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IDENTIFICATION OF UNCORRELATED SOURCES OF EXCITING FORCES BY MEANS OF THE METHODS OF CROSSPOWER SPECTRUM MATRIX DECOMPOSITION INTO SINGULAR VALUES AND EIGENVALUES

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The practical use of the method of the crosspower spectrum matrix decomposition into singular values in problems of the identification of exciting force sources is discussed. These methods were used to identify the sources of the exciting forces acting during the work of a real mechanical system – a collecting press of high level crushing. The method allowed to draw conclusions about the number of uncorrelated exciting forces acting in the system and the points of the exciting force application.

1. Introduction

A commonly used method of identification the uncorrelated sources of exciting forces is the method of square crosspower spectrum matrix decomposition into eigenvalues. The analysis of the mutual relation between the principal eigenvalue spectrum and the autopowers determined at each measuring points allows to make conclusion about the number and places of the force sources application in the system. A similar diagnostic application should have a method based on the analysis of the mutual relation between the principal singular value spectrum, determined by decomposition of the crosspower spectrum matrices into singular values, and the crosspower spectra. The application of the method of decomposition of the crosspower spectrum matrices into singular values allows to reduce the number of calculations. If there are several dominant exciting forces, there is no need to determine all elements of the crosspower spectrum matrices; only some chosen columns and rows should be determined.

In the paper, practical examples of the identification of uncorrelated sources of exciting forces by means of the singular value decomposition method were presented.

In the following discussion SV (singular value) and SVD (singular value decomposition) will be used intensively.

2. Mathematical bases of the procedures of separation of the uncorrelated components from the reciprocal crosspower spectrum matrices

Some of the basic theorems of linear algebra are those describing the decomposition of matrices into eigenvalues (Eq. (1)) and the decomposition of the matrices into singular values (Eq. (2)). The theorem of the matrix decomposition $[A]_{N\times N}$ into eigenvalues [4] states that for each square matrix $[A]_{N\times N}$ there is such a unitary matrix $[U]_{N\times N}$, that

$$[U]^H_{N\times N}[A]_{N\times N}[U]_{N\times N} = [\Lambda]_{N\times N} = \operatorname{diag}(\lambda_1,\ldots,\lambda_n) \tag{1}$$

with: $[U]_{N\times N}$ a unitary matrix containing in its columns the eigenvectors, $[\Lambda]_{N\times N}$ a diagonal matrix containing the eigenvalues in descending order. The following formula can be derived from this theorem

$$[A]_{N\times N} = [U]_{N\times N}[\Lambda]_{N\times N}[U]^H_{N\times N}\,. \tag{2}$$

The theorem of the singular value matrix decomposition [1, 2, 4] says that for each $[A]_{M\times N}$ matrix there are such unitary matrices $[U]_{M\times M}$, $[V]_{N\times N}$, that

$$[U]^H_{M\times M}[A]_{M\times N}[V]_{N\times N} = [\Sigma]_{M\times N} \tag{3}$$

where:

$$[\Sigma]_{M\times N} = \operatorname{diag}(\sigma_1,\sigma_2,\ldots,\sigma_{r+1},\ldots,\sigma_n),$$

if the rank of matrices $[A]_{M\times N}$ equals r, then

$$\sigma_1 \geq \sigma_2 \geq \ldots \sigma_r \quad \text{and} \quad \sigma_{r+1},\ldots,\sigma_n = 0 \quad \text{if} \quad r < n.$$

The σ_1, σ_2, σ_r, σ_l numbers are unanimously defined and are called singular values of the $[A]_{M\times N}$ matrix. The columns of the $[V]_{N\times N}$, $[U]_{M\times M}$ matrices are called sometimes singular vectors of the $[A]_{M\times N}$ matrices.

The following formula can be derived:

$$[A]_{M\times N} = [U]_{M\times M}[\Sigma]_{M\times N}[V]^H_{N\times N}\,. \tag{4}$$

The theorems mentioned above are used to determine the decomposition crosspower spectrum matrices into eigen- and singular values. For a set of $X_{1,\ldots,N}$ mechanical vibration signals measured in a system (in the time domain) it is possible to compute their characteristics in the frequency domain $G_{N\times N}(\omega)$ (crosspower spectra) achieving a crosspower spectrum matrix. The crosspower spectrum is defined as the complex ensemble average of the complex product of the conjugated 1-sided instantaneous spectrum $G_i^*(k)$ and 1-sided instantaneous spectrum $G_j(k)$:

$$\overline{G}_{ij}(k) = \overline{G}_i^*(k)\overline{G}_j(k) \tag{5}$$

where k – frequency index.

The 1-sided instantaneous spectrum is defined as:

$$G_i(k) = \begin{cases} S_i(k) & \text{for} \quad k = 0, \\ 2S_i(k) & \text{for} \quad 1 \leq k \leq N_s/2 - 1, \\ 0 & \text{for} \quad N_s/2 \leq k \leq N_s - 1, \end{cases} \tag{6}$$

where $S_i(k) = F[w(n)x_i(n)]$ is 2-sided spectrum, F – forward discrete Fourier transform operator, $w(n)$ – weighting function (was used Hanning weighting function), $x_i(n)$ – time record, n – time index, N_s – number of samples (was used 2048).

Computing all crosspower spectra we achi ving 3 dimensional crosspower spectrum matrix. This matrix can be considered as a set of square matrix (determined for each frequency). The element $(i, j\omega)$ of this matrix contained value of crosspower spectrum between signal x_i and x_j at frequency ω. In the determined matrix, the non-zero off-diagonal elements indicate a correlation relationship between the corresponding signals.

The application of the eigenvalue matrix decomposition for the analysis of operating response data based on the eigenvalue decomposition of the crosspower matrix (Eq. (6)) [5]:

$$[G_{N\times N}(\omega)] = [U_{N\times N}(\omega)]\,[\Lambda_{N\times N}(\omega)]\,[U_{N\times N}(\omega)]^H. \tag{7}$$

The obtained eigenvalues $[\Lambda_{N\times N}(\omega)]$ of $[G_{N\times N}(\omega)]$ in descending order can be considered as principal component autopower spectra. The principal component spectra are mutually totally uncorrelated (crosspower spectra are zero). The principal autopower spectra, sorted in descending order and plotted as a function of frequency, yield a graphical representation of the rank of the crosspower matrix which indicates the number of incoherent phenomena (principal uncorrelated sources of mechanical vibration) observed in the signal set $S(x_1, x_2, \ldots, x_N)$ at every frequency [5, 6].

Instead of analysing the eigenvalue decomposition of a square matrix, one can use its SVs. The SVD allows to draw conclusions about the number of uncorrelated sources in the same way as the eigenvalue analysis [1].

The decomposition of crosspower spectrum matrices into singular values can be performed by means of the following relationship (Eq. (8)):

$$[G_{M\times N}(\omega)] = [U_{M\times M}(\omega)]\,[\Sigma_{M\times N}(\omega)]\,[V_{N\times N}(\omega)]^H. \tag{8}$$

The method of decomposition of the crosspower matrix into singular values and singular eigenvectors does n require the analysis of the whole (square) crosspower spectrum matrix but to analyse only a matrix composed of chosen columns or rows. The number of analysed columns or rows should be higher than the number of uncorrelated forces acting on the system.

The computation times of decomposition of random generated matrices were determined. The calculations were performed by a microcomputer with the Intel 486DX2/66 processor. The matrix decomposition into eigenvalues was performed by means of the EISPACK package algorithms, while for computation the matrix decomposition into singular values the LINPACK package algorithms were used. The calculation was made using the MATLAB 4.2C programme containing LINPACK and EISPACK procedures. A comparison of the relative computation times of decomposition of complex symmetric matrix $G_{N\times N}$ $(N = 1, 2, \ldots, 100)$ into singular and eigenvalues (without computing eigen and singular vectors) is shown in Fig. 1.

Comparing the computation times of decomposition of the matrix $G_{N\times N}$ into singular and eigenvalues one can say that the method of the singular value matrix decomposition is a less time consuming method. The possibility of performing the SV

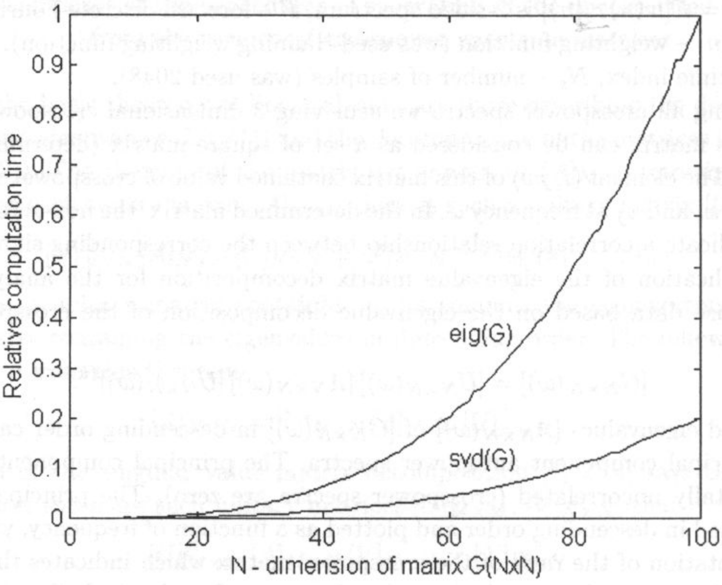


Fig. 1. The comparison of the relative computation time of decomposition of the $G_{N\times N}$ ($N = 1, 2, \dots, 100$) complex symmetrical matrix into singular and eigenvalues.

decomposition of a square matrix containing only chosen columns of crosspower spectra considerably reduces the time of the executed calculations.

3. Test results. Application of the methods of matrix decomposition into singular values to determine the number of uncorrelated sources of exciting forces acting during the operation of the machine

The method depicted above was used to identify the sources of exciting forces acting during the work of a real mechanical system – a collecting press of high level crushing.

The geometrical model of the press with marked measuring points is presented in Fig. 2.

The SV frequency spectrum (calculated for the dominant SV) obtained from the analysis of the crosspower spectrum of signals of vibration acceleration determined during exciting the machine by means of force applied to one of the points (the measuring point 2 in y direction) is shown in Fig. 3.

In order to spot the place of the force application, the correlation coefficients between the frequency spectrum of the highest principal SV ($X(\omega_1, \omega_2, \dots, \omega_n)$) and the autospectrum ($Y(\omega_1, \omega_2, \dots, \omega_n)$) calculated for each measuring point was analysed.

The highest values of the correlation coefficients obtained were those between the principal spectrum and the point of the force application (Fig. 4). The Pearson correla-

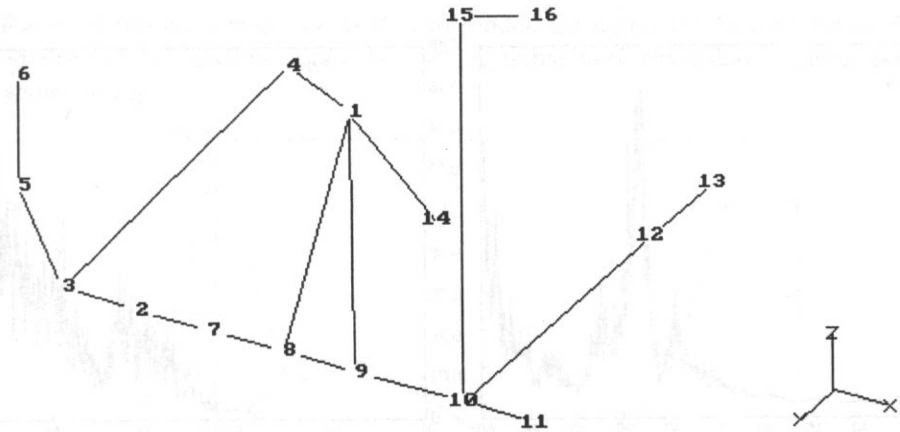


Fig. 2. The geometrical model of the press with measuring points.

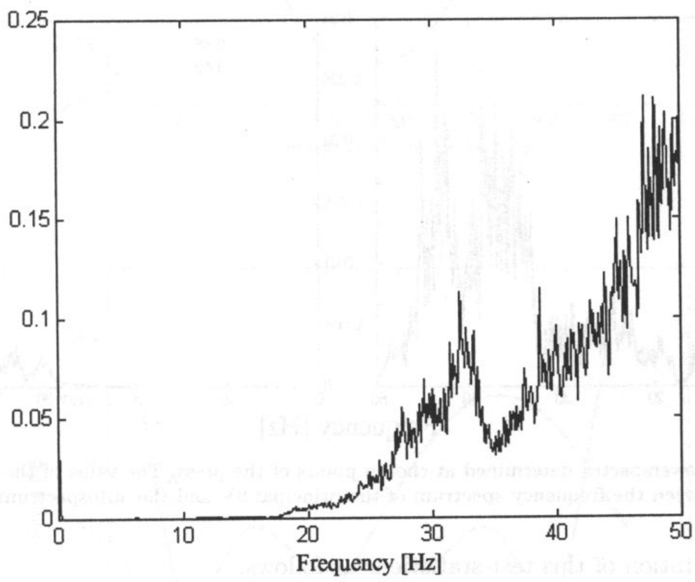


Fig. 3. Frequency spectrum of the principal SV (the SV of the highest value).

tion coefficients r was determined by means of the following relationship:

$$\text{corr} = \frac{S_{XY}}{\sqrt{S_{XX}S_{YY}}} \tag{9}$$

where

$$S_{XY} = \sum_{i=1}^{n} X_i Y_i - \frac{\sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n}, \quad S_{XX} = \sum_{i=1}^{n} X_i^2 - \frac{\sum_{i=1}^{n} X_i^2}{n}, \quad S_{YY} = \sum_{i=1}^{n} Y_i^2 - \frac{\sum_{i=1}^{n} Y_i^2}{n}.$$

The hypotheses associated with this test is: $H_0 : \rho = 0$; $H_A : \rho \neq 0$.

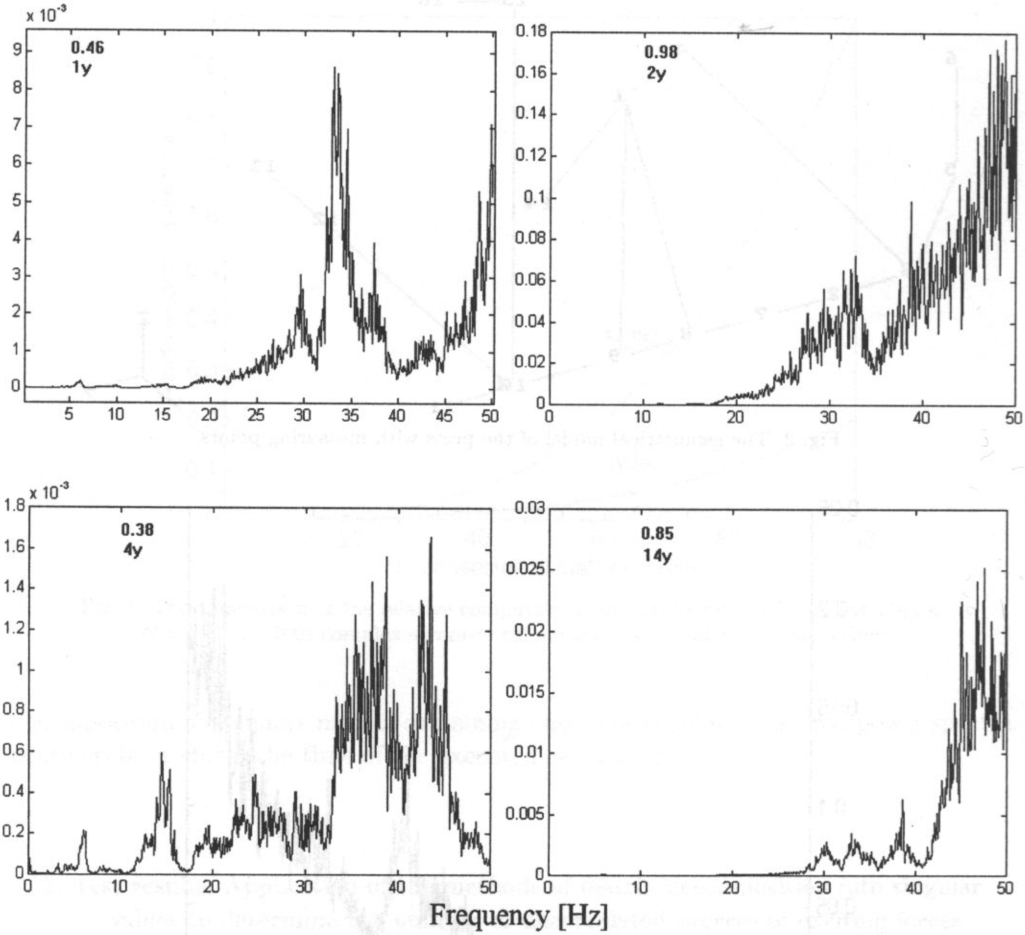


Fig. 4. Autopowerspectra determined at chosen points of the press. The value of the correlation coefficient between the frequency spectrum of the principal SV and the autospectrum are shown.

The computation of this test statistic is as follows:

$$t^* = \frac{\text{corr}\sqrt{n-2}}{\sqrt{1-r^2}} \tag{10}$$

and the rejection region for this test (for $\alpha = 0.05$) are $|t^*| \geq 1.96$.

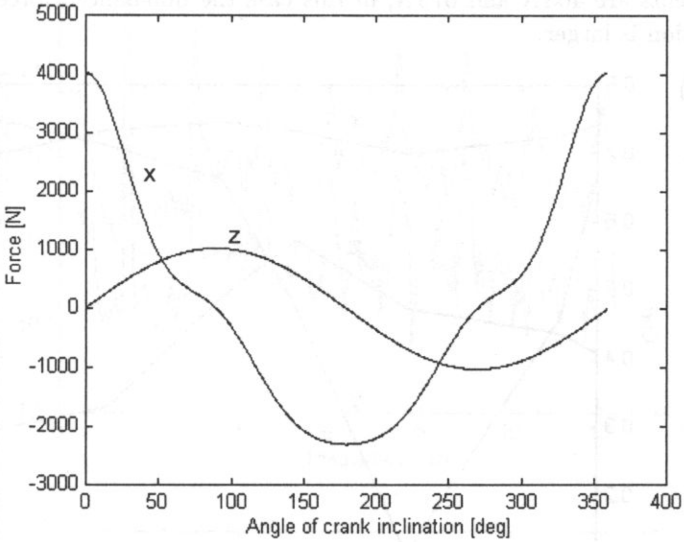
Computed test statistic for correlation coefficients larger than 0.0692 is not in the rejection region, we cannot reject the hypothesis that the data are not linearly related in our example.

The confidence intervals were computed. For correlation coefficients larger than 0.689 the confidence interval was smaller than 0.01 and for correlation coefficients larger than 0.859 the confidence intervals was smaller than 0.005.

Before applying the method of identification of sources and the force application points to analyse the real behaviour of the machine, an analytical model was built [3].

Forces, determined from the analytical model, acting on the bearing crank during the idle run of the machine (the balanced and unbalanced crankshaft – piston system) are shown in Fig. 5.

a)



b)

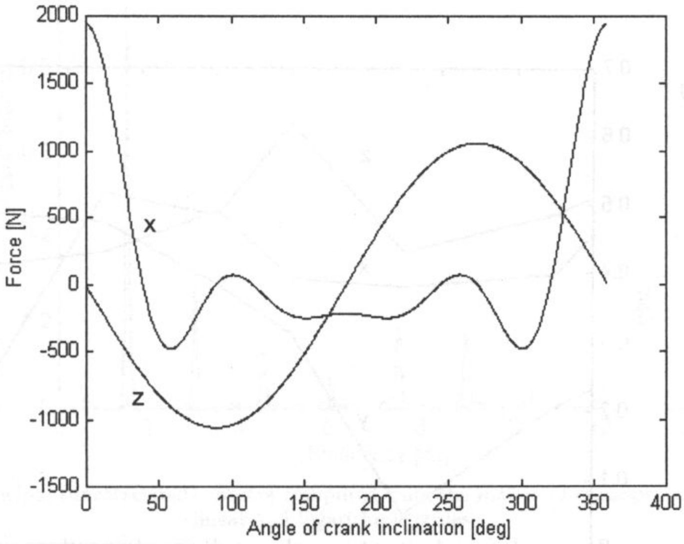


Fig. 5. Forces acting on the crank bearing: x – horizontal component, z – vertical component, a) the unbalanced crankshaft-piston system, b) the balanced crankshaft-piston system.

The analysis of the included charts leads to the conclusion that during the idle run the unbalanced force acting on the bearing crank in the horizontal direction (x) is higher than that acting in the vertical direction (z). Average values of the amplitudes of those forces are 1562 N and 653 N, respectively. After adding a balancing mass in the crankshaft — piston system, the average values of the amplitudes of the unbalanced force components are 408 N and 675 N; in this case the unbalanced force acting in the vertical direction is larger.

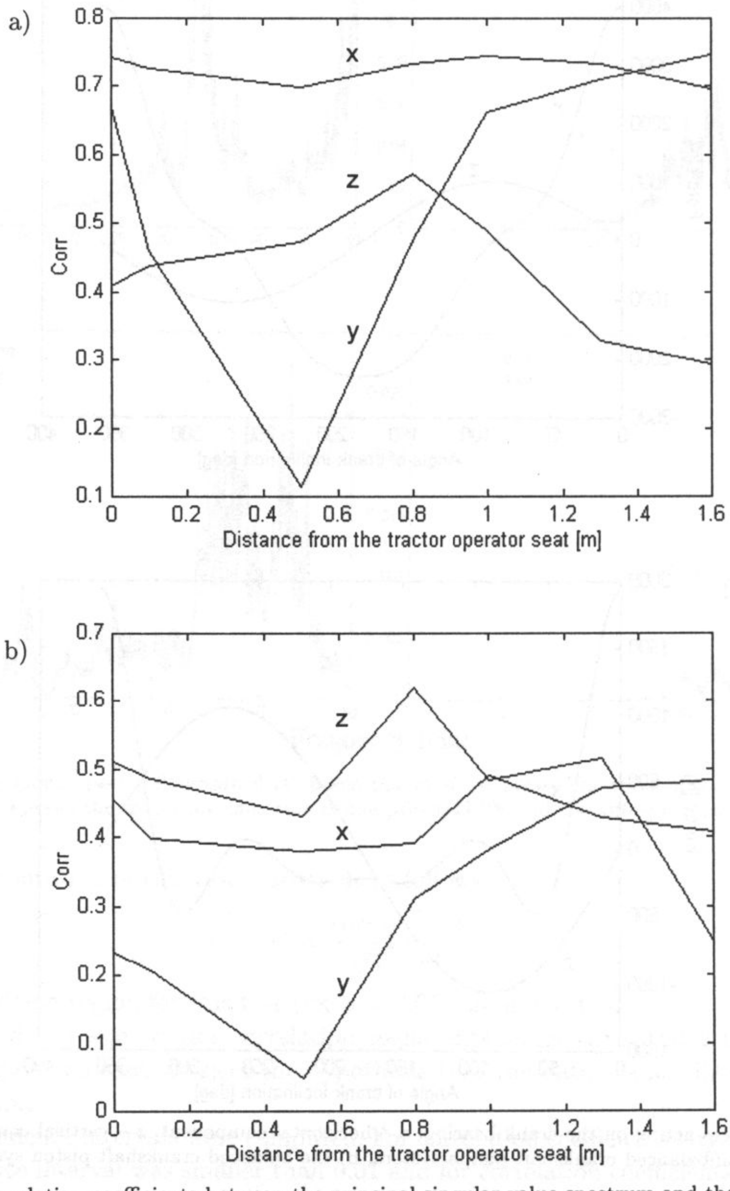
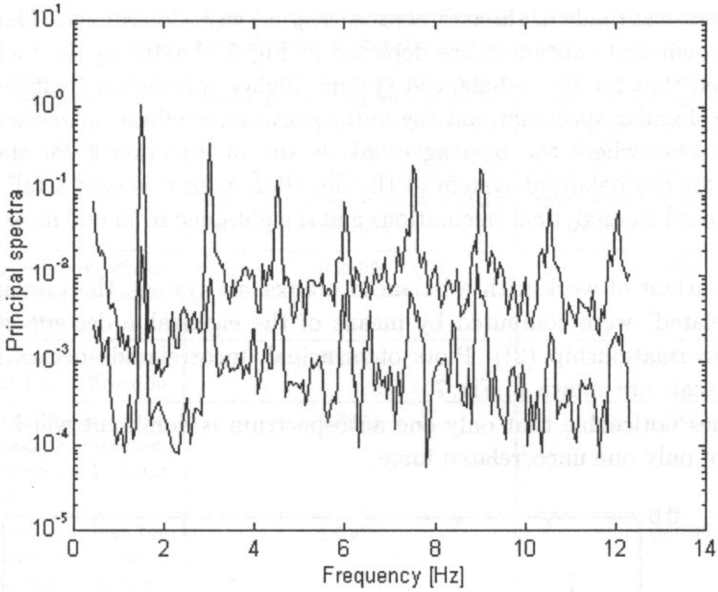


Fig. 6. Correlation coefficients between the principal singular value spectrum and the autopower spectrum measured at important jointing points of the press presented as a function of the distance from the tractor operator seat; a) the unbalanced crankshaft-piston system, b) the balanced crankshaft-piston system.



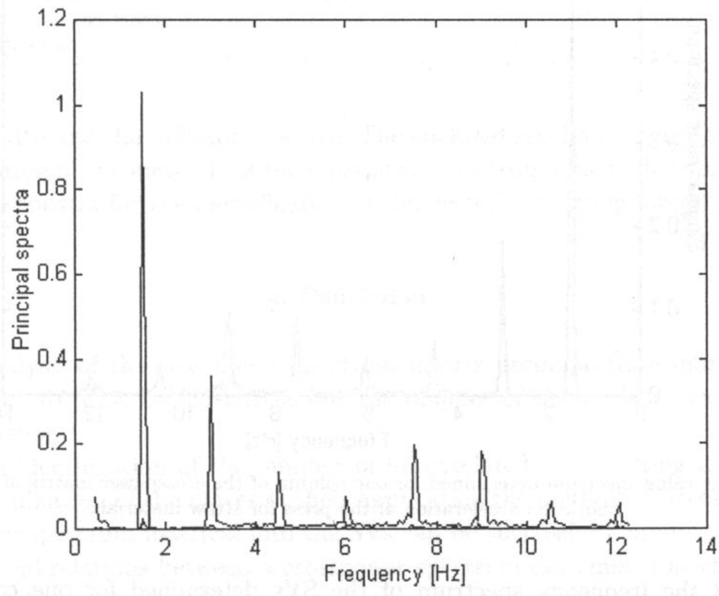


Fig. 7. The principal (uncorrelated) spectra computed from the matrix of crossspectra presented in linear and logarithmic scales.

For both cases (balanced and unbalanced system) measurements of the mechanical vibration were conducted, in joint places of the machine during the idle run.

The correlation coefficients between the principal singular spectrum and the auto spectrum of vibration accelerations measured at seven points of the press (at each point a measurement was made in three directions: x, y, z) were determined. The values of the correlation coefficients obtained are depicted in Fig. 6. Analysing the included results, it can be seen that for the unbalanced system, higher correlation coefficient values are between the singular spectrum and the auto spectrum of vibration accelerations in the place of the case where the bearing crank is: the in direction x for the unbalanced system and for the balanced system in the direction z. In this case a full agreement of the results based on analytical calculations and the obtained from real measurements was achieved.

For the variant of work with a balanced crankshaft system, the principal autospectra (uncorrelated) were computed by means of the eigenvalue decomposition method (according to relationship (2)). Plots of principal uncorrelated spectra in linear and logarithmic scale are shown in Fig. 7.

It becomes noticeable that only one autospectrum is dominant which indicates the occurrence of only one uncorrelated force.

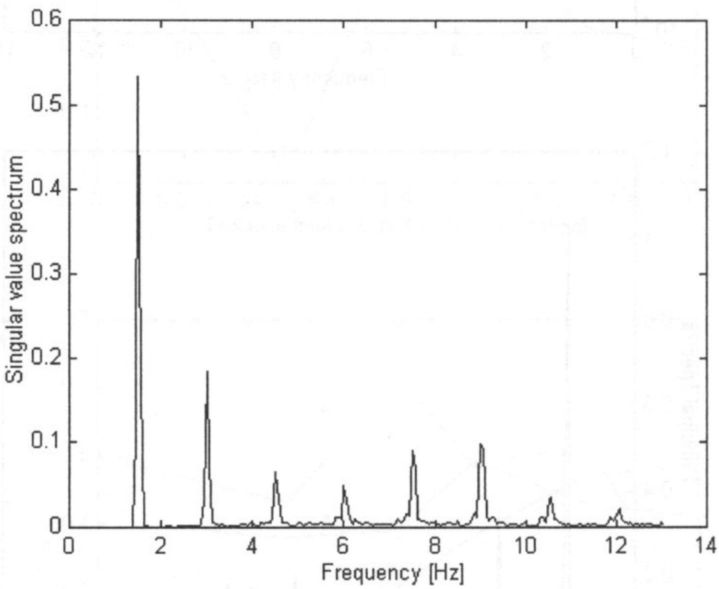


Fig. 8. Singular value spectrum determined for one column of the crosspower matrix of the mechanical vibration acceleration of the press for straw materials.

In Fig. 8 the frequency spectrum of the SVs determined for one column of the crosspower spectrum matrices is shown. A similarity between the principal auto spectrum and the singular one is seen. The correlation coefficients between the autospectra and the principal uncorrelated spectra (computed from the eigenvalues decomposition of the matrix) or the singular value spectrum is depicted in Table 1.

In both cases the highest correlation coefficients were obtained in the connecting-rod in the direction of the pressing force. The closer to the point of the pressing force application was the measuring point, the higher the values of the correlation coefficients

Table 1. Correlation coefficients between the principal spectrum of SVs or eigenvalues and the crosspower spectrum of vibration accelerations.

Point (j) of crosspower (G_{ij}) spectrum measurement (i – case of press gearbox, direction along the machine track)	Correlation coefficient between the crosspower spectrum and the principal spectrum determined by decomposition of the crosspower spectra matrices into eigenvalues	Correlation coefficient between the crosspower spectrum and the SV spectrum determined by decomposition of one column of the crosspower matrices into SVs
Case of press gearbox, direction along the machine track, point 1 in x direction	0.2895	0.602
Case of press gearbox, vertical direction, point 1 in z direction	0.1435	0.4133
Case of the machine near the chamber of pressing, direction along the machine track, point 8 in x direction	0.8041	0.9883
Case of the machine near the chamber of pressing, vertical direction, point 8 in z direction	0.8012	0.9736
Floor under tractor seat, direction along the machine track, point 5 in x direction	0.0117	0.279

between the auto and the principal spectra. The enclosed results confirm the possibility of the application of the methods of the crosspower spectrum matrix decomposition into singular value spectra for the identification of the vector of exciting forces.

4. Conclusions

1. The analysis of the crosspower spectrum matrix decomposition into eigenvalues and SVs allows to draw conclusions about the number of uncorrelated exciting forces acting in a system.

2. For the identification of the number of uncorrelated forces acting in the system and the determination of the places of their application the method of decomposition of the crosspower spectrum matrices into the SVs can be successfully used.

3. Reciprocal relations between a crosspower spectrum determined at chosen points of a system and the principal (uncorrelated) eigen- or singular spectra allow to identify the points of the exciting force application.

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