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Non-linear Water Waves – Experiments and Theory

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Abstract

The paper concerns laboratory experiments on shallow water waves. The waves in our flume were generated as programmed groups, with gentle fading-in and fading-out of amplitudes of the wavemaker's piston's motions. The higher harmonic components have been considered in the motion. Their influence on charactericties of generated waves has also been studied. The measurments were carried out at 6 cross sections along the propagation path; both wave profiles and velocities were recorded. As a description of phenomena, the wave profiles were approximated by harmonic components according to the Stokes solution. It was noted, that parameters of harmonic components change along the propagation path; there are energy transfers between them. The higher harmonic components are more significant for the description of wave profiles, than for the velocities. The results of the experiments were used in verifications in a research program on the theoretical description and numerical algorithm for shallow water waves generated and propagated in a flume.

1. Introduction

Measurement in a wave flume is the most effective and precise method of research on water waves. We can identify in detail the physical properties of wave phenomena and verify the theoretical approach. Very often the waves are created by the motion of a vertical stiff plate which oscillates horizontally (piston). Generally, kinematic parameters of the piston's motion and water waves are different (vertical distribution of displacements and velocities). This causes disturbances in the the vicinity of the wavemaker's piston. Thus, we should control the movement of a piston considering non-linear effects, for creating waves that are close to the ideal, theoretical ones. Moreover, the profile of a flume influences the quality of waves (the length of the flume, geometrical precision, smoothness of walls and bottom, efficiency of wave's dumping on the end of a flume, etc.).

The wave laboratory of the *Institute of Hydro-Engineering PAS in Gdańsk* is equipped with a modern, 64 m long wave flume (Sobierajski 2000). It is 0.6 m wide and 1.4 m high; has glass walls and the bottom is made of aluminium plates; each boasting very smooth surfaces. The dimensional tolerance is 0.5 mm.

Fig. 1. The wave flume

At the end of the flume a very effective wave damper has been installed (the coefficient of wave reflection is close to 0.05), in the form of a porous slope made of plastic elements. The flume is shown in Fig. 1. The laboratory set is equipped with programmable piston-type wavemaker and the motion of the wavemaker's piston is controlled by a computer. It is possible to feed a suitable time series calculated on another computer into the control system. The time series may be a sample function with a random process.

The following problems have been included in our research:

- creation of a proper model (algorithm and numerical program) to control the motion of the wavemaker's piston, as regards the kinematics of the generated waves;
- theoretical analysis and creation of a numerical program to study the propagation and transformation of waves along the flume;
- a wide range of experiments to study the influence of generation methods on the wave parameters and to verify of theoretical descriptions.

In our investigations we focused on shallow water waves, which were the most suitable for piston-type generation carried out in our flume. We have worked with waves of $8 < L/h < 14.8$, where h is the depth of the water and L is the length of the wave.

2. Assumptions and Conditions of Research

The waves in the flume cover a two dimensional problem. The two-dimensional area of the fluid is bordered by: a stable bottom, free surface of water, absorbtion boundary (the end of a flume with wave energy absorber) and moving boundary of generation (the plate of the wavemaker's piston). We assume that the waves, reflected from the absorption boundary, are of insignificant height. If this is not the case measurements have to be stopped before the reflected waves reach the considered region. On the surface of contact, the horizontal components of displacements and velocities of plate's (piston's) points and the adjacent water elements, are the same. The initial conditions of movement are very important. We assume, that at the beginning ($t = 0$), the displacements and velocities of the piston are equal to 0. We assume also that water is incompressible. In the case of

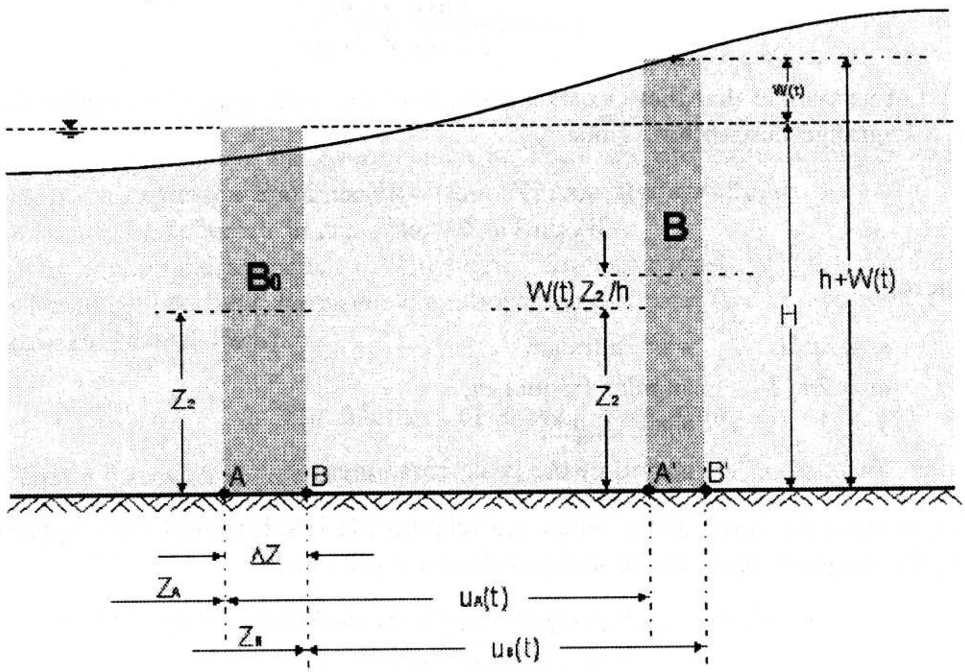

Fig. 2. The mathematical description

shallow water waves, changes of the horizontal components of displacements and velocities along the depth are quite small; thus we assume that their vertical distribution is rectangular. All vertical material planes in the fluid at rest remain as plane and vertical during motion. Such a material description is outlined in Fig. 2. Two vertical lines, crossing through points A and B, situated at the distance ΔZ from each other, represent calm water with depth h. After time t points A and

B will move along Z to their new positions A' and B', covering distances $u_A(t)$ and $u_B(t)$. The water level will rise by $W(t)$. Due to the assumption, that water is incompressible, the area between the lines crossing points A and B is equal to the area bordered by lines that cross poins A' and B' in the actual state and time t. Thus we have (Fig. 2):

$$h\Delta Z = [h + W(t)]\,[DZ + uB(t) - uA(t)]\,. \tag{1}$$

So, the vertical displacements depend on the horizontal displacements as follows:

$$w(t) = -h\frac{[u_B(t) - u_A(t)]/\Delta Z}{1 + [u_B(t) - u_A(t)]/\Delta Z}. \tag{2}$$

When ΔZ tends to zero relation (2) goes over to:

$$w(z,t) = -h\frac{\partial u(Z,t)/\partial Z}{1 + \partial u(Z,t)/\partial Z}. \tag{3}$$

Let us assume that there exists a solution of the Stokes type for regular waves in a Lagrange description. Thus:

$$\begin{aligned} u\,(Z,t) = &-W_1 \sin\,(kZ - \omega t) - W_2 \sin 2\,(kZ - \omega t) + \\ &-W_3 \sin 3\,(kZ - \omega t) + \ldots. \end{aligned} \tag{4}$$

where:

$k = 2\pi/L$ – wave number,
$\omega = 2\pi/T$ – angular frequency,
L – wave length,
T – period of the basic component.

It can easily be verified that when the relation (4) is substituted into expression (3) the standard form of the Stokes solution results in:

$$\begin{aligned} w\,(Z,t) = &A_1 \cos\,(kZ - \omega t) + A_2 \cos 2\,(kZ - \omega t) + \\ &+A_3 \cos 3\,(kZ - \omega t) + \ldots, \end{aligned} \tag{5}$$

where the coefficients A_1, A_2, $A_3 \ldots$ can be calculated from algebraic equations expressed in terms of the coefficients W_1, W_2, $W_3 \ldots$ Displacements of the free surface can be expressed as sum of cosine components and thus when the first term assumes maximum value all the higher terms are also maximum. The horizontal displacements are expressed by the relation (4) as the sum of sine curves. Thus when the first term assumes zero value all the higher components are also of zero value. The measured horizontal displacements of the piston are depicted in Fig. 3 for an interval with almost regular waves. It can be seen that the slope for positive displacements is greater than the absolute value of the slope for

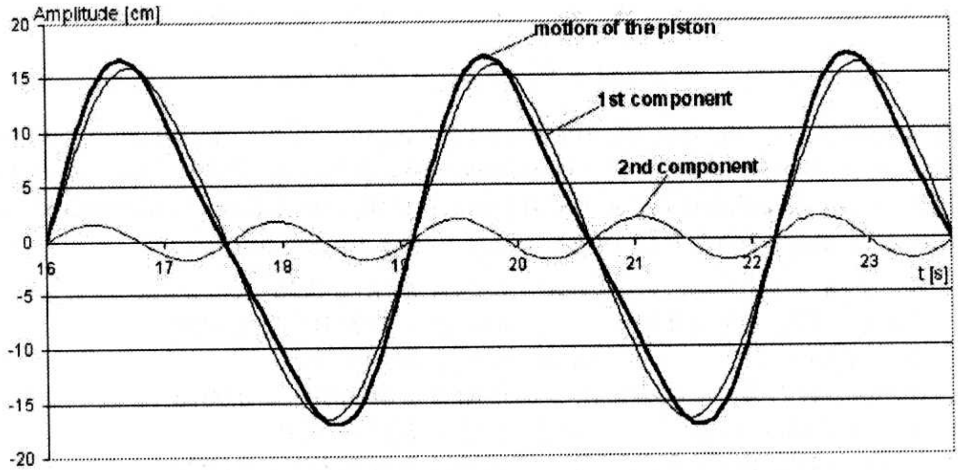

Fig. 3. Measured piston motion composed of two harmonic components

negative value. This means that the velocities towards propagation are greater than the magnitudes of backward velocities. These are properties arising from the assumption expressed by relation (4) that a Stokes type solution exists and if it is true should be reflected in experiments.

The theoretical approach differs slightly from previously, when the solution was based on two basic harmonic components and second free component (Bendykowska, Massel 1988).

3. The Method of Wave Generation

To create waves close to the theoretical description, a special method of generation has been developed (Wilde P., Wilde M. 2000), which affords the following possibilities:

- minimization of disturbances in a water medium, caused by dynamic starting and braking of a wavemaker;
- analysis of relations between the kinematics of the piston's movement and kinematics of the waves generated.

The suitable theoretical model and computer program (Wilde P., Wilde M. 2000) is based on the solution described by Wilde, Kozakiewicz (1993). The program creates files of time series for the assigned wave parameters; then such files are fed into the control system of the wavemaker.

The investigated waves are strongly non-linear. Thus it can be assumed that the piston motion follows the Stokes type solution. The piston corresponds to the material coordinate $Z = 0$ and thus according to the expression for regular waves according to the relation (4) the displacements of the piston should be

$$u(t) = W_1 \sin \omega t + W_2 \sin 2\omega t + W_3 \sin 3\omega t + \cdots . \tag{6}$$

It is possible to calculate W_1, W_2, W_3 for the motion of the wavemaker's piston, in relation to the parameters of a generated wave (Chybicki 2000). The ranges of changes of the calculated ratios W_2/W_1 and W_3/W_1 depending on dimensionless lengths L/h and heights H/h of generated waves are the following:

1) $8 < L/h < 8.1$ and $0.18 < H/h < 0.36$
   $0.021 < W2/W1 < 0.039$ $\quad$ $0.0015 < W3/W1 < 0.0054$
2) $10 < L/h < 10.3$ and $0.15 < H/h < 0.30$
   $0.041 < W2/W1 < 0.071$ $\quad$ $0.0005 < W3/W1 < 0.0016$
3) $12.1 < L/h < 12.5$ and $0.12 < H/h < 0.35$
   $0.061 < W2/W1 < 0.141$ $\quad$ $0.0013 < W3/W1 < 0.0074$
4) $14.2 < L/h < 14.8$ and $0.15 < H/h < 0.35$
   $0.101 < W2/W1 < 0.190$ $\quad$ $0.0063 < W3/W1 < 0.0240$

In the paper by Wilde P. & Wilde M. (2000) an approximate generalization to the case of wave groups (in general irregular waves) is presented. The values of the coefficients W_1 are taken as for regular values. The procedure is written as a power series in complex variables. The result is a first approximation to the solution of the irregular non-stationary case.

The elaborated computer program for wave generation needs the following input data:

- depth of water h,
- length of wave L,
- number of waves in each section of a wave group (a section of fading-in, a section of stability and a section of fading-out),
- tempo of fading-in and fading-out,
- amplitudes of harmonic components (W_1, W_2, W_3),
- sampling frequency.

The fading-in and fading-out sections (characterized by gentle increasing and decreasing of amplitudes) are to avoid disturbances caused by dynamic starting and breaking of the wavemaker's piston. An example of a typical wave group is shown in Fig. 4.

4. Experiments and Results

The following problems have been included in investigations:

- movement of wavemaker's piston,
- wave profiles at 6 points along the flume,

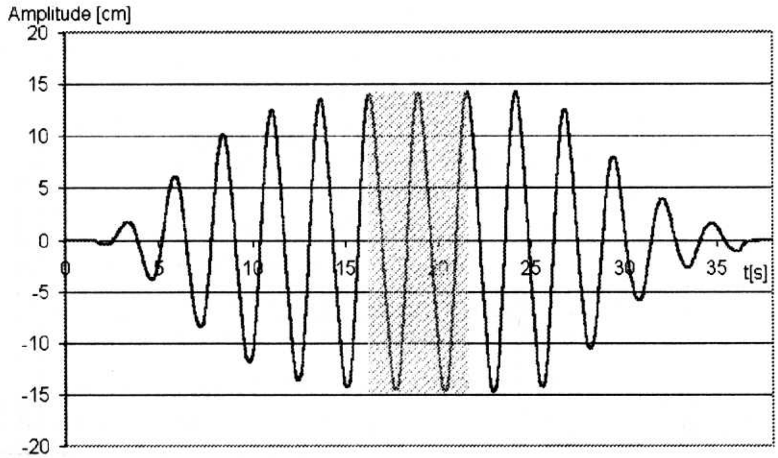

Fig. 4. Typical generation of waves in a group form

– components of the orbital wave velocities at 3 different levels of selected cross section of the flume.

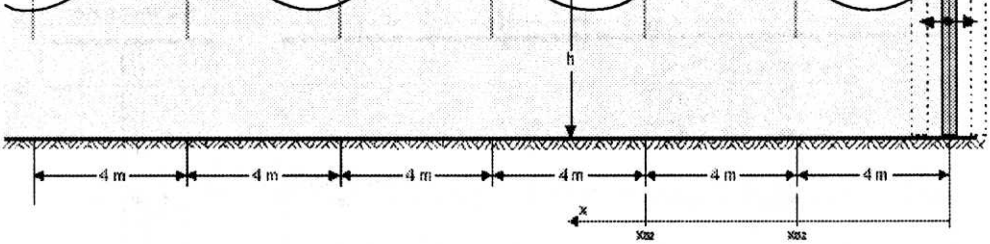

Fig. 5. Location of wave gauges

All the measurements were performed for water depth $h = 60$ cm. The time series of piston motion differed in each input file (wave length amplitudes of components). To create the possibility of stopping the measurements before the small reflected waves reach the gauges, experimental equipment was located in the vicinity of the wavemaker. Displacements of the wavemaker's piston were

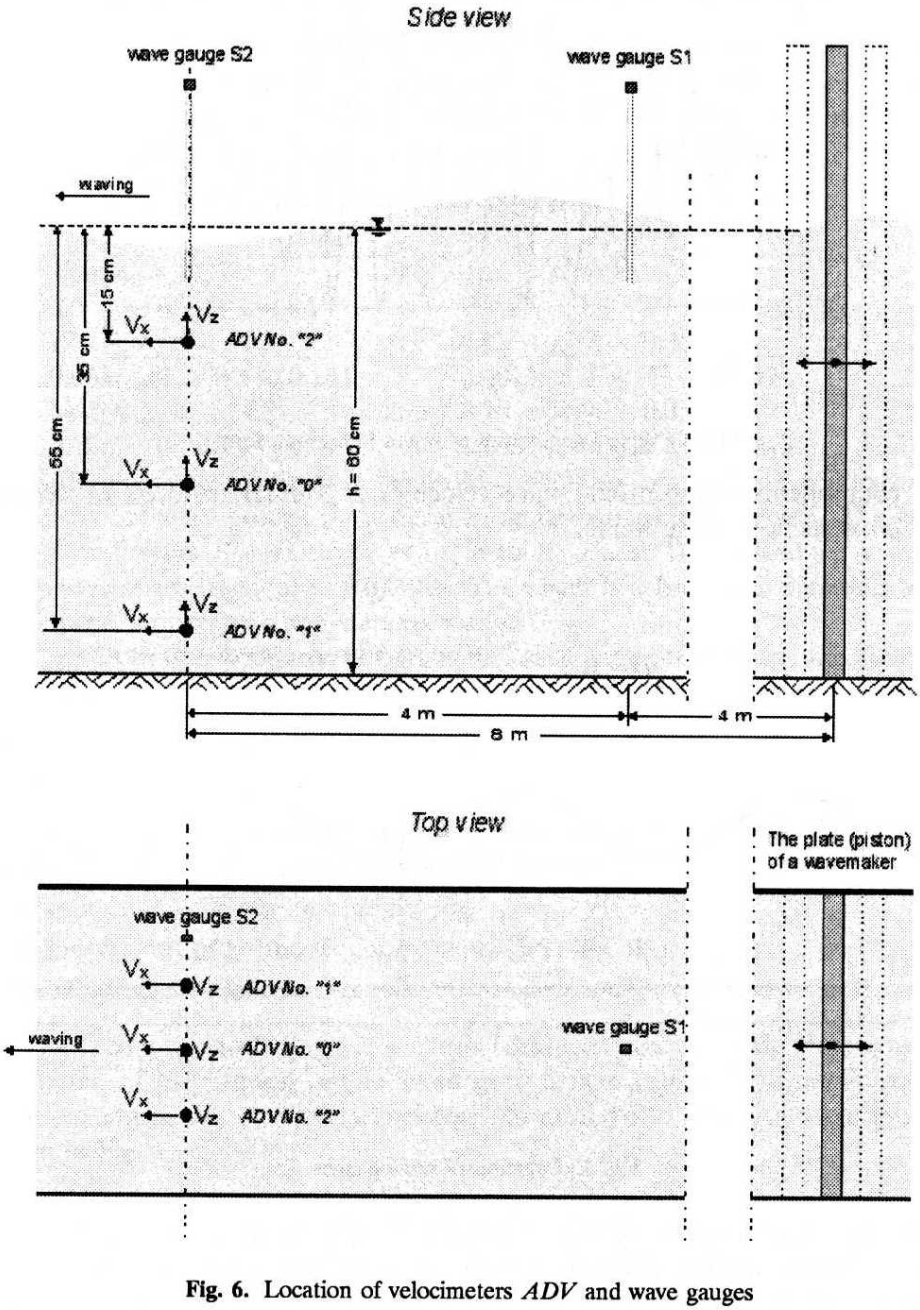

Fig. 6. Location of velocimeters *ADV* and wave gauges

measured by one inductive gauge. Simultaneously, the wave profiles have been measured by 6 wave gauges ($S1$, $S2$, $S3$, $S4$, $S5$, $S5$), placed along the flume, as shown in Fig. 5. They have been spaced at 4 m distances from each other, starting from the wavemaker's piston. To measure orbital wave velocities, *ADV* velocimeters have been used (*Acoustic Doppler Velocimeters,* type 3D, enable us to measure 3 components of the velocity vector). A cross section of velocity measurment was situated 8 m from the wavemaker's piston. The location of *ADV* velocimeters is presented in Fig. 6.

As shown in the picture, velocities were taken at 3 levels; we can describe them by the relation of submersion z to water depth h:

- upper *ADV* velocimeter (No. 2) $-z/h = 0.250$
- middle *ADV* velocimeter (No. 0) $-z/h = 0.583$
- lower *ADV* velocimeter (No. 1) $-z/h = 0.917$

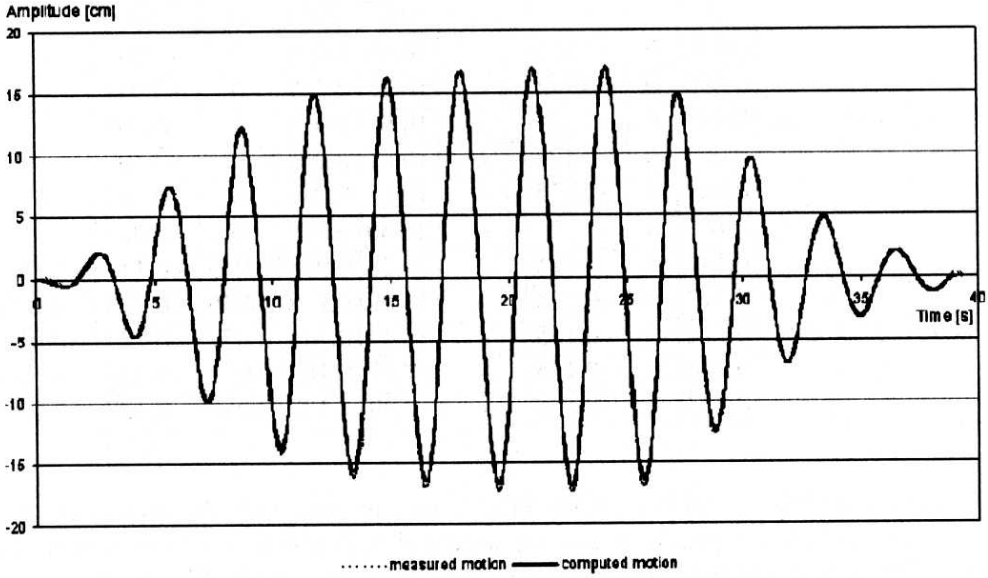

Fig. 7. Measured and computed wavemaker piston motion

Upper *ADV* velocimeter No. 2 was submerged below the wave trough (always remaining in water). In the cross-section where velocities were measured a wave gauge was installed (Fig. 6). Thus at the same point in space the surface displacement and velocities were measured. As mentioned earlier, the motion of the piston is controlled by a time series fed into the wavemaker's hydraulic system. It is important to know how efficient the wavemaker is. The comparison of calculated time series data and measured motion of the piston is shown in Fig. 7. For the assigned parameters of generated wave, the piston's motion was programmed in two ways: as simple harmonic motion ($W_1 \neq 0$, $W_2 = W_3 = 0$) and complex harmonic motion, using 5 different values of W_2/W_1. The piston motion was measured by an inductive gauge. An approximation by a Fourier series was applied to two wave periods from the stationary interval and the three harmonic components were calculated (amplitudes and phase shifts). The phase shift between components with amplitudes W_1 and W_2 is described by the dimensional variables $(t_{p2} - t_{p1})/T$ and $(t_{p3} - t_{p1})/T$. $(t_{p3} - t_{p1})/T$ corresponds to the dimensionless phase shift between the third and first components. The example of programmed and recorded characteristics of piston's motion for two different wave profiles is shown in Table 1.

Table 1. Selected computed and measured characteristics of the wavemaker piston's motion

T [s]	L/H	Computed W_1 [cm]	$\frac{W_2}{W_1}$	$\frac{W_3}{W_1}$	$\frac{t_{p2}-t_{p1}}{T}$	$\frac{t_{p3}-t_{p1}}{T}$	Measured W_1 [cm]	$\frac{W_2}{W_1}$	$\frac{W_3}{W_1}$	$\frac{t_{p2}-t_{p1}}{T}$	$\frac{t_{p3}-t_{p1}}{T}$
2.165	8.04	10.6	0	0	–	–	10.67	0.012	0	−0.080	–
			0.030	0	−0.125	–	10.69	0.026	0	−0.091	–
			0.038	0	−0.125	–	10.70	0.029	0	−0.091	–
			0.045	0	−0.125	–	10.68	0.035	0	−0.098	–
			0.053	0	−0.125	–	10.67	0.042	0	−0.102	–
			0.061	0	−0.125	–	10.65	0.048	0	−0.103	–
3.098	12.53	20.7	0	0	–	–	20.60	0.015	0	−0.020	–
			0.142	0	−0.125	–	19.28	0.098	0.043	−0.113	−0.022
			0.177	0	−0.125	–	18.89	0.118	0.046	−0.120	−0.022
			0.212	0	−0.125	–	18.44	0.144	0.057	−0.122	−0.023
			0.248	0	−0.125	–	17.77	0.168	0.057	−0.120	−0.030
			0.283	0	−0.125	–	17.49	0.197	0.063	−0.127	−0.025

In view of Fig. 7 and data from Tab. 1, we can state that the measured motion of wavemaker's piston is not an identical reproduction of the calculated one. The differences between resultant amplitudes are small (Fig. 7). However, greater differences appear between computed and measured harmonic components of the motion (W_1, W_2, W_2). For stronger nonlinearities (bigger L/h and H/h) the differences are greater. This is due to the interaction of wavemakers with the generated waves. Thus when the waves are calculated for verification by a numerical algorithm it is better to take measured displacements of the piston in the boundary conditions and not the theoretical time series. The characteristics of waves were determined from the measured wave profiles (Fig. 5). For the analysis, the profile of two adjoining waves from the middle section of a wave group (the section of stable profile) was taken (Fig. 3). It was possible to learn, how the

Table 2. Changes of wave harmonic components ($L/h = 8.11$) along the propagation path x/L at various W_2/W_1 of generation

Generation		Wave characteristics						
W_1 [cm]	$\frac{W_2}{W_1}$	$\frac{x_{Si}}{L}$	H_{Si} [cm]	$\frac{2A_i}{H_{Si}}$	$\frac{A_2}{A_1}$	$\frac{A_3}{A_1}$	$\frac{A_4}{A_1}$	$\frac{A_5}{A_1}$
		0.823	23.2	0.803	0.498	0.206	0.089	0.037
		1.646	21.6	0.986	0.149	0.032	0.003	0.005
		2.469	23.7	0.799	0.460	0.176	0.075	0.033
14.27	0.000	3.292	21.7	0.936	0.264	0.083	0.010	0.010
		4.115	21.8	0.895	0.332	0.105	0.038	0.014
		4.938	21.4	0.912	0.295	0.085	0.028	0.019
		0.823	22.5	0.840	0.449	0.160	0.057	0.018
		1.646	21.4	0.974	0.190	0.034	0.002	0.005
		2.469	21.7	0.879	0.404	0.135	0.048	0.016
14.09	0.032	3.292	21.3	0.939	0.261	0.070	0.019	0.008
		4.115	21.5	0.904	0.325	0.100	0.033	0.009
		4.938	20.9	0.929	0.305	0.089	0.027	0.004
		0.823	21.5	0.881	0.432	0.137	0.040	0.003
		1.646	21.3	0.968	0.212	0.040	0.001	0.005
		2.469	21.6	0.889	0.383	0.122	0.041	0.013
14.07	0.041	3.292	21.2	0.939	0.263	0.068	0.018	0.014
		4.115	21.3	0.910	0.320	0.094	0.029	0.008
		4.938	20.8	0.931	0.298	0.085	0.026	0.007
		0.823	21.1	0.905	0.412	0.114	0.029	0.005
		1.646	21.0	0.968	0.231	0.047	0.000	0.014
		2.469	21.6	0.894	0.362	0.112	0.037	0.011
13.95	0.055	3.292	21.2	0.933	0.265	0.181	0.018	0.000
		4.115	21.3	0.905	0.319	0.093	0.029	0.009
		4.938	20.9	0.921	0.290	0.080	0.025	0.009

characteristics of two selected waves change along the flume – by analysing the records from wave gauges $S1 - S6$. The distance of propagation was determined by a multiple of wave length L (ratio x_{si}/L). The heights of waves recorded by each wave gauge Si ($i = 1, 2, \ldots, 6$) were denoted as H_{si}. The measured motions of the wavemaker's piston and related profiles of generated waves – changing along the propagation path – were aproximated using the harmonic components W_i and A_i ($i = 1, 2, \ldots, 6$). For example, the selected results are shown in Tables 2 and 3. The data presented in Tab. 2 are related to the wave characterized by dimensionless length $L/h = 8.11$ and approximately stable height H. The data in Tab. 3 are related to the wave characterized by $L/h = 12.53$ and also approximately stable height. Both types of waves were generated in various ways, changing the values of the second component W_2 in characteristics of piston motion. The case of $W_2 = 0$

(sinusoidal motion) has also been considered. For each of the generation methods, the selected profiles of two adjoining waves – crossing through the gauges from $S1$ to $S6$ (from x_{S1} to x_{S2}) – were approximated applying 5 harmonic components (amplitudes A_i, $i = 1, 2, \ldots, 5$).

Table 3. Changes of wave harmonic components ($L/h = 12.53$) along the propagation path x/L at various W_2/W_1 of generation

Generation		Wave characteristics						
W_1 [cm]	$\frac{W_2}{W_1}$	$\frac{x_{Si}}{L}$	H_{Si} [cm]	$\frac{2A_i}{H_{Si}}$	$\frac{A_2}{A_1}$	$\frac{A_3}{A_1}$	$\frac{A_4}{A_1}$	$\frac{A_5}{A_1}$
20.6	0.000	0.532	24.2	0.774	0.407	0.235	0.105	0.046
		1.064	25.6	0.576	0.874	0.366	0.194	0.095
		1.596	25.3	0.581	0.844	0.405	0.199	0.096
		2.128	23.5	0.781	0.406	0.223	0.084	0.035
		2.660	19.8	1.013	0.146	0.033	0.020	0.000
		3.191	23.0	0.765	0.464	0.230	0.097	0.041
19.28	0.089	0.532	21.8	0.842	0.418	0.142	0.022	0.064
		1.064	24.5	0.664	0.556	0.361	0.175	0.094
		1.596	23.4	0.689	0.632	0.301	0.136	0.068
		2.128	21.3	0.827	0.441	0.144	0.048	0.017
		2.660	20.1	0.915	0.293	0.078	0.025	0.004
		3.191	20.9	0.826	0.424	0.174	0.069	0.027
18.44	0.144	0.532	22.4	0.787	0.418	0.212	0.085	0.046
		1.064	24.8	0.677	0.408	0.345	0.075	0.088
		1.596	21.2	0.784	0.509	0.265	0.128	0.065
		2.128	20.2	0.843	0.465	0.119	0.028	0.024
		2.660	20.2	0.863	0.393	0.130	0.050	0.021
		3.191	19.9	0.853	0.415	0.159	0.060	0.023
17.49	0.197	0.532	23.58	0.714	0.416	0.288	0.160	0.090
		1.064	24.7	0.693	0.289	0.324	0.182	0.092
		1.596	22.2	0.764	0.404	0.248	0.135	0.067
		2.128	19.4	0.841	0.484	0.139	0.115	0.007
		2.660	19.6	0.832	0.488	0.173	0.069	0.027
		3.191	19.7	0.838	0.425	0.166	0.064	0.024

The ratio $2A_i/H_{Si}$ has been calculated. For a linear wave and perfect second order Stokes wave it should be one. The data from Tables 2 and 3 give an insight into the transformation of amplitudes of wave components along the propagation path. The transformations depend upon the choice of the W_2/W_1 ratio in the horizontal piston motion.

Examples of the variability of wave profiles along the propagation path, for dimensionless lengths $L/h = 10.3$ and $L/h = 14.8$ are presented in Figs. 8a and

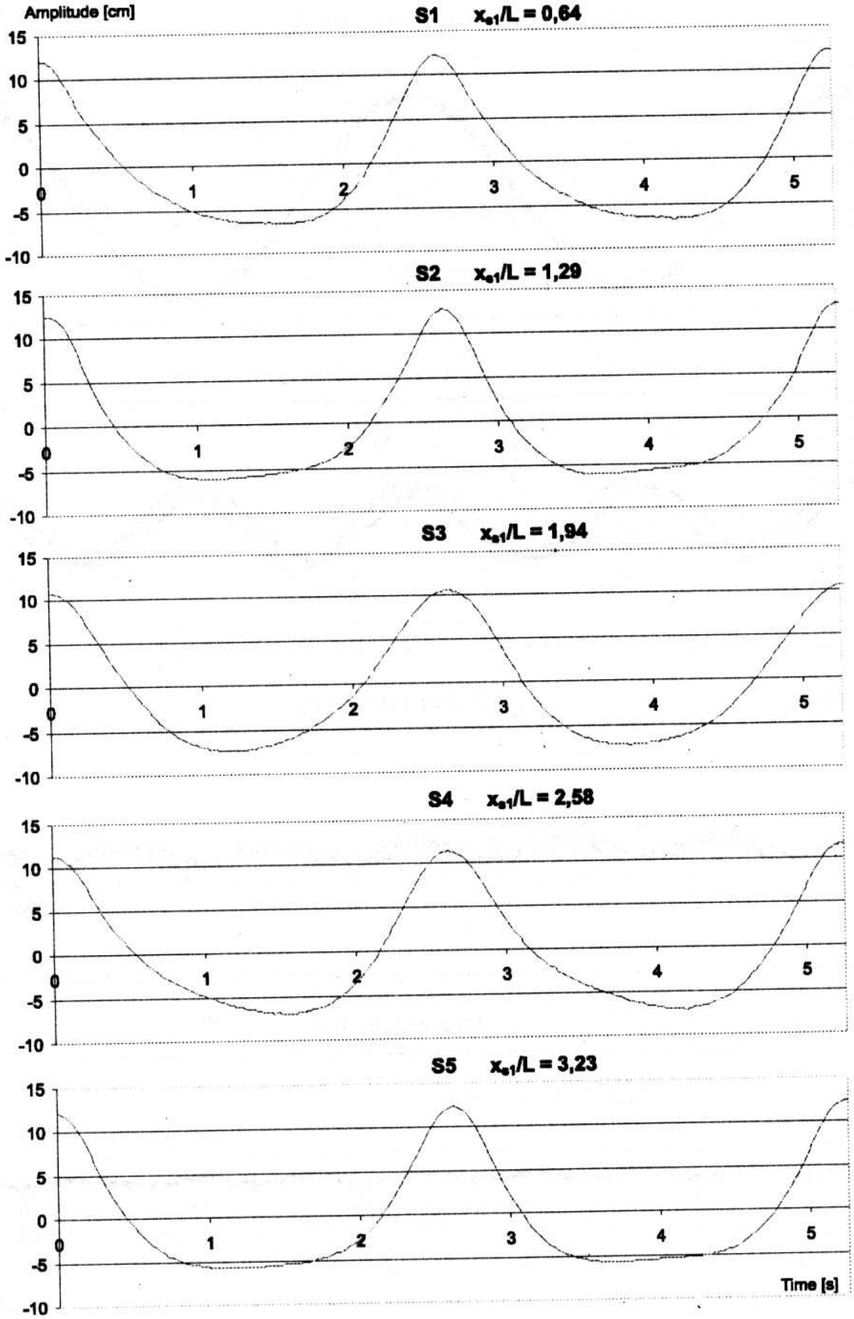

Fig. 8a. The transformation of wave profile ($L/h = 10.3$) on the propagation path

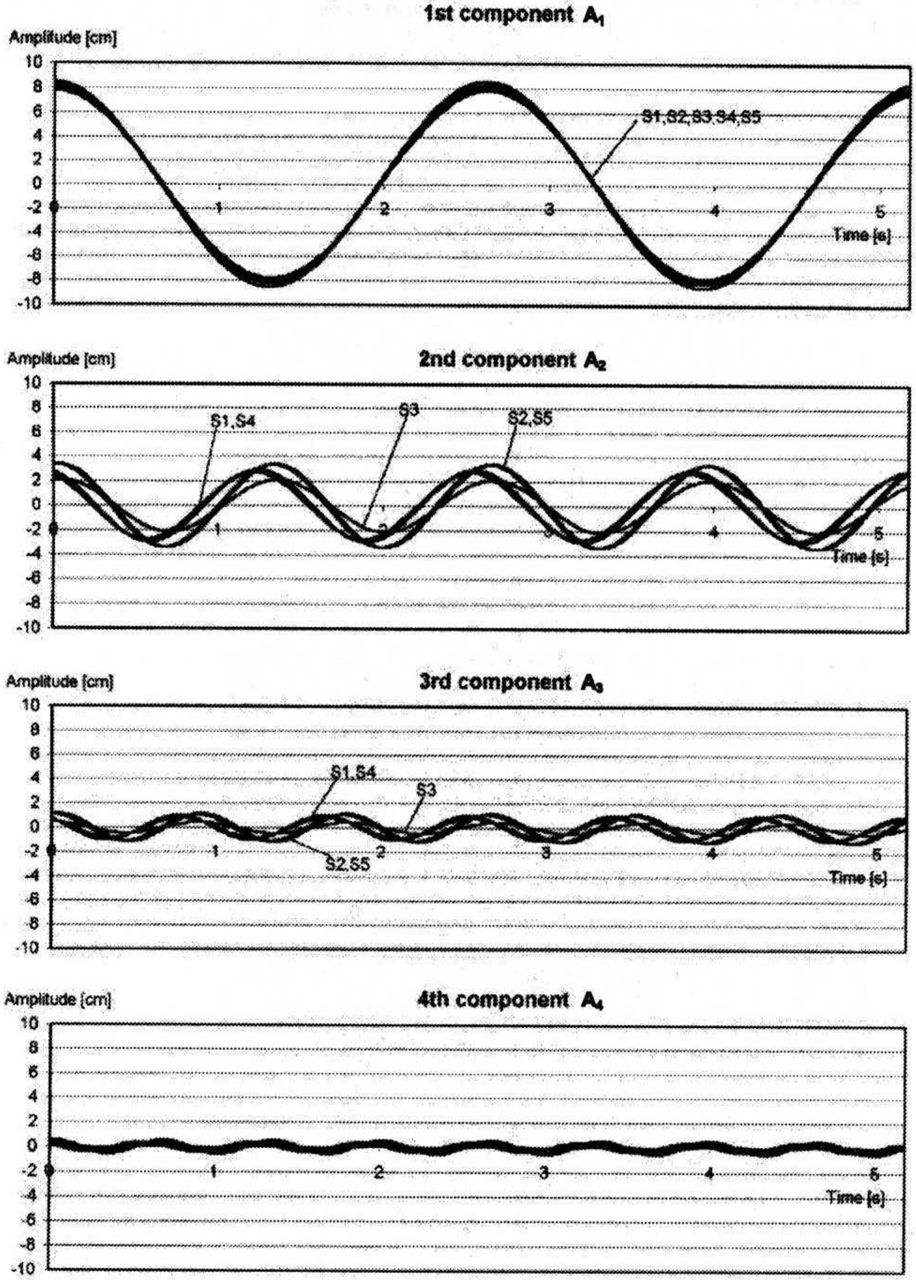

Fig. 8b. The harmonic components of wave profiles $(L/h = 10.3)$ at wave gauges$S1 - S5$

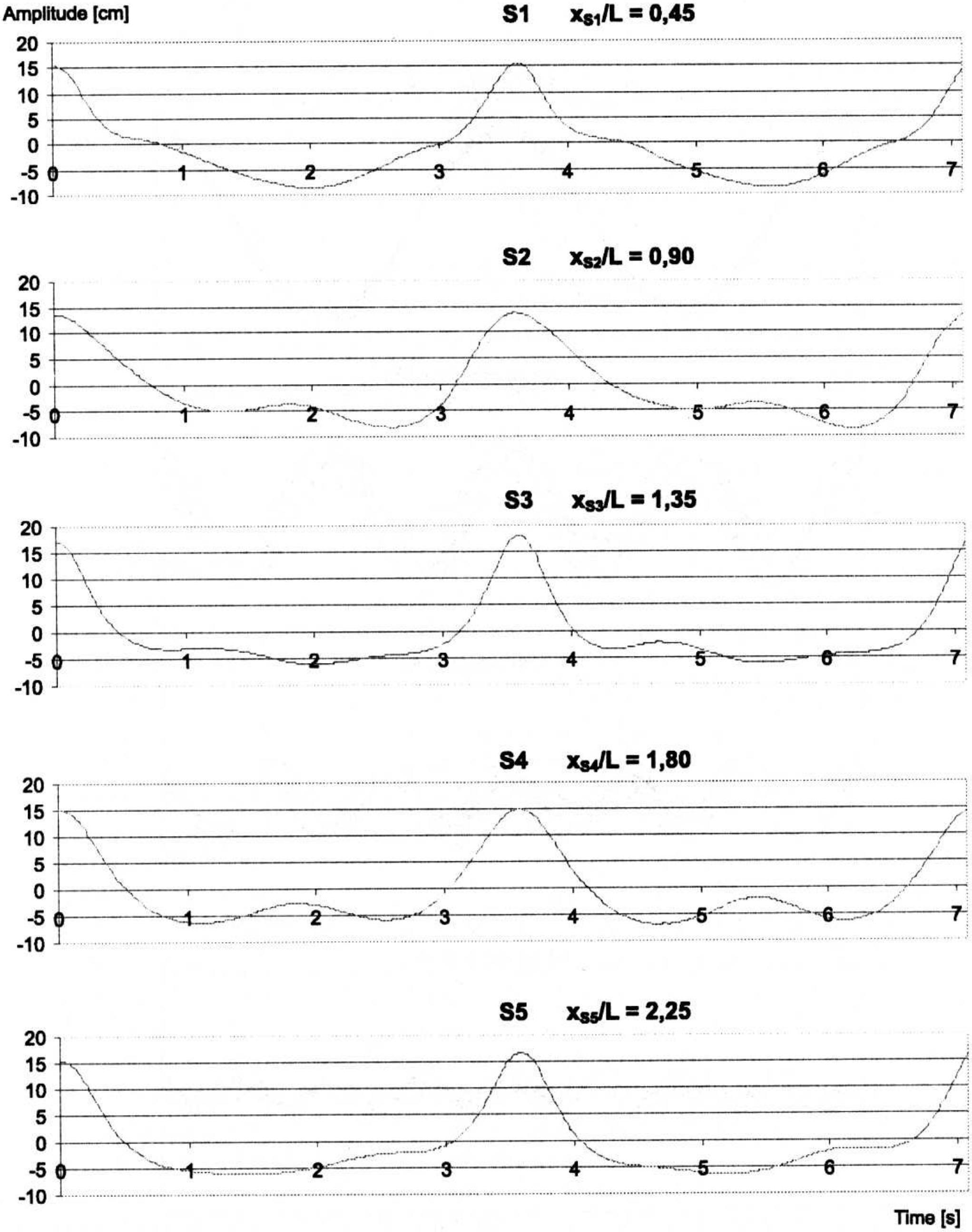

Fig. 9c. The changing profile of wave $L/h = 14.8$ measured on the propagation path

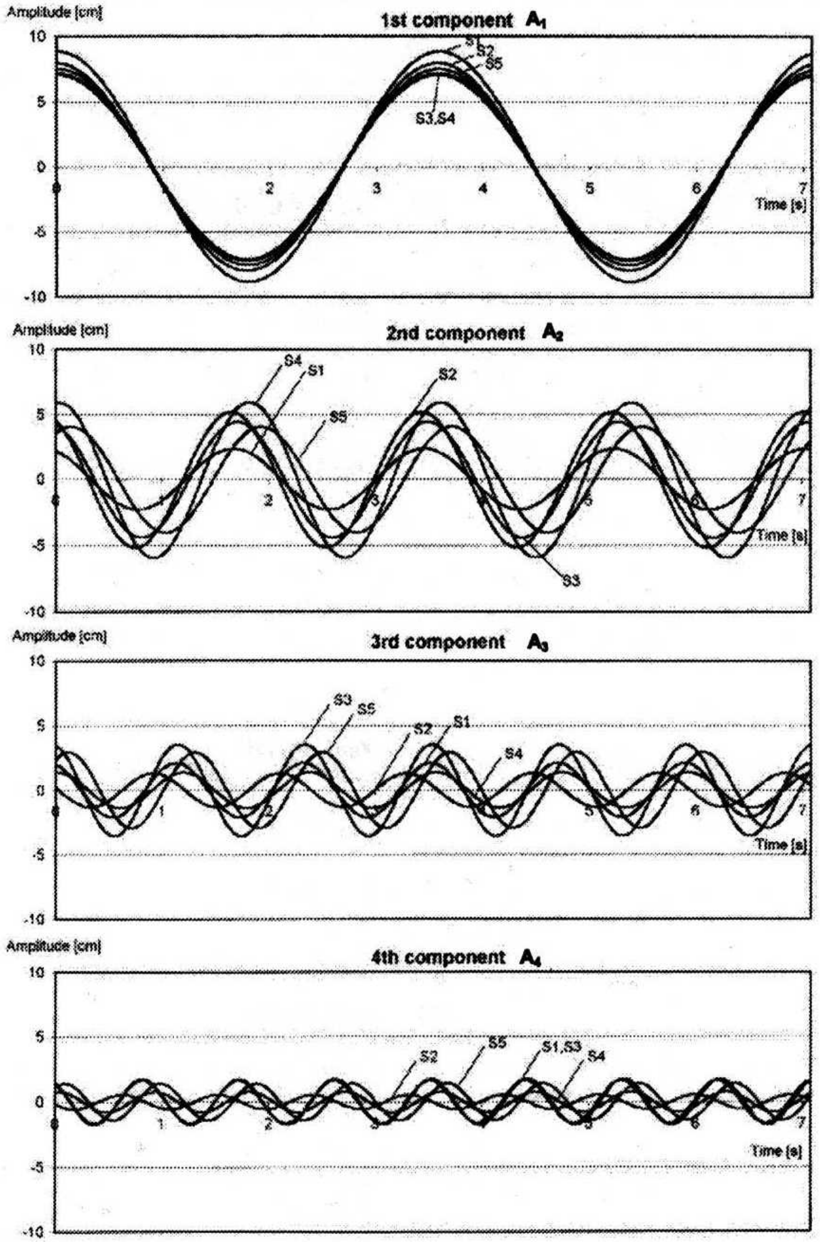

Fig. 9d. The harmonic components of wave profiles $(L/h = 14.8)$ at wave gauges $S1 - S5$

9a. For analysis, the profiles of two adjoining waves from the middle sections (characterized by stable wave height) of wave groups recorded at wave gauges $S1 - S6$ were taken. They were approximated by four harmonic components A_1, A_2, A_3 and A_4. The results of approximation are presented in Figs. 8b and 9b.

Taking into consideration the data presented in Tab. 2 and 3 and charts in Figs. 8a, 8b, 9a and 9b we can note that the amplitudes of harmonic components change along the propagation path. The ratios of these amplitudes change and there are phase shifts between them. We can thus conclude that there are transfers of energy between harmonic components. The more waves are non-linear (bigger L/h and H/h), the more intensive the phenomenon. This depends also on the method of wave generation. The variability of the wave harmonic components is greater in the case of sinusoidal wavemaker piston motion ($W_2 = 0$) and smaller when the motion is complex ($W_2 \neq 0$).

As already mentioned, wave orbital velocities were also investigated (Fig. 6). The problem discussed is two-dimensional, thus only horizontal (V_x) and vertical (V_z) components of the velocity vector are considered. The example of the recorded velocity components of waves characterized by ratio $L/h = 12.07$, generated in group form, is shown in Fig. 10. The results of measurements have proved the property shown in Fig. 10, that in the case of shallow water waves the distribution of the horizontal velocities can be treated as rectangular. The vertical component of velocity behaves differently, classically changing from a maximum value near to the water surface and falling to zero at the bottom. The measured horizontal velocities V_x have also been approximated by harmonic components, according to the Stokes solution (equation 4). The examples of approximation of the horizontal velocity V_x, measured at the level $z/h = 0.25$ and related wave characterized by $L/h = 12.22$ and $H/h = 0.23$, are shown in Fig. 11. In the result of approximation, we obtained three components W_1, W_2, W_3 and the "rest". The correctness of such analysis is shown by the graphic comparison of measured and approximated velocities in Fig. 11. The exemplary results of analysis of horizontal velocities V_x in the relation with wave parameters are given in Tab. 4. The profiles of two waves ($L/h = 12.53$ and $L/h = 14.80$) and the related horizontal velocities V_x at the 3 levels were approximated by 5 components W_1, W_2, W_3, W_4 and W_5. Considering the results given in Tab. 4, it can be stated that higher harmonic components (W_2, W_3, W_4, ...) influence the phenomenon more in the description of wave profile than of the horizontal velocity components V_x. In the description of V_x, the influence of higher harmonic components is the biggest in the upper layer of water and decreases towards the bottom.

As already mentioned, the results of laboratory experiments have also been used to verify the theoretical description and numerical solution for long waves generated in the flume (Wilde, Chybicki 2000). For the programmed (assumed) motion of wavemaker's piston, the numerically computed profiles have been compared with the measured ones. The example of such comparison, for the wave

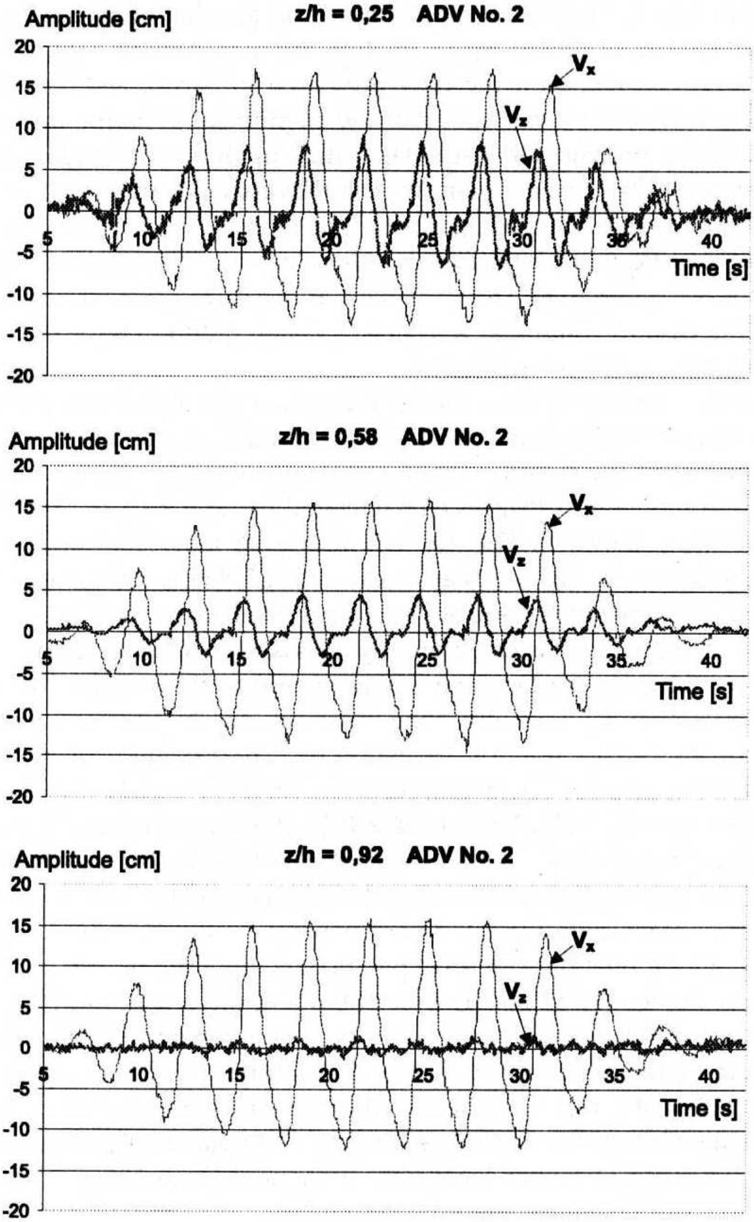

Fig. 10. The example of components V_x and V_z of the wave orbital velocities measured at three different levels z/h $(L/h = 12.07)$

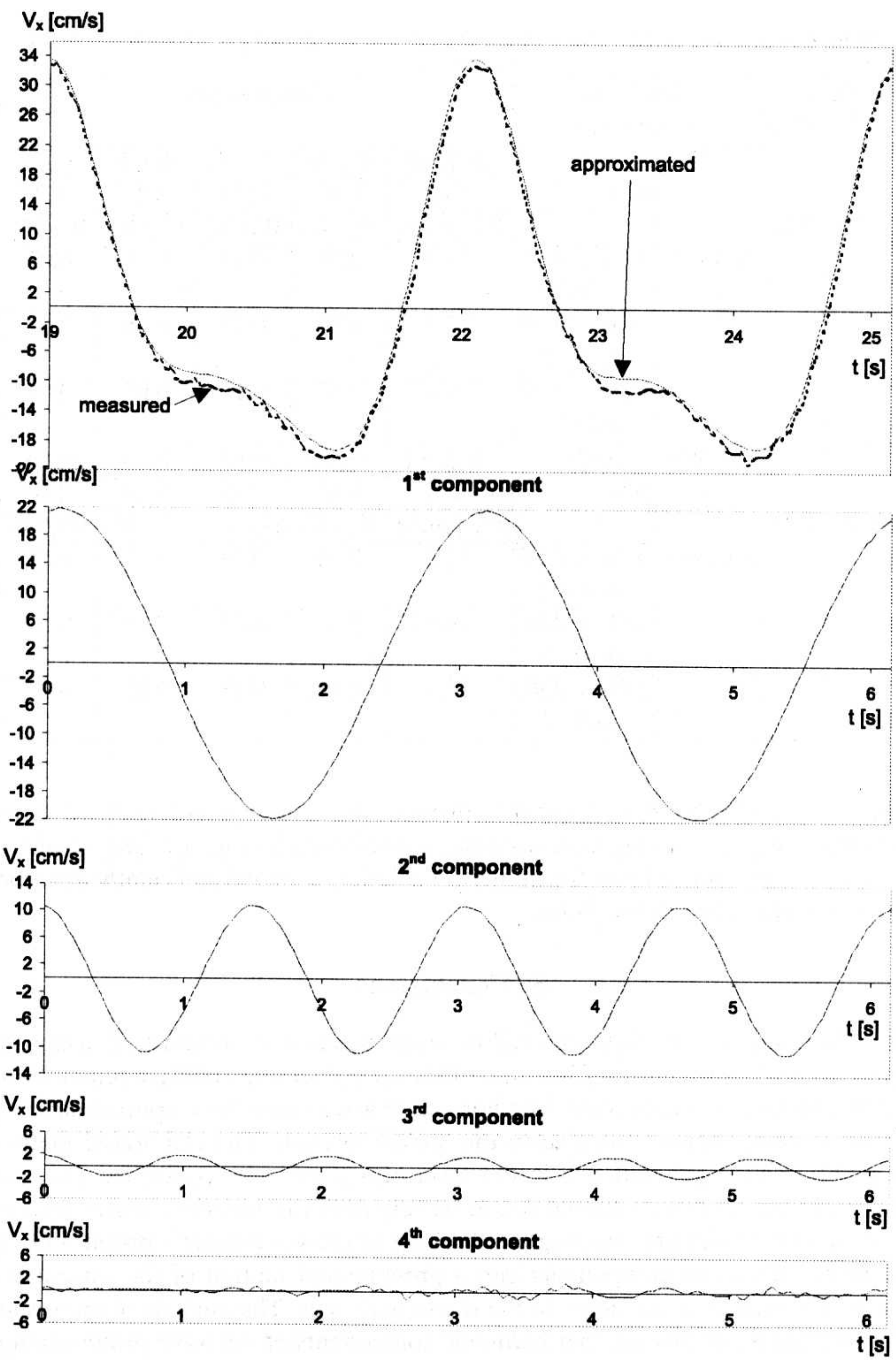

Fig. 11. Comparison of measured and approximated velocity V_x at level $zz/h = 0.25$

(wave $L/h = 12.22$, $H/h = 0.23$)

Table 4. Harmonic characteristics of wave profiles and velocities V_x at different levels z/h

<table>
<tr><th colspan="2">Wave</th><th colspan="2" rowspan="2">Analysed parameter</th><th colspan="5" rowspan="2">Components</th></tr>
<tr><th>L/h</th><th>H/h</th></tr>
<tr><td rowspan="6">12.53</td><td rowspan="6">0.44</td><td colspan="2" rowspan="2">Wave profile (S2)</td><td>A_1 [cm]</td><td>A_2/A_1</td><td>A_3/A_1</td><td>A_4/A_1</td><td>A_5/A_1</td></tr>
<tr><td>7.40</td><td>0.89</td><td>0.39</td><td>0.21</td><td>0.10</td></tr>
<tr><td rowspan="4">Velocity</td><td></td><td>W_1 [cm/s]</td><td>W_2/W_1</td><td>W_3/W_1</td><td>W_4/W_1</td><td>W_5/W_1</td></tr>
<tr><td>$z/h = 0.25$ ADV 2</td><td>28.4</td><td>0.87</td><td>0.28</td><td>0.11</td><td>0.03</td></tr>
<tr><td>$z/h = 0.58$ ADV 0</td><td>27.7</td><td>0.72</td><td>0.19</td><td>0.06</td><td>0.02</td></tr>
<tr><td>$z/h = 0.92$ ADV 1</td><td>26.0</td><td>0.67</td><td>0.17</td><td>0.05</td><td>0.01</td></tr>
<tr><td rowspan="6">14.80</td><td rowspan="6">0.43</td><td colspan="2" rowspan="2">Wave profile (S2)</td><td>A_1 [cm]</td><td>A_2/A_1</td><td>A_3/A_1</td><td>A_4/A_1</td><td>A_5/A_1</td></tr>
<tr><td>7.75</td><td>0.69</td><td>0.46</td><td>0.20</td><td>0.10</td></tr>
<tr><td rowspan="4">Velocity</td><td></td><td>W_1 [cm/s]</td><td>W_2/W_1</td><td>W_3/W_1</td><td>W_4/W_1</td><td>W_5/W_1</td></tr>
<tr><td>$z/h = 0.25$ ADV 2</td><td>31.3</td><td>0.64</td><td>0.38</td><td>0.12</td><td>0.05</td></tr>
<tr><td>$z/h = 0.58$ ADV 0</td><td>29.8</td><td>0.57</td><td>0.28</td><td>0.08</td><td>0.03</td></tr>
<tr><td>$z/h = 0.92$ ADV 2</td><td>28.7</td><td>0.55</td><td>0.25</td><td>0.07</td><td>0.02</td></tr>
</table>

$L/h = 12.37$, generated in a complex manner (W_1 and W_2) and analysed at the 3 points along the propagation distance, is presented in Fig. 12. The results of verification have proved that the numerical solution obtained sufficiently describes the waves generated in the flume.

5. Conclusions

The laboratory experiments on shallow water waves were performed, using the highly accurate wave flume of Institute of Hydroengineering (IBW), equipped with the programmable wavemaker. The non-linear waves have been approximated by harmonic components according to the Stokes solution. The elaborated method of modified wave generation – in the form of a group with controlled fading-in and fading-out sections – turned out to be very useful in minimization of disturbances caused by dynamic starting and braking of the wavemaker's piston. Putting the higher harmonic components into a programmed motion of the wavemaker results in waves, that are closer to the theoretical ones. The analysis of the results of experiments has proved, that harmonic components of the wave profile change along the propagation path. The changes embrace the amplitudes of components,

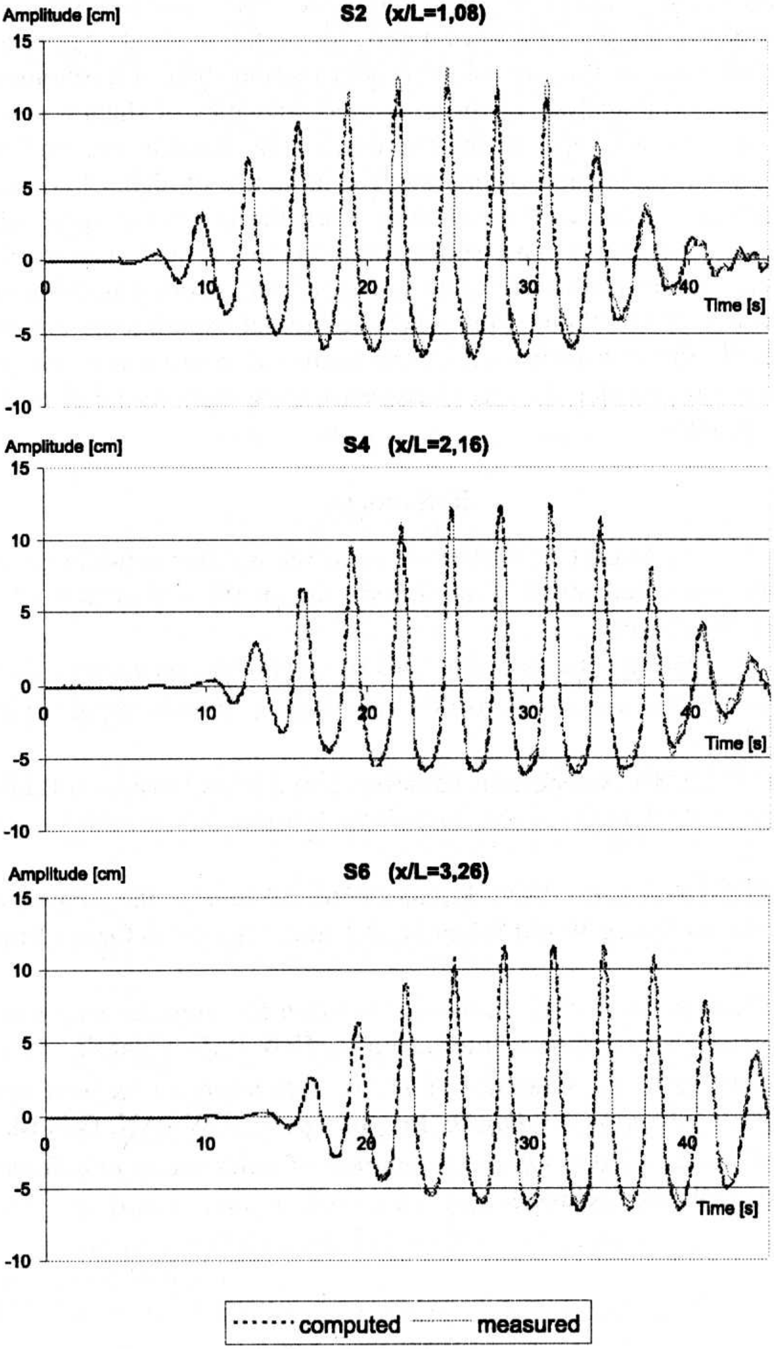

Fig. 12. Comparison of numerically computed and measured wave profiles ($L/h = 12.37$)

their ratios and phase shifts between them. Thus we can state that there is a transfer of energy between the harmonic components. The more nonlinear the wave, the more intensive the phenomenon; it depends on the method of generation also – it's more intensive in the case of sinusoidal motion than in harmonically complex generation. The analysis of the measured velocities of shallow water waves has shown that the assumption that the horizontal velocities are uniform along the depth is a reasonable approximation. The influence of higher harmonics components decreases in horizontal velocities when the bottom is approached. The results of the experiments have shown that the theory and numerical procedure developed in our institute describe sufficiently the strongly non-linear shallow waves generated in the flume. Howevher, additional experiments are needed to study in details the transformation of the harmonic components. At present, it can be stated, that smaller distances between wave gauges would give us a better view of the problem.

References

Bendykowska G., Massel S. (1988), On the theory and experiment of mechanically generated waves, *Proc. Intern. Symp. Wave Research and Coastal Engng*, Hannover 1.

Chybicki W. (2000), *Opracowanie do III rzędu przybliżonego rozwiązania swobodnej powierzchni w opisie Lagrange'a*, Intern. report, IBW PAN, Gdańsk.

Sobierajski E. (2000), Nowoczesny badawczy kanał falowy Instytutu Budownictwa Wodnego PAN w Gdańsku, *Inżynieria Morska i Geotechnika*, nr 2/2000, Gdańsk.

Wilde P., Kozakiewicz A. (1993), *Kalman Filter Method in the Anaysis of Vibrations due to Waves*, World Scientific, Advanced Series on Ocean Engineering – Vol. 6.

Wilde P., Chybicki W. (2000), *Numerical solution for irregular long water waves in Lagrange's description*, Intern. report, IBW PAN, Gdańsk.

Wilde P., Sobierajski E., Sobczak Ł. (2000), *Laboratoryjne badania wpływu generacji na własności fal długich*, Intern. report, IBW PAN, Gdańsk.

Wilde P., Wilde M. (2000), *On the generation of water waves in a flume*, Intern. report, IBW PAN, Gdańsk.