

Determination of Roughness Coefficient of the Underside of Ice Cover

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Abstract

The formation of ice cover in winter on rivers, channels and run-of-reservoirs, changes flow characteristics significantly. Calculations of water surface profiles or discharge along a river reach with ice cover by means of 1-D models, requires detailed values of the roughness coefficients of the river bed and underside of the ice cover. Numerous studies dealing with roughness coefficients of the river bed with free surface flow were carried out in the past, however, very few investigations were performed on the determination of the roughness coefficient of the underside of ice cover. Data of calculations indicate that this coefficient may vary considerably with space and time, depending on the character of the ice cover, flow characteristics and meteorological conditions. In 1-D models roughness coefficient of the bed and of the underside of the ice cover must be combined into one composite roughness coefficient. The paper presents determination of roughness coefficient of the underside of ice cover based on velocity distributions in the Vistula River cross-sections with ice cover. It may be assumed that vertical velocity distribution has a logarithmic character in the regions near the bottom and ice cover. Taking into account this assumption the procedure for the calculation of Darcy-Weisbach friction factor f and Manning roughness coefficient n is proposed, basing on the data of velocity measurements in a given vertical. It was found that roughness coefficient of the underside of the ice cover varies in a much larger range than the bottom roughness coefficient and depends very much on the character of the ice cover.

Key words: ice cover, velocity distribution, Darcy-Weisbach friction factor, Manning roughness coefficient, calculation procedure

Notation

- a, b – coefficients in Eq. 2,
- A – flow cross-sectional area,
- f – Darcy-Weisbach friction factor,
- g – gravitational acceleration,
- i – ice cover thickness,

k	- linear dimension of roughness,
K	- correlation coefficient,
n	- Manning roughness coefficient,
P	- wetted perimeter,
v	- local velocity,
$v_{\max,m}$	- maximum measured velocity,
$v_{\max,c}$	- maximum calculated velocity,
V	- average velocity,
v_*	- shear velocity,
R	- hydraulic radius,
y	- depth measured from the underside of ice cover,
y_0	- depth of zero velocity in logarithmic distribution,
y_{\max}	- depth of the position of maximum velocity,
τ	- shear stress,
κ	- Karman constant,

Subscripts

i	- ice cover,
b	- bottom.

1. Introduction

Formation of ice cover on flowing waters (rivers, channels, run-of-reservoirs) is a very complicated physical process. It depends on meteorological conditions (air temperature, wind speed, humidity, solar radiation) and hydraulic conditions (flow velocity, water depth, water temperature). Ice begins to form when the water temperature drops to 0°C and heat from the water is further transferred to the atmosphere. In river where small velocity, small depth and intensive heat loss occur, usually near the river bank, ice forms quickly and is called static border ice. This ice grows gradually towards the centre of the river, and finally covers the whole width of the river. This type of ice has a smooth undersurface.

In water flowing with high velocity ($V > 0.6$ m/s) there is no possibility for the formation of static ice. Water temperature in such flow is uniform, due to turbulence, over the whole depth. As the result of heat loss to the atmosphere water becomes supercooled (small fraction of Celsius degree) and frazil ice starts to form in the whole water volume. Frazil ice consists of small ice crystals, a few millimeters in dimension. Ice due to a lower density than water flows to the surface. Frazil ice crystals in supercooled water have a tendency to adhere to each other and thus form bigger flocks. On the water surface, they create ice pans

which move with the river current downstream. In some river cross-sections where natural or artificial obstacles may appear, moving ice pans are stopped and freeze together, thus forming a solid ice cover of rough undersurface. Frazil ice may move under the ice cover and settle in some places, thus forming frazil deposits. Sometimes these frazil deposits achieve considerable thickness and are called hanging dams. The ice cover increases its thickness from its bottom down, till the moment when heat flux equilibrium is achieved between water and atmosphere, and thus forms heat insulation between water and the atmosphere.

In wide river channels, stable ice cover does not move in relation to the river bed, however, it floats on the water surface moving in a vertical direction. This is possible due to ice cracks which form near the shore. It can be assumed that the flow under the ice cover is a *quasi pressure flow*, while additional pressure is exerted by the weight of the ice cover.

Formation of ice cover on flowing waters essentially alters the magnitude of the wetted perimeter and hydraulic resistance. The magnitude of the wetted perimeter almost doubles by adding a solid boundary of ice cover and flow resistance increases due to roughness of the underside of the ice cover (Fig. 3). In consequence, there is a change of velocity distribution and increase of flow depth in relation to free surface flow of the same discharge. Calculation of the water surface profile along the river reach or impounding reservoir with ice cover is a very important engineering problem. It is of special significance for high discharges when flood danger exists and the height of flood dykes may not be sufficient. It is also very important in energy channels conveying water to hydro-power plants where limitation of depth may result in smaller discharge thus decreasing the power of the turbines.

Studies of flow with ice cover are relatively limited as compared with the studies of free surface flow, despite the fact that in winter conditions several dangerous floods took place. An additional problem is that flow with ice cover is much more complicated than free surface flow. This is mainly due to the significant variation of ice cover characteristics in space and time.

Flow resistance may be expressed in the form of shear stress τ , Darcy-Weisbach friction factor f or Manning roughness coefficient n . The last one found extensive application in hydraulic calculations. There are formulas which determine the relations between τ , f and n .

2. Velocity Distribution in a Flow with Ice Cover

Measurements of velocity distribution under ice cover in several Vistula cross-sections were carried out. The methodology of such measurements is similar to free-surface measurements. The difference is that in winter, measurements are carried out from the ice cover. First of all the river cross-section is determined, as are positions of consecutive verticals. Then holes in the ice cover are made

at points of measuring verticals and subsequently ice thickness is measured. In cases where frazil deposits are present in a given vertical, its thickness is also determined. Ice thickness and flow velocity are measured from the ice cover, which is thick enough to carry safely people who carry out measurements. In every vertical, velocity is measured at several points by means of a velocity current meter attached to the vertical rigid rod supported on the river bed. The amount of measuring points depends on the flow depth and ice cover configuration. Close to the underside of ice cover, measurements are made more frequently in order to determine the exact velocity profile. Two examples of velocity distribution in Vistula cross-section with ice cover are shown in Figs. 1 and 2 (Majewski 1987). The vertical scale of the figures is distorted in relation to the horizontal scale.

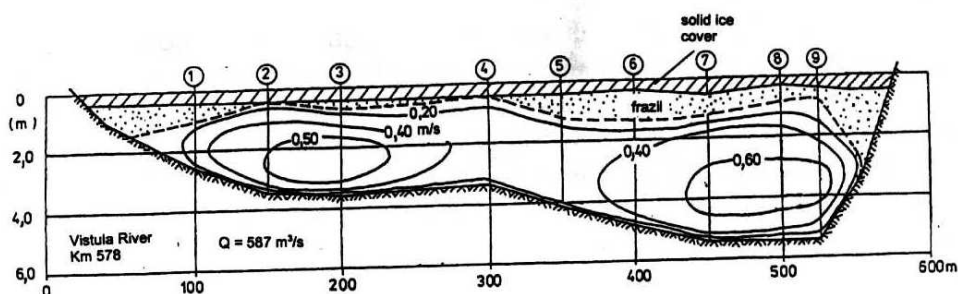


Fig. 1. Velocity distribution in Vistula cross-section (km 578) with ice cover

This cross-section is located on the Lower Vistula. The length of the cross-section exceeds 500 m, and maximum depth is more than 5 m. The whole width of the river is covered with solid ice with a thickness of about 0.5 m. Frazil ice deposits were also detected over the whole width. In some places frazil deposits reach more than 1.0 m. It can be seen that bed morphology and the configuration of the underside of ice cover have a visible influence on velocity distribution. Maximum velocity reached 0.6 m/s and this was in the deepest part of the cross-section. Ice cover characteristics and velocity distribution were measured in 9 verticals. Calculated discharge (Q), based on velocity distribution was $587 \text{ m}^3/\text{s}$.

This cross-section is located in the upper part of Włocławek Reservoir where its length attains nearly 1 km. Measurements of ice cover characteristics and velocity distribution covered only part of the cross-section of the length of about 600 m because in the further part of the cross-section ice cover was not thick enough to carry the measuring team safely. The depth was quite uniform at about 4.0 m. Solid ice cover was of the order of 0.15 m practically without frazil deposits. Velocities were measured in 9 verticals. They were quite uniform over the whole depth. Only close to the left bank more non uniformity was observed. Measurements in both

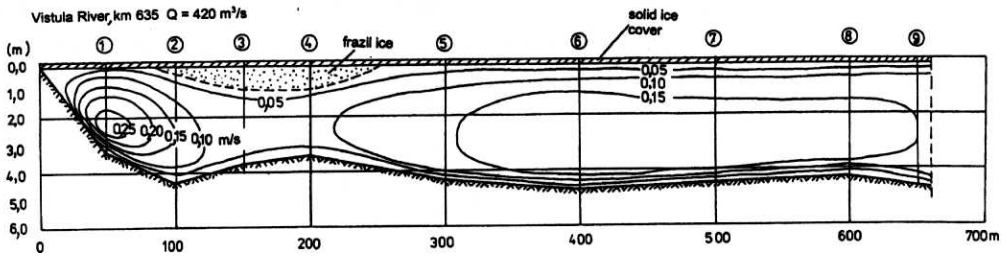


Fig. 2. Velocity distribution in Vistula cross-section (km 635) with ice cover

cross-sections shown in Figs. 1 and 2 were carried out at different times, therefore river discharge was different.

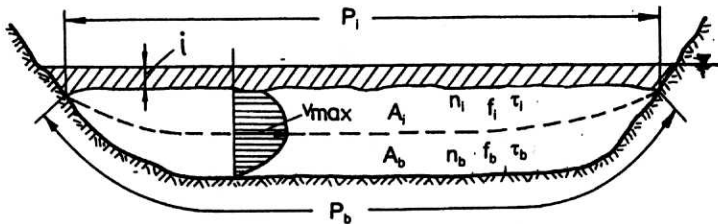


Fig. 3. Schematic presentation of river cross-section with ice cover

The scheme of the channel flow with ice cover is shown in Fig. 3. The whole flow cross-section is divided by means of a dashed line into two parts. The upper part is under the influence of the ice cover and the lower one the river bottom. The dashed line represents the position of maximum velocity. This position depends mainly on the roughness and flow resistance of these two surfaces. The following notation is used in this figure:

- P – wetted perimeter,
- A – cross-sectional area,
- i – ice cover thickness,
- τ – shear stress,
- n – Manning roughness coefficient,
- f – Darcy-Weisbach friction factor,

subscript i – denotes ice cover, and subscript b – river bottom.

Total wetted perimeter $P = P_i + P_b$, total flow area $A = A_i + A_b$, hydraulic radius $R = P/A$.

Velocity distribution depends on the roughness of two surfaces: ice cover and the bottom. Depending on the roughness of these two surfaces, maximum velocity

will be situated at a given depth. With equal roughness of both surfaces maximum velocity will be at half depth. With unequal roughness the position of maximum velocity will move towards a smoother boundary. In both regions logarithmic velocity may be assumed

$$v = \frac{1}{\kappa} v_* \ln \left(\frac{30y}{k} \right), \quad (1)$$

where:

- v – velocity at a given depth,
- κ – Karman constant usually assumed 0.4,
- v_* – shear velocity,
- y – depth measured from the underside of ice cover,
- k – linear dimension of roughness.

Eq. 1 may be presented in a simplified form

$$v = a \ln(y) + b. \quad (2)$$

With the known parameters a and b in Eq. 2 the following values are subsequently calculated:

$$y_0 = \exp \left(\frac{-b}{a} \right), \quad (3)$$

y_0 is the depth of zero velocity under ice cover, which may, to a certain extent, be regarded as the measure of the roughness of the underside of the ice cover,

$$v_{\max,c} = a \ln(y_{\max}) + b \quad (4)$$

$v_{\max,c}$ is the calculated maximum velocity at the depth y_{\max} assuming logarithmic velocity distribution. Integration of logarithmic velocity distribution over the depth leads to the determination of the average velocity V over the depth ($y_{\max} - y_0$)

$$V = \frac{v_{\max,c} \cdot y_{\max}}{y_{\max} - y_0} - a. \quad (5)$$

Assuming that $a = \frac{1}{\kappa} \sqrt{gSR}$

$$V = \sqrt{\frac{8g}{f}} \sqrt{SR}$$

and that

$$y_{\max} - y_0 \cong R,$$

$$\kappa = 0.4$$

it is possible to obtain

$$f_i = 1.28 \left(\frac{v_{\max}}{V} - 1 \right)^2, \quad (6)$$

f_i is the Darcy-Weisbach friction factor for the underside of the ice cover for the vertical in which velocity distribution was measured

$$n_i = \sqrt{\frac{f_i}{8g}} (y_{\max} - y_0)^{1/6}, \quad (7)$$

n_i is the Manning roughness coefficient for the underside of the ice cover, and g is gravitational acceleration.

3. Determination of Roughness Coefficient of the Underside of Ice Cover

The method of determining the roughness coefficient of the underside of the ice cover using velocity distribution was initially proposed by Larsen (1969). It was modified in the Institute of Hydro-Engineering (Majewski et al 1985).

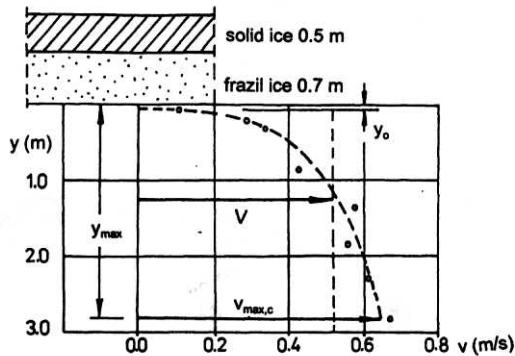


Fig. 4. Example of velocity distribution under the ice cover

Example of velocity distribution under the ice cover is shown in Fig. 4. The following notation is used:

- y_{\max} – depth at which maximum velocity appears under ice cover,
- V – average velocity over the depth y_{\max} ,
- y_0 – depth where the dashed line of calculated velocity distribution intersects the vertical,
- $v_{\max,c}$ – maximum velocity calculated.

Points represent measured velocities. Calculated maximum velocity sometimes does not coincide with the measured value.

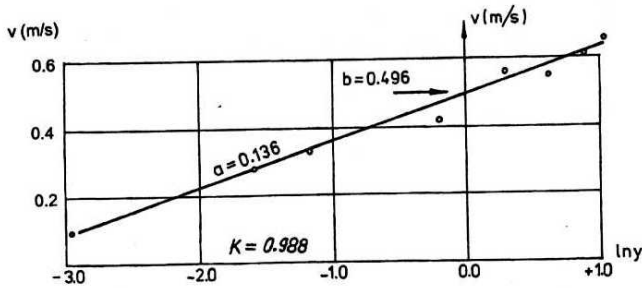


Fig. 5. Scheme for the determination of parameters a and b of the logarithmic velocity distribution

Measured velocities under the ice cover at various depths (Fig. 4) are plotted in the coordinates $v - \ln(y)$. The points should be located along a straight line. Some dispersion of the points results from inaccuracies of velocity and depth measurements. The equation of this line may be determined using the least mean square fit method. Value a in Eq. 2 denotes the slope of the line and the value b – the intersection of the line with the velocity axis. A correlation coefficient K is calculated for each case to indicate the difference between measured velocity and theoretical logarithmic velocity distribution. In this case the value of correlation coefficient $K = 0.988$, thus indicating good coincidence between measured and calculated logarithmic velocity distribution.

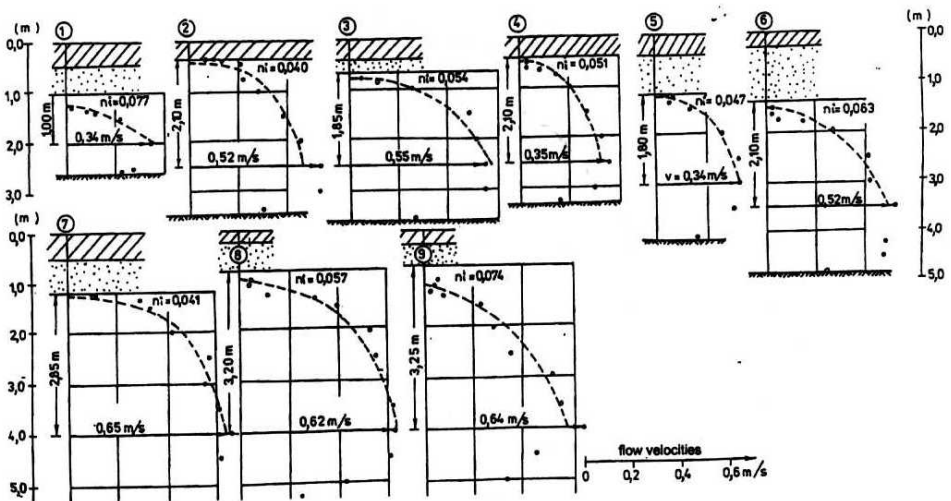


Fig. 6. Velocity distribution in 9 verticals of the Vistula cross-section (Fig. 1)

Measured velocity distribution in 9 verticals of the Vistula cross-section (Fig. 1) is presented in Fig. 6. For each vertical, logarithmic velocity distribution is shown

with a dashed line, maximum measured velocity – $v_{\max,m}$, and calculated Manning roughness coefficient of the underside of ice cover n_i .

Measured and calculated characteristic values for this cross-section are shown in Table 1.

Table 1. Measured and calculated values for Vistula River cross-section (Fig. 1)

Vertical No.	y_{\max} (m)	$v_{\max,m}$ (m/s)	a	b	K	$v_{\max,c}$ (m/s)	V (m/s)	y_0 (m)	f_i	n_i
1	1.00	0.34	0.257	0.357	0.981	0.36	0.22	0.25	0.514	0.077
2	2.10	0.52	0.105	0.376	0.952	0.45	0.36	0.03	0.099	0.040
3	1.85	0.55	0.188	0.460	0.987	0.58	0.42	0.09	0.189	0.054
4	2.10	0.35	0.096	0.252	0.954	0.32	0.24	0.07	0.161	0.051
5	1.80	0.34	0.095	0.280	0.973	0.34	0.25	0.05	0.147	0.047
6	2.10	0.52	0.184	0.339	0.934	0.48	0.33	0.16	0.248	0.063
7	2.85	0.65	0.142	0.487	0.988	0.64	0.50	0.03	0.093	0.041
8	3.20	0.62	0.200	0.410	0.956	0.64	0.47	0.13	0.174	0.057
9	3.25	0.64	0.251	0.274	0.950	0.57	0.38	0.34	0.298	0.074

Measured and calculated characteristic values for Vistula River cross-section shown in Fig. 2 are presented in Table 2.

Table 2. Measured and calculated values for Vistula River cross-section (Fig. 2)

Vertical No.	y_{\max} (m)	$v_{\max,m}$ (m/s)	a	b	K	$v_{\max,c}$ (m/s)	V (m/s)	y_0 (m)	f_i	n_i
1	2.05	0.27	0.097	0.202	0.975	0.27	0.19	0.13	0.219	0.059
2	2.55	0.16	0.039	0.107	0.911	0.14	0.11	0.06	0.134	0.048
3	3.10	0.09	0.080	0.060	0.525	0.07	0.06	0.47	0.024	0.021
4	2.60	0.08	0.032	0.038	0.896	0.07	0.05	0.30	0.316	0.073
5	3.10	0.13	0.031	0.090	0.966	0.12	0.10	0.05	0.112	0.046
6	3.10	0.19	0.024	0.153	0.869	0.17	0.13	0.00	0.074	0.036
7	2.60	0.17	0.033	0.134	0.991	0.17	0.13	0.02	0.074	0.036
8	2.60	0.19	0.026	0.140	0.887	0.17	0.14	0.00	0.043	0.027
9	2.65	0.15	0.011	0.138	0.677	0.15	0.14	0.00	0.009	0.012

The average value of the Manning roughness coefficient of the underside of ice cover for Vistula River cross-section (Fig. 1, Table 1) is $n_i = 0.056$. The range of this coefficient varies from 0.077 to 0.040. This value is much higher than for natural river bed ($n_b = 0.030$). High value of Manning roughness coefficient for the underside of ice cover in this Vistula cross-section (Fig. 1, Table 1) results from the fact that ice cover consisted of refrozen ice pans and a considerable amount of frazil deposits. Average value of the Manning roughness coefficient of the underside of ice cover in the Vistula cross-section presented in Fig. 2

(Table 2) is $n_i = 0.040$ with the range from 0.012 to 0.073. The low value is for solid crystalline ice without any frazil deposits. The highest value is for the vertical 4, with nearly one metre of frazil deposit. It is interesting that for the neighboring vertical with the same amount of frazil ice, the value of n_i is considerably lower.

4. Conclusions

The method for the determination of the roughness coefficient of the underside of ice cover presented in this paper is based on the assumption of logarithmic velocity distribution near this surface. Detailed measurements of velocity distribution under ice cover in several verticals and cross-sections of the Vistula River proved that this assumption is correct. In the paper, the method for the determination of this coefficient is presented. Several difficulties arise during field measurements and calculations:

- it is difficult to estimate the position of maximum velocity, as there are flow instabilities caused by the influence of both rough surfaces (underside of ice cover and river bottom),
- the character of logarithmic velocity distribution has a deficiency, as tangential to the maximum velocity is not vertical,
- the differences between measured and calculated values of maximum velocity vary, however, they usually do not exceed 10% .

Calculated values of the roughness coefficients indicate considerable differences from one point to another. They may differ also with time. The range of extreme values varies for two investigated cross-sections from 0.012 to 0.077. This range is much higher than for free surface flow. Maximum values of the roughness coefficient of the underside of the ice cover are also much higher than roughness coefficients for the river bed.

The last statement indicates that very rough ice cover may cause considerable increase in flow depth. This is of particular importance during high discharges, which may cause overtopping and breaching of flood dykes. This situation appeared during the important winter flood on the Lower Vistula in 1982.

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