Paper

A new methodology of accounting for uncertainty factors in multiple criteria decision making problems

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Abstract—A new approach is proposed to select a predetermined number of "reasonable" (the best in a certain sense) alternatives from the considerable (maybe a vast) set of initial alternatives according to an arbitrary number of optimization criteria and accounting for uncertainty factors. The approach is based on using a special intuitive methodology, developed to account for uncertainty factors when solving such multiple criteria decision making (MCDM) problems. This methodology is based on performing multi-variant computations (MVC) and finding their "stable-optimal" solutions, and it's realized as a multi-level hierarchical system of MVC series. It's possible to use this methodology for solving various real problems.

Keywords—multiple criteria decision making, uncertainty factors, "reasonable" solutions, multi-level hierarchical system of multi-variant computation series, "stable-optimal" solutions, scenarios, Monte Carlo simulations.

1. Introduction

An approach [1–6] is proposed to select *a predetermined number of "reasonable" alternatives* (their set is named RAS) from their *considerable* (maybe vast) *initial set* (ISA) according to multiple criteria, presented by *an arbitrary number of optimization criteria*. This selection is performed as accounting for *uncertainty factors*, inherent in both the considered problem and its solution process.

The solution process considered should include the following basic stages: (1/2) creating the ISA/ISCAV, where the ISCAV, interrelated with the ISA, should reflect the problem solution objectives, expressed by multiple criteria; (3) multi-criteria optimization in the ISCAV/ISA space to reach the RAS by decreasing ISCAV/ISA and accounting for uncertainty factors. The methodology to perform the stages (1/2) is specific to each considered problem, but for the stage (3) a quite *universal intuitive methodology* was developed, based on accounting for uncertainty by performing *multi-variant computations* (MVC) and finding their *"stable-optimal" solutions*. This MVC process is realized as *a multi-level hierarchical system of MVC series*, where each its level includes a totality of *scenarios*, having the same specific nature for this level. These scenarios reflect varying problem conditions and parameters. Such varying allows to account for uncertainty factors using procedures specific to each level of the multi-level hierarchical system considered. So, one type of procedures to account for uncertainty consists of varying the assigned (a priori) different versions of ISA and ISCAV as well as multi-criteria optimization techniques, used in computations for the upper (exterior) levels of the hierarchical system. Other types of such procedures involve some directed or random sorting out of possible values of problem parameters. This corresponds to the system intermediate and interior levels, where these parameters might be considered as any intervals of random variables. In this case a Monte Carlo simulation is used. Another aspect of accounting for uncertainty is using only such versions of multi-criteria optimization techniques, which were especially modified (by us) to account for uncertainty factors.

Generally, the main aspect of accounting for uncertainty in the proposed methodology is use of the multi-level hierarchical system of MVC series itself to reach the required solutions. It is what precisely allows to account for uncertainty factors, having different nature, by a way of finding "stable-optimal" solutions of MVC series, formed for each level of scenarios. Moving this way, subsets of "stable-optimal" alternatives are formed for each level of the system, based on analysis of such "stable-optimal" subsets, obtained before for the preceding (lower) level. In the end of this moving "from bottom to top" of the system, the required RAS is reached as a "stable-optimal" subset for the first (top) level of this hierarchical system. For each suitable real problem, based on processing vast amount of initial information, the *final solution* may be found by a way of the RAS (or the RAS series) analysis (usually, non-formal) using additional qualitative and quantitative criteria and estimates.

Various investigations have been performed to apply the proposed approach to solving several real problems, actual for conditions prevailing in Israel [1–6].

Although several methods were developed to solve various multiple criteria decision making problems, our intuitive approach might be considered as an original one. In our opinion, this approach might be used to solve problems for which other methods are not convenient. With this point of view, we will compare it with the well-known

AHP methodology of Prof. T. Saaty [8, 9]. Concerning this comparison, it's possible, on our opinion, to say the following:

1. At first glance both these methods use the same multi-level structures to solve the appropriate problems. However, we see *different essence of the levels in these structures*:

   – in the AHP, such levels represent only objects (criteria and alternatives), considered as obvious parameters in the solution process;

   – in our approach, these levels reflect the series of multi-variant computations, which are performed to account for uncertainty factors, specific only to the considered level objects, having various nature.

   This is the main difference between both methods and their solution methodologies, leading to the discrepancy between their fields of application.

2. The AHP methods consider, mainly, the MCDM problems with small number of initial alternatives, but our approach is oriented to solve *the MCDM problems with a considerable number of initial alternatives*. The areas of MCDM problems suitable to application of these methods are very different.

3. The AHP is based on *use of expert estimates of objects' priorities, but our approach can practically avoid use of such estimates*. Use of our approach expands possibilities for solving various MCDM problems, but AHP methods (when they can be practically used) can give more reliable results.

4. The AHP considers one top goal and small number of objects (criteria or alternatives) on other hierarchy levels; *in our approach the number of versions (scenarios) on each hierarchy level may be arbitrary*.

5. *It's possible to include the criteria multi-level hierarchy system, used in the AHP, in our solution methodology as well.*

Let's consider further some basic features of the proposed methodology.

2. Calculation process peculiarities

In accordance with an available uncertain situation, the process to solve the considered problem is treated as *a two-step* one. In its *first step*, a "reasonable" alternatives set (RAS) should be selected from all initial alternatives in accordance with joint accounting for multiple criteria, assigned a priori. This first step might be completed also by finding a totality of such RAS, including several ones, that depends mainly on organization of the second step of solving the whole problem. In this *second step, the final solution of the whole problem*, ready to be used in a practical decision making process is found, basing on the RAS analysis obtained, performed basically in a non-formal way using additional qualitative (including subjective) and quantitative information, criteria and procedures.

Thus, the proposed approach allows to *sharply decrease* the amount of information needed for decision making.

This first step includes the following basic stages: (1-2) constructing *the initial sets of alternatives* (ISA) and *criteria assessment vectors* (ISCAV); (3) decreasing the considerable (maybe vast) ISA/ISCAV to the required (usually small) RAS.

The calculation methodologies used to construct the ISA/ISCAV should be specific to each MCDM problem considered, elaborated especially to account for specific features of this problem. These elaborations can have various basic directions and "bottlenecks". In our experience of solving the appropriate problems, we have encountered situations, when the basic information and calculation difficulties were related to the ISA construction as well as when the ISA was formed in obvious and easy way, but the ISCAV construction required considerable efforts.

In our experience with the ISA construction process [1–6], we had very difficult case of forming the vast initial set of alternatives for the problem of power generation system expansion (PGSE) planning [1–3, 5], where each such alternative reflected the dynamic PGSE strategy. The case of implicit assignment of initial alternatives is linked with the problem of stock buying on the stock market [6], where each initial alternative reflects the natural operation of a stock buying.

The ISCAV construction process consists of the following: (a) *assignment of the criteria totality*; (b) *development of the criteria calculation models*, allowing to determine *criteria assessment vector* for each considered alternative, where this vector is represented for this alternative by one numerical value for each alternative from the ISA; (c) *forming the initial set of such vectors* (ISCAV), interrelated with the ISA. For this ISCAV construction process, we can have the opposite situations: (a) there are the natural criteria (economic, technical, reliability, others), where the criteria calculation models are developed, mainly, by using the existing models, methods and procedures (e.g., [1–3, 5]); (b) the necessity is raised to create principally new models to form the ISCAV (e.g., for the above considered stock market problem [6]). We will consider in detail (Section 4) the latter situation "(b)" for the same stock market problem, where a new approach, different from one developed early (see [6]), is described.

To implement stage (3) of the problem solution process, we have developed a quite universal *intuitive solution methodology* [4–6] to reach the RAS by decreasing the ISA. This methodology of *accounting for uncertainty*, applicable to various MCDM problems, is based on performing multi-variant computations (MVC) and finding their "stable-optimal" solutions.

The developed *methodology of accounting for uncertainty* is implemented as *a multi-level hierarchical system of MVC series*. Each *l-level* ($l = 1,\ldots,L$) of this system includes *a totality of l-scenarios*, having the same nature, specific only to this l-level. Such l-scenarios reflect the possible variations of parameters and conditions, corresponding to this l-level. *Each* $l^{\wedge}$*-MVC series*, corresponding to a fixed $l^{\wedge}$-level, reflects a combination of $l^{\wedge}$-scenarios from their totality and generates an appropriate *subset of "stable-optimal" alternatives*. Forming a combination of $l^{\wedge}$-MVC series allows to find the appropriate set of "stable-optimal" subsets, corresponding to this $l^{\wedge}$-level ($l^{\wedge} = 1,\ldots,L$). On a basis of this set processing, a "stable-optimal" subset of the next upper ($l^{\wedge}-1$)-level might be determined using *a special procedure*. Its "key" operations are based on calculating the highest frequencies of entering into this full set for the alternatives from the "stable-optimal" subsets of $l^{\wedge}$-level, forming this set.

Thus, the multi-level hierarchical system of MVC series performance, realizing the calculation stage (3) to reach the resulting RAS, consists of a successive forming of sets of "stable-optimal" subsets for all l-levels ($l = 1,\ldots,L$), beginning with the lowest L-level ($l = L$) and ending with the top l-level ($l = 1$). The resulting (one or several) RAS should be also found as subset (subsets) of "stable-optimal" alternatives, where this finding represents a final operation in the calculation process considered. In a case when several RAS are derived, their non-formal analysis is performed in the second step of the whole solving process in order to find a final solution of the problem.

The *multi-level hierarchical system of MVC series* can have various structures and content, differing by the number of l-levels as well as the accepted *system of l-scenarios*. We have already considered *nine-* and *six-level hierarchical systems* with some variations in their totalities of l-scenarios [4–6].

At present, we use *a six-level hierarchical systems* [4–6] based on *the multi-criteria optimization technique* TOPSIS [7], modified to consider *the criteria weights as random variables* which are presented by the intervals of their *possible values* [1–6]. These values are determined inside these intervals by *Monte Carlo simulations.*

3. Illustration of the six-level hierarchical system performance on the sample

The process of this six-level hierarchical system performance for the simple sample is illustrated in Fig. 1. This process consists of successive forming of all possible l-MVC series ($l = 1,\ldots,6$), from the lowest 6-level ($l = 6$) to the top 1-level ($l = 1$), and determination of the "stable-optimal" subset for each such l-MVC series, reflecting a combination of full Scenarios. Each *full Scenario* is a combination of l-scenarios, taken one at a time for each of all l-levels ($l = 1,\ldots,6$). We will present this calculation process to reach the RAS, performed from "bottom" ($l = 6$) of the hierarchical system to its "top" ($l = 1$), for the conditions of the considered sample.

The initial data of the considered simple sample are presented in Fig. 2, where 40 points reflect all initial alternatives, i.e., the ISA includes 40 i-alternatives (points $\{i = 1-40\}$). Each such i-point (i-alternative) has two coordinates (the criteria values $\{C_{ij},\ j = 1,2\}$), for example in Fig. 2 we can see that 1-point (1-alternative) has the coordinate (criteria) values $\{C_{11} = 1.0,\ C_{12} = 13.5\}$.

We will consider this MCDM problem according to the above mentioned principle, where multi-criteria optimization (MCO) on all l-levels of this six-level hierarchical system is based on finding the following minimal (for all i) scalar sums with the random j-criteria weights $\{W_j,\ j = 1,\ldots,J\}$:

$$\min_{\{i=1,\ldots,I\}} \{C_{i1}W_1 + \ldots + C_{ij}W_j + C_{iJ}W_J\}. \qquad (1)$$

Here, these random variables are presented by the intervals $\{[w_j^{\min}, w_j^{\max}],\ j = 1,\ldots,J\}$ of their possible values $\{w_j,\ j = 1,\ldots,J\}$, where each such value is chosen within its interval using a Monte Carlo simulation.

According to our sample conditions, we have $\{i = 1,\ldots,40;\ I = 40\}$ and $\{j = 1,2;\ J = 2\}$, that leads to the necessity to solve the following minimization problem:

$$\min_{\{i=1,\ldots,40\}} \{C_{i1}W_1 + C_{i2}W_2\}. \qquad (1')$$

The considered six-level hierarchical system of MVC series, applicable to the sample conditions, should include the following l-scenarios for all l-levels (l = 1–6) of this system:

- $l = 1$. Single 1-scenario, representing a version of MCO technique TOPSIS (see [1–3]), modified to consider the criteria weights as random variables and to use Monte Carlo simulations (the appropriate MCDM problem was reflected above by formula (1′).

- l = 2–3. Single 2-, 3-scenarios, reflecting single versions for the ISA/ISCAV, both corresponding to data represented in Fig. 2 and reflecting 40 initial alternatives, having the numbers {points $i = 1,\ldots,40$}, as well as 2 criteria with the numbers $\{j = 1,2\}$.

- l = 4–5. The totalities of 4-, 5-scenarios reflect the accepted (see [6]) 9 versions of possible values' intervals for the random criteria weights W1,W2:
  (1) $\{[0.475, 0.525\}, [0.475, 0.525\}\}$;
  (2) $\{[0.45, 0.55\}, [0.45, 0.55\}\}$; …
  (4) $\{[0.6175, 0.6825\}, [0.3325, 0.3675\}\}$; …
  (9) $\{[0.2975, 0.4025\}, [0.5525, 0.7475\}\}$.

- $l = 6$. The totalities of 6-scenarios represent 90 combinations of possible values of random criteria weights, obtained within the above 9 intervals using the 10 assigned series of Monte Carlo simulations. Each series of 2 Monte Carlo simulations generates

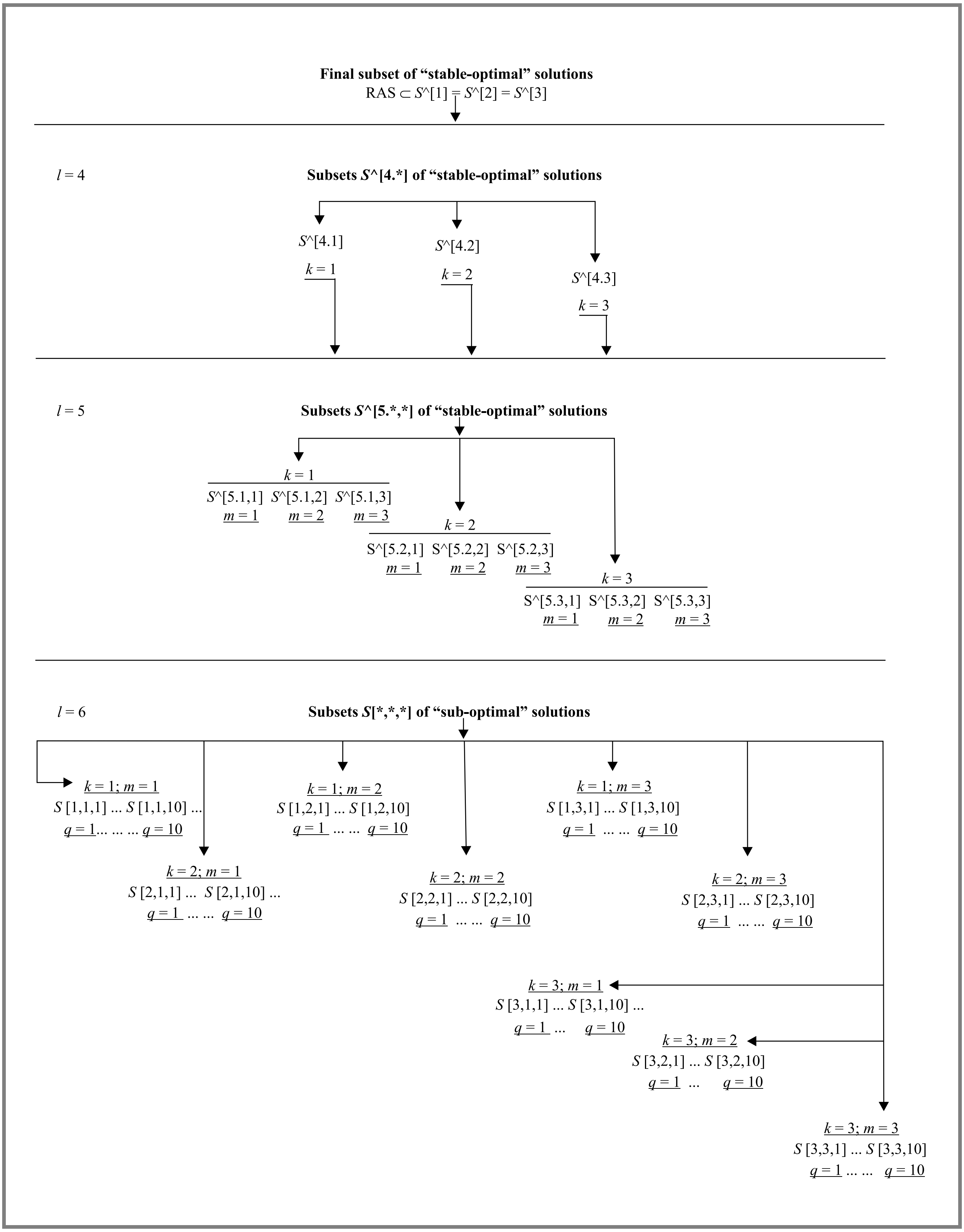

Fig. 1. Results of the calculation process for the considered sample: ninety (90) of "sub-optimal" subsets ($l = 6$), deriving nine (9) "stable-optimal" subsets ($l = 5$), from them–three (3) "stable-optimal" subsets ($l = 4$), and finally–the resulting subset RAS = $S^\wedge[1] = S^\wedge[2] = S^\wedge[3]$.

one combination of 2 weight values $\{w_1, w_2\}$ inside the appropriate intervals $\{[w_1^{\min}, w_1^{\max}],\ [w_2^{\min}, w_2^{\max}]\}$ (e.g., we have such intervals (4) $\{[w_1^{\min} = 0.6175,\ w_1^{\max} = 0.6825],\ [w_2^{\min} = 0.3325,\ w_2^{\max} = 0.3675]\}$ and their inside values $\{w_1 = 0.6643,\ w_2 = 0.3412\}$).

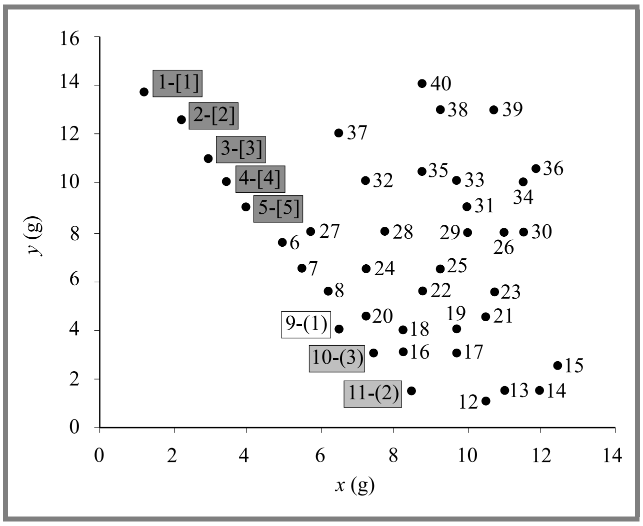

Fig. 2. Points and their coordinates reflecting ISA and ISCAV.

Consideration of all l-scenarios mentioned above allows to form the totality of 90 full Scenarios $\{1, 1, 1, k, m, q\}$, corresponding to single l-scenarios for three of the first l-levels (l = 1–3), K ($k = 1, \ldots, 3$; $K = 3$) 4-scenarios, M ($m = 1, \ldots, 3$; $M = 3$) 5-scenarios and Q ($q = 1, \ldots, 10$; $Q = 10$) 6-scenarios.

Each full Scenario defines a *mono-optimization problem* (1′) and its solution–a subset, including a predetermined number of "sub-optimal" alternatives. Analysis of sets of such subsets allows to find the "*stable-optimal*" *subsets*. All this forms the solution process to reach the RAS, implementing the considered six-level hierarchical system performance for this sample. This solution process is presented in Fig. 1.

According to Fig. 1, the above mentioned solution process includes the following *operations*:

a) Choosing the first full Scenario–combination $\{1, 1, 1, 1, 1, 1\}$, reflecting 1-scenarios taken one at a time for each of all l-levels (l = 1–6) $\{k = 1,\ m = 1,\ q = 1\}$; we form a *mono-optimization sub-problem* (1′) using 2 Monte Carlo simulations to determine the values of scalar sums for 40 (the number of alternatives) *criteria assessment vectors*.

b) A predetermined number (e.g., $N = 12$) of minimal values are selected among these scalar sums, that allows to form the subset $S[1, 1, 1, 1, 1, 1]$ or $S[1, 1, 1]$ (in Fig. 1) of *sub-optimal alternatives*, including only their numbers (e.g., $\{11, 9, \ldots\}$).

c) In the same way, by varying all ten 6-scenarios and leaving unchanged l-scenarios for all upper five l-levels (l = 1–5), we determine the set of "sub-optimal" subsets $\{S[1, 1, 1], \ldots, S[1, 1, 10],\ k = 1,\ m = 1,\ q = 1, \ldots, 10\}$. It defines a MVC series, for which a *subset* $S^{\wedge}[5.1, 1]\{l = 5,\ k = 1,\ m = 1\}$ of "stable-optimal" alternatives is determined using the special procedure.

d) Repeating the preceding operations, while varying all three 5-scenarios $\{l = 5,\ m = 1, 2, 3\}$ and leaving unchanged l-scenarios for all upper four l-levels (l = 1–4), a set of "stable-optimal" subsets $\{S^{\wedge}[5.1, 1],\ S^{\wedge}[5.1, 2],\ S^{\wedge}[5.1, 3],\ l = 5,\ k = 1,\ m = 1, 2, 3\}$ is determined. This allows to find (on a basis of this set analysis) the "stable-optimal" subset $S^{\wedge}[4.1]$ $\{l = 4,\ k = 1\}$, corresponding to the next 4-level.

e) Continuing this process, we can define the set of "stable-optimal" subsets $\{S^{\wedge}[4.1],\ S^{\wedge}[4.2],\ S^{\wedge}[4.3],\ l = 4,\ k = 1, 2, 3\}$ and for it–the "stable-optimal" subset $S^{\wedge}[3]$ $\{l = 3\}$, corresponding to the next 3-level. Since we have only single version for each of three upper l-levels (l = 1–3), their "stable-optimal" subsets are the same ones: $S^{\wedge}[3] = S^{\wedge}[2] = S^{\wedge}[1]$. This is *the resulting RAS*, including 12 ($S = 12$) i-points/"reasonable" alternatives $\{9, 11, 10, 8, 20, 7, 16, 12, 5, 6, 13, 18\}$, which are picked out in Fig. 2.

We would like to underline that in this RAS, all selected "stable-optimal" alternatives gain their priorities on a basis of *frequency of their presence* in all "stable-optimal" subsets, obtained for the preceding l-level, as well as accounting for the *sum of places*, which they have in these subsets.

Thus, the result we obtain for this sample is not an obvious one (with general positions). It becomes more illustrative if we locate all 40 alternatives-points in the Euclid space and estimate their distances to the Origin of the coordinates. We see in Fig. 2 that the best alternatives–points 9, 11, 10 are the nearest to the Origin of the coordinates.

4. The implementation for a problem of buying on the stock market

This implementation has two main purposes: (1) to apply our approach to the problem, where good statistic data allowing to reach the "reasonable" solutions using the proposed methodology are available; (2) to demonstrate the possibility of overcoming the difficulties of the ISCAV modeling in case of a real problem, where it's required to apply analytical methods of such modeling since it isn't possible to construct any natural (implicit) optimization criteria. Such an attempt was made early [6], but in this paper a new approach is demonstrated, where another totality of such criteria is considered, linked with other approach to construct a greater part of them.

4.1. Statement of the problem

The following problem is considered: *to select* (in the current T-day) a holding of stock, including a predetermined number of stocks, proper for buying on the stock market for the next prognosis $(T+1)$-day. Such selection is performed on a basis of processing the appropriate statistical data, where such data are considered for the period $[1,\ldots,T]$ of T days, as well as for expert estimates' use. All this concerns two parameters of stock market process: *deal sums, stock prices*. Thus, the problem solution purpose is to determine a proper quantity of each type of stock to be bought for the prognosis $(T+1)$-day.

These resulting proper stocks might be selected in accordance with one of two *purposes*: (a) to be sold on the days $(t = T+1, T+2, \ldots)$, nearest to the prognosis $(T+1)$-day (this is the *speculative Model A*); (b) to be kept for a long period as a part of decision makers' available capital (the *keeping Model B*).

The accent on this statistical data processing is more convenient in a framework of the *Model A* use; applying the *Model B* should be based, first of all, on using the expert estimates of production conditions for the enterprises, whose stocks are bought. However, the principal peculiarity of the considered problem, connected with the competition of great quantity of stocks, seriously limits the possibility to take such expert estimates for all considered stocks. Accounting for it, we are more closely focused on using the *Model A*, considering quite long period of the appropriate statistical data, especially if this period is characterized by "a stable behaviour" of the stock market considered. In this situation it's possible to expect that these observed statistical data are a sum of various aspects, affecting the "stock market behaviour of each considered stock", like the status of production, psychology, interaction of stock market buyers and sellers, etc.

The choice of this problem to apply the proposed approach of MCDM accounting for uncertainty is caused, first of all, by availability of required initial (statistical) data as well as of specific methodological difficulties to apply such approach to this real situation. Such difficulties were related mainly to criteria modeling and ISCAV construction.

4.2. Methodological peculiarities of ISA construction

For the problem considered, the operations necessary to construct the ISA have not been of our main methodological interest, since: (a) an initial alternative concept is very implicit–a stock itself is such alternative; (b) it was not needed to develop the special calculation procedures to construct a vast ISA. The latter (case (b)) is explainable by the measurable quantity of stocks, which can be considered in each existing stock market. In our opinion, this situation is very different from the one we met solving the problem [1–3, 5], with a practically non-measurable quantity of initial alternatives.

Thus, in the considered problem of stock buying the ISA (each its version) may be presented as the set $\{i = 1,\ldots,I\}$ of i-alternative's numbers.

4.3. Methodological peculiarities of criteria modeling (ISCAV construction)

The main methodological peculiarity of ISCAV construction in the considered problem is related to *non-implicit character of criteria*, which should express the optimization process in the problem. This aspect differentiates this problem from others (e.g., see [1–3, 5]), where there is a full possibility to assign (a priori) such natural criteria (e.g., economic, reflecting profit maximization or expenditure minimization; environmental–pollution minimization, etc.). Thus, the modeling process for the present problem is linked with necessity to perform some analytical research to express the required optimization criteria. Such approach allowed to define some basic concepts for modeling of various types of such criteria, providing the choice of "reasonable" stocks for buying on the prognosis $(T+1)$-day in order to sell them in the future in a short/long time (*Model A* and *Model B*).

4.3.1. Constructing the criteria, related to estimates of stock deal sums (DS) values [6]

The i-stocks, having the greatest expected *prognosis* (for $(T+1)$-day) *absolute values* $\{A^{pr}(i, T+1), i = 1,\ldots,I\}$ for *deal sums* (DS), are preferable in both *Model A* and *Model B*. Such confirmation is based on the following sentence: the stocks with the greatest values $A^{pr}(i, T+1)$ might be in great demand on $(T+1)$-day and few following days. Expected values $\{A^{pr}(i, T+1), i = 1,\ldots,I\}$ are determined on a basis of the appropriate *statistical data processing* to define their trends as well as by an implicit assigning of expert estimates. The first way corresponds to *Model A*, since it allows to estimate the conditions for selling the stocks on the days nearest to the $(T+1)$-day; the second expert way is more convenient to *Model B*, since experts might predict more long-term tendencies. However, this expert way is difficult to implement for a large quantity of stocks. In such case, the first trend way might also be used for the *Model B*, if we could determine reliable long-term trends. In both cases (for *Model A* and *Model B*), when the trends might be used to find the required prognosis, various trend types are found, and the required values $\{A^{pr}(i,t), i = 1,\ldots,I;\ t = T+1, T+2,\ldots\}$ are determined as the weighted values of these trends' continuations. Further, we will consider the criteria for *Model A* only.

Thus, we use the *maximization* (on all i-stocks, $i = 1,\ldots,I$) *of absolute prognosis* (on $T+1$-day) DS *values* $A^{pr}(i, T+1)$ as the *Criterion 1*, formulated as follows:

$$\max_{\{i=1,\ldots,I\}} \{C_{i1}\} = \max_{\{i=1,\ldots,I\}} \{A^{pr}(i, T+1)\}, \qquad (2)$$

where these values $\{A^{pr}(i,\ T+1),\ i = 1,\ \ldots,\ I\}$ are determined by constructing various types of trends and weighting these trend prognosis (on $T+1$-day) values. Let's consider the appropriate method.

These *trends* should be calculated on a basis of statistical data processing. The period of statistical data, intended for such processing, could be arbitrary, and its optimal duration could be established after many tests.

At present, the following 6 *types of trends* may be used to obtain the following prognosis (for any t-day) values for all i-stocks ($i = 1,\dots,I$), based on statistical data processing for the period $[1,\dots,t-1]$: (1) *Linear (Lin)* $A^{(1)}(t)$, (2) *Exponential (Exp)* $A^{(2)}(t)$, (3) *Logarithmic (Log)* $A^{(3)}(t)$, (4) *Polynomial (Pol) 3rd (third) order* $A^{(4)}(t)$, (5) *Power (Pow)* $A^{(5)}(t)$, (6) *Hyperbolic (Hpr)* $A^{(6)}(t)$.

When all these trend prognosis values $\{A^{(j)}(t),\ j = 1,\dots,6\}$ are obtained, the required *prognosis* (*for any t-day*) values $A_i^{pr}(t)$ of the *Criterion 1* are calculated for all i-stocks ($i = 1,\dots,I$) as follows:

$$A^{pr}(i,T+1) = A^{(1)}(i,t)w^{(1)} + A^{(2)}(i,t)w^{(2)} + \dots + A^{(5)}(i,t)w^{(5)} + A^{(6)}(i,t)w^{(6)}, \qquad (3)$$

where the weight values $\{w^{(1)}, w^{(2)}, w^{(3)}, w^{(4)}, w^{(5)}, w^{(6)}\}$ are assigned by experts or calculated using appropriate models. In both cases, we can't consider these weights as random variables.

For the latter case, the *special heuristic multi-step procedure* was developed early [6] to calculate *weights* $\{w^{(1)}, w^{(2)}, w^{(3)}, w^{(4)}, w^{(5)}, w^{(6)}\}$, using the same statistical data. This procedure is based on finding the values $A^{pr}(i,T+1)$ from formula (3) for the days $\{t = T+1-k,\ 0 < k \ll T\}$, preceding the basic $(T+1)$-prognosis day. In this case, the weight values, needed for (3), should reflect the validity of all trend-prognosis values $A^{pr}(i,T+1)$ by the following way of their calculation for all preceding t-days, corresponding to the assigned (a priori or in the calculation process itself) integer numbers k, and of the comparison of these calculated values with the appropriate actual (statistical) data:

Step 1: Stepping k^*-days back from the prognosis $(T+1)$-day (where $0 < k^* \ll T$ is a fixed integer number), we determine the DS values $\{A^{(j)}(i,t^*),\ j = 1,..,6\}$ from (3) for the chosen prognosis t^*-day ($t^* = T+1-k^*$) and each i-stock ($i = 1,\dots,I$), according to all 6 trends considered.

Step 2: Comparing these values $\{A^{(j)}(i,t^*),\ j = 1,\dots,6\}$ with the appropriate fact (statistical) DS values $A^{fact}(i,t^*)$, we can calculate the *aberration values* $D^{(j)}(i,t^*)$ as follows:

$$D^{(j)}(i,t^*) = \left|[A^{(j)}(i,t^*) - A^{fact}(i,t^*)]\right| / A^{fact}(i,t^*), \quad j = 1,\dots,6;\ \ i = 1,\dots,I. \qquad (4)$$

Step 3: Comparing the derived aberration values (4) with the *limits* $L(t^*)$, assigned (a priori) for deal sums (DS) and t^*-day, we select the "good" i-stocks, having these absolute aberration values lower than limits $(D^{(j)}(i,t^*) < L(t^*))$. The *numbers* (*quantity*) *of "good" i-stocks*, corresponding to each considered j-trend and reflecting its validity, are designated as $\{N^{(j)}(t^*),\ j = 1,\dots,6\}$.

Step 4: We determine the weights $\{w^{(j)}(t^*),\ j = 1,\dots,6\}$ from (3) as follows:

$$w^{(j)}(t^*) = N^{(j)}(t^*) \big/ \left[N^{(1)}(t^*) + N^{(2)}(t^*) + \dots + N^{(5)}(t^*) + N^{(6)}(t^*)\right], \quad j = 1,\dots,6. \qquad (5)$$

This four-step procedure might be repeated for several (R) t-days $\{t = t^1,\dots,t^R\}$, corresponding $(t = T-k)$ to the assigned values $k\{k = k^1,\dots,k^R\}$. Thus, we obtain R totalities of weights (5). On their basis, we can find the following *weighted average weights* to obtain the criterion values $A^{pr}(i,T+1)$, according to (3):

$$w^{(j)}[R] = d(t^1)w^{(j)}(t^1) + d(t^2)w^{(j)}(t^2) + \dots + d(t^R)w^{(j)}(t^R), \quad j = 1,\dots,6, \qquad (6)$$

where $\{d(t^1),\dots,d(t^R),\dots\}$ are the assigned (a priori) weights of t-days and sum of these weights' sum for all such days should be equal to 1. This leads to the following formula:

$$w^{(1)}[R] + w^{(2)}[R] + \dots + w^{(6)}[R] = 1\,. \qquad (7)$$

It's possible to exclude some j-trends from consideration, if it is possible to take a priori their weights $w^{(j)} = 0$ in formula (3). Besides this, we could vary the totalities of $\{w^{(j)}[R],\ j = 1,\dots,6\}$ from (6), (7), considering various totalities of j-trends as well as k-days $\{k = k^1,\dots,k^R\}$.

To estimate the quality of the derived DS prognosis absolute values $\{A^{pr}(i,T+1),\ i = 1,\dots,I\}$, they are compared with the DS fact absolute values $\{A^{fact}(i,T),\ i = 1,\dots,I\}$, taken for the last T-day from the appropriate statistical data. The *minimization of relative estimates* $A^{pr\wedge}(i,T+1)$, reflecting for each i-stock the ratio of absolute value of this pair values' difference to the last (fact) of them, is *Criterion 2*:

$$\min_{\{i=1,\dots,I\}}\{C_{i2}\} = \min_{\{i=1,\dots,I\}}\{A^{pr\wedge}(i,T+1)\},$$

$$A^{pr\wedge}(i,T+1) = \left\{\left|(A^{pr}(i,T+1) - A^{fact}(i,T))\right| \big/ A^{fact}(i,T)\right\}, \quad i = 1,\dots,I. \qquad (8)$$

Use of this *Criterion 2* is based on the following principle: when the value $A^{pr\wedge}(i,T+1)$ is less, the appropriate DS prognosis absolute value $A^{pr}(i^*,T+1)$ may be considered more reliable.

4.3.2. Constructing the criteria, connected with estimates of stock prices values

The approach to form the criteria for stock prices (SP) for *Model A* should be, in principle, different from the one for DS considered above. If we consider *Model A*, oriented on the stock buying for the prognosis $(T+1)$-day and their selling for the next t-days $\{t = T+2, T+3, \ldots\}$, we should account for the sinusoidal character of changing the SP values during the whole period of SP values observation (the days $[1,\ldots,T]$). This sinusoidal pattern is dictated by the nature itself, of stock buying on stock market when the SP falls, volumes of such stocks buying increase, and this tendency remains up to the day of reaching SP minimum. After this, another picture is observed, when the SPs go up, and holders of these stocks begin to sell such stocks, that leads again to SPs fall, and so on. Accounting for it, in a framework of *Model A*, it's expedient to buy those stocks, which is expected when the current minimum of its SP sinusoid is close to the prognosis for $(T+1)$-day. Such aspect might be reflected by *Criteria 3–5*, having very specific methodology of their construction, based on accounting for sinusoidal pattern of the initial data used.

In accordance with the sinusoidal character of changes to SP values, the set of all "*j-sinusoidal Hills*" $\{H[i,j], j = 1,\ldots,J[i]\}$, corresponding to each i-stock, is defined as one, reflecting the SP values, given on a totality of time j-intervals $\{\left[Tl[i,j], Tr[i,j]\right],\ j = 1,\ldots,J[i]\}$, measured in integer numbers of days of the period $[1,T]$. Each such j-interval includes t-days from interval $[1,T]$, arranged between the *left/right bounds* $(Tl[i,j] < Tr[i,j])$ of this j-interval. These bounds (for $j = 1,\ldots,J[i]$) corresponding to the neighboring minimal SP values, observed for the period $[1,T]$, are subject to the following conditions:

$$1 \le Tl[i,1] < Tr[i,1] < Tl[i,2],\ldots, Tl[i,j] < Tr[i,j],\ldots$$
$$\ldots, Tl[i,J[i]] < Tr\left[i,J[i]\right] \le T\,. \qquad (9)$$

Let's consider *a heuristic algorithm* of these *j-intervals constructing*, defined for each i-stock and based on considering the appropriate indicators, where $t[j]$ is the number of current day [j-interval] in the observed period $[1,\ldots,T]$ of statistical data, $SP[t]$ is the SP value for this t-day, and $Dif[t] = SP[t] - SP[t-1]$:

1. Establishing the *initial conditions*: $Dif[1] = 0$, $t = 2$, $j = 1$, $Dif[2] = SP[2] - SP[1]$.

2. Going to the next $(t+1)$-day with calculation $Dif[t+1] = SP[t+1] - SP[t]$.

3. Performing the joint analysis of signs for the differences $Dif[t+1], Dif[t]$; the following cases may be:

   3.a) If $Dif[t+1] > 0$, $Dif[t] \le 0$, we fix the upper limiting day $Tc = t+1$ for this *cycle* 3.a of finding the first preceding $(Tc-t1)$-day, when the value $Dif[Tc-t1] < 0$ $(t1 = 1,\ldots,Tc-1)$. If this value is reached, the right bound-day $Tr[i,j] = t$ of j-Hill is established to reflect the current minimal SP value in the sinusoid considered (for i-stock), and we go to its next $(j+1)$-Hill with fixing its left bound-day $Tl[i,j+1] = t+1$. Thus, the j-interval $\left[Tl[i,j], Tr[i,j]\right]$ is completed. However, in the process of executing this cycle, we can encounter two particular *cases*:

   3.a1) $Dif[Tc-t1] > 0$ for some value $t1$ $(t1 = 1,\ldots,Tc-2)$;

   3.a2) $Tc - t1 = 1$, i.e., we reached in this cycle the beginning of sinusoid (the 1-day).

   In both these cases (3.a1) and (3.a2), operations of *cycle* 3.a end without completion of the interval, and we go to operation 2 for the next $(t+2)$-day.

   3.b) If $Dif[t+1] < 0$, $Dif[t] \ge 0$, we execute the *cycle* 3.b (it's similar to the *cycle* 3.a) of finding the first preceding $(Tc-t1)$-day, when $Dif[Tc-t1] > 0$ $(t1 = 1,\ldots,Tc-1)$. If this is reached, we fix the day $T\max[i,j] = t$ of maximal SP value for j-Hill with saving j-Hill and its other indicators without changes. In this case we can also encounter two particular *cases*:

   3.b1) $Dif[Tc-t1] < 0$ for some value $t1$ $(t1 = 1,\ldots,Tc-2)$ and execution of *cycle* 3.a is stopped on $(Tc-t1)$-day without establishing $T\max[i,j]$;

   3.b2) *cycle* 3.b ends, reaching $(Tc–t1{=}1)$-day, and we fix the day $T\max[i,j=1] = t$.

4. After completion of all these 3 operations and for all other cases of relationship between $Dif[t+1]$ and $Dif[t]$, we go to the next $(t+2)$-day of the period $[1,\ldots,T]$ $(t < T)$ to perform the next operation 2.

We bring the following appropriate *parameters* into operations to construct the *Criteria 3–5*:

– The *summary distances* $DsSHw[i]$ or $DsSHp[i]$ (both in days) *of all j-intervals or their part*, corresponding to the full period $[1,T]$ or its continuous part, including the assigned number $K(>1)$ of j-intervals from the fixed j^*-interval to the $J[i]$-interval $(j^* = J[i]-K)$. It's calculated as follows:

$$DsSHw[i] = Tr[i,J[i]] - Tl[i,1] + 1,$$
$$DsSHp[i] = Tr[i,J[i]] - Tl[i,j^*] + 1,$$
$$i = 1,\ldots,I\,. \qquad (10)$$

– The *average distances* $DsAHw[i]$, $DsAHp[i]$ (both expressed in days, maybe with 0.1 day resolution), determined as the quotient from division of the summary distances (10) on the appropriate numbers of j-intervals, forming these summary distances:

$$DsAHw[i] = DsSHw[i] / J[i],$$
$$DsAHp[i] = DsSHp[i] / K,$$
$$i = 1, \ldots, I. \quad (11)$$

– The *residual distance* $DsRHl[i]$ (in days) reflects the *last non-completed* $(J[i]+1)$-*Hill* (*-interval*), including all t-days after $Tr[i,J[i]]$ (the $J[i]$-day of the last minimal SP value) up to the final T-day of the period $[1,T]$, where:

$$DsRHl[i] = T - Tr\big[i,J[i]\big], \quad i = 1, \ldots, I. \quad (12)$$

Using these parameters, we can define *Criteria 3–5* as follows:

1. According to the formulas (9)–(11), the *average distances* $DsAHw[i]$, $DsAHp[i]$ from (11) are calculated for all i-stocks considered. By comparing them, the *relative coefficient* $CfAwp[i]$, characterizing their closeness, is calculated as the quotient from division of the absolute value of these average distances' difference on their maximal value; the appropriate formula is as follows:

$$CfAwAp[i] = \big|\big(DsAHw[i] - DsAHp[i]\big)\big| /$$
$$/ \max\big\{DsAHw[i], DsAHp[i]\big\}, \quad i = 1, \ldots, I. \quad (13)$$

2. The *final average distance* $DsAHF[i]$, determined as weighted sum of both average distances from (11), should reflect (as well as the *Criterion 3* itself) the prediction of the expected day $Tr\big[i,J[i]+1\big]$ (maybe with 0.1 day resolution) of current minimal SP value (this prognosis day of $(J[i]+1)$-SP minimum should be the next one after the last fact day of SP minimum). Thus, $DsAHF[i]$ is calculated as:

$$DsAHF[i] = DsAHw[i] \cdot WAw + DsAHp[i] \cdot WAp,$$
$$i = 1, \ldots, I, \quad (14)$$

where the expert weights WAw, WAp ($WAw + WAp = 1$) are assigned according to the following principle: *when making the prognosis for the nearest future it's more important to take into account what happened in the last fact days*. According to it, we take $WAp > WAw$ (it's possible to take: $WAw = 0.4$, $WAp = 0.6$).

3. We formulate the *Criterion 3*, based on the above considered principles, as *minimization* (on all totality of i-stocks) *of closeness for the i-stock expected* $(J[i]+1)$-*day of its SP sinusoid minimum to the accepted prognosis* $(T+1)$-*day*. This expected $(J[i]+1)$-day is calculated (maybe in with 0.1 day resolution) for each i-stock as the end of its last non-completed $(J[i]+1)$-sinusoidal Hill $H\big[i,J[i]+1\big]$:

$$Tr\big[i,J[i]+1\big] = Tr\big[i,J[i]\big] + DsAHF[i],$$
$$i = 1, \ldots, I, \quad (15)$$

where: $Tr\big[i,J[i]\big]$ is the last fact day of SP minimum, $DsAHF[i]$ is from (14).

Thus, the *Criterion 3* is minimization of the value, calculated as the corrected (using (13)) absolute value of difference between the rated day from (15) and the prognosis $(T+1)$-day:

$$\min_{\{i=1,\ldots,I\}}\{C_{i3}\} = \min_{\{i=1,\ldots,I\}}\{C_3^{\sin}[i,T+1]\},\ C_3^{\sin}[i,T+1]$$
$$= \big|\big(Tr[i,J[i]+1] - T - 1\big)\big| \cdot (1 + CfAwAp[i])\},$$
$$i = 1, \ldots, I. \quad (16)$$

4. The *Criterion 4* should reflect *the accordance between the last fact non-completed* $(J[i]+1)$-*sinusoidal Hill* $H\big[i,J[i]+1\big]$ *and the prognosis of* $(T+1)$-*day*. This accordance may be estimated as follows: we find the *day* $T\max\big[i,J[i]+1\big]$ of maximal value on this last non-completed Hill, and the *Criterion 4* is considered as minimization of relative difference between this maximal day and this *Hill initial day* $Tr\big[i,J[i]\big]$; thus, the *Criterion 4* value is calculated as the quotient from division of this difference on this Hill full distance $DsRHl[i]$ (see (12)):

$$\min_{\{i=1,\ldots,I\}}\{C_{i4}\} = \min_{\{i=1,\ldots,I\}}\{C_4^{\sin}[i,T+1]\},\ C_4^{\sin}[i,T+1]$$
$$= \big(T\max\big[i,J[i]+1\big] - Tr\big[i,J[i]\big]\big) / \big(T - Tr\big[i,J[i]\big]\big),$$
$$i = 1, \ldots, I. \quad (17)$$

Such minimization approach is based on the following principle: if the relative value $C_4^{\sin}[i,T+1]$ is small, it is possible to expect the great distance $(T - T\max\big[i,J[i]+1\big])$, i.e., the next (out $[1,\ldots,T]$) SP minimum value could be expected as one closer to the prognosis of $(T+1)$-day. Thus, such stocks are good for buying (only with these positions) on this $(T+1)$-day. However, if the required value $T\max\big[i,J[i]+1\big]$ isn't found on the Hill $H\big[i,J[i]+1\big]$, we assume the *Criterion 4* value as follows:

$$C_4^{\sin}[i,T+1] = 1 + Eps[i], \quad i = 1, \ldots, I, \quad (18)$$

where the values $\{Eps1[i] > 0, i = 1, \ldots, I\}$ are some expert estimates $Eps[i]$ (or they may be accepted as $\{Eps[i] = 1/\big(T - Tr\big[i,J[i]\big]\big), i = 1, \ldots, I\}$).

5. The *Criterion 5* estimates *the difference between the fact and rated values of SP maximal value on the last*

non-completed Hill $H\left[i, J[i]+1\right]$. It may be accepted as an additional estimation of agreement between the fact and rated data. If there is the fact maximum $T\max\left[i, J[i]+1\right]$, the *Criterion 5* value is calculated by defining the absolute value of difference between the value $T\max\left[i, J[i]+1\right]$ and the rated maximal value of this Hill, derived using a half of distance $DsAHF[i]$ from (14), where this difference is corrected using the value $CfAwAp[i]$ from (13). Thus, we have the following *minimization Criterion 5*:

$$\min_{\{i=1,\ldots,I\}} \{C_{i5}\} = \min_{\{i=1,\ldots,I\}} \{C_5^{\sin}[i, T+1]\}, \qquad (19)$$

where:

$$C_5^{\sin}[i, T+1] = \left|\left(T\max\left[i, J[i]+1\right] - Tr\left[i, J[i]\right] - 0.5DsAHF[i]\right)\right| \cdot (1+CfAwAp[i])$$

$$i = 1, \ldots, I. \qquad (19')$$

If $T\max\left[i, J[i]+1\right]$ does not exist, the *Criterion 5* value is accepted as in (18):

$$C_5^{\sin}[i, T+1] = \left|\left(T + Eps1[i] - Tr[i, J[i]] - 0.5DsAHF[i]\right)\right| \cdot (1+CfAwAp[i]),$$

$$i = 1, \ldots, I, \qquad (20)$$

where the values $\{Eps1[i] > 0,\ i = 1, \ldots, I\}$ are some expert estimates.

4.3.3. Illustrative sample of *Criteria 1–5* constructing

We will illustrate the above considered methods of *Criteria 1–5* constructing on the real sample of data for one stock (3-stock) of the holding "Tel-Aviv-100" on the Israeli stock market. The appropriate statistical data for this stock are considered for the period 11/11/01–01/01/02, which included 32 working days ($T = 32$, $t = 1, \ldots, 32$). The prognosis day ($T + 1 = 33$) corresponds to 02/01/02. All these data present the deal sums (DS), shown in Fig. 3, and the stock prices (SP), presented in Table 1.

Construction of *Criterion 1*, reflecting the prognosis (for 33-day) of absolute value $A^{pr}(3, 33)$ for deal sums (DS), is performed according to (3), where only the linear (Lin) $A^{(1)}(3, 33)$, logarithmic (Log) $A^{(3)}(3, 33)$ and polynomial (Pol) 3rd order $A^{(4)}(3, 33)$ trends are taken into account (it corresponds to $w^{(2)} = 0$, $w^{(5)} = 0$, $w^{(6)} = 0$, $w^{(Ex)} = 0$ in (3)). These trends for 3-stock are presented in Fig. 3, where they are shown by bold (Lin), stroke (Log) and dotted (Pol) lines. In this Fig. 3, we can see that their prognosis (for 33-days) values are very close to Log- and Pol-trends ($A^{(3)}(3, 33) = 3757.9$; $A^{(4)}(3, 33) = 3754.4$), but they differ from Lin-trend ($A^{(1)}(3, 33) = 5029.1$). According to it and the closeness of values $A^{(3)}(3, 33)$ and $A^{(4)}(3, 33)$ to the known (statistics for 02/01/02) fact DS values ($y^{fact}(3, 33) = 3840.8$), we can expertly assign, to realize calculations according to formula (3) for this 3-stock, the following weight values: $w^{(1)} = 0.2$, $w^{(3)} = w^{(4)} = 0.4$ (we would like to underline that such assigning is performed conditionally accounting only for this situation and for this 3-stock). Using these weight values and in accordance with (3), we calculate this *Criterion 1* value $C_{31} = A^{pr}(3, 33) = 5029.1 \cdot 0.2 + 3757.9 \cdot 0.4 + 3754.4 \cdot 0.4 = 4010.7$ ($= 1.04\, y^{fact}(3, 33)$ is very closed to the fact).

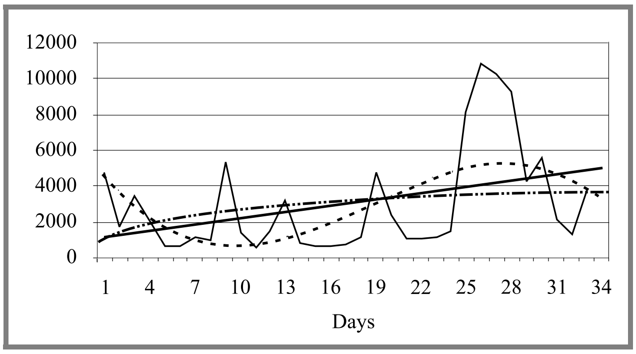

Fig. 3. Lin (bold), Log (stroke) and Pol (dotted line) trends ($j = 1, 3, 4$) and statistic curve for DS of 3-stock.

Using the derived value of *Criterion 1* and the fact DS value for the T-day $A^{fact}(3, 32) = 1293.7$, the *Criterion 2* value is calculated according to formula (8): $C_{31} = A^{pr\wedge}(3, 33) = |(A^{pr}(3, 33) - A^{fact}(3, 32))/A^{fact}(3, 32)| = |(4010.7 - 1293.7)|/1293.7 = 2.10$.

We illustrate construction of *Criteria 3–5* through the analysis of 3-stock price (SP) sinusoidal data, presented in Table 1. Such construction is performed according to (9)–(20) and the special procedure described before. The procedure defines the basic parameters of the criteria C_{33}–C_{35} construction, reflecting the "j-sinusoidal Hills" $\{H[3, j],\ j = 1, \ldots, J[3]\}$ and their j-intervals $\{[Tl[3, j], Tr[3, j]],\ j = 1, \ldots, J[3]\}$. So, according to operation 1 in this procedure, we establish the following initial conditions: $t = 2$, $j = 1$, $Dif[2] = SP[2] - SP[1] = 3283 - 3290 = -7$ (see Table 1). Executing operations 2–3, we find $t = 3$, $Dif[3] = 63$, and the case 3.a ($Dif[3] > 0$, $Dif[2] \le 0$) is established. Performing the appropriate cycle ($t1 = 1$, $Dif[t - t1] = Dif[2] < 0$), we fix the first minimal SP value for $t = 2$ ($Tr[3, 1] = 2$) and go to the next 2-Hill $H[3, 2]$ ($j = 2$, $Tl[3, 2] = 3$). Through the operation 4 going to the next day $t = 4$, we repeat again the operations 2–3, reaching $Dif[4] = -127$ and the case 3.b ($Dif[4] < 0$, $Dif[3] > 0$), where the maximal SP value in 2-interval is fixed for $t = 3$ ($T\max[3, 2] = 3$). Continuing this process, we define the following full totality of j-intervals for this 3-stock: $[1, 2]$, $[3, 7]$, $[8, 12]$, $[13, 17]$, $[18, 22]$, $[23, 24]$, $[25, 26]$, $[27, 32]$, including the following t-days of maximal SP val-

Table 1
SP values for 3-stock and their differences $Dif[t]$ for all t-days ($t = 1, \ldots, 32$)

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
SP	3 290	3 283	3 346	3 219	3 144	3 097	3 040	3 050	3 437	3 424	3 417	3 388	3 478	3 427	3 368	3 36
$Dif[t]$		−7	63	−127	−75	−47	−57	10	387	−13	−7	−29	90	−51	−59	−12
t	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
SP	3 310	3 361	3 489	3 482	3 415	3 387	3 404	3 399	3 412	3 226	3 261	3 315	3 412	3 406	3 353	3 330
$Dif[t]$	−46	51	128	−7	−67	−28	17	−5	13	−186	35	54	97	−6	−53	−23

ues: $\{1,3,9,13,19,23,25,29\}$. Thus, we can obtain the values needed for (9)–(20): $J[3]=7$, $Tl[3,1]=1$, $Tr[3,7]=26$, $DsSHw[3] = 26-1+1 = 26$, $K = 3$ (it's assigned a priori), $j* = 7-3 = 4$, $DsSHp[3] = Tr[3,7]-Tl[3,4]+1 = 14$, $DsAHw[3] = DsSHw[3]/J[3] = 26/7 = 3.714$, $DsAHp[3] = DsSHp[3]/K = 14/3 = 4.667, DsRHl[3] = T-Tr[3,7] = 32-26 = 6$ (for non-completed 8-interval [27,32]), $CfAwAp[3] = |(DsAHw[3]-DsAHp[3])|/\max\{DsAHw[3], DsAHp[3]\} = |3.714-4.667|/4.667 = 0.204$. According to (11), (14) and the above accepted expert weights ($WAw = 0.4, WAp = 0.6$), we find the values $DsAHF[3] = 3.714 \cdot 0.4 + 4.667 \cdot 0.6 = 4.286, Tr[3,8] = 26 + 4.286 = 30.286$.

Thus, in accordance with the above calculated parameter values using formulas (16)–(20) and accounting for the SP maximum $T\max[3, 8]$ availability in the last non-completed 8-interval $[27, 32]$, the following *Criteria 3–5* values are calculated:

$$C_{33} = C_3^{\sin}[3,33] = |(30.286-33)| \cdot 1.204 = 2.714 \cdot 1.204 = 3.268;$$

$$C_{34} = C_4^{\sin}[3,33] = (29-26)/(32-26) = 3/6 = 0.5;$$

$$C_{35} = C_5^{\sin}[3,33] = |(29-26-0.5 \cdot 4.286)| \cdot 1.204 = 1.032.$$

We could perform some preliminary analysis of these criteria values obtained $\{4010.7, 2.10, 3.268, 0.5, 1.032\}$ with the intent to estimate quality of these results in accordance with these criteria. As is evident from the foregoing, the concordance with the fact data for the prognosis 33-day period is very good (104%) for the *Criterion 1* and very bad (210%) for the *Criterion C2*. The first characterizes the good reflection of general tendencies of DS statistical data by the chosen totality of trends and their weight values, the second–the sharp fall of fact DS on day 32 (see Fig. 3). The good values of *Criteria 3–5* characterize a good estimate of current SP minimum, calculated taking into account the fact 3-sinusoid "behaviour" in the average and in the last t-days.

4.3.4. Peculiarities of the six-level hierarchical system performance for the problem considered

The above considered (Section 3) six-level hierarchical system of MVC series is applied, at present, to reach the required "reasonable" solutions for the problem of stock buying on the stock market. We will not comment here on the aspects of this application contents and results, but we will accent only the methodological aspects of this application. With these positions we will consider the peculiarities of: (a) forming this hierarchical system; (b) performance of the appropriate computations.

To solve the problem considered, we accepted performance, in principle, of the same six-level system that was presented above (Section 3). The totalities of possible scenarios for all 6 levels ($l = 1, \ldots, 6$) of this six-level system of MVC series as applied to this problem of stock buying are as follows:

- $l = 1$. Only one 1-scenario is accepted, intended to use the same modified TOPSIS method [1–3], based on considering the following *scalar goal function* (the partial case of the above general function (1))

$$\min_{\{i=1,\ldots,I\}} \{(-C_{i1}W_1) + C_{i2}W_2 + C_{i3}W_3 + C_{i4}W_4 + C_{i5}W_5\}, \tag{21}$$

  where the first component, reflecting the maximizing *Criterion 1*, is considered with the sign "-".

- $l = 2$. Various 2-scenarios correspond to different ISA versions, whose variation might reflect the change of the observed stock groups, where this changes may include: (a) types of stocks; (b) period of observation; (c) number of considered stocks inside the same group, etc.

- $l = 3$. At present, we consider only one 3-scenario, reflecting the *Criteria 1–5*, presented in (21).

- $l = 4-5$. Various combinations of 4- and 5-scenarios should reflect different versions of weights of possible value j-intervals $\{[w_j^{\min}, w_j^{\max}],\ j = 1,\ldots,5\}$, whose construction is linked with assigning (a priori) their "central points" $\{w_j^{\wedge}, j = 1,\ldots,5\}$ (4-scenarios) and their quite standard surroundings by the bounds (5-scenarios). In a framework of multi-variant computations performed, we consider different versions of such "central points", reflecting: (a) expert estimates, where the weight $w_1^{\wedge}$ of *Criterion 1* is more preferable than others ($w_1^{\wedge} > w_j^{\wedge},\ j = 2,\ldots,5$) and the weight $w_2^\wedge$ of *Criterion 2* is the least one; (b) estimates, opposite to the preceding case (a); (c) the uncertain situation ($w_1^\wedge = w_j^\wedge = w_3^\wedge = w_4^\wedge = w_5^\wedge = 0.2$). The totality of 5-scenarios used presents 4 types of interval bounds (see [6]), surrounding these "central points" on: (1)5%, (2)10%, (3)15%, (4)20% (e.g., we consider the interval $[0.95w_j^\wedge,\ 1.05w_j^\wedge]$).

- $l = 6$. The quantity of 6-scenarios depends upon the assigned number of Monte Carlo simulation series (each such series includes 5 Monte Carlo simulations according to the number of criteria). This quantity might be considerable according to our wishes (e.g., it's varied from 10 to 100 to estimate the sensitivity of results to such variations).

Thus, the above considered variations of 2-, 4-, 5- and 6-scenarios lead to a considerable number of multi-variant computations, related to reaching different RAS totalities. Their analysis allows, in principle, to research the influence of varying the problem conditions on the final solutions, but this analysis of varied results have led to the necessity to perform extensive research before the development of pithy (rich in content) methodology for behaviour on stock market. However, the research already performed shows *viability of our MCDM approach as applied to this real problem.*

5. Conclusions

An original, intuitive methodology to solve various real MCDM problems is proposed. This methodology reflects the approach, focused primarily on accounting for uncertainty factors in the process of selecting a predetermined number of "reasonable" alternatives from their considerable (maybe vast) initial set in accordance with an arbitrary number of optimization criteria, considered jointly as a multiple criteria. The methodology allows to take into account the uncertainty factors of different nature in a framework of multi-level hierarchical system of multi-variant computation series.

At present, the main purpose of promotion of this methodology is the research connected with application of this approach to real MCDM problems. The "bottlenecks" of such application may result from creating the initial sets of alternatives or criteria assessment vectors (ISA or ISCAV). From this point of view, the problem considered in this paper is a very suitable one, since it reflects the rich statistical data, which could be used to apply the proposed approach. Besides, this problem is connected with a lack of implicit optimization criteria, creating difficulties in ISCAV construction.

Thus, the main purpose of this paper was to show the possibility of applying the quite general methodology of MCDM accounting for uncertainty to real problem proposed, using the quite reliable initial data regarding the construction of initial alternatives and multiple criteria (the ISA/ISCAV). An important aspect of this application is related to *modeling the totality of non-implicit criteria* (ISCAV), based on reliable statistical data processing.

Other aspects of modeling the problem of stock buying on the stock market are linked to practical usage of the appropriate calculation results. Here, we can suggest two directions of work:

1) developing the methods of successful "behaviour" on stock market in buying the "reasonable" stocks;

2) accumulating the experience in researching such "behaviour" by performance of multi-variant computations to estimate the influence of various factors on the choice of "reasonable" solutions.

At present, we carry out work in the second direction, performing a lot of multi-variant computations while varying the problem conditions and parameters. In this way, we hope to start the "self-education" process, leading in the future to a development of successful "behaviour" on the stock market in buying the "good" stocks.

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