and should not hinder the analytical calculations too much on the other. In literature two forms of this formula are offered. One is suggested by Meyer (1985, 1986) as the relationship describing the system's reaction to the unknown extortion.

$$Ky(y) = \kappa_1 H \cdot V_0 \cdot \exp\left(-a\frac{y}{H}\right). \tag{35}$$

Parameter a is that which includes wind stresses.

This relationship has a certain disadvantage: the coefficient does not depend on the way in which the wind changes the tachoida's shape. A more complicated formula was therefore suggested later (Coufal, Meyer 1998):

$$Ky(y) = \kappa_2 \cdot H \cdot \sqrt[4]{\frac{\tau_w^2 + \tau_b^2}{\rho_2}} \cdot \exp\left(-a\frac{y}{H}\right). \tag{36}$$

This relationship better reflects the production of turbulence by wind stresses at the surface. Application of this relationship comes up against big analytical difficulties, therefore the formula (35) seems to be satisfactory for the purpose of practical evaluations.

The second form of the formula is assumed classically according to Prandtl:

$$Ky(y) = m_1 \cdot \sqrt{\frac{\tau_b}{\rho}} \cdot y + m_2 \cdot \sqrt{\frac{\tau_w}{\rho}} \cdot (H - y) + K_0. \tag{37}$$

The examples have been shown in Figs. 3 and 4:

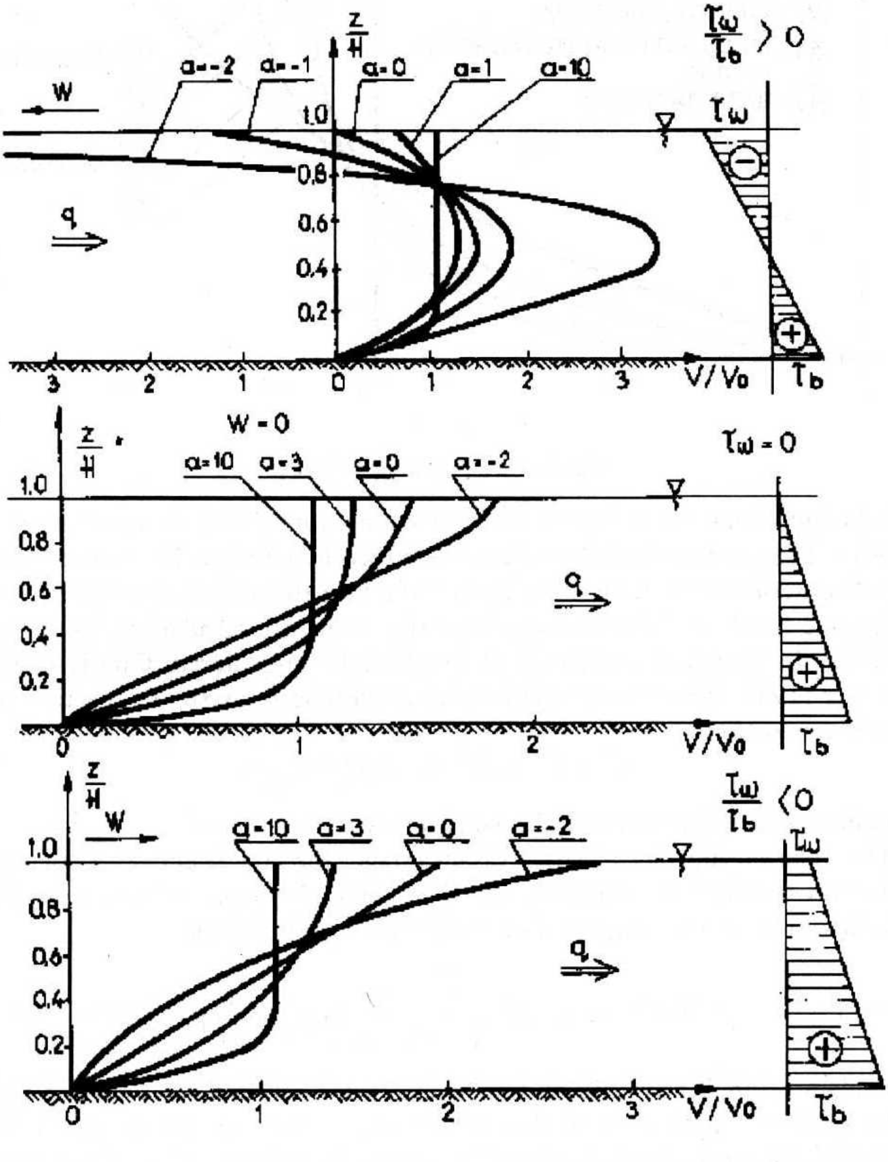

Fig. 3. Various shapes of tachoidas by Meyer (1985)

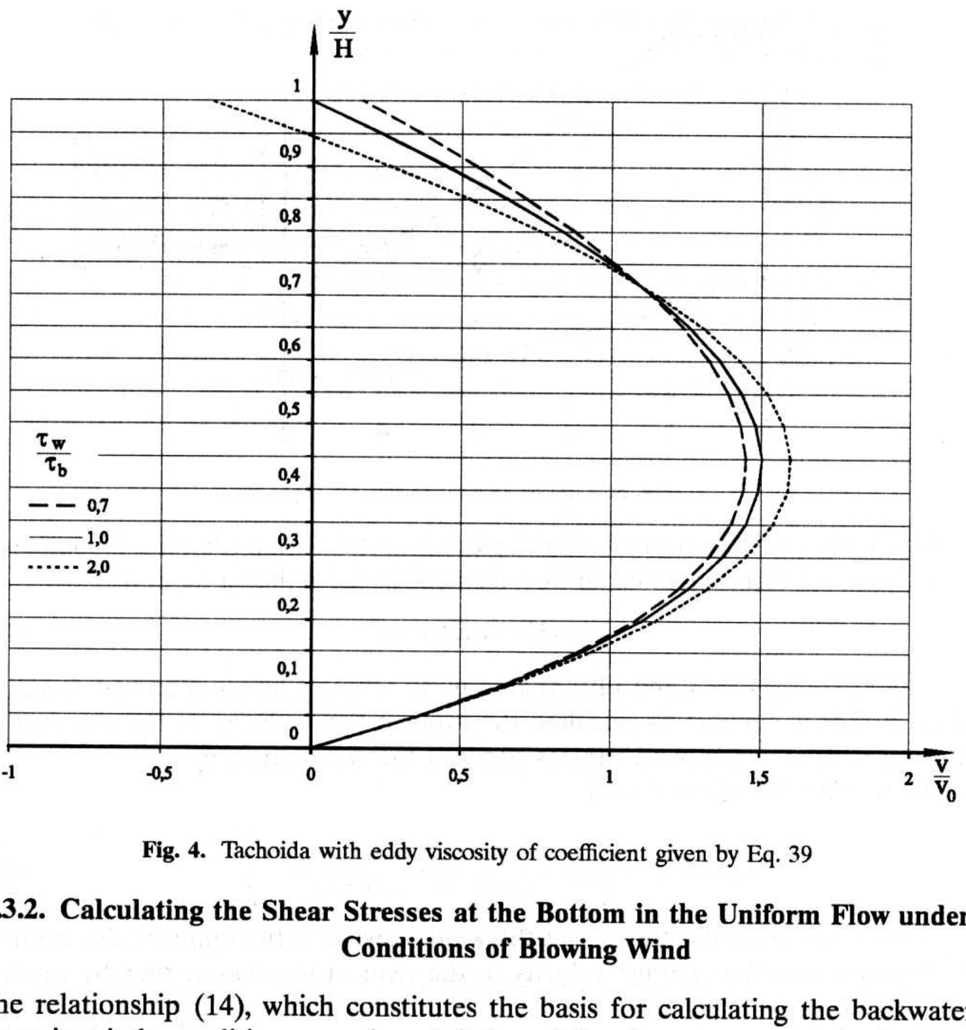

Fig. 4. Tachoida with eddy viscosity of coefficient given by Eq. 39

3.3.2. Calculating the Shear Stresses at the Bottom in the Uniform Flow under Conditions of Blowing Wind

The relationship (14), which constitutes the basis for calculating the backwater curve in windy conditions, requires defining of the shear stresses at the bottom, which is opposite to the water movements. These stresses should take into consideration not only elements of water motion, but also wind stresses on the water surface. These relationships will be obtained as follows. The repeated integration of the formula (17) enables obtaining of the relationship, which describes the average flow velocity, and then:

$$\kappa_1 \cdot \rho \cdot V_0^2 = \tau_b \cdot D_b(a) - \tau_w \cdot D_w(a) \tag{38}$$

where $D_w(a)$ and $D_b(a)$ were defined in Meyer's previous elaboration (1985).

As we should obtain Chezy's formula from the relationship (38) for $\tau_w = 0$, we can calculate that:

$$\kappa_1 = \frac{g}{C^2} \cdot D_b(a). \tag{39}$$

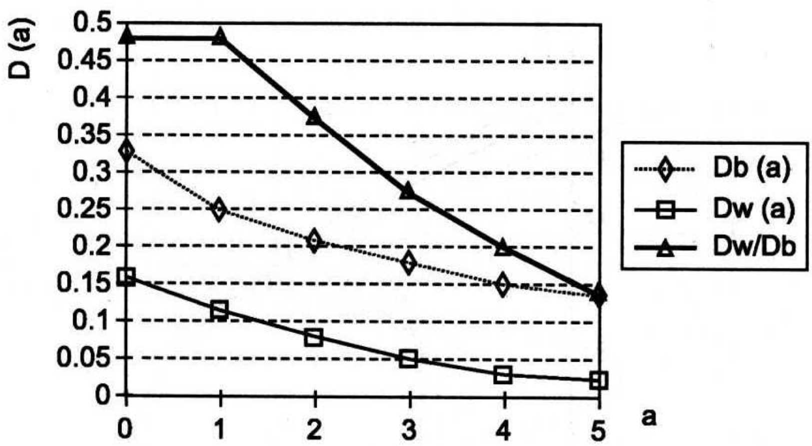

Fig. 5. Graphs $D_w(a)$, D_b(a), D_w/D_b

One more conclusion can be drawn from the relationship (38), if it is compared with the equilibrium equation of flow element in the uniform motion:

$$g\rho HI = \tau_w + \tau_b. \tag{40}$$

Equations (38) (39) and (40) enable us to obtain Chezy's modified formula, which makes it possible to calculate the average flow velocity in the riverbed in uniform motion, when wind stresses occur at the water surface.

Thus, after Buchholz (1989):

$$V_0 = C\sqrt{H\cdot}\left[I_d - \frac{\tau_w}{\rho g H}\cdot\frac{D_b(a) + D_w(a)}{D_b(a)}\right]^{1/2}. \tag{41}$$

This relationship illustrates that the wind blowing in the opposite direction to the flow reduces the average velocity in the river cross-section, thereby causing water swelling.

The relationship which describes the shear stresses at the bottom as the function of flow and wind stresses, is of key importance for analysis of the energetic equation (14). We obtain from the formula (38):

$$\tau_b(V_0, w) = \frac{\kappa_1}{D_b(a)}\cdot\rho\cdot V_0^2 + \tau_w\cdot\frac{D_w(a)}{D_b(a)}. \tag{42}$$

This relationship is substituted to relationship (14) resulting in:

$$\frac{\partial}{\partial x}\left(\alpha\frac{V_0^2}{2g}\right) = -\frac{\partial Rz}{\partial x} + \kappa_3\cdot H\cdot\frac{\partial^2}{\partial x^2}\left(\frac{V_0^2}{2g}\right) - \frac{V_0^2}{C^2 H} - \frac{\tau_w}{\rho g H}\cdot\frac{D_w + D_b}{D_b}. \tag{43}$$

This is the basic equation, which generalises the mechanism of backwater curve with wind influence at the water surface and rapid changes of riverbed cross-section.

4. Equation of the Backwater Curve in a Rectangular Channel

4.1. Basic Solution

The above relationship (43) is an equation of the swelling curve in a river in its most general form. Wind stresses on the free surface of the water and big changes of riverbed cross-sections are included in this equation. In order to apply this equation practically, it must be completed with elements transforming it into differential, linear one of the second order, which would present the relationship: depth, distance. The aim of the present paper is to explain the mechanism of wind shear stress influence at a backwater curve. The rectangular cross-section of the river and linear bed slope has therefore been assumed. We must therefore use the flow continuity equation:

$$V_0 = \frac{Q}{H(x) \cdot B} \quad \text{where} \quad Q = \text{const.} \tag{44}$$

location of water level ordinate in the river:

$$Rz = Id \cdot x + H(x), \tag{45}$$

the constant value for Chezy's formula can be described by Manning's formula:

$$C = \frac{1}{n}\,[H(x)]^{1/6}, \tag{46}$$

wind stresses:

$$\tau_w = \kappa_w \cdot \rho \cdot W|W| \quad \text{where} \quad W = W(x). \tag{47}$$

The boundary conditions of the second order equation are defined in the general case by giving:

$$\text{for } x = 0 \; H = H_0 \; \text{ and } \; \frac{dH}{dx} = I_0. \tag{48}$$

For $x \to \infty$ the differential equation (43) has the following form:

$$\frac{\partial Rz}{\partial x} = I_d = +\frac{V_0^2}{C^2 H} + \frac{\tau_w}{\rho g H} \cdot \frac{D_w + D_b}{D_b}. \tag{49}$$

After transformation the relationship (49) results in the previously obtained relationship (41), which defined the average flow velocity of uniform motion when the wind is blowing.

The equation in form of (43) enables extorting the assumed slope (e.g. $I_0 = 0$) on the swelling curve at its lower edge. It is important when the river flows into

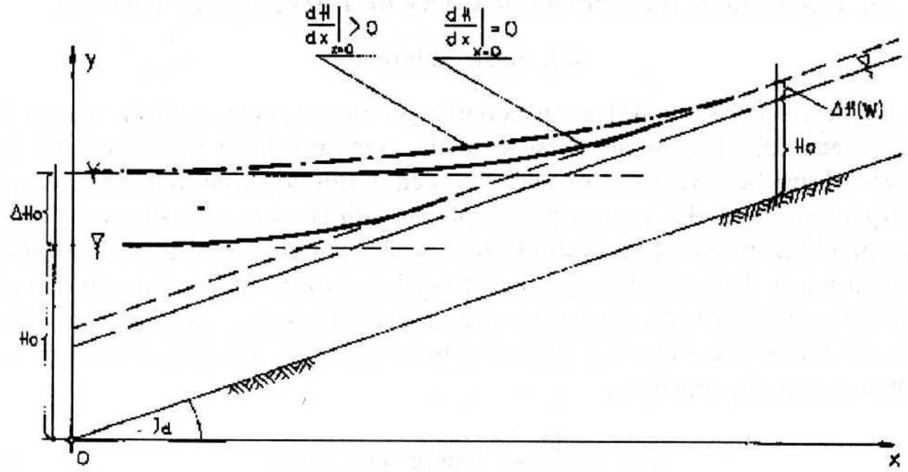

Fig. 6. Wind backwater curve concept

a reservoir. Shapes of backwater curves in windy conditions have been shown in the Fig. 6.

For further analysis it is convenient to present the basic equation (43) in the dimensionless form (Pacewicz, Meyer 1985). We put:

$$v_o = \frac{q}{H}, \quad H_{kr}^3 = \frac{\alpha q^2}{g}, \quad H_0^3 = \frac{q^2}{c^2 I_d}, \quad H_G^3 = \frac{q \cdot K_x}{g}, \tag{50}$$

and so-called wind slope:

$$I_w^* = \frac{\tau_w}{\rho g H_0} \cdot \frac{D_b(a) + D_w(a)}{D_b(a)}. \tag{51}$$

After substituting assumed dimensionless values to the basic of equation motion (43), we obtain :

$$\frac{dH}{dx} = -I_d \frac{1 - \frac{H_0^3}{H^3}}{1 - \frac{H_{Kr}^3}{H^3}} - \frac{\frac{H_G^3}{H^2} \cdot \frac{d^2H}{dx^2}}{1 - \frac{H_{Kr}^3}{H^3}} - I_w^* \cdot \frac{\frac{H_0}{H}}{1 - \frac{H_{Kr}^3}{H^3}}. \tag{52}$$

The equation (52) contains two additional terms as compared with the classic theory of Rühlmann and Tolkmitt. One of them contains the second derivative of depth and the other – so-called wind slope. The equation term containing the second derivative represents so-called eddy viscosity in the longitudinal direction – along the river. In practice it denotes the influence of riverbed cross-section rapid changes upon water level shape. This term generalises the curve equation with so-called river slope at the lower boundary. The classic solution of the swelling curve – resulting from the differential equation of the first order (linear) – requires knowing only the river depth as the lower boundary.

The water level slope in this place (e.g. at the dam) can be calculated from the formula (52) assuming $H_0 = 0$; $I_w^* = 0$. We then will obtain:

$$\left.\frac{dH}{dx}\right|_{x=0} = -I_d \cdot \frac{1 - \frac{H_0^3}{H^3(0)}}{1 - \frac{H_{Kr}^3}{H^3(0)}} \neq 0. \tag{53}$$

In conditions of a river estuary it is difficult to assume that the slope amounts as much as the formula (53). It can be expected that this slope is given (measured) or it can be approximated as:

$$\left.\frac{dRz}{dx}\right|_{x=0} = 0, \tag{54}$$

due to very slow water motion and substantial dimensions of the river-bed cross-section. The full solution of the equation (52) is not the subject of the present elaboration. The author would like to focus mainly on wind influence upon the backwater curve. The calculation results of the simplified swelling curve equation are therefore presented further as $H_G = 0$, and so:

$$\frac{dH}{dx} = -I_d \frac{1 - \frac{H_0^3}{H^3}}{1 - \frac{H_{Kr}^3}{H^3}} - I_w^* \cdot \frac{\frac{H_0}{H}}{1 - \frac{H_{Kr}^3}{H^3}} \tag{55}$$

related to the conditions of the estuarial Odra River section. The section from the estuary, in which river water levels are subjected to distinct influences of sea level and wind changes, is understood in this elaboration as the estuarial Odra River section. This is the section that reaches from the Szczecin Lagoon to Gozdowice (often even further). In order to calculate the swelling curve in the Lower Odra River in a shorter manner, the following simplifications have been assumed:

- river bottom is a straight line with slope I_d,
- riverbed cross-section is rectangular,
- wind along the river is stable (wind velocity and direction).

In case of such simplifications, different swelling curves in the Lower Odra River have been obtained for the assumed boundary conditions (water level in the Szczecin Lagoon, Trzebież limnigraph and water flow in Gozdowice) as well as for different winds. The calculation results have been presented in the figures (according to Libront 1999).

From the presented schemes of the wind backwater curve in the Lower Odra River it results that this solution depends very much on the boundary conditions (water level at Trzebież and water flow at Gozdowice). Moreover, these solutions indicate that the range of swelling curve, i.e. wind backwater, reaches the Gozdowice cross-section and often even further. One can therefore speak of the wind influence upon the limnigraphic read-outs in the Gozdowice cross-section and upon their interpretation.

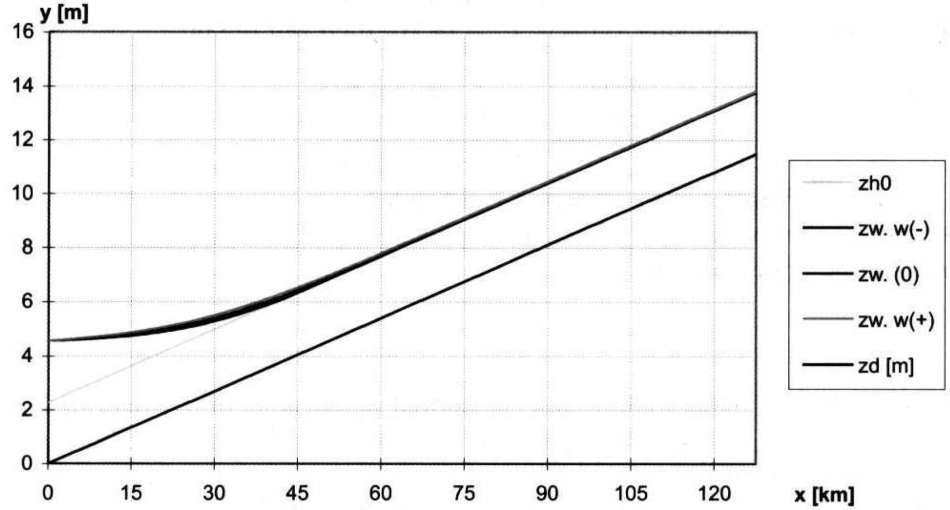

Fig. 7. Wind influence upon the water level within wind backwater ($Q = 250$ m^3/s, $H_p = 4.5$ m, $I_d = 0.00009$, $\kappa'_w = 2 \cdot 10^{-6}$) (Libront 1999)

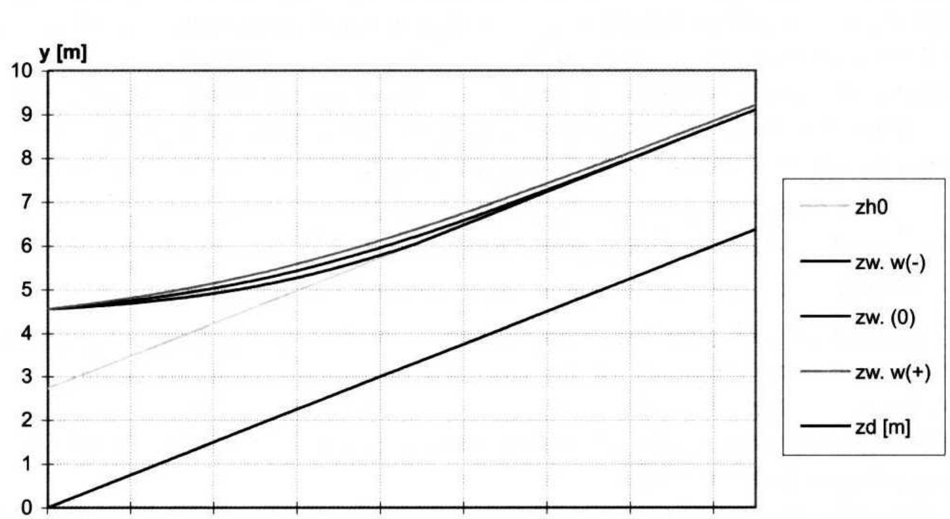

Fig. 8. Wind influence upon the water level within wind backwater ($Q = 250$ m^3/s, $H_p = 4.5$ m, $I_d = 0.00005$, $\kappa'_w = 2 \cdot 10^{-6}$) (Libront 1999)

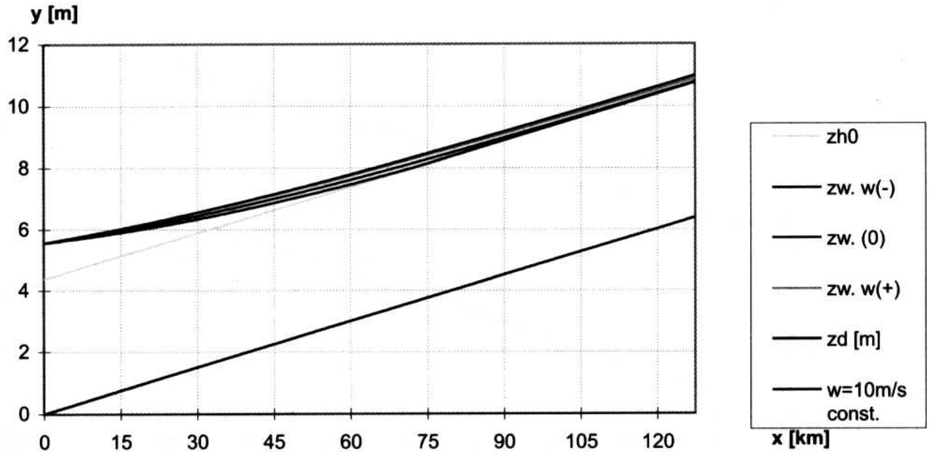

Fig. 9. Wind influence upon the water level within wind backwater ($Q = 550$ m^3/s, $H_p = 5.5$ m, $I_d = 0.00005$, $\kappa'_w = 2 \cdot 10^{-6}$) (Libront 1999)

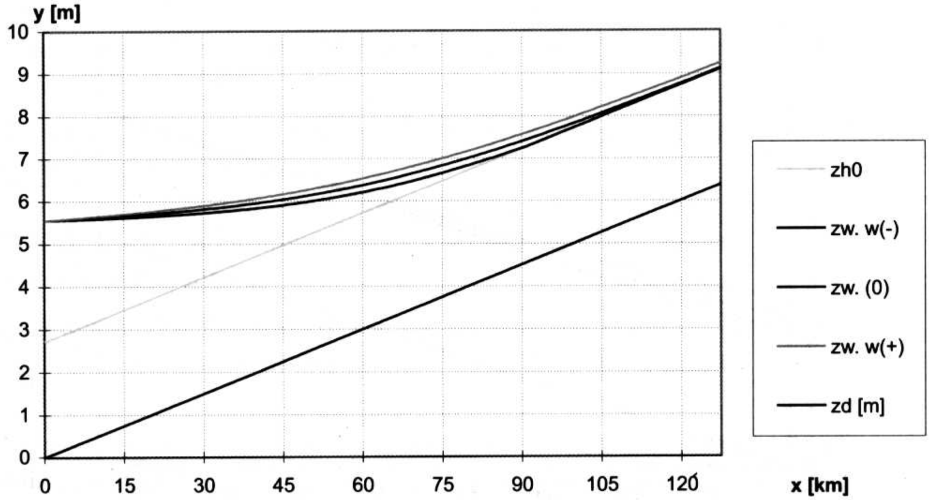

Fig. 10. Wind influence upon the water level within wind backwater ($Q = 250$ m^3/s, $H_p = 5.5$ m, $I_d = 0.00009$, $\kappa'_w = 2 \cdot 10^{-6}$) (Libront 1999)

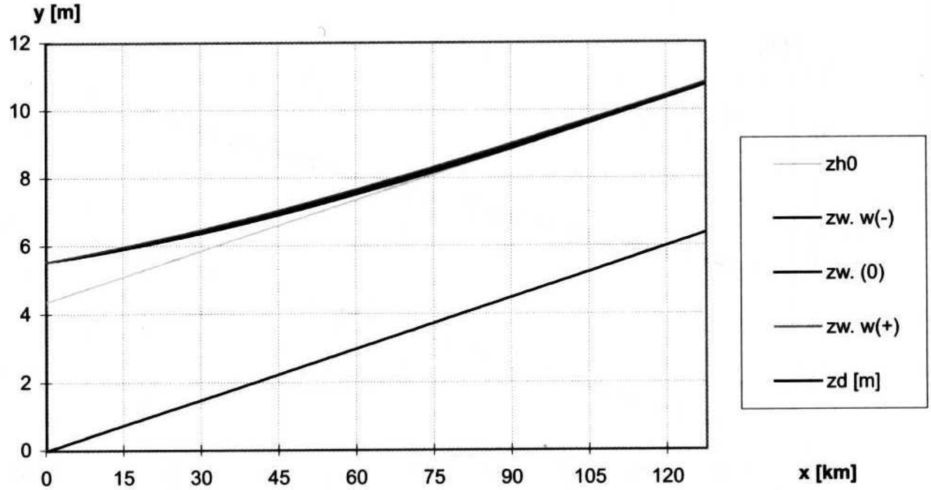

Fig. 11. Wind influence upon the water level within wind backwater ($Q = 550$ m^3/s, $H_p = 5.5$ m, $I_d = 0.00005$, $\kappa'_w = 10^{-6}$) (Libront 1999)

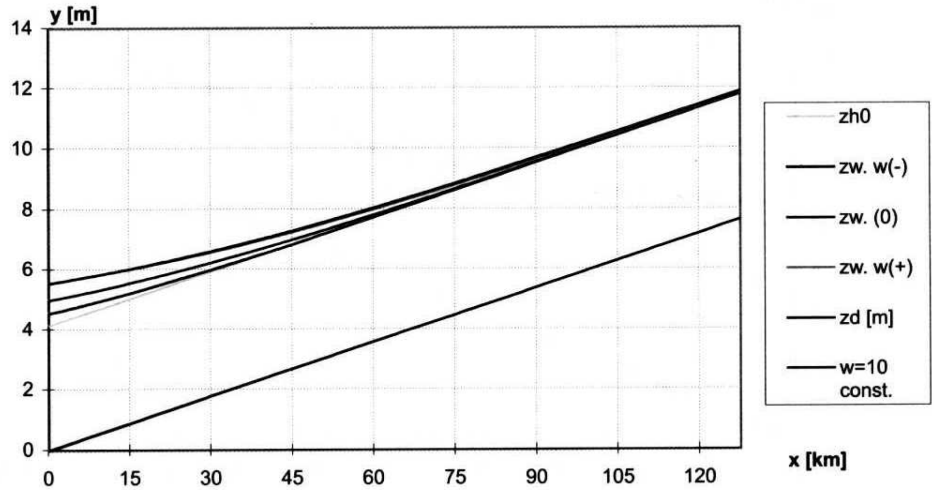

Fig. 12. Wind influence upon the water level within wind backwater ($Q = 550$ m^3/s, $H_p = 5.55$ m for wind (+) and 4.55 for wind (−) $I_d = 0.00006$, $\kappa'_w = 10^{-6}$) (Libront 1999)

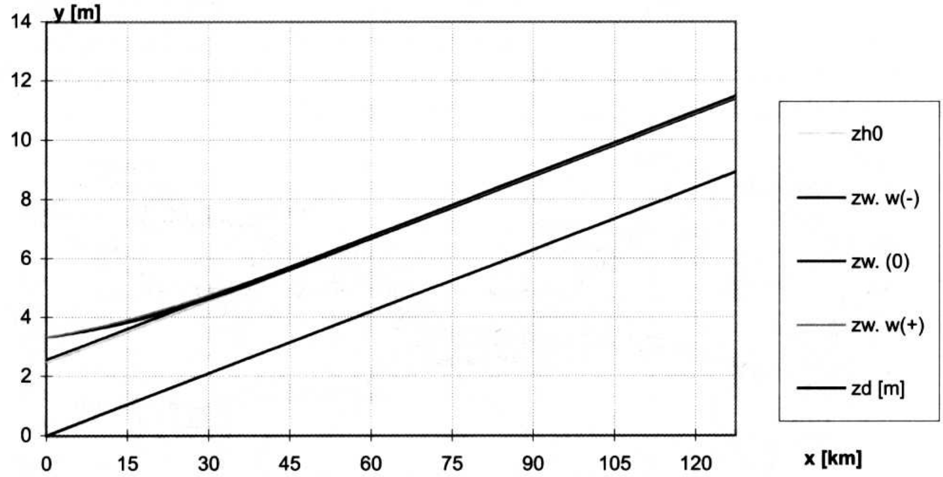

Fig. 13. Wind influence upon the water level within wind backwater ($Q = 250$ m^3/s and 264 m^3/s, $H_p = 3.3$ m, $I_d = 0.00007$, $\kappa'_w = 2 \cdot 10^{-6}$) (Libront 1999)

4.2. Solution Analysis of Boundary Conditions

Wind is the basic factor in shaping the backwater curve. The correct solution of the problem (Meyer 1985) requires knowledge the wind field, i.e. the spatial distribution of wind force and direction in the entire analysed section of the Odra River. Researches on the wind field in the analysed section lead to two basic conclusions (Libront 1999):

- Water level at Trzebież, which is the lower boundary condition, is a function of wind velocity direction,
- Velocity of northern wind, which causes wind swellings, decreases very quickly at the Świnoujście–Trzebież section and then shows a tendency towards stabilisation.

The longitudinal distributions, which are the analysis result of wind field along the Odra River for northern winds, can be assumed according to the following formulae:

This function has the following form for northern winds (Libront 1999):

$w = 30.519 \cdot (l + 10)^{-0.331}$ for wind velocity of 15 m/s at Świnoujście,
$w = 31.362 \cdot (l + 10)^{-0.3844}$ for wind velocity of 14 m/s at Świnoujście,
$w = 23.102 \cdot (l + 10)^{-0.33}$ for wind velocity of 12 m/s at Świnoujście,
$w = 17.958 \cdot (l + 10)^{-0.2834}$ for wind velocity of 10 m/s at Świnoujście,
$w = 32.394 \cdot (l + 10)^{-0.5132}$ for wind velocity of 9 m/s at Świnoujście,
$w = 28.07 \cdot (l + 10)^{-0.5278}$ for wind velocity of 8 m/s at Świnoujście,
$w = 7.11 \cdot (l + 10)^{-0.1683}$ for wind velocity of 7 m/s at Świnoujście,

$w = 12.25 \cdot (l + 10)^{-0.31}$ for wind velocity of 6 m/s at Świnoujście,
$w = 11.857 \cdot (l + 10)^{-0.3598}$ for wind velocity of 5 m/s at Świnoujście,
In these formulae l denotes the distance in kilometres from Świnoujście. These changes have been shown graphically in the figure 15.

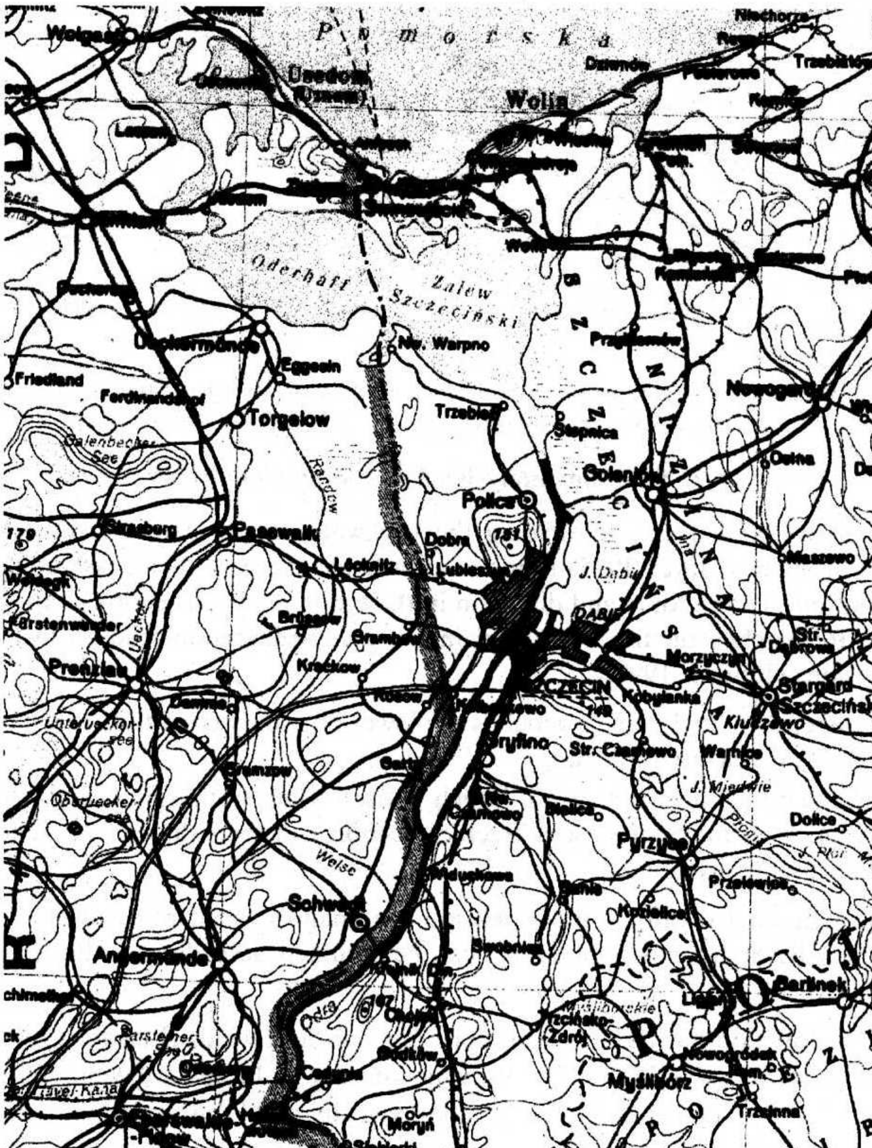

Fig. 14. Map of the Lower Odra River

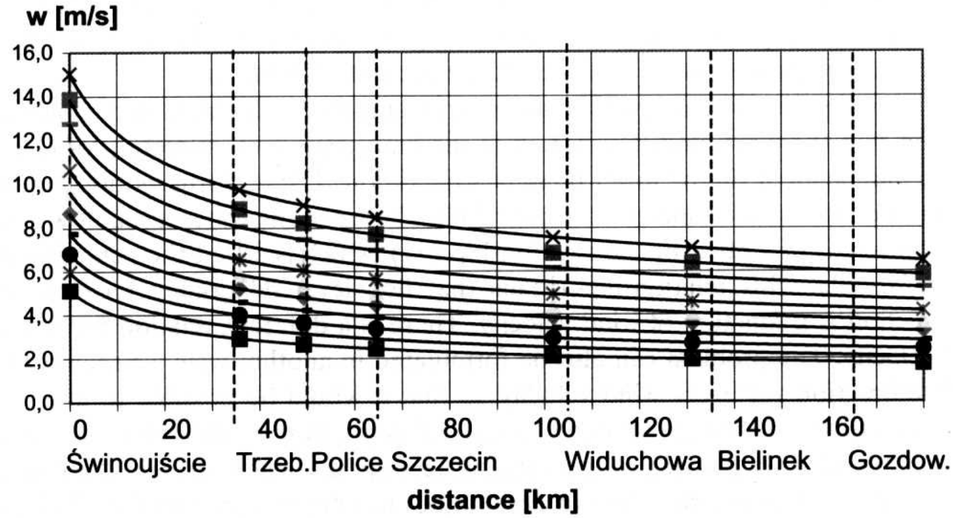

Fig. 15. Changes of wind velocities along the valley of the Lower Odra River (Libront 1999)

The first of the elements i.e. water level at Trzebież as a function of wind direction and velocity, is a problem, which is colloquially called wind swelling in the Szczecin Lagoon. Many authors have taken up this problem. Libront (1999) describes this relationship for the north wind in the form of the following diagram (Fig. 16).

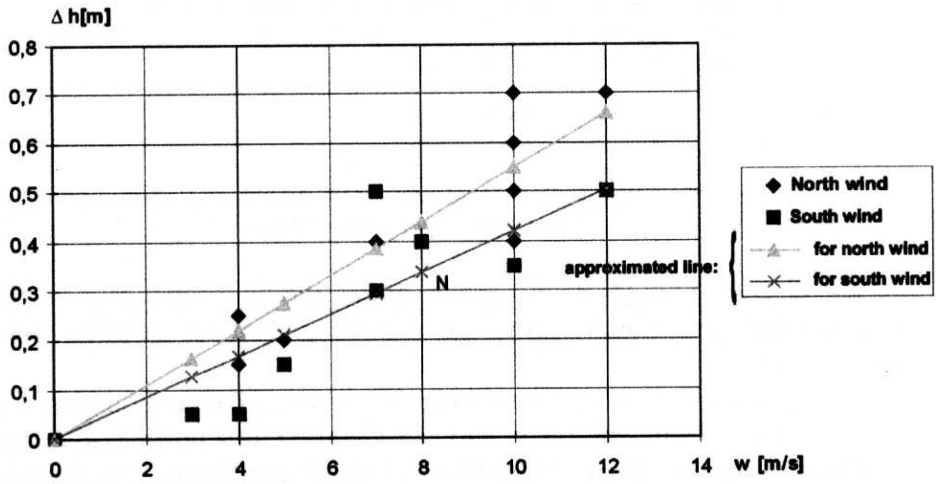

Fig. 16. The measured and calculated values of swellings caused by wind at Trzebież (Libront 1999)

Such an analysis was carried out by Gautam, Holz, Meyer (1999) applying the theory of neuronial network. The obtained relationships are not linear, and the quantity of variables describing the water level in Trzebież is bigger: sea level, wind, atmospheric pressure and water flow along the Odra River. Detailed analysis of this phenomenon is not the subject of the present elaboration.

The limnigraph in Gozdowice is the next boundary condition. It is commonly assumed that there is a uniform motion in the limnigraphic cross-section of Gozdowice independent on the lower boundary condition. The uniform motion in the Gozdowice cross-section means that the relation between the water level, flow and wind stresses is described by the relationship (41), which is called the generalised Chezy formula. This relationship means that flow-level relation in Gozdowice cross-section is a function of wind velocity. There is a different relation for every wind velocity. The problem can also be formulated in another way: we search for depth correction for every wind velocity so that it would be possible to apply a universal curve. The universal curve is one of depth-flow relation for the case, when the wind is not blowing $w = 0$. This correction has been shown in the figure 17. It results that in practice this correction does not depend on flow for winds of less than 10 m/s (Libront 1999).

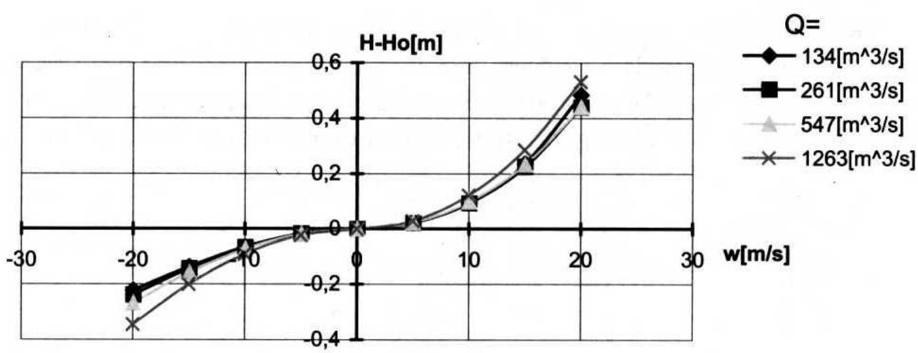

Fig. 17. Changes of average depth at Gozdowice under the influence of wind (for I_b equal to water level slope) (Libront 1999)

The diagram $Q = Q\,(Rz, W)$, which makes the flow dependent on the water level ordinate and wind velocity, can also be applied for practical evaluations (Libront 1999).

If the above relationships describing the upper and lower boundary condition as the function of wind are taken into consideration, it is possible to compare the swelling curve in the Lower Odra River in windy conditions. The situation from 7th September 1996 was given as the example here referring to Libront (1991). In this case the north wind was measured 10 m/s and the water level at Gozdowice – 6.34 mNN. After some necessary corrections – $Q = 519$ m^3/s. The wind swelling in the Szczecin Lagoon – $\Delta h = 0.55$ m. The ordinal condition of depth at Trzebież – $H_T = 13.30 + 0.55 = 13.85$ m.

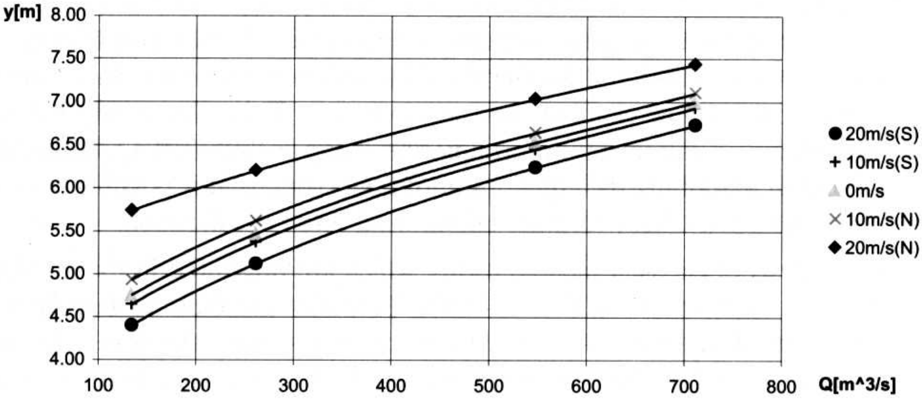

Fig. 18. Flow curves in Gozdowice in windy conditions (Libront 1999)

The results of analytical calculations and backwater ordinates have been measured and presented in the figure 19.

Very good agreement of the measured and calculated values can be noted in this figure.

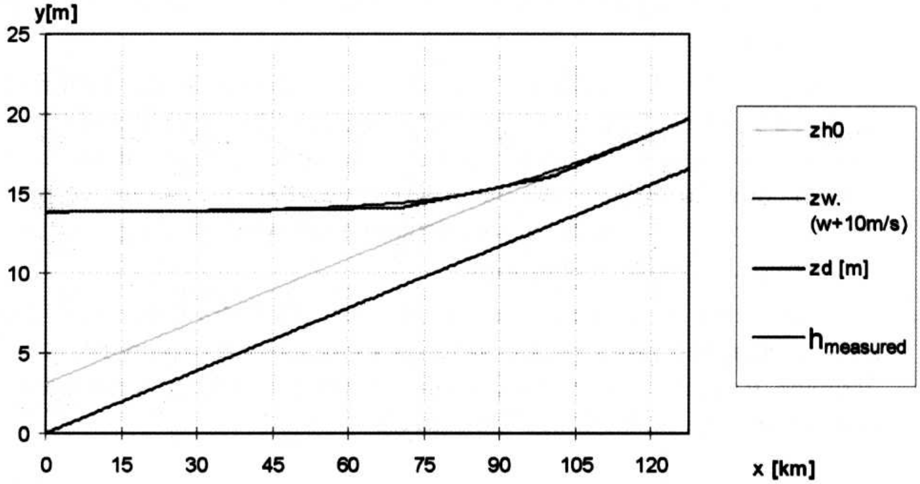

Fig. 19. The calculated and measured swelling curves in the Odra River mouth (Libront 1999)

5. Conclusions

5.1. In the elaboration there an analysis of the swelling curve in the river when wind stresses occur at the water surface has been presented. The analysis is based upon the hydrodynamic equations in windy conditions.

5.2. Reynold stresses' tensor. After taking Boussinesq's hypothesis into consideration, this distribution allows defining of the vertical distribution of river flow velocities (tachoida) and finding the relationship between the shear stresses at the bottom, flow elements and wind stresses. This relationship has been derived for conditions of uniform motion. After including the additional relationship describing the equilibrium of flow element, it is possible to obtain the generalised Chezy formula taking wind into consideration.

5.3. The analysis of classical tachoid forms, which are presented in the literature, leads to the conclusion that so-called Prandtl's tachoids do not fulfil the condition of $\tau_{xy}(H) = -\tau_w = 0$, as there is no wind. Moreover this tachoida does not fulfil the bottom condition $V(0) = 0$. Hence modification of Prandtl's tachoids is suggested to meet these conditions. The necessity for this arises not only from the fact that the eddy viscosity exceeds considerably the kinematic viscosity at the bottom, but there must be $\tau_{xy}(H) = 0$ at the water surface when here is no wind.

5.4. Also presented are tachoidas in conditions in which no wind stresses appear on the water surface. The wind blowing in the direction opposite to the flow of water reduces the flow velocity in the superficial layers. Very strong wind can give rise to superficial reverse currents. This flow continuity requires water swelling.

5.5. The analysis of wind swelling in the river has lead to the derivation of the so-called wind swelling curve (wind backwater). The equation of this curve includes not only wind influence, but also rapid changes of the riverbed cross-section. In order to enable practical calculations, evaluative program has been prepared, which allows calculating the wind swelling in conditions of longitudinally changeable wind.

5.6. The evaluations, which were made for the estuarial Odra River cross-section, indicate that the range of the wind backwater reaches even 100 km from Trzebież. It means that the wind backwater curve should be considered while interpreting the limnigraphic indications.

References

Buchholz W. (1989) *Wpływ wiatru na przepływy w ujściach rzek*, Wyd. Instytutu Morskiego, Gdańsk.

Coufal R., Meyer Z. (1998) Wpływ wiatru na spiętrzenie wód powodziowych w ujściu Odry, *Materiały Szkoły Hydrauliki - "Współczesne problemy hydrauliki wód śródlądowych"*, Wyd. IBW PAN Gdańsk, Zawoja 14–18 września.

Coufal R., Meyer Z., Roszak A. (1999) Model sortowania rumowiska w rozwidleniu rzecznym i jego weryfikacja dla przepływu wielkich wód w rejonie dolnej Odry, *7 Konferencja "Współczesne Problemy Hydrotechniki"*, Wrocław.

Libront D. (1999) *Wpływ prędkości wiatru zmieniającej się wzdłuż koryta dolnej Odry na stany i przepływy wody*, Ph.d. Thesis, Technical University of Szczecin.

Meyer Z. (1986) Vertical Circulation Density Stratified Reservoir, *Encyclopaedia of Fluid Mechanics*, Gulf Publishing Co. New Jersey.

Meyer Z. (1985) Mechanics of bottom density Wedges, *Archiwum Hydrodynamiki*, No. 2.

Meyer Z., Pacewicz F. (1984) Wpływ lokalnych zmian zwierciadła wody na kształt krzywej spiętrzenia, *Materiały konferencji "Wybrane problemy hydrotechniczne ujścia Odry do Bałtyku"*, IM Szczecin.

Prandtl L. (1956) *Dynamika przepływów*, PWN Warszawa.

Puzyrewski R., Sawicki J. (1998) *Podstawy mechaniki płynów i hydrauliki*, PWN Warszawa.