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Hydraulic Investigation of Venturi Flumes

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Abstract

Venturi flume is a very popular flow measuring device. A particularly attractive version of such a device is a standing-wave-flume, as this enables determination of liquid discharge by one-point depth measurement only. However, it is necessary to underline, that some essential simplifications must be introduced into the theoretical description of this case. The paper contains the results of the control investigations of six existing Venturi flumes, installed in three functionning sewage-disposal-plants in Poland. It was shown, that some important mistakes can be committed, when somebody plans such a flume. These mistakes worsen the flow measurement accuracy. In conclusion it was stated that the standing-wave-flume must be designed very carefully, with the use of gradually-varied flow equations. An alternative, which is worth using, is a Venturi flume with two-point depth measurement.

Notation

b – throat width,
B – width,
B_0 – initial width,
C – empirical constant,
C_s – empirical constant,
Fr – Froude number,
g – gravity acceleration,
h – centre–throat depth,
h_c – critical depth,
h_L – hydraulic loss,
H – depth,
H_0 – initial depth,
i_f – friction slope,
i_0 – bottom slope,

L – length,
n – empirical exponent,
n_M – Manning coefficient,
n_t – theoretical exponent,
p_{atm} – atmospheric pressure,
Q – discharge,
R_H – hydraulic radius,
v – mean velocity,
x, y, z – Cartesian coordinates,
ξ – coeffcient of hydraulic loss,,
μ – discharge coefficient.

1. Introduction

The Venturi flume is a relative of the Venturi tube, which can serve as the water-meter in pipelines. It has several important advantages – very simple construction, relatively low cost, reliability. Such a flume does not contain moving parts and consumes quite a small amount of the hydraulic head, hence it is a very convenient measuring device, especially for polluted liquids. Its accuracy would seem to be not higher than 5% (Engineering Hydraulics 1961).

From the geometrical point of view, there are two main kinds of the Venturi flumes:

- flat-floor measuring flume (i.e. the "proper" Venturi flume (Engineering Hydraulics 1961) – see Fig. 1a);
- throated-flume with a hump (so-called "Parshall–flume" (Parshall 1926) – see Fig. 1b).

The theory of the Venturi flumes is a classical chapter of hydraulics and is one of the most fundamental applications of the Bernoulli equation (Douglas 1996, Fannelop 1994), which for two characteristic flume cross-sections (AA – inflow, BB – throat, see Fig. 1) can be written as follows:

$$\frac{v_A^2}{2g} + \frac{p_{atm}}{\rho g} + H_0 = \frac{v_B^2}{2g} + \frac{p_{atm}}{\rho g} + h + h_L. \tag{1}$$

The energy loss h_L is usually expressed by the empirical coefficient, so we have:

$$h_L = \xi \frac{v_B^2}{2g}. \tag{2}$$

Making use of the continuity equation:

$$Q = v_A B_0 H_0 = v_B b h \tag{3}$$

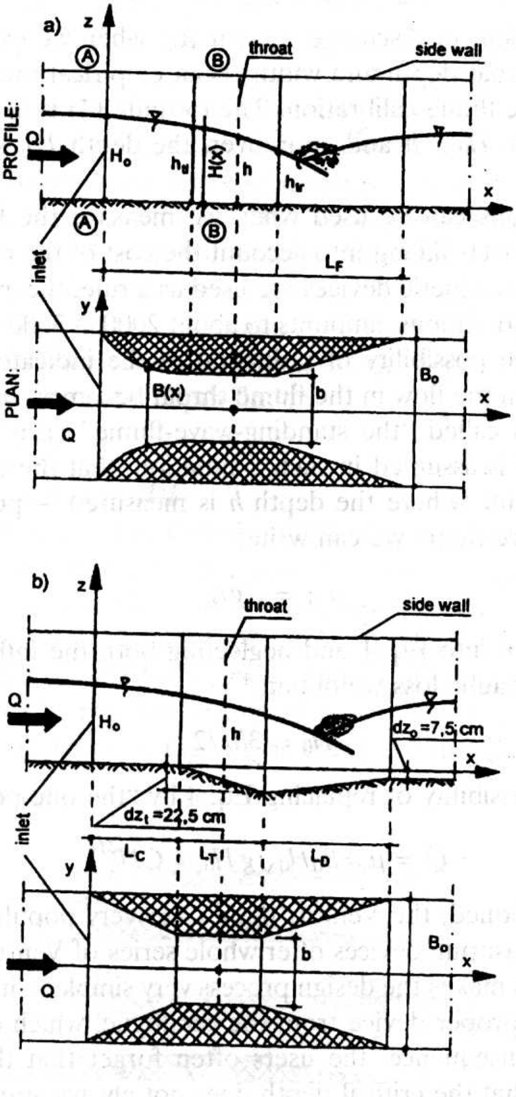

Fig. 1. General scheme of the Venturi (a) and Parshall (b) flume

after some rearranging, we obtain:

$$Q = \mu_H B_0 H_0 \sqrt{2h(H_o - h)} \tag{4}$$

or:

$$Q = \mu_h b h \sqrt{2g(H_o - h)}. \tag{5}$$

The effective coefficient of discharge μ_H (or μ_h, when we express the discharge Q in terms of the throat depth and width) is an empirical multiplier, which must be determined by the flume calibration. The formula (4) is more convenient than (5), as usually $B > b$, $H_0 > h$ and, moreover, the depth H_0 is much more stable than h.

The latter relations can be used when we measure the liquid depth in two points (inlet and throat). Taking into account the cost of the each depth indicator (ultrasonic or electromagnetic devices are used as a rule; the average price of such a unit, in European conditions, amounts to about 2000 – 3000 USD), the investors very often accept the possibility of reduction of the indicators number. Such a possibility exists when the flow in the flume throat becomes supercritical ($h < h_c$). Whole the device is called "the standing-wave-flume" (Engineering Hydraulics 1961) in this case. It is assumed in such a situation, that the critical flow appears in the measuring point (where the depth h is measured – point BB in Fig. 1a). For the standing-wave-flume we can write:

$$v_B = \sqrt{g h_c}. \tag{6}$$

Substituting Eq. 6 into Eq. 1 and neglecting both the influence of the initial velocity and the hydraulic loss we obtain:

$$H_0 = 3h_c/2 \tag{7}$$

which affords the possibility of replacing Eq. 4 by "the one-point formula":

$$Q = \mu_{ef} B_0 H_0 \sqrt{g H_0} = C H_0^{3/2}. \tag{8}$$

As already mentioned, the Venturi flumes are very popular. So popular, that the producers of measuring devices offer whole series of Venturi flume types. The typification of flumes makes the design process very simple – its essence consists in the selection of the proper device from the catalogue, which can be done almost automatically. In consequence, the users often forget that the relation (7) is a simplified one and that the critical depth does not always appear in the centre of the flume throat. In fact, the position of the point where $h = h_c$ changes together with the flow discharge Q. One can accept the situation, when $h_{tl} < h < h_{tr}$ (Fig. 1a), however, sometimes the flow in the throat is subcritical along the whole the flume $h > h_c$, but none the less – the discharge is calculated by means of Eq. 8. This problem is so essential, that it is worthwhide analysing some hydraulic aspects of the Venturi flumes. Such an analysis is presented in this paper.

2. Examples of Venturi Flumes Investigated

General remarks

All considerations presented in this paper are related to the flow parameters, experimentally determined for six Venturi flumes which were installed in three sewage disposal plants (SDP) in Poland – in Gdynia–SDP (FG), Tczew–SDP (FT) and Malbork–SDP (FM). Each unit was designed as a standing-wave-flume. The initial stream was always subcritical $h > h_c$. Inflowing channels were rectangular in shape (Figs. 2, 5, 8). The time-variability of the discharge was typical of sewage treatment plants, viz. was unsteady, but its changes were so slow, that might be neglected during each series of measurements.

The control measurements of the flow parameters were performed using a flow meter, made by the German firm NIVUS (ultrasonic measurement of the discharge and piezometric indicator of depth). The accuracy of this device, determined experimentally in the Hydraulic Laboratory of the Gdańsk Technical University, is 2.5%. During normal exploitation, the depth of waste water in each flume is measured by the echo-depth-finder.

The Venturi flume in Gdynia–SDP

Two objects investigated in the Gdynia–SDP, were designed individually. Their shapes can be placed somewhere between the typical Venturi flumes proposed by the German enterprise Nivus (Venturihalbschalen-Typenubersicht), and the Polish series of types - UNICLAR–77 (UNIKLAR–77). Dimensions of these flumes are shown in Fig. 2 (FG1 – inlet channel of crude waste water, FG2 – outflow of purified medium). The results of control measurements, i.e. the set of experimental points $Q(H_0)$ – in Fig. 3 (for FG1) and in Fig. 4 (for FG2).

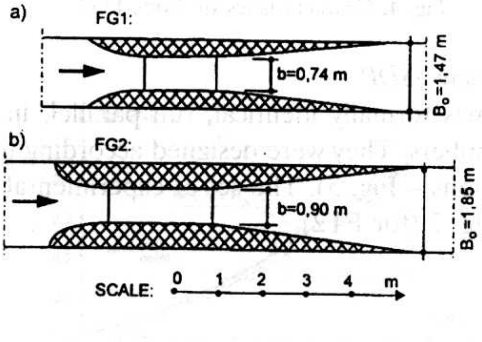

Fig. 2. The Venturi flumes in Gdynia – SDP

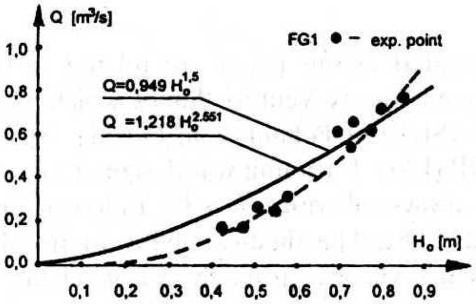

Fig. 3. Characteristics of flume FG1

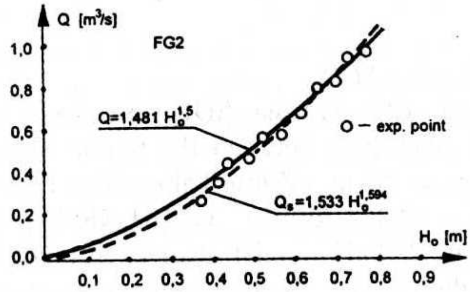

Fig. 4. Characteristics of flume FG2

The Venturi flume in Tczew–SDP

Here, two Venturi flumes formally identical, run parallel, in two inlet channels, just behind the grit chambers. They were designed according to some vague Czech series of types (dimensions – Fig. 5). The set of experimental points $Q(H_0)$ – see Fig. 6 (for FT1) and Fig. 7 (for FT2).

The Venturi flume in Malbork–SDP

The dimensions of both FM1 and FM2 flumes (see Fig. 8) were determined according to UNIKLAR-77. The control measurements data – see Fig. 9 (FM1) and Fig. 10 (FM2).

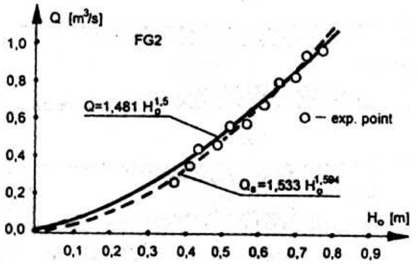

Fig. 5. The Venturi flume in Tczew – SDP

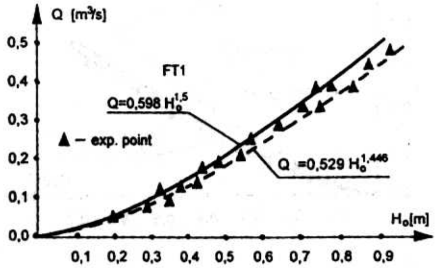

Fig. 6. Characteristics of flume FT1

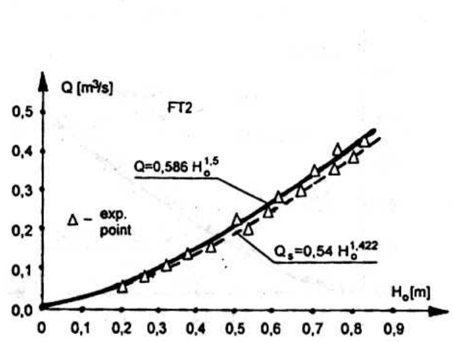

Fig. 7. Characteristics of flume FT2

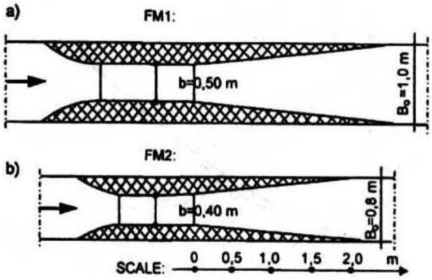

Fig. 8. The Venturi flume in Malbork – SDP

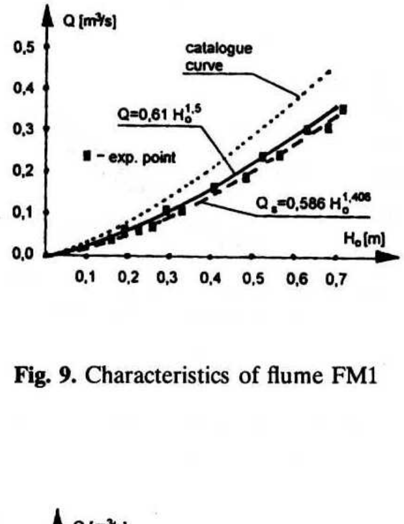

Fig. 9. Characteristics of flume FM1

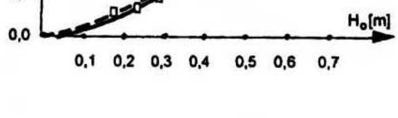

Fig. 10. Characteristics of flume FM2

3. Hydraulic Analysis of Tested Flumes

Determination of characteristic curves

As already mentioned, all investigated units were constructed as standing-wave-flumes, which means that their characteristic curves should be described by Eq. 8. So, in the first order, the experimental data described above, were fitted by the proper functions, using the standard least square method. The following characteristic relations were obtained ($[H_0]$ = m, $[Q]$ = m^3/s):

$$\mathbf{FG1}: Q = 0.949 H_0^{1.5} \tag{9}$$

$$\mathbf{FG2}: Q = 1.481 H_0^{1.5} \tag{10}$$

$$\mathbf{FT1}: Q = 0.598 H_0^{1.5} \tag{11}$$

$$\mathbf{FT2}: Q = 0.586 H_0^{1.5} \tag{12}$$

$$\mathbf{FM12}: Q = 0.610 H_0^{1.5} \tag{13}$$

$$\mathbf{FM2}: Q = 0.658 H_0^{1.5} \tag{14}$$

Comparing each characteristic curve (from the set given by the above equations) with the respective configuration of experimental points (Figs. 3, 4, 6, 7, 9 and 10) we can state the differentiated level of conformity between the theoretical equations (in which the exponent is equal to 1.5) and measured values. In order to obtain a more precise evaluation of this conformity, the experimental data were fitted once again, by means of the less theoretical but more general relation (QH_0), which contains the optional exponent n:

$$Q_s = C_s H_0^n. \tag{15}$$

After the standard calculations, the following were obtained :

$$\mathbf{FG1}: Q_s = 1.218 H_0^{2.551} \tag{16}$$

$$\mathbf{FG2}: Q_s = 1.533 H_0^{1.594} \tag{17}$$

$$\mathbf{FT1}: Q_s = 0.529 H_0^{1.446} \tag{18}$$

$$\mathbf{FT2}: Q_s = 0.540 H_0^{1.422} \tag{19}$$

$$\mathbf{FM1}: Q_s = 0.586 H_0^{1.406} \tag{20}$$

$$\mathbf{FM2}: Q_s = 0.622 H_0^{1.387} \tag{21}$$

Determination of the Froude number

The discrepancy between the parameters of the Eq. 8 (C and $n_t = 1.5$) and Eq. 15 (C_s and n) gives rise to the suspicion, that some of the investigated flumes were not properly designed and as a result – some units do not work as standing-wave-flumes. In order to find this out, the Froude number Fr_t for the centre of each flume throat was calculated, according to the formula:

$$Fr_t(Q) = \frac{Q}{(bh\sqrt{gh})}. \tag{22}$$

The throat depth h for each flume was measured during control investigations (together with Q and H_0). The obtained functions $Fr_t(Q)$ are presented in Fig. 11.

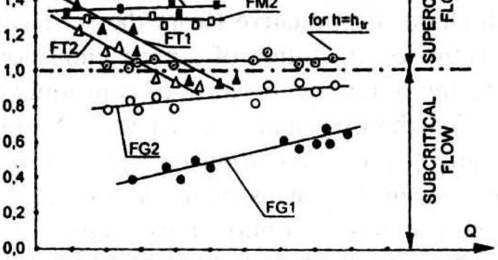

Fig. 11. Froude numbers for the investigated flumes .

Application of the catalogue characteristic curves

Only two flumes were selected from the catalogue of types series. The others were designed individually, so the catalogue characteristic curves $Q(H_0)$could be analysed only for flumes FM1 and FM2. The proper lines are shown in Figs. 9 and 10 (dotted lines).

Discussion of results

The set of data, presented above, enables the formulation of some interesting remarks, both with respect to the practical functionning of investigated flumes, and the general theory of Venturi flumes.

Analysis of the conformity of empirical (Eq. 15), theoretical (Eq. 8) and catalogue (for two flumes FM1 and FM2) characteristics, considered together with the function $Fr_t Q$ for the flume throat, leads to the following statements:

1. the flow along the flume can be subcritical ($h > h_c$), even if it was intended to be a standing-wave-flume;
2. even if the condition $Fr_t > 1$ is fulfiled, it is often much better to replace the exponent $n_t = 1.5$ in Eq. 8 by the empirically identified parameter n (usually n differs from 1.5);
3. catalogue characteristics should be applied after very careful analysis of the flow conditions.

Ad. 1

It can be seen in Fig. 11, that the worse conformity between theoretical and empirical lines – $Q(H_0)$ and $Q_s(H_0)$ – goes together with the low Froude number. Especially for the flume FG1 (Eqs. 9 and 16, Fig. 3), for whole the investigated range of discharge, we have $Fr_t < 1$. For the flume FG2 in turn, the Froude number lies slightly below unity, but the conformity of the characteristics with the set experimental points (Fig. 4) is quite fair. Probably it results from the fact, that in this case the critical conditions of flow are reached in the terminal part of the throat, so for $h = h_{tr}$ (Fig. 1) the Froude number exceeds unity (see the additional line in Fig. 11). As already underlined in the introduction, only theoretically is the condition $h = h_c$ fulfiled in the centre of the throat (when the Eqs. 6 and 7 are valid). In practice the critical depth appears at different points of the throat, depending on the fluid discharge. Such a situation can be observed for flume FG2.

In the case of units FT1 and FT2, the supercritical flow (when $h < h_c$) appears only for lower discharges, as for the higher values of Q the outflow is submerged, and the Froude number falls below unity. This conclusion is confirmed by the lines in Figs. 6 and 7 (where the experimental points lie closer to the characteristics for the lower values of Q).

And finally, flumes FM1 and FM2 both work properly (Figs. 9, 10 and 11).

Ad. 2

In the case of FG1 the exponent $n = 2.551$ in Eq. 16 differs considerably from the theoretical value $n_t = 1.5$ (Eq. 9). This is most probably the consequence of the already proved fault in this channel, which works as a subcritical-flow-meter

($h > h_c$), whereas the discharge is computed by Eq. 8, as if it was a supercritical-flow-meter ($h < h_c$).

In the other cases the differences between n and $n_t = 1.5$ are not very serious. The reason, apart from unavoidable measuring errors, is probably the fact, that Eq. 7 is of an approximated character.

Ad. 3

Usability of the catalogue characteristics can be evaluated only for two flumes FM1 and F2 (as only these two units were selected from the catalogue). For unit FM2, quite good conformity of this curve with the set of experimental points was obtained (Fig. 10), whereas for FM1 – conformity is not too good. This is probably an effect of the difference between the required (taken from the catalogue) and real conditions of flow. It can be a serious problem, as usually the measuring devices must be composed within the existing and/or planned technological diagram and it can be difficult to fulfil the producer's conditions (or these conditions may be forgotten). It seems that the Venturi flumes, even selected from the typical catalogue, should be considered as individual devices and should always be calibrated when ready. This individual character of the Venturi flumes can also be seen in Figs. 6 and 7, where we have two apparently different curves, although both flumes were designed as identically.

4. Determination of the Free–Surface Profile

As we can see, the Venturi flume (although simple in construction and reliable during exploitation) is not so simple from the hydraulic point of view. This means that the designer of such a device should not plan its shape automatically, using the standard catalogue only, but should devote more time to the task.

The conditions of flow along the flume depend, among other things, on the initial depth of the liquid H_0 (for $x = 0$, see Fig. 1). This depth results from the parameters of the upper channel (inlet), which should be in conformity with the requirements of the typical flume producer. However, when the flume is individually designed (or when the designer has to change its typical dimensions), it seem to be necessary to use more precise mathematical tools, than the technical instructions only.

The flow in the flume, generally speaking, is of an unsteady and non-uniform character. However, in practice the time-dependence of the flow parameters can be neglected, and the free-surface profile along the flume described by the gradually-varied flow equation (8):

$$\frac{dH}{dx} = \frac{i_0 - i_f}{1 - Fr^2}. \qquad (23)$$

As a matter of fact, Eq. 23 describes the gradually varied flow, whereas in the Venturi flume we can observe a zone of rapidly varied motion. However, some investigations show (e.g. Abbot 1979, Szydłowski 1999) that we can neglect this discrepancy in such situations and accept the Eq. 23 (otherwise we would have to take into account the vertical structure of the velocity field and the analysis would be much more difficult).

The Venturi flume bottom slope $i_0 = 0$, and the friction slope can be computed from the following relation:

$$i_f = \frac{Q^2 n_H^2}{B^2 H^2 R_M^{4/3}}. \tag{24}$$

The two following evident relations describe the hydraulic radius:

$$R_H(x) = \frac{B(x)H(x)}{[B(x) + 2H(x)]} \tag{25}$$

and the Froude number:

$$Fr^2(x) = \frac{Q^2}{[g B^2(x) H^3(x)]}. \tag{26}$$

Eq. 23 can easily be integrated along the flume, for the following initial condition:

$$H(x = 0) = H_0. \tag{27}$$

A very important element is proper choice of the Manning coefficient n_M. Taking into account the standard empirical juxtapositions one can suspect, that for waste water this coefficient should be close to $n_M = 0.02$, but after some numerical experiments it was stated, that the best coincidence between the calculated and measured free-surface profiles can be obtained for $n_M = 0.03$.

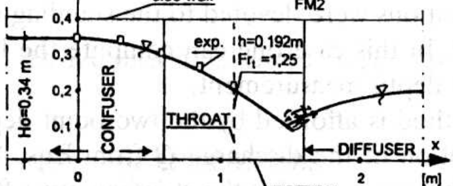

Fig. 12. Free-surface profile for flume FG1

Two exemplary shapes of this profile are shown in Fig. 13 (for the flume FG1, where $Q = 0.841$ m^3/s and $H_0 = 0.865$ m; the function $B(x)$ is shown in Fig. 2a)

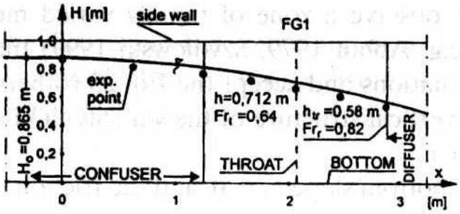

Fig. 13. Free-surface profile for flume FM2

and in Fig. 12 (for the flume FM2, where $Q = 0.132\ \mathrm{m}^3/\mathrm{s}$ and $H_0 = 0.34$ m; the flume width $B(x)$ – Fig. 8b). The numerical calculations were performed for the step $\Delta x = 0.005$ m.

As can be seen, the conformity between the calculated and measured depths is quite acceptable. The difference at the end of the throat of the flume FG1 amounts to $\Delta H = +3.9\%$ (for $h_{tr} = 0.58$ m, when $Fr_r = 0.82 < 1.00$; as mentioned, this flume works as a subcritical channel) and $\Delta H = -6.3\%$ for the centre of the flume FM2 (when $h = 0.192$ m and $Fr_t = 1.25 > 1.00$; the condition given by Eq. 7 in this case is satisfied about 0.20 m ahead of the throat centre, as $h = 0.192$ m $< 2H_0/3 = 0.227$ m.

Certainly, this conformity could be easily improved, by means of the Manning coefficient correction, but the purpose of these calculations was not the individual identification of this parameter for each flume, but the more general description of the liquid free-surface along the flume. The obtained results (Figs. 12 and 13) prove, that the equation of the gradually-varied flow can be an effective tool for the Venturi flume dimensioning. Moreover, these calculations can be performed for a longer distance, including the inlet channel.

5. Two-Point Measurements

Our previous considerations were devoted to the standing-wave-flumes, for which Eq. 7 can be satisfied. In this case one can compute the liquid discharge on the basis of the one-point depth measurement.

An alternative method is afforded by the two-point depth measurements (H_0 and h) and determination of the discharge Q from Eqs. 4 or 5. This alternative can be applied in each case – not for the standing-wave-flume only, but also for the subcritical-flow-Venturi-flume ($h > h_c$).

Among the six cases, described above, only two flumes (FG1 and partly FG2) were not properly designed, therefore for these two units the "two-point characteristic curves" were determined. According to Eq. 4, we obtained:

$$\mathbf{FG1}: Q = 2.704h\sqrt{H_0 - h}, \tag{28}$$

$$\mathbf{FG2}: Q = 3.878h\sqrt{H_0 - h}. \tag{29}$$

The proper diagrams $Q = Q(H_0, \Delta H = H_0 - h)$ are shown in Figs. 14 and 15.

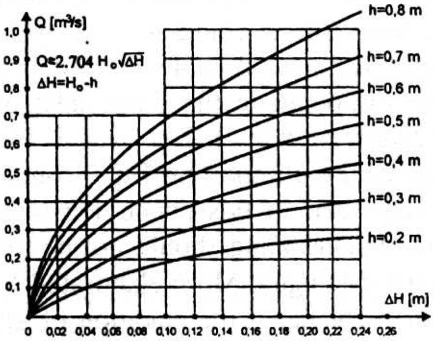

Fig. 14. Two-point characteristics for flume FG1

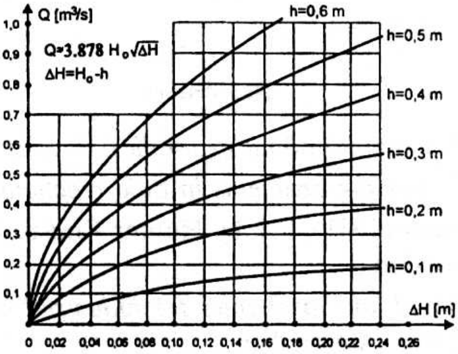

Fig. 15. Two-point characteristics for flume FG

6. Conclusions

The Venturi flumes are very useful (and thus very popular) devices which serve for liquid discharge measurement. They are especially attractive for polluted liquids (e.g. – waste water).

The Venturi flumes are so commonly applied, that their users seem to fall into the routine, selecting proper sizes of these devices. In particular, one can automatically adjust the flume, using the catalogue of the series of types.

Most frequently these devices are designed as standing-wave-flumes, when the discharge can be computed by means of one-point depth measurement. The main reason for this situation is probably the desire to reduct financial expenses.

Designing the Venturi flume, mistakes can be made. Improper inlet conditions and geometrical deviations, can seriously worsen the accuracy of measurements. The hydraulic theory of these flumes contains some essential simplifications.

As a result, many Venturi flumes do not work in the supercritical flow regime (although users measure the depth in the flume inlet only, believing that their device works as a standing-wave-flume). But even if the critical velocity in the flume throat can be exceeded, the important Eq. 7 contains some simplifications.

The investigations, described in this paper, led to the conclusion, that the process of Venturi flume dimensioning should not be reduced to the simple application of the producer's catalogue. More elaborated hydraulic tools should be used for this purpose, in particular – the equation of gradually-varied-flow (Eq. 23).

This equation cannot be solved analytically, so a computer must be used during the integration, which makes the designing process more complex. Having this fact in mind, investors should always consider the possibility of "two-point discharge measurement". Installation of the second depth indicator, although it increases the total cost of the object, can simplify its hydraulic description and give much better measurements accuracy.

References

Abbot M. B. (1979), *Computational hydraulics*, Pitman, London.

Douglas J. F. (1996), *Solving problems in fluid mechanics*, Longman, Harlow.

Engineering Hydraulics (1961), Ed. by Hunter Rouse, John Wiley & Sons Inc., New York & London.

Fannelop T. K. (1994), *Fluid mechanics for industrial safety and environmental protection*, Elsevier, Amsterdam.

Parshall R. L. (1926), The improved Venturi flume, *Trans. ASCE*, Vol. 89.

Szydłowski M. (1999), Rozwiązywanie równań nieustalonego przepływu wody z odskokami hydraulicznymi, *Mat. XIX Ogólnopolskiej Szkoły Hydrauliki*, Frombork, 20–24.09.1999.

Uniklar–77 – the Venturi flumes series of types, Centrum Techniki Komunalnej, Warszawa, Poland.

Ven Te Chow (1952), *Open-channels hydraulics*, McGraw Hill, New York.

Venturihalbschalen-Typenubersicht, NIVUS Kontrollgerate GmbH, Eppingen, Germany.