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Kinematic Model of Solute Transport in Stream Networks: Example with Phosphate Retention in Morsa Watershed, Norway

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Abstract

A theoretical description of reactive solute transport in a network of stream channels is derived by convoluting unit solutions based on a physical representation of transport and topographical information of the distributions of solute load as well as pathways. The theory is applied to a generic analysis of the phosphate export in Morsa watershed due to the load from 620 individual households with a local wastewater treatment. Essential factors for the phosphate export is filtering of the water in stream-bed sediments through a distribution of hyporheic flow paths of various lengths. This generic study indicates that a significant portion of phosphate is retained in the hyporheic zones for a long time. The 90% recovery time following a hypothetical remediation action in the households is expected to be in the order of one decade.

Key words: solute transport, watersheds, stream network, hyporheic zone, transport model

1. Introduction

Wetlands are important means of reducing the nutrient content of wastewater and limit eutrophication of recipient waters. Wetlands can be constructed in connection with ordinary wastewater treatment plants, constructed as separate treatment systems (Kadlec and Knight 1996) or be natural parts of aquatic systems with a sufficient capacity for receiving nutrient loads. This study concerns the self-purification of phosphorus loads from individual households in a stream-network in Morsa watershed close to Oslo, Norway.

An investigation of the environmental status of Morsa stream networks reveals severe eutrophication. Nutrients originate from agriculture, municipal (partially treated) wastewater, runoff from other areas with agricultural and individual households (Lyche Solheim et al 2001). Retention of phopshorus in stream sediments and lakes reduces the load to downstream systems. Such retention and self-purification has implications for assessing remedial actions and time-lags of temporal changes in environmental status.

This study focuses on the response in downstream water systems due to changes of the load from individual households in Morsa. The phosphorus load from individual households accounts for about 15% of the total anthropogenic phosporus load and is twice as large as that of the municipal wastewater. Important questions are, thus, what is the time-lag before a remedial action at the households can be noticed in recipient waters, what is the degree of retention of phosphorus in the stream network, and how important are remedial actions focused on specific sources relative to different loads.

In the first tentative approach, a generic modelling study is performed and reported here. Reasons for a generic analysis are to provide principal insight into the above-mentioned questions and as planning of further field investigations, especially, for prioritisation of financial resources. The current data-base, connected to a geographic information system, contains information on spatial distribution of households, discharge, spatial phosphorus loads, etcetera.

A theoretical and practical difficulty is to integrate the spatially distributed information of watershed parameters and load distribution in terms of a model that takes into account the most salient transport processes. Integration (convolution) models have been developed to be able to predict the geomorphological instantaneous unit hydrograph (GIUH) (Snell and Sivapalan 1994, Franchini and Connell 1996, and Rodriguez-Iturbe and Rinaldo 1997) and inert transport (Simic and Destouni 1999), but not reactive transport. Reactive transport is significantly affected by the solute-water interaction between surface and subsurface, the hyporheic zone, (Bencala and Walters 1983) and sorption to solid surfaces, which contributes significantly to retard transport (Wörman 2000, Johansson et al 2001).

Following the approach of e.g. Snell and Sivapalan (1994), the problem can be divided into a one probability density function (PDF) for transport distances, reflecting the topographic information such as distribution of pathways, and a response function for transported mass. Here we will employ the method of Wörman et al (2002) to capture the filtering impact of water exchange between the streams and the hyporheic zone and sorption in the subsurface.

2. Mathematical Formulation of Solute Transport in Stream Networks

2.1. Transport with Solute Filtering in a Distribution of Hyporheic Flow Paths

Individual households in the Morsa watershed are normally equipped with septic tanks or local sand filter treatment and, therefore, the phosphorus load that reaches the stream system exists predominantly in terms of phosphate (approximately 90%). Phosphate is either dissolved in the water or adsorbed in different carrier particles (Fig. 1).

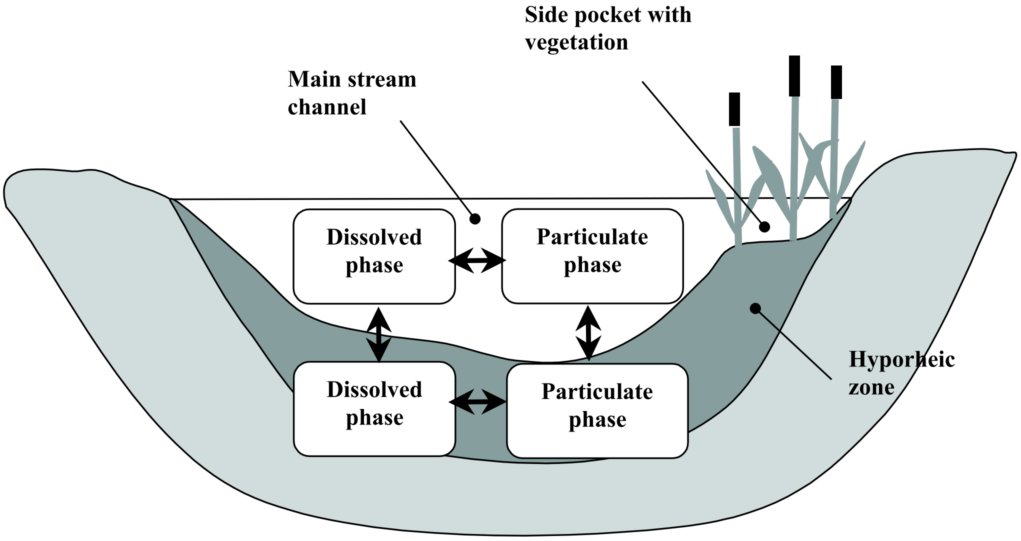

Fig. 1. Schematic of stream cross-section with hyporheic zone, main stream channel and side pockets with vegetation. Boxes indicate mass phase species accounted for in the interpretative model and arrows indicate mass transfer

Organic compounds, bacteria and other phosphorus forms are assumed to be negligible. Further, for linear problems, we can analyse the transport from the individual households independent of other sources and apply the superposition principle to obtain a complete description of the phosphorus transport. Conservation of the phosphate mass along a single one-dimensional stream branch yields

$$\frac{\partial c_d}{\partial t} + \frac{1}{A_T}\frac{\partial (AUc_d)}{\partial x} - E\frac{\partial^2 c_d}{\partial x^2} = \frac{J_s}{(1+K_d)}, \tag{1}$$

where

t – time [s],
x – longitudinal coordinate [m],
A_T – the cross-sectional area of the main stream including side pockets [m^2],

A – the cross-sectional area of the main stream excluding side pockets (Fig. 1),

U – flow velocity in the main stream [m/s],

$(Q = U/A), Q$ – discharge [m^3/s],

E – main stream dispersion coefficient [m^2/s],

$K_d = c_a/c_d|_e$ – ratio between particulate (e.g. adsorbed) and dissolved phase species in stream water, subscript e denotes 'evaluated at equilibrium',

J_s – flux of dissolved solute mass per unit stream-length between surface water and subsurface [kg m^{-2} s^{-1}].

If it is assumed that the advection velocity in the side pockets is assumed to be zero, a consequence of the exchange with side pockets is a reduction of the effective advection velocity (Wörman 1998). This can be manifested in the formulation in terms of a retardation factor, $K_R = A_{sp}/A$, which means that $A_T = A(1 + K_R)$, where A_{sp} is the area of the side pockets.

The exchange of the dissolved phase is caused by pressure variation along the bed surface of biotic (Huettel et al 1996) or hydro-mechanical origin (Elliott and Brooks 1997 a and b) or obstacles like tree branches or stones. The sum of in- and outwards solute flux can be given as $J_s = P/A_T\left[-V_{z|z=0}\, c_d + (V_z g_d)|_{z=L}\right]$, where V_z – effective flow velocity into and out of the stream bed [m/s], L – length of hyporheic pathway, P – wetted perimeter [m], g solute mass per unit volume of water in the hyporheic zone [kg/m^3] (Wörman et al 2002).

An important effect of the water and solute flux with the bed is the filtering of solute element by means of sorption on solid surfaces. In terms of a kinetic formulation and a linear adsorption isotherm, the governing transport equations for porous media are gerenally given in the form (Comans and Hockley 1992, Cunningham et al 1997a and b, Reichle et al 1998, Smith et al 2000)

$$\frac{\partial g_d n}{\partial t} + \frac{\partial V_z g_d}{\partial z} + k_\tau(K_B g_d n - g_a n) = 0, \tag{2}$$

$$\frac{\partial g_a n}{\partial t} - k_\tau(K_B g_d n - g_a n) = 0, \tag{3}$$

where n – porosity, $K_B - -g_p/g_{d|e}$ and k_r – rate coefficient [s^{-1}]. These equations are based on the assumption that sorption is the only relevant process on the time scale under consideration here. Since the turnover times for aquatic life is usually between days to a year, biological consumption could be relevant to take into account for corresponding intermediate time scales.

Wörman et al (2002) introduced the idea to combine a residence time distribution for the transport pathways (with length L and transit time $T = L/V_z$) with the description of the hyporheic flux and longitudinal transport in the stream.

Several independent empirical observations, as well as analysis using the 'pumping model' (Elliott and Brooks 1997a, Packman et al 2000) supported this formulation. In particular log-normal T-PDF was found consistent with the observations of in-stream and in-bed processes. Hence, the log-normal T-PDF was applied in this study.

2.2. Convoluting Solution for Stream Network

A model of the solute transport from a watershed can be formulated, basing on the load contribution in the distribution of points of the stream network by treating each response according to (1)–(3) independently. We can imagine that the load is defined in terms of a large number of Dirac delta functions (Dirac pulses) that are released in points of the network and distributed over time. Solutions are then sought by convolution, which is permissible for linear differential equations.

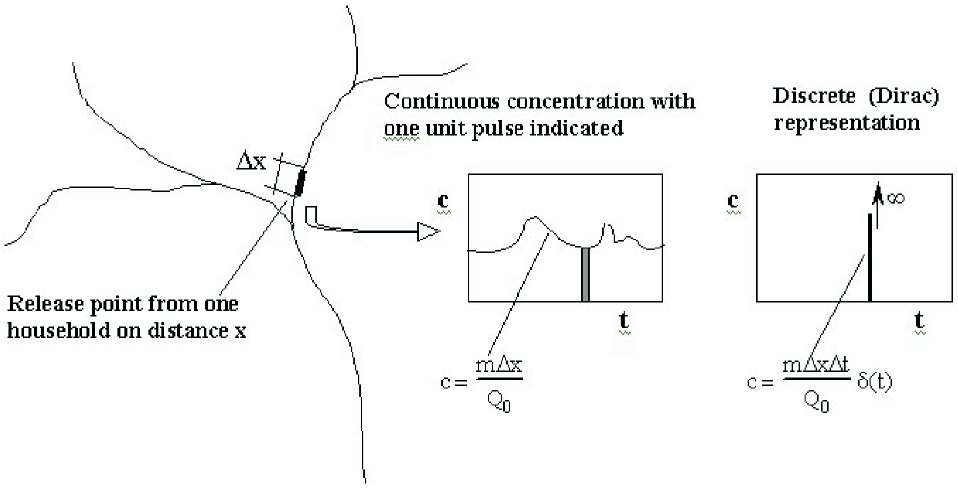

Fig. 2. Network of streams and two options of representing the concentration variation in a section Δx in the stream due to the solute fall-out in this section under the time increment Δt

If the mass released within the spatial interval Δx of the stream is mixed instantaneously with the water, the concentration in this interval will be $[m\Delta x/Q_0]$ (Fig. 2), where m is the mass of solute that falls-out per time and space unit [kg s^{-1} m^{-1}] and Q_0 is the discharge in the stream at the release point [m^3 s^{-1}]. A discrete representation of the concentration would be

$$c(x=0,t) = \frac{m\Delta t\,\Delta x}{Q_0}\delta(t), \tag{4}$$

where $\delta(t)$ – Diracs delta distribution in time [s^{-1}]. The solution to (1)–(3) for such a boundary condition can be found from either numerical techniques or in closed-form and will be denoted by $\delta c_v(x, t)$ [kg/m^3]. More specifically we choose to write

$$\delta c_v = m\Delta t \Delta x / Q_0 \delta S_v, \tag{5}$$

where δS_v is the unit solution to the Dirac pulse and reflects only the stream processes.

Furthermore, we use a probability density function (PDF) of the transport distances, the discharge points from individual households to the exit section of the watershed, $W(x)$; $x \in [0, X]$. The solution can thus be expressed as a double convolution over space and time

$$C_v(t) = L \int_0^X \int_0^t \frac{\overline{m}(x, \tau)}{\overline{Q}_0(x)} \delta \overline{S}_v(x, t - \tau) d\tau W(x) dx, \tag{6}$$

where L – total length of all stream branches in which the total mass M is distributed. Over-bar denotes a weighted average typical for transport distance x.

3. Information on Processes and Data from Morsa Watershed

The Morsa watershed as defined in Fig. 3 is 690 km^2 and consists of approximately 19% of agricultural land, 80% coniferous forest, 1% water surfaces and 0.1% municipal run-off (hard) surfaces. The landscape is of the boreal type with moraine deposits and partly clayey soils on flat areas. The phosphorus load from the watershed is dominated by run-off from agricultural land as is clear from Table 1. However, this study is focused on the phosphorus load in the watershed from the 620 individual households in the watershed upstream of Hogfors, but excluding those of the lake districts Langen, Våg and Mjær. Due to the differences in phosphorus phases in the various sources and their different composition, their spreading follows different patterns and requires different analyses. For instance, phosphorus from agriculture is generally generated in a particulate form and transport is predominantly by erosion. Local wastewater treatment is generally by means of sand filters that retain phosphorus through sorption. The load from each sand filter to the stream system is generated in the form of phosphate and estimated to be 3.51 g/day on average with a variation coefficient of 1.21.

Geographical information is included in a computerised system and used to derive transport distances PDF from the discharge points of the households to the river section at Hogfors (Fig. 4). The transport distance PDF can be weighted with phosphorus load, but the difference is marginal in Fig. 4 and for the final results accounted for in Section 4.

The annual arithmetic mean discharge at Hogfors based on statistics between 1976 to 2002 is 4.55 m^3/s with a coefficient of variation of 2.76 (maximum discharge in this period was 78 m^3/s). This large variation in hydrology can be dealt with

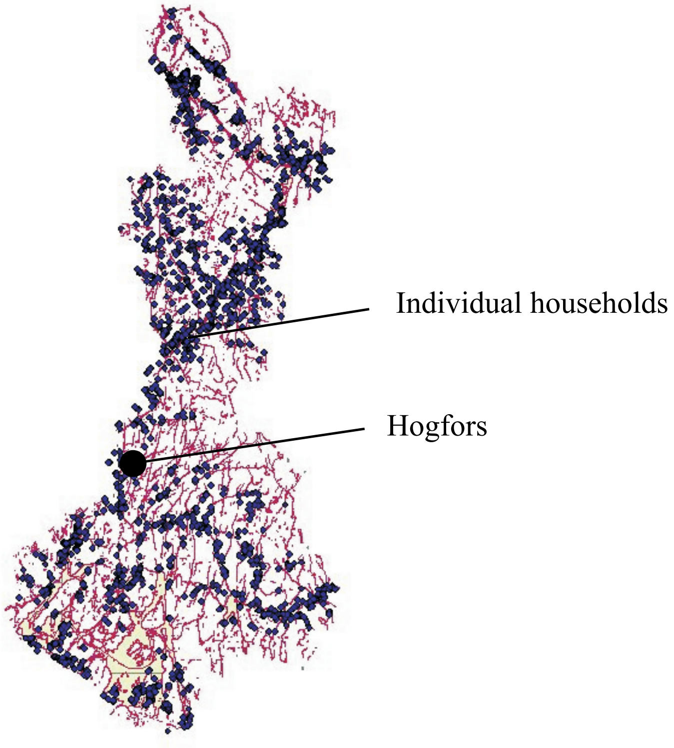

Fig. 3. Morsa Watershed with stream network indicated by red curves. Blue points denote households and black marker denotes Hogfors

by assuming the flow being quasi-stationary for sufficiently short (when flow does not change) and long time periods (much longer than flow travel time) in the derivation of δS_v. As the solution (6) is inversely proportional to discharge, it was considered a reasonable approximation to use a constant harmonic mean value of the discharge, i.e. 0.82 m^3/s.

A simplification accepted in this first tentative approach to analysing the phosphorus export from the Morsa stream network, is to use spatially constant values typical of the network. A reason for doing so is that the dimensionless parameters which appear in the final solution (e.g. Wörman 1998) do not vary as much as the individual parameters. This is due to the cross-covariance between properties. For instance, the investigation by Wörman et al (2002) showed that the retardation factor $[(P/A)V_z T]$ did not exhibit an obvious spatial trend, though a spatial heterogeneity in its value was apparent. Therefore, we used a constant water depth $h = A/P = 1.1$ m, constant stream flow velocity $u = 0.2$ m/s, and constant values of $W/(1 + K_D) = 3.96e - 6$ m/s and $T = 26180$ s. The latter two parameters were considered to be typical for the geomorphology and Froude number prevailing in

Table 1. Local contribution of Total-P (kg per year) from different sources as distributed on sub-catchments (Source: Lyche Solheim et al 2001)

Subcatchment	Agriculture[1]	Individual households	Municipal wastewater	Other sources[2]	Total
1. Langen	106	320	3	475	904
2. Våg and Mjær	425	246	276	427	1374
3. Hobølelva-øvre	3762	588	159	707	5216
4. Kråkstadelva	2823	243	48	465	3579
5. Hobølelva-nedre	1173	197	33	347	1750
6. Mørke-Veidalselva	703	107	6	535	1351
7. Såbyvannet-Svinna	1165	249	53	681	2148
8. Local contribution to Vansjø-Storefjorden	402	103	3	732	1240
8. Total to Vansjø-Storefjorden	10559	2053	581	4369	17562
9. Local contribution to Vansjø-Vanenfjorden	557	171	316	667	1711
10. Mosseelva	24	17	250	69	360
Total	11140	2241	1147	5105	19633

[1] "natural background" from run-off is subtracted,

[2] including "natural background" in run-off water froma agricultural areas.

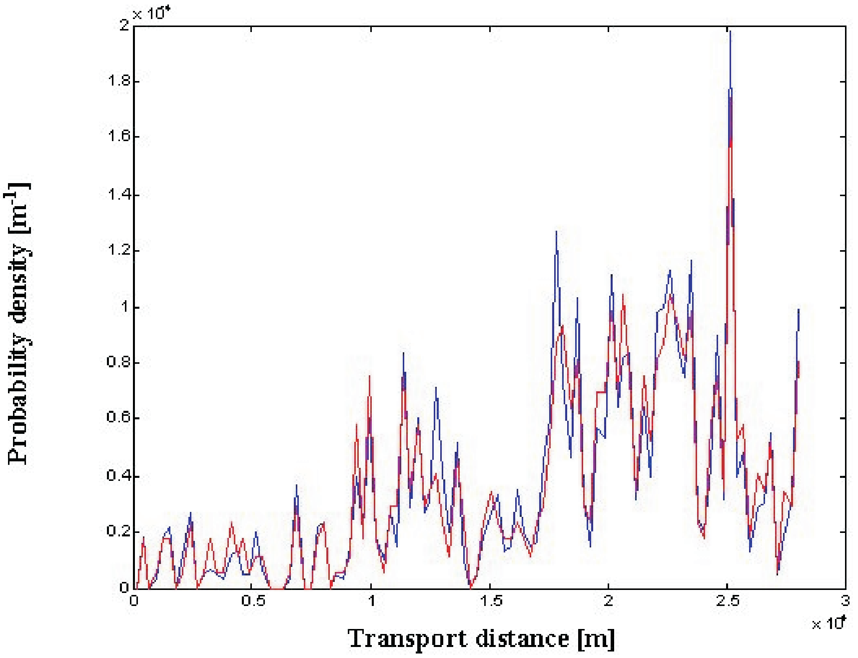

Fig. 4. Probability density function (PDF) for transport distance between households in Morsa Watershed and exit at Hogfors. Blue curve represents the PDF weighted by household load and red curve represents the PDF with equal weight given to each household

Morsa and based on the constitutive relationships of Wörman et al (2002). The discharge was also assumed to be constant.

Furthermore, the sorption properties of phosphorus in the bed sediments of the stream system were taken to be $k_r = 10^{-6}\ \mathrm{s}^{-1}$ and $K_B = 3{,}000$. The K_B-value can be considered to be a lower limit for the phosphorus sorption capacity in stream sediments and is used to estimate the lower limit of the retention characteristics of phosphorus in the Morsa stream network. For instance, Sakadevan and Bavor (1998) investigated the adsorption isotherms for different materials, some of which were extremely favourable with respect to sorption, and found for the linear range of sorption that K_d in units [l/g] varied between 3 and 200. This suggests that K_B could vary between 3,000 and 300,000 (taking a density of $\sim$ 1,600 g/l).

4. Analysis of Phosphorus Retention

Two scenarios were analysed using a numerical implementation of the model described above. These analyses comprised both the short-term system response due to a Diracs delta pulse and long-term response due to the wash-out of the Dirac pulse, as well as the asymptotic equilibrium approach under consideration of a sudden and sustained rise in the load condition. The first case is important as it contains all relevant information and can be used in the convolution integral (6) for any type of load situation (defined by $m(x,t)$).

Figure 5 shows the concentration response in three different special cases. The dotted curve is derived by considering only in-stream processes (advection and dispersion) and is the expected response function for water according to the model procedure of Snell and Sivapalan (1994). The dashed curve shows how the hyporheic exchange significantly changes the response curve by filtering the water through the subsurface. The solid curve is the predicted response for phosphorus with also being taken into account sorption to solid surfaces in the hyporheic zone. Approximately 40% of the phosphorus pulse is retained in the hyporheic zone after two days and retained there for a long time. The corresponding retentions for water and inert solutes are 0.5% and 1%, respectively.

The wash-out of the retained phosphorus is slow, as can be seen in Fig. 6, where the pulse response is shown for a period of 100 days. After three months, only about 32% of the phosphoprus mass originally retained in the hyporheic zone has left the stream network. The slow transport and filtering of sorbing solutes are emphasised by Fig. 7 which shows the asymptotic approach of the equilibrium state due to a sustained load from 620 individual households. Even after one decade, there is a significant change in concentration levels which also indicates that the full effect of remediation actions can be expected only after several years or even decades.

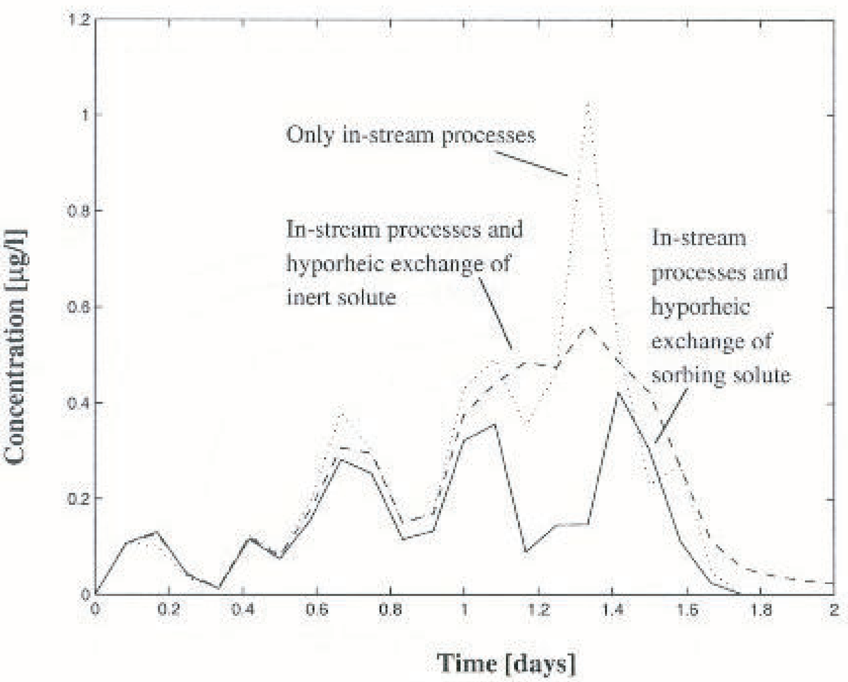

Fig. 5. Response curves at outflow section (Hogfors) in Morsa stream network due to a temporal (short) pulse from each household (2 hours). Solid curve represents phosphorus response with account taken to filtering through the hyporheic zone and sorption, dashed curve is the predicted water response due to the hyporheic exchange and the dotted curve is the response with account only to in-stream processes (advection and dispersion)

5. Discussion and Conclusions

This paper outlines a new model approach to analysis of the export of solute elements from stream networks which is based on convoluting unit solutions for the topography typical to the watershed. Similar model approaches have previously been developed to describe the geomorphological instantaneous unit hydrograph and also for transport of inert solutes. This study focuses on a sorbing solute, phosphate, and provides clear indications of the importance of taking into account the solute mass exchange between surface and subsurface water. In particular, sorbing solutes will be filtered and retained in the hyporheic zone for significant periods of time.

The case study of the transport of phosphate load from 620 individual households in Morsa watershed indicates that about 40% of a unit pulse is retained in the stream bed sediments upstream of Hogfors. The subsequent wash-out of the retained phosphorus mass is relatively slow, depending on sorption characteristics.

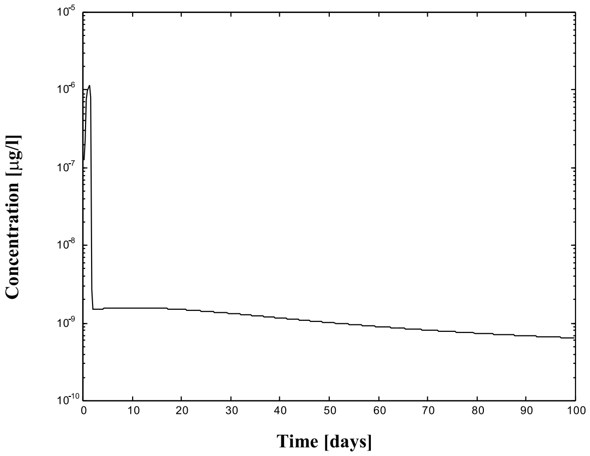

Fig. 6. Response curve at outflow section (Hogfors) in Morsa stream network due to a temporal (short) pulse from each household (2 hours). The flat tail of the response is due to the slow wash-out of phosphorus retained by filtering in the hyporheic zone

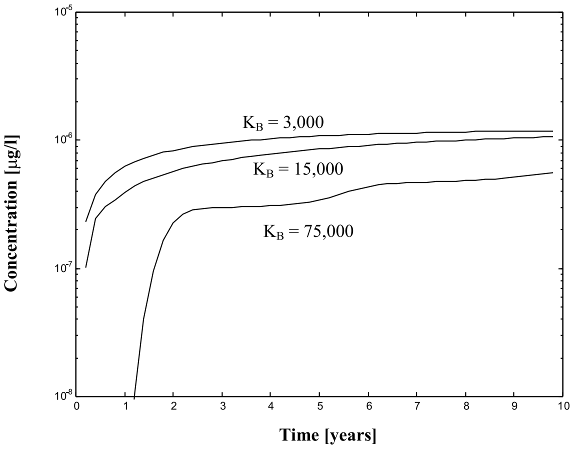

Fig. 7. Response curve at outflow section (Hogfors) in Morsa stream network due to a suddenly introduced and sustained load from 620 individual households in Morsa watershed. A distribution coefficient of $K_B = 3{,}000$ corresponds to a plausible lower limit for the Morsa stream-bed sediments

A reduction of the phosphate load from individual households to Morsa stream system will give about 90% of its full environmental improvement in a few years. If K_B equals 3,000 the recovery time is expected to be about 2.5 years, whereas if K_B is larger we can expect a longer recovery period (cf. Fig. 7). The K_B-value of 3,000 can be considered to be a lower limit for the phosphorus sorption capacity in fine-grained stream-bed sediments. As the P-load from the individual households stands for 15% of the P-load of anthropogenic origin (Table 1), only a minor environmental improvement can be expected from reducing the load of individual households compared with remediation actions in agriculture. The P – transport from agricultural land is primarily caused by erosion of particulate matter, which does not directly follow the model formulation applied here for the hyporheic exchange.

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References

Bencala K. E., Walters R. A. (1983), Simulation of Solute Transport in a Mountain Pool-and-riffle Stream: a Transient Storage Model, *Water Resources Res.*, 19(3), 718–724.

Comans R. N. J., Hockley D. E. (1992), Kinetics of Cesium on Illite, *Geochemica et Cosmochimica Acta*, Vol. 56, 1157–1164.

Cunningham J. A., Werth C. J., Reinhard M., Roberts P. V. (1997a), Effects of Grain-Scale Mass Transfer on the Transport of Volatile Organics through Sediments: 1. Model Developments, *Water Resources Res.*, 33(12), 2713–2726.

Cunningham J. A., Werth C. J., Reinhard M., Roberts P. V. (1997b), Effects of Grain-Scale Mass Transfer on the Transport of Volatile Organics through Sediments: 1. Column Results, *Water Resources Res.*, 33(12), 2727–2740.

Elliott A. H., Brooks N. H. (1997a), Transfer of Nonsorbing Solutes to a Streambed with Bed Forms: Theory, *Water Resources Res.*, 33(1), 123–136.

Elliott A. H., Brooks N. H. (1997b), Transfer of Nonsorbing Solutes to a Streambed with Bed Forms: Laboratory Experiments, *Water Resources Res.*, 33(1), 137–151.

Franchini M., O'Connel P. E. (1996), An Analysis of the Dynamic Component of the Geomorphologic Instantaneous Unit Hydrograph, *Journal of Hydrology*, 175, 407–428.

Huettel M., Ziebis W., Forster S. (1996), Flow-induced Uptake of Particulate Matter in Permeable Sediments, *Limnol. Oceanogr.*, 41(2), 309–322.

Johansson H., Jonsson K., Forsman J., Wörman A. (2000), Retention of Conservative and Sorptive Solutes in Rivers – Simultaneous Tracer Experiments, *The Science of the Total Environment*, 266(2001), 229–238.

Kadlec R. H. & Knight R. L. (1996), *Treatment Wetlands*, CRC Press LLC, USA.

Lyche Solheim A., Vagstad N., Kraft P., Løvstad Ø., Skoglund S., Turtumøygard S., Selvik J.-R. (2001), *Remedial Action Analysis for Morsa*, Report by Norsk Institutt for vannforskning, Publ. NIVA, ISBN 82-577-4016-0 (in Norvegian).

Packman A. I., Brooks N. H., Morgan J. J. (2000), A Physicochemical Model for Colloid Exchange between a Stream and a Sand Streambed with Bed Forms, *Water Resources Res.*, 36(8), 2351–2361.

Reichle R., Kinzelbach W., Kinzelbach H. (1998), Effective Parameters in Heterogeneous and Homogeneous Transport Models with Kinetic Sorption, *Water Res. Research.* 34(4), 583–594.

Rodriguez-Iturbe I., Rinaldo A. (1997), *Fractal River Basin*, Cambridge University Press, U.S.

Sakadevan K., Bavor H. J. (1998), Phosphate Adsorption Characterisation of Soils, Slags and Zeolite to be Sued as Substrates in Constructed Wetlands, *Water Resources Res.*, 32(2), 393–399.

Simic E., Destouni G. (1999), Water and Solute Residence Times in a Catchment: Stochastic Mechanistic Model Interpretation of O–18 Transport, *Water Resources Res.*, 35(7), 2109–2119.

Smith J. T., Comans R. N. J., Ireland D. G., Nolan L., Hilton J. (2000), Experimental and in situ Study of Radiocaesium Transfer across the Sediment-Water Interface and Mobility in Lake Sediments, *Applied Geochemistry*, Vol. 15, 833–848.

Snell J. D., Sivapalan M. (1994), On the Geomorphological Dispersion in Natural Catchments and the Geomorphological Unit Hydrograph, *Water Resources Res.*, 30(7), 2311–2323.

Wörman A. (1998), Analytical Solution and Time Scale for Transport of Reactive Solutes in Rivers and Streams, *Water Resources Res.*, 34(10), 2703–2716.

Wörman A., Packman A., Jonsson K., Johansson H. (2002), Effect of Flow-Induced Exchange in Hyporheic Zones on Longitudinal Transport of Solutes in Streams and Rivers, *Water Resources Res.*, 38(1), 2: 1–15.