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The Investigation of Riverbed Erosion in a Mountainous River

Marta Łapuszek

Institute of Water Engineering and Water Management, Cracow University of Technology, ul. Warszawska 24, 31-155 Cracow, Poland, e-mail: mlapusz@smok.wis.pk.edu.pl

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Abstract

The channel, floodplain and terraces are formed primarily through the erosion, transport and deposition of sediment by streamflow. The aim of the paper is to establish the changes at the gauging sections caused by riverbed erosion. The erosion process is evaluated on the basis of minimal annual water stages in gauging station. Minimal annual water stages correspond to the changes of river-bed level. On the basis of 90 years of low stages for gauging stations: Krościenko and Żabno on the Dunajec River, the erosion trends and intensity of cross-section changes are established. The computation results are compared with the quantity of real changes at gauging sections which are measured in different years. The problem presented in this paper is important in engineering. The knowledge of river-bed changes is necessary when designing water intakes, bridges, etc. It is also connected with determination of the location of underground water in the adjacent area, which is very important for evaluation of the water regime in the soil.

Keywords: mountain rivers, riverbed erosion

1. Introduction

Erosion which occurs in mountain rivers is observed in the catchment areas and riverbeds. All natural forces (wind, temperature changes, precipitation) lead to sediment movement compatible with the gravity in order to level the terrain. Areal slope erosion and bottom and bank erosion yield sediment which the rivers transport from the various source areas to its mouth. The intensity of riverbed erosion is the result of many factors: climatic (temperature, precipitation type, seasonal duration and occurrances), geological (types of soil and their distribution, land configuration), soil properties (grain-size distribution, erosivity), hydrological (infiltration rate, type of flow), vegetation cover, etc. The products of erosion, which occurs in the catchment area, finally enter the streams and rivers. Depending on the hydrodynamic conditions in the riverbed, these products are transported down the river (Bartnik 1992).

There are many forms of erosion (Graf 1971). This paper is limited to the process which occurs in riverbeds. Bed erosion is a highly natural process which has been observed since time immemorial. It has been intensified by human activity – not always proper – during the last decades. The most important factors are:

- deforestation which reduces the retention ability of a drainage area (Dynowska 1971),
- hydraulic engineering (Parzonka 1993, Ratomski 1992),
- extraction of coarse material (gravel and cobble) (Osuch 1968).

The aim of the paper is to establish the main trends of changes at the gauging stations caused by riverbed erosion in the Carpathian rivers, observed over a long period of time. The process of riverbed erosion can be considered as a stochastic process, hence the investigation of the intensity of riverbed erosion presented in this paper is based on the method of statistical analysis.

The consideration is based on the minimal and mean annual water stages. The assumption is that the minimal annual water stages correspond with the changes in the riverbed level. Also applied is the method of establishing a forecast of riverbed erosion.

The data series analysis leads to the establishing of previous trends and the forecast of riverbed erosion in each gauging station. The statistical model treats each particular annual water stage as a random variable. The data series are obtained by hydrological observations which have been carried out in the gauging station of a particular river.

The problem presented in this paper is one of the most important in water engineering. The knowledge of riverbed changes is necessary for future needs and the uses of the river system (designing water intakes, bridges and other water engineering constructions). This knowledge is also connected with the locating of underground water in an adjacent area which is very important for the evaluation of water relations in the soil. The future study of the erosion process and sediment transport is necessary. It should supply more complete data and afford the opportunity to prepare a more precise riverbed level forecast.

2. Description of the Experimental Catchment Area

The experimental catchment area is one of the largest mountain drainage basins of southern Poland. In this region the topography of drainage catchments is highly varied. The division of the Carpathian river courses is as follows: the upper courses (with average slopes: $7^0/_{00}$–$15^0/_{00}$), the middle and lower courses (with average slopes: $2.5^0/_{00}$–$0.5^0/_{00}$).

In the Carpathian rivers a great variety of water stages is observed. It is due to many factors. Heavy rainfalls, which usually last for a very short time cause a rapid rise in water in streams and mountain rivers. Additional reasons for a rapid rise in water in a short time are: melting snow and impermeability or low permeability of the ground in the Carpathian area. In the mountainous area one can also observe the reduction of evaporation from the surface of the terrain. All these factors give rise to a rapidly-growing flow, which is observed especially in spring and early summer. During the dry season or the long-lasting snow-cover on rivers, a low-flow period can be observed.

The investigation of main riverbed erosion trends is carried out at 8 gauging stations of the Dunajec River (Kowaniec, Krościenko, Gołkowice, Nowy Sącz, Rożnów, Czchów, Zgłobice, Żabno). The Dunajec River is a mountainous tributary of the Vistula River and forms one of the largest drainage catchment areas of the Upper Vistula Basin. The total catchment area is 6798 km^2, the length of the hydrographic network being 4847 km. The Dunajec River has 186 mountain tributaries, the division of its course being: the upper course (with average slope: $13^0/_{00}$), middle course (with average slope: $3^0/_{00}$), and lower course (with average slope: $1^0/_{00}$). In the Dunajec River a considerable variety of discharge is observed. During floods the discharge can amount to $Q = 1000$ m^3/s in the upper course, and about 3500 m^3/s in the lower (Dynowska, Maciejewski 1991).

3. Method of Analysis

The erosion process is evaluated on the basis of minimal water stages at gauging stations of the Dunajec River. This investigation can be carried out on condition that minimal annual water stages correspond to the changes of the riverbed level (Punzet 1993, 1994).

3.1. Justification of the Above Thesis

To prove the above thesis, appriopriate computations were carried out (Łapuszek 1999) at 3 gauging stations (Kowaniec, Nowy Sącz, Zgłobice). On the basis of the rating curves the height of a water surface which corresponds with the discharge $Q = 0$ has been established. The height of the water surface above the natural bed (h) is calculated by the simple equation:

$$h = H - B \ \text{[cm]} \tag{1}$$

where:

- h – height of water surface above the natural bed in year T [cm],
- H – low annual water stage in year T [cm],
- B – stage at zero discharge [cm].

The obtained data series have some points of discontinuity, due to lack of rating curves in some years. However, this has no influence on the results of the computation. In order to obtain the trends of the mentioned height of a water h

over a long period of time (about 50 years), this data series is approximated by linear interpolation. The method of computation is the same as for the equations which express the intensity of riverbed erosion (Section 3.3).

The results of the computation indicate that the relation between height of a water surface above the natural bed in year and time over a period of some years is almost constant at the three gauging stations mentioned above (Fig. 1). It means that the relation between height of a water surface h and time T does not change although the changes of the riverbed level over a longer period can be observed.

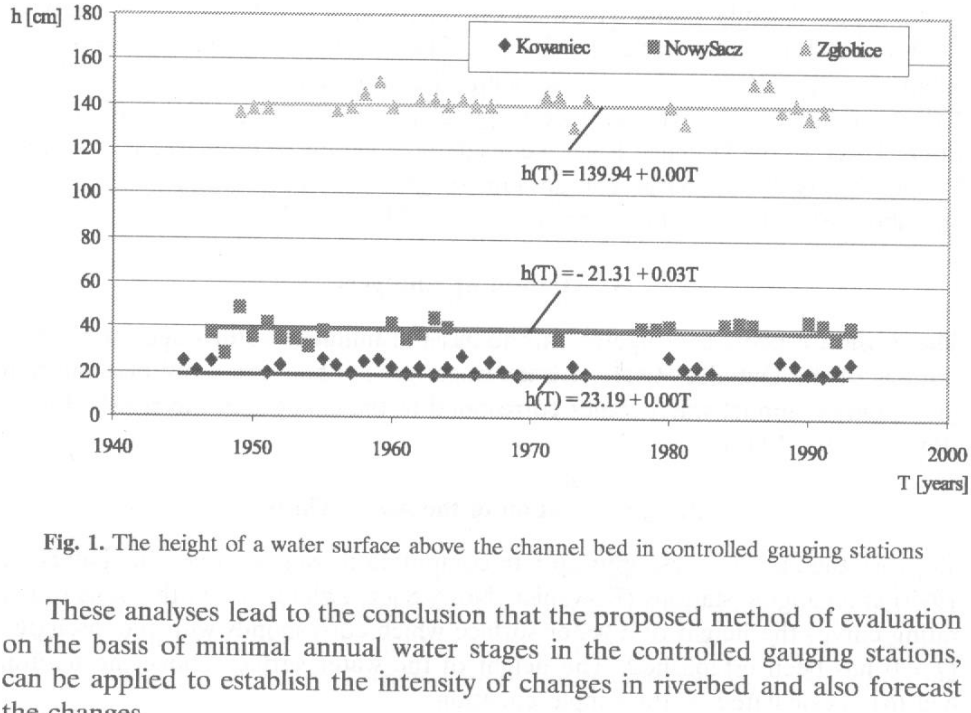

Fig. 1. The height of a water surface above the channel bed in controlled gauging stations

These analyses lead to the conclusion that the proposed method of evaluation on the basis of minimal annual water stages in the controlled gauging stations, can be applied to establish the intensity of changes in riverbed and also forecast the changes.

3.2. Method of Computation

The course of low annual water stages in the eight gauging stations has been analysed (Punzet 1996). In a particular gauging station this data series for each determined time interval is approximated by linear interpolation. The investigated equation (3) gives the relation between water stage H and time T and also illustrates the main trends of changes in the gauging station over a long time period. The equation determining the level of the bed at any time is expressed as:

$$H_d = H_z + H_i(T) \tag{2}$$

where:

H_d – average level of riverbed in year T (metres above sea level),

H_z – assumed datum level (metres above sea level),

T – year of observation,

$H_i(T)$ – approximated equation (3) obtained by:

$$H_i(T) = aT + b. \tag{3}$$

The intensity of changes in time is expressed by the coefficient for T variable.

A few tests have been carried out to divide the data series into time intervals. The final division is the best solution for the least square method of approximation.

In order to confirm the validity of the computation, the functions determining the level of the bed at any time, are verified on the basis of the comparison of:

- the quantity of real changes of the channel cross-section, which are measured in different years,
- the changes of rating curves observed during this period.

On the basis of the measured amount of eroded material for the assumed time intervals, the real amount is determined. The observations of the changes of rating curves in the selected years also show the changes in the riverbed level which have been observed over a long period of time. The real values of riverbed erosion obtained have been compared to the suitable coefficients for T variables, which describe the main trends of changes in gauging station over a particular time interval. On the basis of the obtained equations which describe the riverbed changes, the forecast of riverbed erosion in the station observed is established.

3.3. Determination of the Equations which Express the Intensity of Riverbed Erosion

The investigation of riverbed erosion has been carried out with the assumption that minimal annual water stages correspond with the changes in the riverbed level. Hence, the obtained data series is divided into time intervals, in which the equation (3) is approximated by linear interpolation. The equation mentioned (in each i-time interval) is expressed as:

$$H_i(T) = E(H\,|T) = \alpha_i T + \beta_i \tag{4}$$

where:

H – low annual water stage [cm],

T – year of observation.

The assumption is that T and H are dependent variables and are expressed by normal bivariate distribution. Coefficients α_i and β_i are the real values of coefficients of the linear regression. In the following description of the computational method the i index is omitted.

Determination of the equation (3) mentioned for the bivariate testing data series: $\{(T_j, H_j)\} = \{(T_1, H_1), (T_2, H_2), \ldots (T_n, H_n)\}$ in a particular time interval is performed in the following way (Fisz 1967):

- estimation of the equation parameters (a and b),
- carrying out the test of significance for the coefficient of linear regression,
- establishing the confidence interval for the estimated coefficient of linear regression.

The coefficients of linear regression are established by the least square method:

$$F(a,b) = \sum_{j=1}^{n} \left[H_j - (aT_j + b)\right]^2 = \min . \tag{5}$$

After the following computation:

$$\frac{\partial F(a,b)}{\partial a} = \sum_{j=1}^{n} 2[H_j - aT_j - b](-T_j) = 0, \tag{6}$$

$$\frac{\partial F(a,b)}{\partial b} = \sum_{j=1}^{n} 2[H_j - aT_j - b](-1) = 0, \tag{7}$$

introducing the symbols of arithmetic mean ($\overline{T}, \overline{H}, \overline{TH}$), and carrying out simple calculations, we obtain the equations which describe the empirical coefficients of linear regression:

$$a = \frac{\overline{TH} - \overline{T} \cdot \overline{H}}{T^2 - (\overline{T})^2}, \tag{8}$$

$$b = \overline{H} - a\overline{T}. \tag{9}$$

The estimated equation of linear regression is:

$$H_i(T) = aT + b. \tag{10}$$

The test of significance $H_0(\alpha = 0)$ for coefficient of linear regression α between the variables (T, H) has been performed basing on the assumption that these variables are subordinated by normal bivariate distribution. In this case, the statistics which are subordinated by two-dimentional Student's distribution, are used for testing previous hypothesis H_0:

$$t^* = \frac{\alpha}{s_R}\sqrt{\sum_{j=1}^{n}(T_j - \overline{T})^2} \tag{11}$$

where:

s_R – deviation to the linear regression:

$$s_R = \sqrt{\frac{1}{n-2}\sum_{j=1}^{n}\left[H_j - (aT_j - b)\right]^2}. \tag{12}$$

For the significance level $\gamma = 0.05$ and n-2 degrees of freedom the critical value t_γ is chosen on the basis of the table of two-dimentional Student's distribution. This critical value t_γ is compared to the value of statistics t^*. Satisfying inequality: $|t^*| < t_\gamma$ means that variations H and T are independent. In opposite case: $|t^*| > t_\gamma$, means that variations H and T are dependent. The quantity measure of fitting the investigated regression line to the experimental points is obtained by the following formula:

$$S^2_{RH} = \frac{1}{n}\sum_{j=1}^{n}\left[H_j - (aT_j + b)\right]^2. \tag{13}$$

This minimizes the sum of mean square deviation from the regression line: $H_i(T) = aT + b$.

This value is the empirical equivalent of the remainder variance: $\frac{(E[\mathrm{var}(H|T)])}{\mathrm{var}H}$ hence it gives the precision of the true regression representation by the investigated regression line. The quantity assessment of the regression obtained is given by confidence intervals. For assessment of confidence level $1-\gamma$ the confidence interval is established by:

$$a - t_\gamma \frac{s_R}{\sqrt{\sum_{j=1}^{n}(T_j - \overline{T})^2}} < \alpha < a + t_\gamma \frac{s_R}{\sqrt{\sum_{j=1}^{n}(T_j - T)^2}} \tag{14}$$

where:

a – coefficient of regression obtained by the least square method for function (3),

t_γ – variable of t-Student distribution obtained for $n-2$ degrees of freedom and established confidence coefficient $1-\gamma$.

The quantity assessment of the regression is given for the obtained equations, which are the assessment basis for the riverbed erosion forecast, by the confidence zone. This confidence zone is situated between the curves given by the following formula:

$$H_{\pm}(T) = H(T) \pm t_{\gamma}(n-2)s_R \sqrt{\frac{1}{n} + \frac{(T_j - \overline{T})^2}{\sum\limits_{j=1}^{n}(T_j - \overline{T})^2}}. \tag{15}$$

The computation is carried out for the observed data series in each gauging station. This data series is divided into time intervals in which the functions mentioned are established. The linear regression describes the tendency of riverbed erosion.

3.4. The Forecast of Riverbed Erosion

The estimation of future riverbed erosion is elaborated on the basis of the linear regression obtained for each gauging station. The linear regression is a good tool for riverbed erosion forecasting for those phenomena for which one can obtain enough data. The data must be stable in time, hence, the equations (10) mentioned are chosen in order to satisfy these conditions. This means that in the time interval of selected equations the process of riverbed erosion is observed continually but without very rapid changes in the riverbed level. This model should be verified, after which riverbed erosion can be estimated. The forecast is elaborated for the assumed confidence coefficient $1-\gamma = 0.95$ according to the following formula (Fisz 1967):

$$P\left\{\hat{H}_p - t_{\gamma} D\left(\hat{H}_p\right) < H_p < \hat{H}_p + t_{\gamma} D\left(\hat{H}_p\right)\right\} = 1 - \gamma \tag{16}$$

where:

$\hat{H}_p$ – forecast for linear regression given by the formula:

$$\hat{H}_p = a T_p + b \tag{17}$$

where a and b are expressed by formulae: (8), (9),

H_p – the estimated future value of low annual water stage [cm],

$D(\hat{H}_p)$ – mean error of forecast given by the formula:

$$D(H_p) = s_R \sqrt{1 + \frac{1}{n} + \frac{(T_p - \overline{T})^2}{\sum\limits_{j=1}^{n}(T_j - \overline{T})^2}} \tag{18}$$

where: T_p – year.

The length of selected time intervals differs for each gauging station. It depends on the following factors (Łapuszek 1999, Węglarczyk 1998):

- length of the data series used as the basis for the forecast,
- value of the mean error,
- possibility of the occurring of a new flow conditions connected with water engineering activity in the riverbed.

The forecast proposed in the paper is elaborated for a short period, between 5-10 years in each gauging station. The flow condition could change over the years, therefore the forecast should be verified every few years.

4. Analysis of the Results

The 90-year data series in each gauging station are divided into time intervals. For each particular interval the linear regression is established and describes the tendency of riverbed erosion. The proposed division of time intervals depends upon the possibility of verification of the obtained results and the observed tendency of the course of annual water stages.

Computation has been carried out for 8 gauging stations, but in this paper we present the results of two: Krościenko and Żabno.

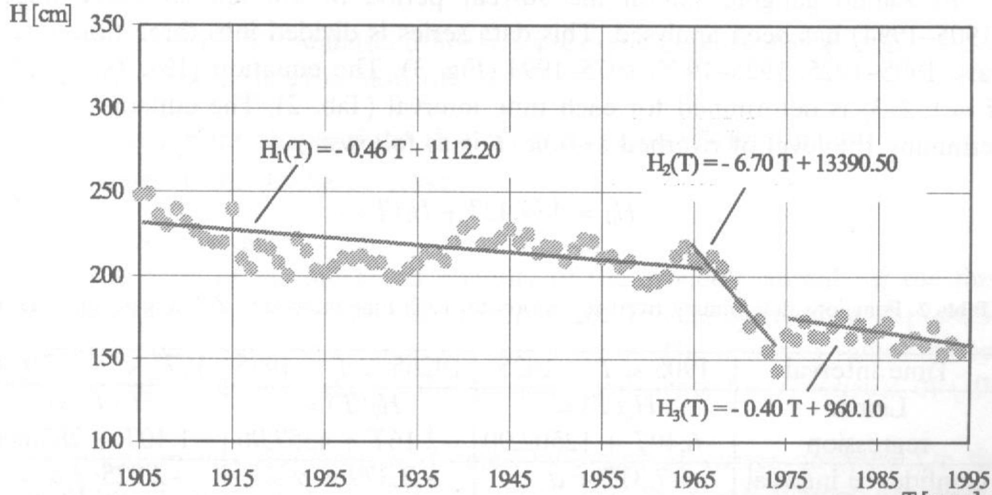

Fig. 2. The low annual water stages and linear regressions in Krościenko gauging station

In Krościenko gauging station the 95-year period of low annual water stages (1900–1994) has been analysed. This data series is divided into three time intervals: 1900–1965, 1965–1975, 1975–1994 (Fig. 2). The relation between water stage H and time T is given by equation (10): $H_i = f(T)$, $(i = 1, 2, 3)$. It is determined for each time interval (Tab. 1). The equation (2) determining the level of riverbed at time T is as follows:

$$H_d = 413.455 + H_i(T). \tag{19}$$

The intensity of changes in time is expressed by the coefficient for T variable. It is easy to note that the elementary depressions of the bed are adequate to the following time intervals: 0.5 cm, 6.7 cm, 0.5 cm per year respectively (Tab. 1).

Table 1. Functions determining riverbed erosion for each time interval in Krościenko gauging station

Time interval	$1900 < T < 1965$	$1965 < T < 1975$	$1975 < T < 1994$
Linear regression	$H_1(T) = -0.46T + 1112.20$	$H_2(T) = -6.70T + 13390.50$	$H_3(T) = -0.40T + 960.10$
Confidence interval of coefficient α	$-0.543 < \alpha < < -0.381$	$-8.596 < \alpha < < -4.821$	$-0.824 < \alpha < < -0.016$
t_γ	1.98	2.201	2.086
Value of statistic $\|t^*\|$	4.648	3.936	2.496
Correlation coefficient R_{TH}	−0.714	−2.488	−0.475

In Żabno gauging station the 90-year period of low annual water stages (1905–1994) has been analysed. This data series is divided into three time intervals: 1905–1925, 1925–1975, 1975–1994 (Fig. 3). The equation (10): $H_i = f(T)$, $(i = 1, 2, 3)$ is determined for each time interval (Tab. 2). The equation (2) determining the level of riverbed at time T is as follows:

$$H_d = 174.427 + H_i(T). \tag{20}$$

Table 2. Functions determining riverbed erosion for each time interval in Żabno gauging station

Time interval	$1905 < T < 1925$	$19265 < T < 1975$	$1975 < T < 1994$
Linear regression	$H_1(T) = -6.40T + 12502.90$	$H_2(T) = -2.16T + 4357.90$	$H_3(T) = -1.40T + 2850.60$
Confidence interval of coefficient α	$-7.317 < \alpha < < -5.482$	$-2.379 < \alpha < < -1.95$	$-2.025 < \alpha < < -0.779$
t_γ	2.080	2.021	2.086
Value of statistic $\|t^*\|$	14.598	17.501	4.695
Correlation coefficient R_{TH}	−3.349	−2.929	−1.106

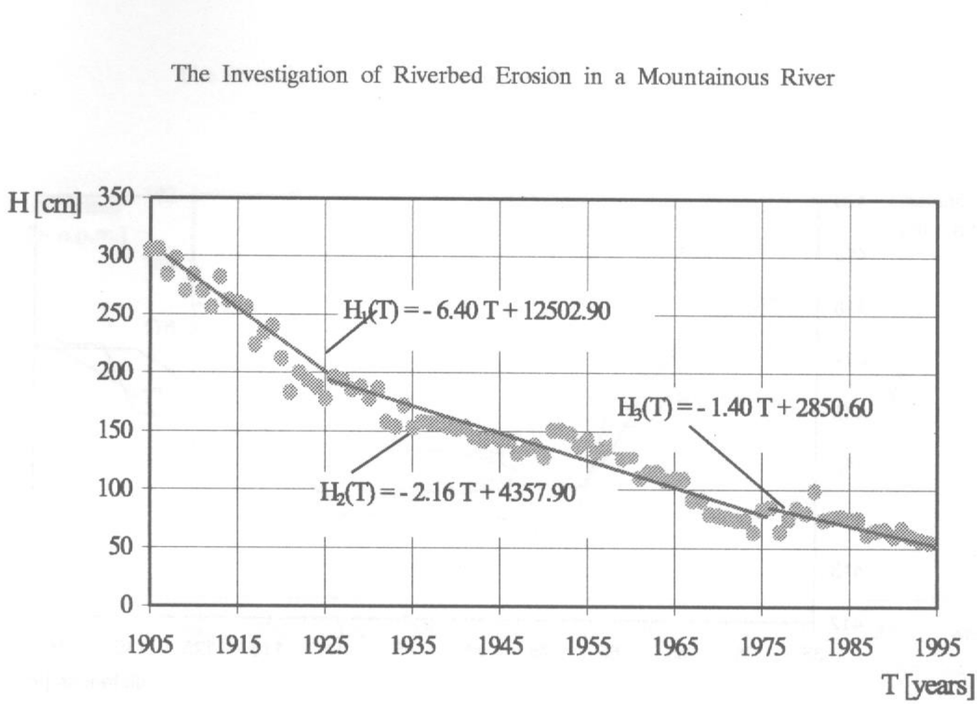

Fig. 3. Low annual water stages and linear regressions in Żabno gauging station

The intensity of changes in time is expressed by the coefficient for T variable. It is easy to note that the elementary depressions of the bed are adequate to the following time intervals: 6.4 cm, 2.1 cm, 1.4 cm per year respectively (Fig. 3).

5. Verification of the Results

The verification of the established linear regressions in particular gauging station has been carried out on the basis of:

- changes in the shape of the channel cross-section (width, depth),
- changes in the discharge curves

observed over the years.

On the basis of the measured quantity of the eroded material for the time intervals the real quantity has been determined. The mean riverbed depression per year is established for a particular time interval. The obtained values in each time interval are compared with the suitable coefficients for T variable of (10): $H_i(T)$.

Verification has also been performed by discharge curves analyses. On the basis of the observed changes of the curves in time, the mean riverbed depression per year has been established and compared with the suitable coefficients for T variable of (10): $H_i(T)$ in each time interval (Łapuszek 1999).

In some cases the results of computation can be verified only by this method, due to the lack of suitable discharge curves in the measured gauging stations.

In Krościenko gauging station the results of computations are verified by comparing the observed changes of the shape of the channel cross-section in time, in

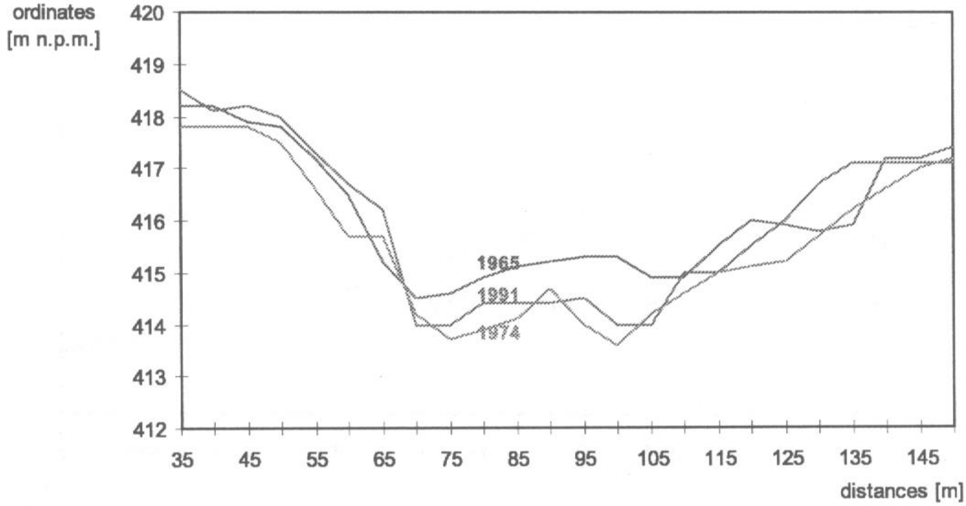

Fig. 4. Changes in the shape of the channel cross-section in Krościenko gauging station

two time intervals (1965–74, 1974–91) (Fig. 4). The average riverbed depression was 9.2 cm per year from 1965 to 1974, and from 1974 to 1994 the depression was 0.8 cm. In the years 1900–65 the established results were verified by the observed changes in curves of discharge. The mean depression during that time interval was 0.6 cm per year. The results of verification (Tab. 3) confirm the correctness of this assumption.

Table 3. The intensity of riverbed erosion in Krościenko gauging station

Time interval	Years	Total erosion [m^2]	Mean riverbed depression obtained by: [cm/year]		
			Changes of		$H_i(T)$
			cross-sections	discharge curves	
1900–1965	66	–	–	0.6	0.5
1965–1974	10	45.75	9.2	–	6.7
1974–1991	18	4.45	0.8	–	0.5

Equations (Tab. 2) for $H_1(T)$ and $H_2(T)$ in Żabno gauging station are verified on the basis of the observed changes in the shape of the channel cross-section (Fig. 5). In this computation the following values of mean year depression: 6.2 cm and 2.6 cm respectively (Tab. 4) were obtained. Equation (Tab. 2) for $H_3(T)$ is verified by the discharge curves analysis. The mean year depression obtained in this way equals 0.5 cm (Tab. 4).

In both cases presented the results obtained independently are significantly similar to the value for coefficient of T variable in approximative equations. It

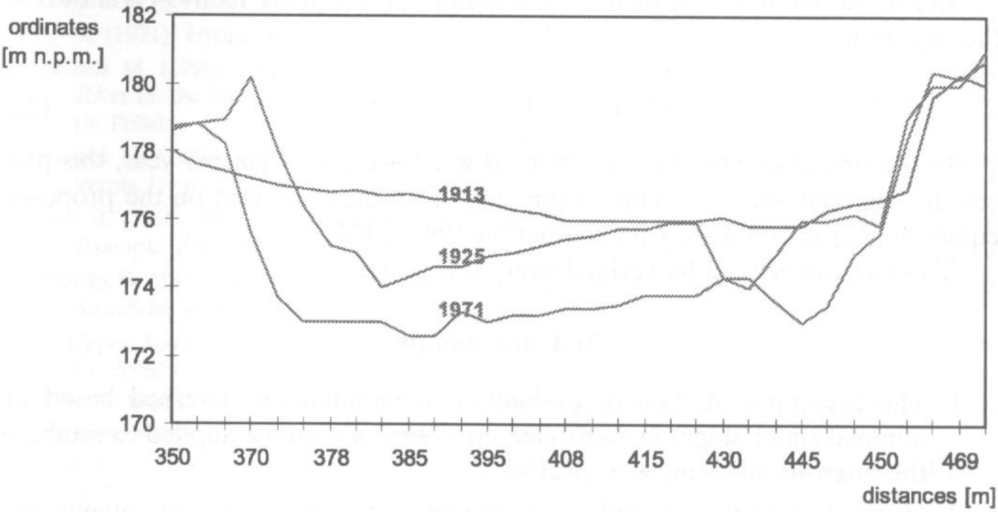

Fig. 5. Changes in the shape of the channel cross-section in Żabno gauging station

Table 4. The intensity of riverbed erosion in Żabno gauging station

Time interval	Years	Total erosion [m^2]	Mean riverbed depression obtained by: [cm/year]		
			Changes of		$H_i(T)$
			cross-sections	discharge curves	
1931–1925	13	80.0	6.2	–	6.4
1925–1971	47	129.0	2.6	–	2.2
1971–1994	24	–	–	1.9	1.4

confirms the correctness of the assumptions. Hence the method proposed in this paper is suitable for riverbed changing forecast.

6. Riverbed Erosion Forecasting

According to the assumptions formulated in Section 3.3, the forecast of riverbed erosion has been established for eight gauging stations of the Dunajec River. This paper presents the forecast elaborated for Krościenko and Żabno gauging stations.

In Krościenko the intensity of riverbed erosion for the years 1975–94 is given by the following equation:

$$H_3(T) = -0.40T + 960.10. \tag{21}$$

The above formula shows the mean erosion value to equal 0.4 cm per year. Taking into account also the value of the mean error, the forecast estimated by the performed equation (2) for 10 years (1995–2004) and no longer can be proposed.

In Żabno the mean riverbed depression during the years 1975–94 is shown by the equation:

$$H_3(T) = -1.40T + 2850.60. \tag{22}$$

In the period mentioned, the riverbed was lowered 1.4 cm per year, this process having continued in a fairly stable way. The forecast based on the proposed equation (22) is established for the period 1995–2004.

This forecast should be verified every few years.

7. Conclusions

1. The presented method of evaluation of mountainous riverbed based on minimal water stages in controlled cross-sections can be applied to establish the intensity of changes in riverbeds.
2. Verification of the established linear regressions in a particular gauging section is carried out independently on the basis of:
   - changes in the shape of the channel cross-section,
   - changes in the discharge curves.

   The results of verification confirm the validity of the assumption, thus the method proposed in the paper can be used for riverbed changing forecasts.
3. The intensity of changes in time expressed by the coefficient for T variables is significantly similar to the value established by the analysis of changing discharge curves. It suggests that the intensity of riverbed erosion in time can be established by the changes of discharge curves in time.
4. The forecast of riverbed erosion in mountain rivers can be elaborated on the basis of the linear regression obtained. However, this should be verified every few years in view of the possibility of some rapid changes of riverbeds connected with water engineering activity or floods.
5. Further studies on the erosion process in mountain rivers is necessary in order to obtain complete data which could enable a more precise forecast to be prepared. Knowledge of riverbed changes is necessary when designing water intakes, bridges and other water engineering constructions. It is also connected with the locating of underground water table in an adjacent area which is very important for the evaluation of water relations in the soil.

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