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Influence of Visco-Elasticity on Pressure Wave Velocity in Polyethylene MDPE Pipe

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Abstract

Many of the pipe materials, like polyethylene (MDPE, HDPE) or polypropylene (PP), turn out to be visco-elastic. The materials in particular, prove to have strongly time-dependent strain properties. As the propagation velocity of pressure waves primarily depends on the fluid bulk modulus and the elasticity of the pipe wall, the above material property has considered the influence in unsteady pipe flow (e.g. in water hammer). The pressure velocity in visco-elastic pipes depends on the pressure oscillation frequency. The applicability of the method of MOC characteristics is limited in the sense that purely elastic strain behaviour has to be assumed. In order to take into account the special properties of visco-elastic pipes, the parameters of the Kelvin-Voigt elements, as well as a complex Young's modulus (dynamic modulus of elasticity) are used to described the pressure velocity. The use of the complex wave velocity seems to be especially useful. However, the velocity should be measured during unsteady water flow. The experiments carried out show that pressure disturbance velocity in MDPE pipe strongly depends on the length of the pipe-line. The velocity increases noticeably when the length of the pipe decreases.

Keywords: water hammer, pressure wave velocity, visco-elasticity, polyethylene MDPE pipe

Notations

A – cross-sectional area of pipe,
c – pressure wave propagation velocity,
D – inside diameter of pipe,
D_0 – outside diameter,
E – elastic modulus of the pipe wall,
E^* – dynamic modulus of the pipe wall,
e – thickness of the pipe wall,
J – creep compliance of visco-elastic material,

J' – storage compliance,
J'' – loss compliance,
K – bulk modulus of liquid,
L – length of pipe,
p – pressure,
Q – discharge,
R – linearized friction factor,
T – water hammer period,
t – time,
v – flow velocity,
ε – strain,
κ – coefficient taking into account the effect of pipe support conditions,
λ – friction factor,
μ – Poisson's coefficient,
ρ – liquid density,
σ – stress,
τ – relaxation time,
ω – angular frequency.

1. Introduction

For the analysis of unsteady pipe flow various methods are available. These include the method of characteristics (MOC) with adaptable grid and frequency-response-based-methods are widely used (Streeter & Wylie 1998, Szymkiewicz 1975). To describe the water hammer phenomenon the method of characteristics is particularly often used because of the simplicity of its governing equations and its suitability for numerical calculations. For one-dimensional flow, from the fundamental fluid mechanics equation of continuity:

$$Q\frac{\partial \rho}{\partial x} + A\frac{\partial \rho}{\partial t} + \rho\left(\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t}\right) = 0, \tag{1}$$

and the equation of motion (equilibrium of forces):

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \lambda\frac{v\ |v|}{2D} = X, \tag{2}$$

the characteristic equations which describe unsteady pipe flow are easily derived (Evangelisti 1969, Mitosek 2001):

$$\frac{1}{\rho c}\frac{dp}{dt} + \frac{dv}{dt} + \lambda\frac{v\ |v|}{2D} = X \ \text{ for } \ \frac{dx}{dt} = v + c, \tag{3}$$

$$-\frac{1}{\rho c}\frac{dp}{dt}+\frac{dv}{dt}+\lambda\frac{v\,|v|}{2D}=X \text{ for } \frac{dx}{dt}=v-c, \tag{4}$$

where:

- v – the flow velocity,
- c – the pressure wave propagation velocity,
- p – the pressure,
- ρ – the fluid density,
- λ – the friction factor,
- X – the horizontal component of the body force per unit of liquid (for horizontal pipe-line $X = 0$).

The application of these hyperbolic differential equations assumes a perfectly elastic fluid and a purely elastic stress-strain behaviour of the pipe wall. However, experimental investigations show that pressure waves propagating along the pipe fast decrease and are smoothed (Mitosek & Kodura 2000, Wichowski 2002). As, especially for the water hammer phenomenon, the experimental results do not correspond to the results computed under the assumption of purely elastic strain properties (Hooke's low), this phenomenon cannot be explained solely by the friction of the viscous liquid. The effect is rather caused by a special strain property of the pipe material – visco-elasticity. This means that the elasticity behaviour may also be influenced by the stress history (Ferry 1980, Janson 1995).

Many of the pipe materials, like polyethylene (MDPE, HDPE), polyvinyle chloride (PVC) or polypropylene (PP), prove to be visco-elastic. The materials in particular turn out to have strongly time-dependent strain properties. The crucial feature of these pipes is the high deformability of the pipe wall (especially plasticized PVC), due to liquid stream pressure change (Limmer 1974, Mitosek 1993).

It should be mentioned that material such as steel or concrete have to be classified as weakly visco-elastic in certain circumstances (Franke & Seyler 1983).

As unsteady pipe flow is primarily governed by the bulk modulus and strain properties of the pipe wall, the influence of visco-elasticity is also important.

The high deformability of the plastic pipes means lower velocity c of pressure wave propagation, as well as decrease of maximal pressure rise $\Delta p_{\max}$ at water hammer in a straight pipe-line:

$$\Delta p_{\max} = \pm \rho c v_0, \tag{5}$$

where v_0 is the steady flow velocity before e.g. a sudden closure of the downstream valve.

The visco-elasticity property of the pipe material results in the pressure wave velocity depending on an angular frequency ω of pressure oscillation:

$$\omega = \frac{\pi}{T} \ \text{[rad/s]}, \tag{6}$$

where T means a period of water hammer (return time).

For a pipe-line of length L, the period$T$ is equal to:

$$T = \frac{2L}{c}. \tag{7}$$

Visco-elastic substances do not respond with an immediate constant strain proportional to the stress applied, as is the case for purely elastic materials. From the initial state the visco-elastic material gradually expands with time to a state predetermined by the load. Similarly, gradual changes are observed in the case of decay of the stress. This means that the deformation of visco-elastic pipe wall does not merely depend on liquid stream pressure but is also a function of time. The retarded wall deformation is evaluated from the creep compliance of the wall material.

One of the physical models with equivalent time-dependent properties is the Kelvin-Voigt Element (Fig. 1).

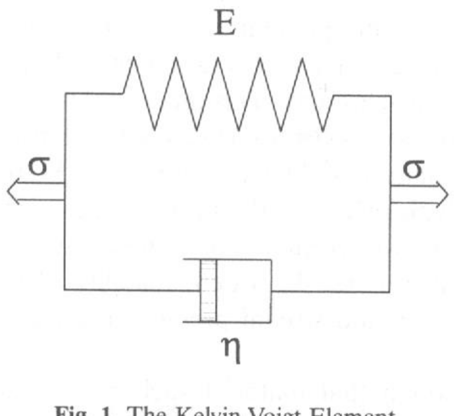

Fig. 1. The Kelvin-Voigt Element

The element consists of a purely elastic spring E and, parallel to the spring, a suppressor η. The damping element is proportional to the velocity of the changing of the distortion. The values of E and η describe the strain property of the element under dynamic load of a specific frequency. In order to describe the behaviour for all frequencies, the model is extended to an infinite series of Kelvin-Voigt Elements. The time-dependent elongation of all elements is described by Franke & Seyler (1983):

$$\varepsilon(t) = \frac{\sigma}{E}\left[1 - \int_0^{\infty} f(\tau)\, e^{-t/\tau}\, d\tau\right], \tag{8}$$

where

- $\varepsilon(t)$ – time-dependent strain,
- σ – stress,
- τ – the relaxation time of each element,
- $f(\tau)$ – the spectrum of the relaxation time.

The relaxation time is defined as the fraction η/E. The function $f(\tau)$ can be determined by means of stress-strain measurements performed on a sample of the material (Schwarzl 1970).

Generalized Kelvin-Voigt model can be written in the form (Lisheng Suo & Wylie 1990):

$$J(t) = J_0 + \sum_{i=1}^{n} J_i \left[1 - \exp\left(-\frac{t}{\tau_i}\right)\right], \tag{9}$$

where J is the function of creep compliance.

The values of parameters J_0, J_l, and τ_l are calibrated from the test data obtained on a rheo-vibration apparatus.

2. Influence of Visco-Elasticty on Pressure Wave Velocity

Omitting changes of wall thickness due to pressure rises, a pressure wave propagation velocity for the purely elastic properties of the pipe wall may be calculated from Korteweg's formula (Mitosek 2001, Streeter & Wylie 1998):

$$c = \sqrt{\frac{\frac{K}{\rho}}{1 + \kappa \frac{KD}{Ee}}}, \tag{10}$$

where:

- K – the bulk modulus of liquid,
- D, e – the inside diameter and wall thickness of the pipe respectively,
- κ – the coefficient introduced to take into account the effect of pipe support conditions, as well as that of the wall thickness (Parmakian 1963).

For thin-walled pipe the coefficient is equal to:

– for a pipe anchored at the upper end and without expansion joints:

$$\kappa = 1.25 - \mu,$$

– for a pipe anchored against longitudinal movement throughout its length:

$$\kappa = 1 - \mu^2,$$

– for a pipe with expansion joints:

$$\kappa = 1 - 0.5\,\mu,$$

where μ means Poisson's coefficient.

However, for plastics, a value of the modulus of elasticity E depends on speed of extension of the material (Ashby 1998). During fast extension, the modulus significantly increases as compared with the slow one. This means that to use the equation (10) for plastic pipes, the influence of frequency of pressure oscillation on the modulus E should be known.

The effect of visco-elastic properties is modelled through a frequency-dependent wave velocity. By incorporating the creep compliance into the unsteady continuity and motion equations, Meißner (1977) derived the velocity for oscillating pressure wave propagation in a thin-walled visco-elastic pipe:

$$c = \sqrt{\frac{\frac{2}{\rho}}{\sqrt{\left[\left(\frac{1}{K} + J'\frac{D}{e}\right)^2 + \left(J''\frac{D}{e}\right)^2\right]\left[1 + \left(\frac{R}{\omega}\right)^2\right]} + \frac{1}{K} + J'\frac{D}{e} - \frac{R}{\omega}J''\frac{D}{e}}}, \tag{11}$$

where:

ω	–	circular frequency,
J', J''	–	the storage and loss compliances of the wall material,
R	–	linearized friction factor.

The linearized friction factor R is defined as (Franke & Seyler 1983):

$$R = \frac{\lambda}{2D}\frac{\int\limits_0^{2T}\int\limits_0^{L} v^2 dx\, dt}{\int\limits_0^{2T}\int\limits_0^{L} |v|\, dx\, dt}. \tag{12}$$

Neglecting the influence of friction the equation of the velocity for an oscillating pressure wave propagation may be written in the form:

$$c = \sqrt{\frac{\frac{2}{\rho}}{\sqrt{\left[\left(\frac{1}{K} + J'\frac{D}{e}\right)^2 + \left(J''\frac{D}{e}\right)^2\right]} + \frac{1}{K} + J'\frac{D}{e}}}. \tag{13}$$

The values of the material specific parameters J' and J'' are determined by the equations (Schwarzl 1970):

$$J' = J\left(\frac{2\pi}{\omega}\right) - 0.86\left[J\left(\frac{4\pi}{\omega}\right) - J\left(\frac{2\pi}{\omega}\right)\right], \tag{14}$$

$$J'' = 2.12\left[J\left(\frac{2\pi}{\omega}\right) - J\left(\frac{\pi}{\omega}\right)\right]. \tag{15}$$

The creep compliance $J(t)$ is a function of time. The function has to be experimentally determined for each visco-elastic material.

Meißner (1977) has given the following equations:

- for a plasticized PVC pipe ($D = 32$ mm, $e = 4$ mm), and stream temperature of 20°C:

$$J(t) = 1.36610^{-7}\left[\ln\left(\frac{t}{3} + 1\right)\right]^{0.1632} \quad [\mathrm{Pa}^{-1}], \tag{16}$$

- for an unplasticized PVC (PVC-U) pipe ($D = 57$ mm, $e = 3$ mm), and stream temperature of 20°C:

$$J(t) = 3.0610^{-10} + 3.5010^{-12} t^{0.23} \quad [\mathrm{Pa}^{-1}]. \tag{17}$$

The function of the velocity $c(\omega)$ for oscillating pressure wave propagation in plasticized PVC pipe is given in Fig. 2, and in unplasticized PVC pipe – in Fig. 3.

As could have been easy to foresee the influence of pipe wall visco-elastic properties on the pressure wave velocity is particularly high for the plasticized PVC pipe.

An alternative approach to deal with an estimation of pressure wave velocity and hydraulic transients in visco-elastic pipes is presented in the paper by Lishen Suo and Wylie (1990). The authors show that Korteweg's equation (10) may be applied to visco-elastic pipes. The equation is then expressed as:

$$c = \sqrt{\frac{\frac{K}{\rho}}{1 + \kappa\frac{KD}{E^*e}}} = \sqrt{\frac{\frac{K}{\rho}}{1 + \kappa\frac{KDJ^*}{e}}}, \tag{18}$$

where $E^*(\omega) = E'(\omega) + i\,E''(\omega) = 1/J^*(\omega)$ is the complex Young's modulus (the dynamic modulus of elasticity).

In as much as $E^*(\omega)$, or $J^*(\omega)$, is a complex-valued function of frequency, Eq. (18) yields a complex-valued, frequency-dependent wave velocity.

The Eq. (18) permits the direct utilization of the parameter $E^*(\omega)$ of visco-elastic material, which is generally the data most readily available from material tests. The material test data, namely the storage modulus $E'(\omega)$ and the loss tangentE'/E'', obtained from rheo-vibration apparatus, form the basis for

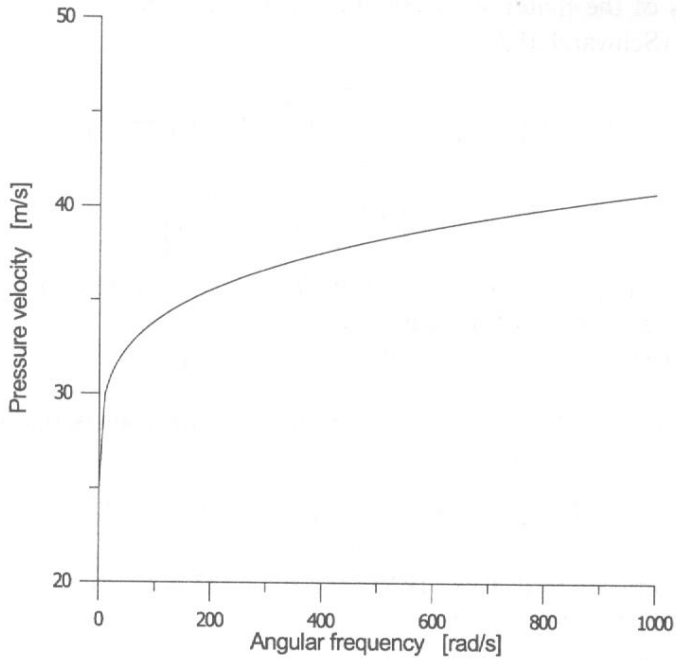

Fig. 2. The function of the pressure wave velocity $c(\omega)$ in plasticized PVC pipe

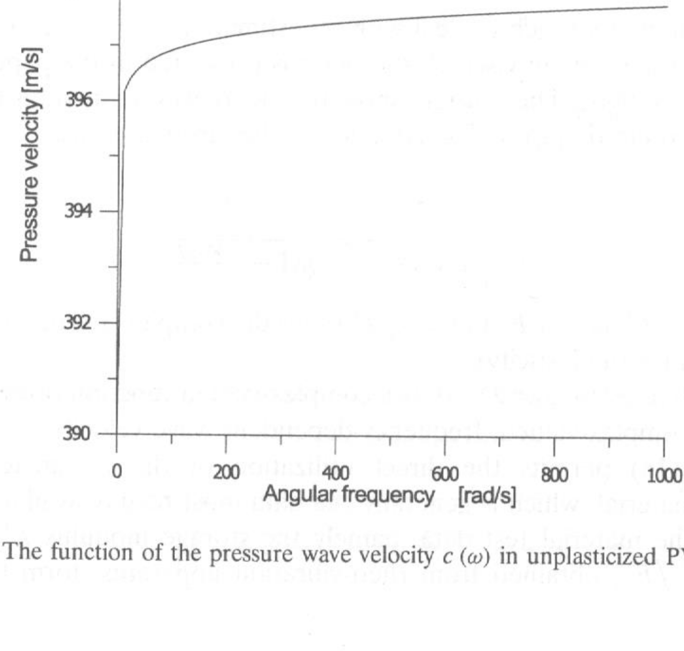

Fig. 3. The function of the pressure wave velocity $c(\omega)$ in unplasticized PVC pipe

calculating the modulus E^* (Güney 1983). However, the absolute value of complex Young's modulus $|E^*|$, calculated in the above way, was even 40% smaller than the real physical value of the dynamic modulus (Lisheng Suo & Wylie 1990).

Both equations presented by Meißner and equations given by Lisheng Suo and Wylie show explicitly that pressure wave velocity for visco-elastic pipes depends on frequency ω. But the frequency depends on a period of water hammer (a return time) T (Eq. 6), and the period – on the length L of the pipe-line (Eq. 7). Consequently, it is possible to estimate the pressure velocity not only as a function of the angular frequency $c(\omega)$, but also as a function of the pipe-line length $c(L)$. The function may be easily determined on the basis of measured characteristics of water hammer.

3. Pressure Wave Velocity in Polyethylene MDPE Pipe

The water hammer phenomenon was investigated experimentally in a medium density polyethylene (MDPE; SDR $\approx$ 11, PN = 10) pipe with outside diameter of D_0 = 50 mm and wall thickness e = 4.6 mm. The symbol SDR represents the standard dimensional ratio, i.e. SDR = D_0/e, and PN represents nominal pressure (bar). The value SDR is the same for MDPE pipes of different diameter but for the same PN. The medium density polyethylene pipes are commonly used in water pipe networks. The tested pipe-lines of different length L were fed from a large pressure reservoir (Fig. 4). The tests involved unsteady water flow resulting from a sudden closure of a ball valve mounted at the end of the pipe. Pressure characteristics $p(t)$ at two points located at the exhaust valve and in the middle part of the pipe-line were measured. Pressure was recorded by means of a measuring system consisting of strain gauges, an amplifier and a computer. All the tests referred to simple water hammer. The experiments were performed at a water stream temperature of 281 $\pm$ 2 K. One should stress that the temperature strongly influences the mechanical properties of the polyethylene pipes (Franke & Seyler 1983, Janson 1995, Pezzinga 2002).

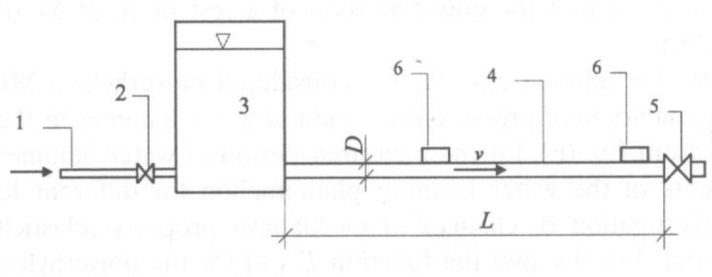

Fig. 4. Measuring stand: 1 – supply pipe, 2 – pressure reducing valve, 3 – reservoir, 4 – pipe-line, 5 – ball valve, 6 – strain gauge

The Figs. 5 and 6 show the recorded characteristics $p(t)$ at the outlet valve during water hammer (L = 24 and 120 m). The pressure wave velocity c was calculated from Eq. (7) for measured period of water hammer T.

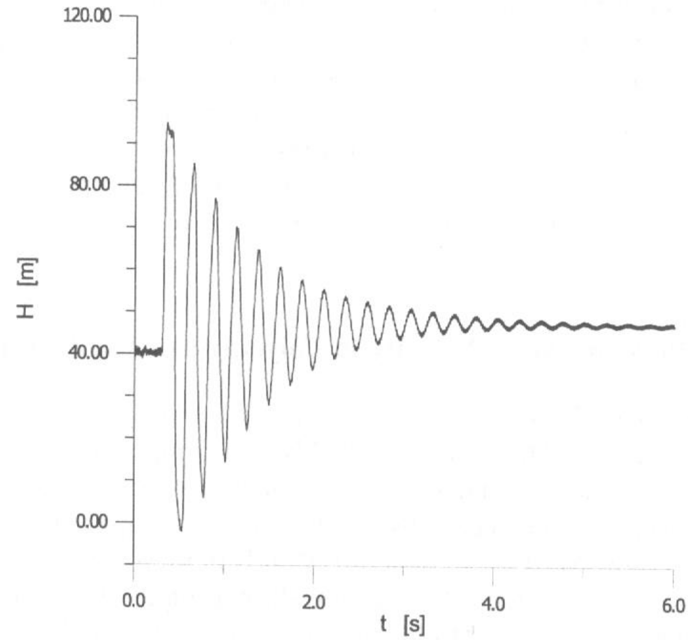

Fig. 5. The recorded characteristic $p(t)$ at the outlet valve during water hammer L = 24 m

The measurement results for MDPE pipe, as a function $c(L)$ for the range: 6 m $\leq L \leq$ 150 m, were approximated with a polynomial of third degree:

$$c = 4.4710^{-5}L^3 - 9.5610^{-3}L^2 - 0.084\,L + 397.6. \tag{19}$$

The regression curve was determined with determination coefficient R^2 = 0.9988 and standard error S_e = 3.2 m/s. The function $c(L)$ is shown in Fig. 7. Additionally, a curve $c(E_0)$, calculated from Korteweg's equation (10) for Young's modulus of polyethylene MDPE $E_0 = 8 \times 10^8$ Pa, is presented in the figure. The modulus E_0 was estimated for slow extension of a test piece of MDPE (Ashby 1998, Janson 1995).

Fig. 8 shows the function c(ω) for the considered polyethylene MDPE pipe. The angular frequency ω of pressure oscillation at water hammer in the pipe-line was calculated from Eq. (6), for the measured period of water hammer T.

Measurements of the water hammer phenomenon for different length of a pipe enable investigation of changes of mechanical property (elasticity) of the pipe wall material. Fig. 9 shows the function $E^*(\omega)$ for the polyethylene MDPE. The dynamic modulus of elasticity E^* is calculated from Eq. (18) for known values of pressure wave velocity c and for $\kappa = 1 - \mu^2$, where Poisson's coefficient

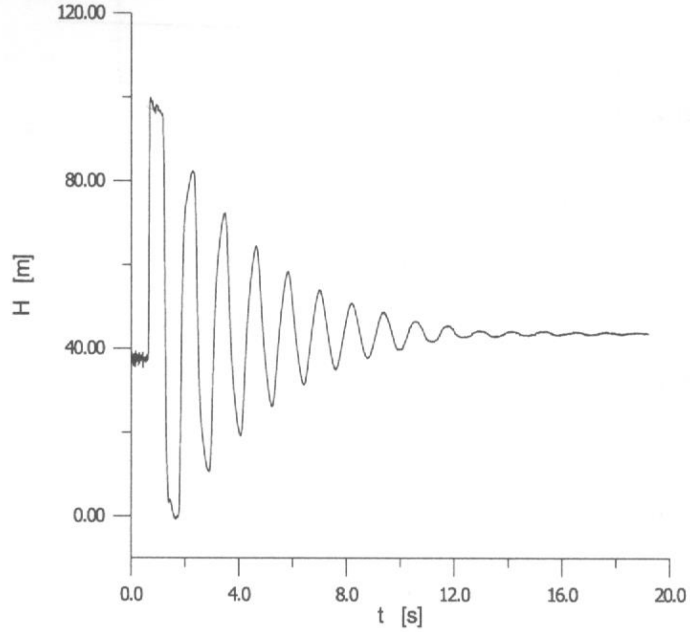

Fig. 6. The recorded characteristic $p(t)$ at the outlet valve during water hammer $L = 120$ m

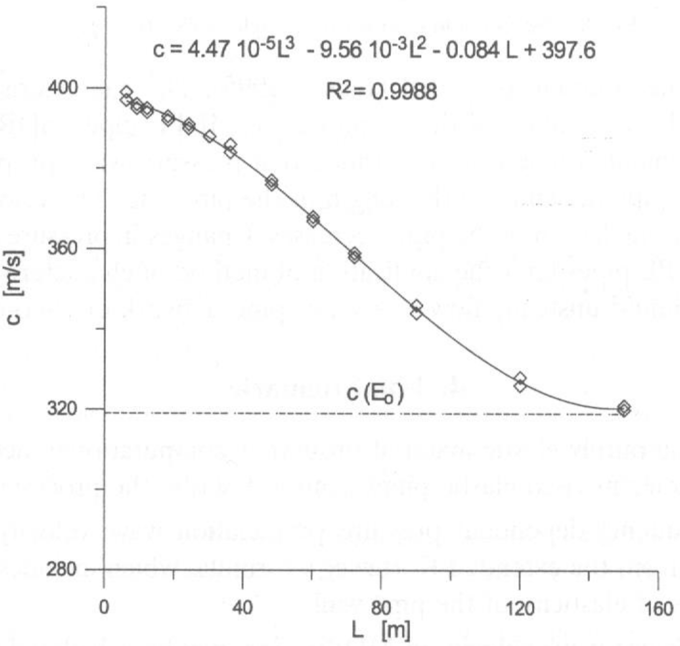

Fig. 7. The function $c(L)$ for polyethylene (MDPE) pipe

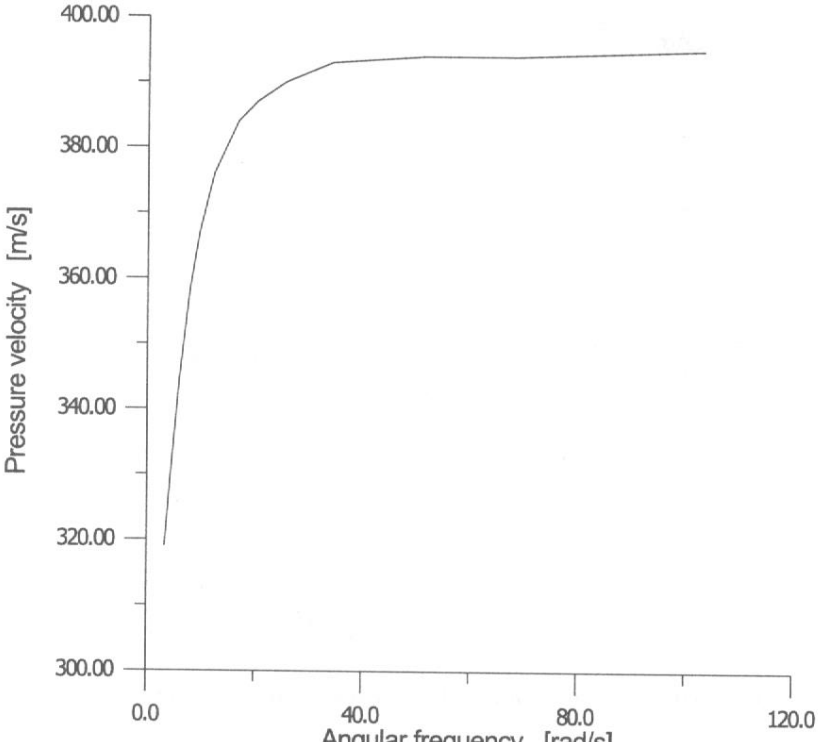

Fig. 8. The function c(ω) for polyethylene (MDPE) pipe

for polyethylene is equal to $\mu \approx 0.4$ (Janson 1995). The characteristic presented in Fig. 9 enables estimation of the velocity c for MDPE pipes: SDR ≈ 11, PN10.

The experiments carried out confirmed that pressure wave propagation velocity in MDPE pipe depends on the length of the pipe-line. The velocity distinctly increases when the length of the pipe decreases. Changes in pressure wave velocity c in PVC and PE pipes limit the application of method of characteristics MOC for a solution of liquid unsteady flow in a water-pipe network of visco-elastic pipes.

4. Final Remarks

1. Assuming purely elastic material properties computational methods for water hammer in visco-elastic pipes cannot describe the process correctly.
2. The frequency-dependent pressure propagation wave velocity may be calculated from the extended Korteweg's formula, which includes the dynamic modulus of elasticity of the pipe wall.
3. The pressure wave velocity in MDPE pipe may be calculated as a function of the length of the pipe-line. The velocity increases when the length of the pipe decreases.

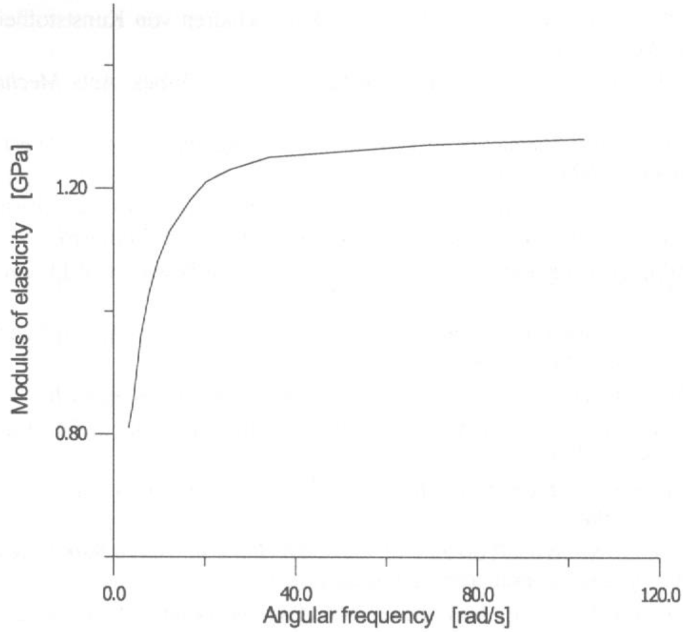

Fig. 9. The function $E^*(\omega)$ for polyethylene (MDPE) pipe

4. The value of pressure wave velocity in MDPE pipe, particularly for short pipe-lines, is distinctly higher than calculated, using Korteweg's equation, for the Young's modulus estimated for slow extension of the pipe material.
5. The application of MOC for the solution of system of differential equations of motion and continuity, used to describe an unsteady flow in water-pipe networks made of visco-elastic pipes, is limited.

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