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A TRIAL OF EVALUATION BASED ON MULTIPLICATIVE MULTIVARIATE MODEL FOR NON-STATIONARY ROAD TRAFFIC NOISE

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Since the non-stationary fluctuation of actual acoustic environment like the road traffic noise within a long time interval is usually caused not by the change of merely additional external noise but by the essential change of internal factors themselves, it is reasonable to give as a mathematical model of this non-stationary fluctuation not an additive model but a multiplicative model. In this paper, mainly from a methodological viewpoint, first, the characteristic function method of Mellin transform type for the above multiplicative model is introduced and a new unified expression form of the probability density function within a long time interval is derived in an expansion form based on the probability density function within a short time interval of local stationary type as its first expansion term with distribution parameters reflecting hierarchically various types linear and nonlinear correlation information.

1. Introduction

In the actual road traffic in a big city, many kinds of cars run on the roadway. Then, many traffic factors, such as the number of cars, the mixture ratio of various types of car, the headway interval between successive two cars and the power level fluctuation of each car are statistically invariant in an average style within a short time interval but fluctuate non-stationarily within a long time interval. Therefore, within a long time interval, the measured sound wave of road traffic noise fluctuates randomly, multiformly and non-stationarily. Accordingly, for its noise evaluation, it is important to find the probability distribution form of noise fluctuation directly connected with the evaluation index L_x ($(100 - x)$ percentile) often employed in the actual evaluation of road traffic noise not only within a short time interval but also within a long time interval. In this paper, from the above viewpoint, a new evaluation method is proposed to predict the probability distribution form of the traffic noise fluctuation within a long time interval based on the one within a short time interval of locally stationary type.

First, the theoretical model of the relationship between the sound wave of the traffic noise within a long time interval and the one within a short time interval should be considered. Since this non-stationarity is originally evaluated as the relative change from a basic stationary fluctuation, it is first noteworthy that the property of non-stationarity can not appear without the existence of original basic traffic flow itself. That is, because this non-stationarity comes not from an addition mechanism of external factors (for example, any background noise) but from the temporal fluctuation of internal factors within a long time interval, which are statistically invariant (i.e., locally stationary) on an average within a short time interval, it would be reasonable to employ the multiplicative model rather than the additive model as a temporal fluctuation of this non-stationary traffic noise. After all, mathematically, for such actual non-stationary phenomena, the mutliplicative model is considered as the central subject of this study and a new unified theory of evaluating the probability distribution is given by introducing the characteristic function of Mellin transform type, especially from a methodological viewpoint. More concretely, the objective probability distribution of non-stationary traffic noise fluctuation within a long time interval is expressed in a hierarchical expansion form by taking the probability distribution of local stationarity within a short time interval, of which expansion coefficients reflect concretely the non-stationary statistics (i.e., linear and nonlinear correlations of higher order) in the form of relative change from the basic fluctuation within a short time interval for mixture ratio of car type and number of cars (see Appendixes I and II).

Finally, the effectiveness of the proposed method is experimentally confirmed too by applying it to the actually observed data of non-stationary road traffic noise fluctuation.

2. Theoretical consideration

2.1. Modelling of non-stationary road traffic noise

Let us consider the traffic noise problem caused by cars moving along two lanes. As is well-known, the instantaneous A-frequency weighted sound intensity E generated by many cars running on the roadway can be expressed theoretically as follows [1, 2]:

$$E = \sum_{i=1}^{n_1} W_{1i} f(d_{1i}, \delta) + \sum_{i=1}^{n_2} W_{2i} f(d_{2i}, \delta), \tag{2.1}$$

where W_{1i} – A-frequency weighted sound power of the i-th heavy car, W_{2i} – A-frequency weighted sound power of the i-th light car, n_1 – number of the heavy cars, n_2 – number of the light cars, $f(d_{ji}, \delta)$ – sound propagation characteristic reflecting surrounding environmental factors, d_{ji} – distance between sound source (i-th car of the j-th type) and observation point, δ – internal factor except d_{ji} in the sound propagation characteristic.

From now on, although d_{ji} depends on the lane, we assume that d_{ji} in each lane is identical (see Appendix III). Here, since the sound level meter with a time constant in the actual measurement gives some averaging effect and the purpose of this study is originally the stochastic evaluation of non-stationary fluctuation within a long time interval, by taking an expectation of Eq. (2.1) within a local time interval under the assumption of local stationarity, binomial distribution assumption for number of heavy cars, n_1, and number of light cars, n_2, with mixture ratio of the heavy car, θ_1, and mixture ratio of the light car, θ_2, $(\theta_1 + \theta_2 = 1)$, and Poisson distribution assumption for total number of cars, n $(= n_1 + n_2)$, with its mean value N in an observed interval, the mathematical macro-model within a long time interval is first derived [2, 3]:

$$\langle E\rangle = N\left(\theta_1\langle W_{1i}\rangle\langle f(d_1,\delta)\rangle + \theta_2\langle W_{2i}\rangle\langle f(d_2,\delta)\rangle\right), \tag{2.2}$$

where $\langle\ \rangle$ denotes an average operation within a local time interval and d_j instead of d_{ji} is employed since d_{ji} $(i = 1, 2)$ are mutually independent and distribute in the same form. Especially from the above macroscopic viewpoint, the strict stationarity within a local time interval is not necessarily needed (see Appendix I). It is natural that within a long time interval, not only N and θ_j but also $\langle W_j\rangle$ and $\langle (d_j,\delta)\rangle$ would fluctuate owing to non-stationary property of internal factors [4]. Next, by choosing an adequate local time interval as a standard time interval of local stationarity and using a mean value of total number of cars, N_0, and mixture ratio of car types, θ_{oj}, Eq. (2.2) is rewritten as follows:

$$\begin{aligned}\langle E\rangle &= \frac{N}{N_0}\frac{\theta_1\langle W_{1i}\rangle + \theta_2\langle W_{2i}\rangle}{\theta_{01}\langle W_{1i}\rangle + \theta_{02}\langle W_{2i}\rangle}\cdot N_0\left(\theta_{01}\langle W_{1i}\rangle\langle f(d_1,\delta)\rangle + \theta_{02}\langle W_{2i}\rangle\langle f(d_2,\delta)\rangle\right)\\ &= \frac{N}{N_0}\frac{\theta_1 + \theta_2 a}{\theta_{01} + \theta_{02}a}\cdot N_0\left(\theta_{01}\langle W_{1i}\rangle\langle f(d_1,\delta)\rangle + \theta_{02}\langle W_{2i}\rangle\langle f(d_2,\delta)\rangle\right),\end{aligned} \tag{2.3}$$

where $a = \langle W_{2i}\rangle/\langle W_{1i}\rangle$.

Here, the term $\langle f(d_j,\delta)\rangle$ is eliminated since d_1 and d_2 are independent and distribute in the same form. Now, after defining new stochastic variables Z, X, U_1 and U_2 as

$$\begin{aligned} Z &= \langle E\rangle, \qquad X = N_0(\theta_{01}\langle W_{1i}\rangle\langle f(d_1,\delta)\rangle + \theta_{02}\langle W_{2i}\rangle\langle f(d_2,\delta)\rangle),\\ U_1 &= \frac{N}{N_0}, \qquad U_2 = \frac{\theta_1 + \theta_2 a}{\theta_{01} + \theta_{02}a},\end{aligned} \tag{2.4}$$

we can have directly the following multiplicative model with three stochastic variables [5]:

$$Z = U_1 U_2 X. \tag{2.5}$$

By corresponding X and Z to the local stationary traffic noise and the non-stationary traffic noise, respectively, this model relates a local stationary traffic noise within a short time interval to the non-stationary one within a long time interval.

2.2. General representation of probability distribution of multiplicative model with stochastic multi-variables

Now, by mathematically generalizing Eq. (2.5), the following multiplicative model with stochastic multi-variables is considered:

$$Z = U_1 U_2 \ldots U_M X, \tag{2.6}$$

where $U_1, U_2, \ldots, U_M$ and X are arbitrary random processes fluctuating only within a positive amplitude region and mutually correlated each other. Here, let us derive the probability distribution expression of the stochastic process described by Eq. (2.6). Using the joint probability density function $P(Z, X, U_1, \ldots, U_{M-1})$ of $Z, X, U_1, \ldots, U_{M-1}$ we have the marginal probability density function $P(Z)$ of Z from its definition as

$$P(Z) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty P(Z, X, U_1, \ldots U_{M-1})\, dX\, dU_1 \ldots dU_{M-1}. \tag{2.7}$$

By denoting a joint probability density function of Z and X conditioned by $U_1, \ldots, U_{M-1}$ by $P(Z, X|U_1, \ldots, U_{M-1})$, and a joint probability density function of $U_1, \ldots, U_{M-1}$ by $P(U_1, \ldots, U_{M-1})$, Eq. (2.7) can be rewritten in the form:

$$P(Z) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty P(Z, X|U_1, \ldots, U_{M-1}) \cdot P(U_1, \ldots, U_{M-1})\, dX\, dU_1 \ldots dU_{M-1}. \tag{2.8}$$

Here, from the probability measure-preserving transformation, it follows that

$$P(Z, X|U_1 \ldots U_{M-1}) = P(U_M, X|U_1 \ldots U_{M-1}) \left| \frac{\partial(U_M, X)}{\partial(Z, X)} \right| = \frac{P(U_M, X|U_1 \ldots U_{M-1})}{X U_1 \ldots U_{M-1}}. \tag{2.9}$$

Substituting Eqs. (2.6) and (2.9) into Eq. (2.8), we have

$$P(Z) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty \frac{P(U_1, \ldots, U_{M-1})}{X U_1 \ldots U_{M-1}} \cdot P\left(\frac{Z}{X U_1 \ldots U_{M-1}}, X|U_1, \ldots, U_{M-1} \right) dX\, dU_1 \ldots dU_{M-1}. \tag{2.10}$$

Furthermore, from the following fundamental relationship of probability [6]:

$$P(U_M, X|U_1 \ldots U_{M-1}) = P(U_M|X, U_1 \ldots U_{M-1}) P(X|U_1 \ldots U_{M-1}), \tag{2.11}$$

Eq. (2.10) becomes

$$P(Z) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty \frac{P(U_1, \ldots U_{M-1})}{X U_1 \ldots U_{M-1}} P(X|U_1, \ldots, U_{M-1}) \cdot P\left(\frac{Z}{X U_1 \ldots U_{M-1}} |X, U_1, \ldots, U_{M-1} \right) dX\, dU_1 \ldots dU_{M-1}. \tag{2.12}$$

Here, for the probability density function $f(x)$ of random variable X, let us introduce the characteristic function of Mellin transform type [7, 8, 9] defined by

$$M_x(s) = \mathsf{m}\{f(x)\} = \int_0^\infty x^{s-1} f(x)\, dx. \tag{2.13}$$

Applying the above characteristics function to $P(Z)$, we have

$$M_Z(s) = \int_0^\infty \int_0^\infty \dots \int_0^\infty \Bigg[\int_0^\infty \left(\frac{Z}{XU_1 \dots U_{M-1}} \right)^{s-1} \cdot P\left(\frac{Z}{XU_1 \dots U_{M-1}} | X, U_1, \dots, U_{M-1} \right) \frac{dZ}{XU_1 \dots U_{M-1}} \Bigg]$$
$$\cdot (XU_1 \dots U_{M-1})^{s-1} P(X|U_1, \dots, U_{M-1})\, P(U_1, \dots U_{M-1})\, dX dU_1 \dots dU_{M-1}\,. \tag{2.14}$$

If we denote that

$$M_{U_M}(s|X, U_1, \dots, U_{M-1}) = \int_0^\infty \left(\frac{Z}{XU_1 \dots U_{M-1}} \right)^{s-1} \cdot P\left(\frac{Z}{XU_1 \dots U_{M-1}} | X, U_1, \dots, U_{M-1} \right) \frac{dZ}{XU_1 \dots U_{M-1}}\,, \tag{2.15}$$

then Eq. (2.14) is rewritten to

$$M_Z(s) = \int_0^\infty \int_0^\infty \dots \int_0^\infty (XU_1 \dots U_{M-1})^{s-1} M_{U_M}(s|X, U_1, \dots, U_{M-1}) \cdot P(X|U_1, \dots, U_{M-1}) P(U_1, \dots, U_{M-1})\, dX dU_1 \dots dU_{M-1}\,. \tag{2.16}$$

With the use of the fundamental relationship of probability:

$$P(X|U_1, \dots, U_{M-1})\, P(U_1, \dots, U_{M-1}) = P(X, U_1, \dots, U_{M-1}) = P(U_{M-1}|X, U_1, \dots, U_{M-2})\, P(X, U_1, \dots, U_{M-2}), \tag{2.17}$$

we have from Eq. (2.16)

$$M_Z(s) = \int_0^\infty \int_0^\infty \dots \int_0^\infty (XU_1 \dots U_{M-2})^{s-1} \cdot \left[\int_0^\infty U_{M-1}^{s-1} P(U_{M-1}|X, U_1, \dots, U_{M-2})\, M_{U_M}(s|X, U_1, \dots, U_{M-1})\, dU_{M-1} \right] \cdot P(X, U_1, \dots, U_{M-2})\, dX dU_1 \dots dU_{M-2}\,. \tag{2.18}$$

After repeated applications of similar mathematical procedures for Eq. (2.18) and employing the fundamental relationship:

$$P(X, U_1, \dots, U_{M-j}) = P(U_{M-j}|X, U_1, \dots, U_{M-j-1}) \cdot P(X, U_1, \dots, U_{M-j-1}), \qquad (j = 2, \dots, M-1), \tag{2.19}$$

we can obtain (see Appendix IV)

$$M_Z(s) = \int_0^\infty X^{s-1} P_X(X) \left[\int_0^\infty U_1^{s-1} P(U_1|X) \left[\dots \left[\int_0^\infty U_{M-1}^{s-1} P(U_{M-1}|X, U_1, \dots, U_{M_2}) \right.\right.\right.$$
$$\left.\left.\left. \cdot M_{U_M}(s|X, U_1, \dots, U_{M-1})\, dU_{M-1} \right] \dots \right] dU_1 \right] dX. \qquad (2.20)$$

Now, for the purpose of finding an explicit expression of $P(Z)$ from Eq. (2.20), let us expand $M_{U_M}(s|X, U_1, \dots, U_{M-1})$ in a Taylor series around $s = 1$ as

$$M_{U_M}(s|X, U_1, \dots, U_{M-1}) = \sum_{i_M=0}^{\infty} A_{i_M}(X, U_1, \dots, U_{M-1})(s-1)^{i_M}. \qquad (2.21)$$

Substituting Eq. (2.21) into Eq. (2.20) and changing the order of the integration and the summation, we have

$$M_Z(s) = \int_0^\infty X^{s-1} P_X(X) \left[\int_0^\infty U_1^{s-1} P(U_1|X) \left[\dots \right.\right.$$
$$\left[\sum_{i_M=0}^{\infty} (s-1)^{i_M} \int_0^\infty U_{M-1}^{s-1} P(U_{M-1}|X, U_1, \dots, U_{M-2}) \right.$$
$$\left.\left.\left. \cdot A_{i_M}(X, U_1, \dots, U_{M-1}) dU_{M-1} \right] \dots \right] dU_1 \right] dX, \qquad (2.22)$$

where, after denoting an average operation of random variable U_m conditioned by $X, U_1, \dots, U_{M-1}$ as $\langle\ |X, U_1, \dots, U_{M-1}\rangle_{U_M}$, the expansion coefficient can be expressed

$$A_{i_M}(X, U_1, \dots, U_{M-1}) = \frac{1}{i_M!} \left\langle (\ln U_M)^{i_M} | X, U_1, \dots, U_{M-1} \right\rangle_{U_M}. \qquad (2.23)$$

After employing the Taylor series expansion around $s = 1$ in each integration of U_{M-1} $(i = 2, \dots, M-1)$, we obtain (see Appendix V)

$$M_Z(s) = \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} \dots \sum_{i_M=0}^{\infty} (s-1)^{i_1+i_2 \dots + i_M} \mathrm{m}\{A_{i_1 i_2 \dots i_M}(X) P_X(X)\}, \qquad (2.24)$$

where

$$A_{i_i i_2 \dots i_M}(X) = \frac{1}{i_1! i_2! \dots i_M!}$$
$$\cdot \left\langle (\ln U_1)^{i_i} \langle (\ln U_2)^{i_2} \dots \langle (\ln U_M)^{i_M} | X, U_1, \dots, U_{M-1} \rangle_{U_M} \dots | X, U_1 \rangle_{U_2} | X \right\rangle_{U_1}. \qquad (2.25)$$

Here, using the fundamental relationship of the Mellin transformation [9, 10]:

$$(-1)^n (s-1)^n \mathrm{m}\{f(X)\} = \mathrm{m}\left\{ \left(\frac{d}{dX} X \right)^n f(X) \right\}, \qquad (2.26)$$

we can obtain the unified expansion expression of the probability density function from the inverse Mellin transformation of Eq. (2.24) as

$$P(Z)=\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\cdots\sum_{i_M=0}^{\infty}(-1)^{i_1+i_2+\ldots+i_M}\cdot\left(\frac{d}{dX}X\right)^{i_1+i_2+\ldots+i_M}\left\{A_{i_1 i_2\ldots i_M}(X)P_X(X)\right\}\big|_{X=Z}, \tag{2.27}$$

where $A_{i_1 i_2 \ldots i_M}(X)$ is given by Eq. (2.25).

As stated in Sec. 2.1, the road trafic noise intensity can be described by Eq. (2.5) and then, from the Eq. (2.27), we can express the objective probability density function of Z within a long time interval as

$$P(Z)=\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}(-1)^{i_1+i_2}\left(\frac{d}{dX}X\right)^{i_1+i_2}\left\{A_{i_1 i_2}(X)P_X(X)\right\}\big|_{X=Z}, \tag{2.28}$$

where

$$A_{i_1 i_2}(X)=\frac{\langle(\ln U_1)^{i_1}\langle(\ln U_2)^{i_2}|X,U_1\rangle_{U_2}|X\rangle_{U_1}}{i_1!i_2!}. \tag{2.29}$$

3. Experimental consideration

The actual road traffic noise has been observed on a national road in suburbs of a big city and the proposed method has been applied to the problem of predicting the probability distribution form directly connected with noise evaluation indexes L_x ($x = 50, 5, 95, \ldots$) for the non-stationary traffic noise level fluctuation within a long time interval based on the local stationarity.

In this experiment, the sound noise level has been measured at every five second interval over four hours and forty minutes by using a digital sound level meter under the actual situation as shown in Fig. 1.

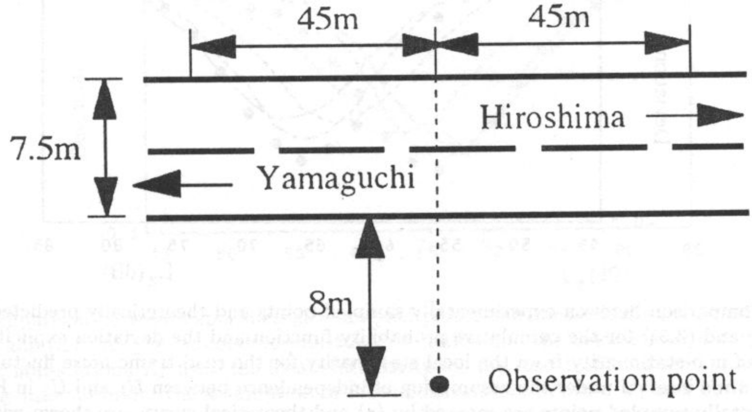

Fig. 1. Measuring situation of road traffic noise.

Before computing Eqs. (2.28) and (2.29), we have chosen five seconds, as the basic local time interval in this experiment, according to the usual type measurement in a JIS standard [11]. Then, the probability density function $P_X(X)$ within a short time interval of locally stationary type for a road traffic noise could be approximated with a logarithimic normal distribution [12]:

$$P_X(X) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\sigma X} \exp\left\{ \dfrac{(\ln X - \mu)^2}{2\sigma^2} \right\} & (X \geq 0), \\ 0 & (X < 0), \end{cases} \tag{3.1}$$

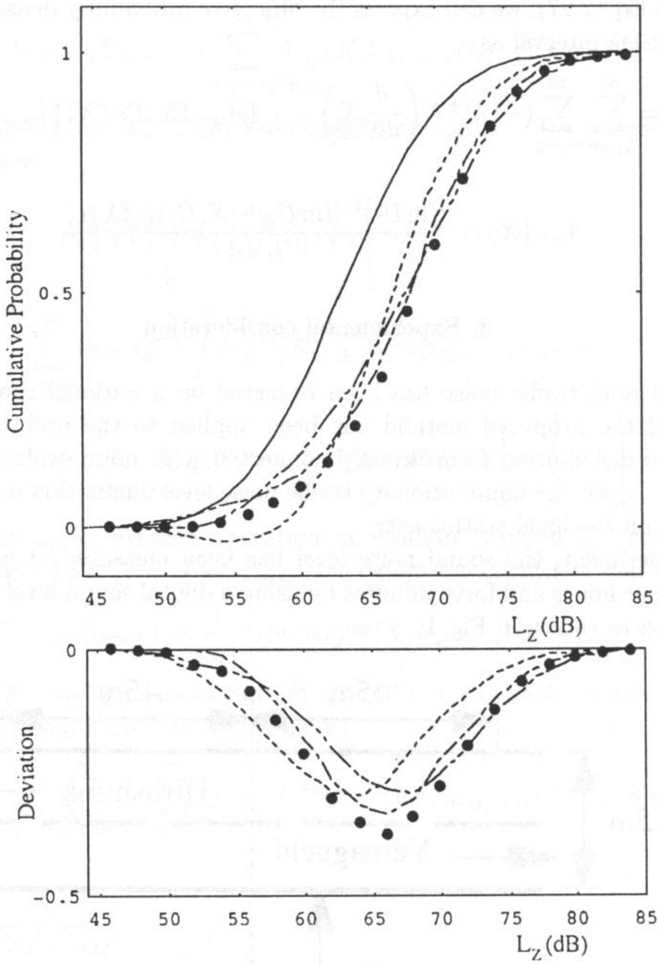

Fig. 2. A comparison between experimentally sampled points and theoretically predicted curves (see Eqs. (2.28) and (3.3)) for the cumulative probability function and the deviation explicitly reflecting the effect of non-stationarity from the local stationarity for the road traffic noise fluctuation within a long time interval under the assumption of independence between U_1 and U_2 in Eq. (3.3). Experimentally sampled points are marked by (•) and theoretical curves are shown with degree of approximation $i_1 + i_2$ [$i_1 + i_2 = 0$ (—), $i_1 + i_2 = 1$ (- - -), $i_1 + i_2 = 2$ (- · -), $i_1 + i_2 = 3$ (- · · -)].

where two distribution parameters μ and σ have been determined by employing the well-known method of moment as follows:

$$\begin{aligned} e^{\mu+\sigma^2/2} &= \langle X \rangle, \\ e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right) &= \langle (X-\langle X \rangle)^2 \rangle. \end{aligned} \tag{3.2}$$

Clearly, U_1 and U_2 are independent of X, hence Eq. (2.29) reduces to

$$A_{i_1 i_2}(X) = \frac{\langle (\ln U_1)^{i_1} (\ln U_2)^{i_2} \rangle_{U_1,U_2}}{i_1! i_2!}. \tag{3.3}$$

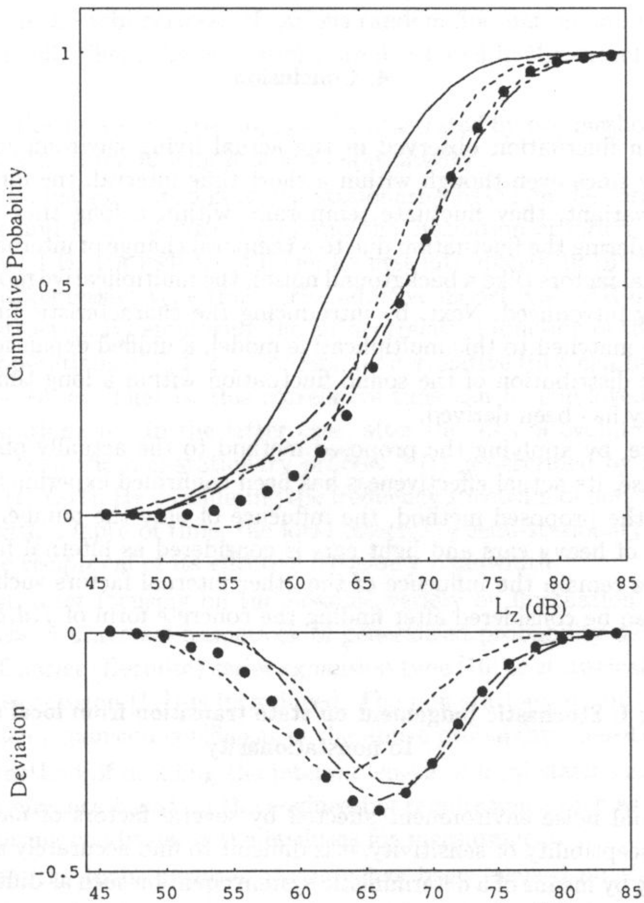

Fig. 3. A comparison between experimentally sampled points and theoretically predicted curves (see Eqs. (2.28) and (3.3) for the cumulative probability function and the deviation explicitly reflecting the effect of non-stationarity from the local stationarity for the road traffic noise fluctuation within a long time interval under the assumption of mutual correlation between U_1 and U_2 in Eq. (3.3). Experimentally sampled points are marked by (•) and theoretical curves are shown with degree of approximation i_1+i_2 [$i_1+i_2=0$ (—), $i_1+i_2=1$ (– – –), $i_1+i_2=2$ (– · –), $i_1+i_2=3$ (– · · –)].

We have calculated the theoretical probability distribution under the following two assumptions: i) U_1 and U_2 are statistically independent, ii) U_1 and U_2 are statistically correlated. Figure 2 shows a comparison between experimentally sampled points and theoretically evaluated curves for the cumulative probability function and the deviation from the first expansion term of local stationarity under the assumption that U_1 and U_2 are independent. Figure 3 shows a comparison between experimentally sampled points and the theoretically evaluated curves of the cumulative probability function and the deviation from the first expansion term of local stationary type under the assumption that U_1 and U_2 are statistically correlated. In both cases, the theoretical probability distribution curves agree well with experimental results but the latter case gives slightly better result than the former.

4. Conclusion

The random fluctuation observed in the actual living environment shows usually nonstationarity since even though within a short time interval, the internal factors are statistically invariant, they fluctuate temporally within a long time interval. In this paper, by considering the fluctuation due to a temporal change of internal factors with no additive external factors (like a background noise), the multiplicative model has first been mathematically introduced. Next, by introducing the characteristic function of Mellin transform type matched to this multiplicative model, a unified expansion expression for the probability distribution of the sound fluctuation within a long time interval of the non-stationarity has been derived.

Furthermore, by applying the proposed method to the actually observed data of a road traffic noise, its actual effectiveness has been confirmed experimentally too.

Finally, in the proposed method, the influence of only the number of cars and the mixture ratios of heavy cars and light cars is considered as internal factors. As future problems, there remain the influence of the other internal factors such as weather and so on. These can be considered after finding the concrete form of $f(d, \delta)$.

Appendix I: Stochastic judgement on state transition from local stationarity to nonstationarity

For an actual noise environment affected by several factors of nature, community and human susceptibility or sensitivity, it is difficult to find accurately an internal law of nonstationarity by means of a deterministic system equation such as differential equation. In this case, exactly it may be said that there are no rationally optimal methods to divide a whole nonstationary process into some intervals of local stationarity in the actual environmental phenomena. Furthermore, it needs to construct some methodology for predicting not only the lower order moments statistics (such as mean value and variance) but also the higher order moment statistics and/or a whole probability density function form from which any noise evaluation indices L_x can be obtained.

There seem to be two basic attitudes to deal with the above nonstationarity, from bottom up and top down ways of viewpoint:

1. After extracting only the frame work of the objective random time series by smoothing its various fluctuation patterns; a certain kind of time series model of additive, multiplicative or mixed type is first introduced and then their parameters are estimated. Hereafter, all of discussions on the objective fluctuation pattern and/or stochastic behaviour are given in various ways on the basis of the above artificial time series model, even in a special case of evaluating the end of its probability density function curve.

2. Whenever the internal law of the object process is known or unknown, first one introduces a universal framework of probability distribution expression, which can minutely deal with the arbitrariness of various random fluctuation and its possible fluctuation pattern itself. Then, the discussions are developed in the extent of its universal framework.

Furthermore, the above two attitudes can be illustrated by two methods of evaluation criterion deciding the interval length of local stationarity:

Case A: The evaluation criterion of non-stationarity based on the correlation method by G.E.P. Box and G.M. Jenkins [13] and the evaluation criterion of semi-stationarity function by M.B. Priestley [14] can be first introduced. In the former case, the framework of autoregressive type time series additive model (e.g., AR or ARMA model [15]) is first constructed. By employing the autocorrelation function of the data sequence after difference operations of $0, 1, 2, \ldots$ orders, its correlative time can be used to decide the order of time series. That is, this correlative time can be employed as the interval length of local stationarity. In the latter case, after the idea of evolutionary spectra is first introduced, then the non-stationary process $X(t)$ is described by a multiplicative model $X(t) = C(t)X_0(t)$. By considering the frequency spectrum of only factor $C(t)$ fluctuating slowly with a lapse of time, the local interval of semi-stationary can be roughly estimated by the reciprocal of its effective frequency bandwidth.

Case B: First, after focusing on the possible variety on fluctuation pattern of non-stationary process, a universal framework of generalized probability density expression (such as Gram–Charier (Hermite) series expansion type [16] or statistical Laguerre series expansion type expression [17]) is introduced. The non-stationarity of the phenomenon is reflected in each expansion coefficient of the above probability density expression.

As another method of deciding the interval length of local stationarity, this interval can be set up in advance based on the engineering requirement, such as a JIS standard, the actual measuring condition or the facilities for measurement.

In this paper, the main purpose of study has been focused on the following two points:

1. To propose some theoretical guideline on how to evaluate the probability distribution for the state transition from local stationarity to non-stationarity of a whole process, once after arbitrarily establishing the intervals of local stationarity.

2. To confirm experimentally the validity of the above theoretical method by applying it to the actual environment.

Appendix II: Two complementary analysis methods for non-stationary stochastic phenomena [18, 19]

As the analysis of non-stationary stochastic fluctuation, it can be very often seen that the analysis is performed systematically based on the law of system dynamics as seen very often in many stochastic systems theories, once after this dynamical law is first established in the form of the time evolution form. However, the proposed method in this paper based on some statistical series expansion is quite different from this usual method.

In general, for the analysis of the non-stationarity property, the methods can be classified contrastively in two types as follows:

I. If the system dynamics can be first represented in a form of some time evolution form (for example, in terms of differential equation) by focusing on only the temporal relationship among the successively processed state values at more than 2 time points, it is usual to find systematically its whole solution in a lapse of time based on this dynamical law. Let us call this a method A.

II. However, even in this method A, it is inavoidable for finding the complete solution along a time axis to reflect aditionally the static state information such as an initial state value, terminal state value and/or instantaneous state values at a particular time point upon the above system dynamics in time axis. That is, to grasp completely the objective phenomenon with a lapse of time, both informations of dynamic and static sides must be considered inevitably. From this point of view, there can be proposed the method of focussing on the static information side between the above two sides which are typically contrastive each other. For example, first, one can establishe some mathematical framework of the universal expression form with unknown parameters based on the basic and obvious static information of state only at each time section. Afterward, one must try to reflect concretely the dynamic characteristic of state on these parameters in process of time. Let us call this a method B.

In comparison with the usual method A, the method B has distinctly the following characteristic properties:

1. First, by neglecting the dynamical characteristic latent among instantaneous state values in time succession, only the static information of instantaneous state values at each time section is abstracted to analyze the objective phenomena by finding the appropriate method such as the multivariate analysis. So, it has an advantage to find some mathematical framework in the universal expression form which is not affected by the system dynamical law in a time evolution form, even if this system dynamics is not given in advance in a peculiar expression form. Here, it is no essential problem whether this dynamical law is complex or not.

2. To analyze the various stochastic phenomena especially in a unified form, it is inevitably necessary to employ some open style expression form rather than some closed style expression form. For instance, as an example of the open style expression form, it can be considered to employ the series expansion type probability expression with appropriate number of distribution parameters. In this particular case, it has an advantage that the objective stochastic phenomena can be investigated hierarchically and moreover

it can be discussed in advance that what type of higher stochastic concept is necessary and how mathematically it should be introduced.

3. In the method A, if the system dynamics is first given in the form of n-th order differential equation, the infinitesimal quantities more than $(n+1)$-th order must be neglected even though they are latently necessary information. So, the optimality of employing the above representation form becomes the first problem depending on how the objective phenomenon is complicated and manifold. For example, in the first order state vector stochastic differential equation with white noise input, sometimes the second order infinitesimal quantities become some kind of problem and it is well-known that the difference between Ito-type and Stratonovich-type stochastic differential equations comes out. Anyway, it has surely an advantage that if employing the law of system dynamics in the method A as a clue of analysis, some succesive and systematic procedure for analysis planning can be easily designed and the algorithm for its calculation process can be successively constructed. On the contrary, in the method B, instead of first acquiring the universality which is independent of individuality of specific dynamical law (proper to the objective system) in the time evolution style based on the system dynamics, there remains the next problem on how to reflect this time evolution property on the universal framework of static style introduced in advance. Generally, if any more valuable information concerning to the state evolution progress of system dynamics can be found, the present problem in the method B becomes undoubtedly definite more and more by successively reflecting these constraint informations afterward.

Appendix III: Traffic flow model in two lanes

In the main part of this paper, for practical use, a model transforming the traffic flows in two lanes to the traffic flow in one lane equivalently is derived. Here, let us derive more accurate model directly derived from traffic flows in two lanes. By splitting heavy cars into ones moving along the first and the second lanes, $n_1 = n_1^* + n_1^{**}$, and heavy cars moving along the first and the second lanes $n_2 = n_2^* + n_2^{**}$, Eq. (2.1) can be rewritten as

$$E = \sum_{i=1}^{n_1^*} W_{1i} f(d_i^*, \delta) + \sum_{i=1}^{n_2^*} W_{2i} f(d_i^*, \delta) + \sum_{i=1}^{n_1^{**}} W_{1i} f(d_i^{**}, \delta) + \sum_{i=1}^{n_2^{**}} W_{2i} f(d_i^{**}, \delta).$$

By choosing an adequate local time interval as a standard time interval of local stationarity in each line and using a mean value of number of cars, N_0^*, and its mixture ratio of car types, θ_{oj}^*, for the first lane and a mean value of number of cars, N_0^{**}, and its mixture ratio of car types, θ_{oj}^{**}, for the second lane, the following macro model can be derived:

$$\begin{aligned}\langle E\rangle = {} & \frac{N^*}{N_0^*}\frac{\theta_1^* + \theta_2^* a}{\theta_{01}^* + \theta_{02}^* a} \cdot N_0^* \left(\theta_{01}^* \langle W_{1i}\rangle \langle f(d_1^*, \delta)\rangle + \theta_{02}^* \langle W_{2i}\rangle \langle f(d_2^*, \delta)\rangle\right) \\ & + \frac{N^{**}}{N_0^{**}}\frac{\theta_1^{**} + \theta_2^{**} a}{\theta_{01}^{**} + \theta_{02}^{**} a} \cdot N_0^{**} \left(\theta_{01}^{**} \langle W_{1i}\rangle \langle f(d_1^{**}, \delta)\rangle + \theta_{02}^{**} \langle W_{2i}\rangle \langle f(d_2^{**}, \delta)\rangle\right).\end{aligned}$$

After defining new stochastic variables $Z, X^*, U_1^*, U_2^*, X^{**}, U_1^{**}, U_2^{**}$ as

$$\begin{aligned}
Z &= \langle E \rangle, \qquad X^* = N_0^* \left(\theta_{01}^* \langle W_{1i} \rangle \langle f(d_1^*, \delta) \rangle + \theta_{02}^* \langle W_{2i} \rangle \langle f(d_2^*, \delta) \rangle \right), \\
U_1^* &= \frac{N^*}{N_0^*}, \qquad U_2^* = \frac{\theta_1^* + \theta_2^* a}{\theta_{01}^* + \theta_{02}^* a}, \\
X^{**} &= N_0^{**} \left(\theta_{01}^{**} \langle W_{1i} \rangle \langle f(d_1^{**}, \delta) \rangle + \theta_{02}^{**} \langle W_{2i} \rangle \langle f(d_2^{**}, \delta) \rangle \right), \\
U_1^{**} &= \frac{\langle N^{**} \rangle}{N_0^{**}}, \qquad U_2^{**} = \frac{\theta_1^{**} + \theta_2^{**} a}{\theta_{01}^{**} + \theta_{02}^{**} a},
\end{aligned}$$

the following model can be obtained:

$$Z = Z_1 + Z_2 ,$$

where

$$Z_1 = U_1^* U_2^* X^*, \qquad Z_2 = U_1^{**} U_2^{**} X^{**}.$$

Here, the probability density functions of Z_1, Z_2 are given by using Eq. (2.27). Since Z_1, Z_2 are statistically independent, the probability density function of Z can be obtained as the convolution of the probability density functions of Z_1, Z_2. Since X^*, X^{**}, Z_1 and Z_2 cannot be confirmed, this model can not be confirmed experimentally.

Appendix IV: Derivation of Eq. (2.20)

Equation (2.20) can be derived by making use of Eq. (2.19) in the following way:

$$\begin{aligned}
M_z(s) &= \int_0^\infty \int_0^\infty \cdots \int_0^\infty (X U_1 \dots U_{M-2})^{s-1} \\
&\left[\int_0^\infty U_{M-1}^{s-1} P(U_{M-1} | X, U_1, \dots, U_{M-2}) M_{U_M}(s | X, U_1, \dots, U_{M-1})\, dU_{M-1} \right] \\
&\cdot P(U_{M-2} | X, U_1, \dots, U_{M-3}) P(X, U_1, \dots, U_{M-3})\, dX dU_1 \dots dU_{M-2} \\
&= \int_0^\infty \int_0^\infty \cdots \int_0^\infty (X U_1 \dots U_{M-3})^{s-1} \left[\int_0^\infty U_{M-2}^{s-1} P(U_{M-2} | X, U_1, \dots, U_{M-3}) \right. \\
&\left. \left[\int_0^\infty U_{M-1}^{s-1} P(U_{M-1} | X, U_1, \dots, U_{M-2}) M_{U_M}(s | X, U_1, \dots, U_{M-1})\, dU_{M-1} \right] dU_{M-2} \right] \\
&\cdot P(U_{M-3} | X, U_1, \dots, U_{M-4}) P(X, U_1, \dots, U_{M-4})\, dX dU_1 \dots dU_{M-3} \\
&= \dots = \int_0^\infty X^{s-1} P_X(X) \left[\int_0^\infty U_1^{s-1} P(U_1 | X) \left[\cdots \right. \right. \\
&\left. \left. \left[\int_0^\infty U_{M-2}^{s-1} P(U_{M-2} | X, U_1, ..., U_{M-3}) M_{U_M}(s | X, U_1, ..., U_{M-1})\, dU_{M-1} \right] dU_{M-2} \right] \cdots \right] dU_1 \right] dX.
\end{aligned}$$

Appendix V: Derivation of Eq. (2.25)

From the definition of the expansion coefficient in a Taylor's series expansion around $s=1$ of $M_{U_M}(s|X,U_1,...,U_{M-1})$, $A_{i_M}(X,U_1,...,U_{M-1})$ can be derived as follows:

$$A_{i_M}(X,U_1,...,U_{M-1}) = \frac{1}{i_M!}\left.\frac{d^{i_M}}{ds^{i_M}}M_{U_M}(s|X,U_1,...,U_{M-1})\,dU_M\right|_{s=1}$$

$$= \frac{1}{i_M!}\frac{d^{i_M}}{ds^{i_M}}\left.\int_0^\infty U_M^{s-1}P(U_M|X,U_1,...,U_{M-1})\,dU_M\right|_{s=1}$$

$$= \frac{1}{i_M!}\left.\int_0^\infty \frac{d^{i_M}}{ds^{i_M}}U_M^{s-1}P(U_M|X,U_1,...,U_{M-1})\,dU_M\right|_{s=1}$$

$$= \frac{1}{i_M!}\left.\int_0^\infty (\ln U_M)^{i_M}U_M^{s-1}P(U_M|X,U_1,...,U_{M-1})\,dU_M\right|_{s=1}$$

$$= \frac{1}{i_M!}\int_0^\infty (\ln U_M)^{i_M}P(U_M|X,U_1,...,U_{M-1})dU_M = \frac{1}{i_M!}\left\langle(\ln U_M)^{i_M}|X,U_1,...,U_{M-1}\right\rangle_{U_M}.$$

Next, let us define $M_{U_{M-1},...,U_M}(s|X,U_1,...,U_{M-2})$ as follows

$$M_{U_{M-1},...,U_M}(s|X,U_1,U_{M-2}) = \int_0^\infty U_{M-1}^{s-1}P(U_{M-1}|X,U_1,...,U_{M-1})A_{i_M}(X,U_1,...,U_{M-1})\,dU_{M-1}.$$

$M_{U_{M-1}U_M}(s|X,U_1,...,U_{M-2})$ is expanded in a Taylor's series expansion around $s=1$ and its expansion coefficient is denoted as $A_{i_{M-1}i_M}(X,U_1,..,U_{M-2})$. The expression of $A_{i_{M-1}i_M}(X,U_1,...,U_{M-2})$ can be obtained as follows:

$$A_{i_{M-1}i_M}(X,U_1,...,U_{M-2})$$

$$= \frac{1}{i_{M-1}!}\frac{d^{i_{M-1}}}{ds^{i_{M-1}}}\left.\int_0^\infty U_{M-1}^{s-1}P(U_{M-1}|X,U_1,...,U_{M-2})A_{i_M}(X,U_1,...,U_{M-1})\,dU_{M-1}\right|_{s=1}$$

$$= \frac{1}{i_{M-1}!}\left.\int_0^\infty \frac{d^{i_{M-1}}}{ds^{i_{M-1}}}U_{M-1}^{s-1}P(U_{M-1}|X,U_1,...,U_{M-2})A_{i_M}(X,U_1,...,U_{M-1})\,dU_{M-1}\right|_{s=1}$$

$$= \frac{1}{i_{M-1}!}\left.\int_0^\infty (\ln U_{M-1})^{i_M-1}U_{M-1}^{s-1}P(U_{M-1}|X,U_1,...,U_{M-2})A_{i_M}(X,U_1,...,U_{M-1})dU_{M-1}\right|_{s=1}$$

$$= \frac{1}{i_{M-1}!}\left\langle(\ln U_{M-1})^{i_{M-1}}A_{i_M}(X,U_1,...,U_{M-1})|X,U_1,...,U_{M-2}\right\rangle_{U_{M-1}}$$

$$= \frac{1}{i_{M-1}!i_M!}\left\langle(\ln U_{M-1})^{i_{M-1}}\langle(\ln U_M)^{i_M}|X,U_1,...,U_{M-1}\rangle_{U_M}|X,U_1,...,U_{M-2}\right\rangle_{U_{M-1}}.$$

By defining $M_{U_{M-2}U_{M-1}U_M}(s|X,U_1,\ldots,U_{M-3}),\ldots,M_{U_1,\ldots U_{M-2}U_{M-1}U_M}(s|X)$ successively and denoting the expansion coefficient of their Taylor's series around $s=1$ as $A_{i_{M-2}i_{M-1}i_M}(X,U_1,\ldots,U_{M-3})$, $\ldots$, $A_{i_1 i_2\ldots i_M}(X)$, respectively, the above similar procedure leads to Eq. (2.25).

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