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ACOUSTO-OPTIC PROPERTIES OF PIEZOELECTRIC INTERFACIAL WAVES

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An analysis is given of acousto-optic coupling associated with piezoelectric interfacial waves for a great number of crystal cuts. Results of numerical calculations are presented of appropriate coupling coefficients in relation to wave parameters for lithium niobate and quartz. It is found that, for some cuts, the coefficients are quite large (over 4% for lithium niobate). It is also found that high acousto-optic coupling is usually accompanied by high piezoelectric coupling.

1. Introduction

The acousto-optic effect is used in many electronic devices with surface acoustic wave, e.g. in deflectors or travelling diffraction gratings. Replacing the surface acoustic wave (SAW) by the piezoelectric interfacial wave (PIW) in such devices is attractive for two reasons. First, piezoelectric coupling of PIW is roughly two times greater than that of SAW. So, generation of PIW is easier and strain of the medium is higher, which should translate into higer acousto-optic coupling. Second, since PIW propagates inside the medium, it is less affected by the environment.

In the paper, we investigate the acousto-optic coupling of PIW for two piezoelectric media which differ much in acousto-optic properties: lithium niobate and quartz. A numerical survey of the two media is made for a great number of crystal cuts. PIW parameters and acousto-optic coupling coefficients are calculated for each cut. Then crystal cuts of high coupling (piezoelectic and/or acousto-optic) are selected.

2. Wave properties

PIW is a surface wave which propagates along a perfectly conducting plane embedded in a homogeneous piezoelectric medium. In the system of coordinates (x, y, z), let the plane be given by the equation $z = 0$. We assume that the electro-mechanical field depends on time as $\exp(j\omega t)$, that it is independent of y (the wave propagates in the x direction), and that the dependence on x is given by the factor $\exp(-j\omega r x)$ where r

is the slowness of the wave. In this case, the electro-mechanical field equations can be reduced to a system of eight first-order ordinary differential equations, as described in Ref. [1].

Let $i, j = 1, 2, 3$ and $(x_i) = (x, y, z)$. The following field variables will be used: particle displacement u_i, electric potential ϕ, surface force $T_i = T_{3i}$ (where T_{ij} is the stress tensor), and normal (to the conducting plane) component D_3 of the electric displacement D_i. We have

$$\frac{d}{dz} F_K = -j\omega r H_{KL}(r) F_L, \tag{1}$$

where $K, L = 1, \ldots, 8$ and $(F_K) = (j\omega r u_i, j\omega r\phi, T_i, D_3)$. We adopt notations and conventions of Ref. [1], in particular, the convention of summing over repeated indices. For real r, which we assume, the matrix H_{KL} is real and non-symmetric. It depends on material constants: elastic tensor c_{ijkl}, piezoelectric tensor e_{kij}, dielectric tensor ε_{ki}, and mass density ρ.

The solution to Eq. (1), which satisfies appropriate boundary conditions at the conducting plane, can be obtained by assuming that it depends on z as $\exp(-j\omega sz)$ where s is the slowness of the wave in the z direction. This leads to the system of eight linear algebraic equations

$$H_{KL}(r) F_L = q F_K\,, \tag{2}$$

where $q = s/r$. After solving the eigenvalue problem defined by Eq. (2) we find the solution, separately in the upper and lower half-space, as a linear combination of four eigenwaves with coefficients determined by the boundary conditions [1]. At the conducting plane, the amplitude of the solution F_K is a linear combination of four eigenvectors.

3. Acousto-optic coupling

Let us introduce the tensor $\eta_{ij} = \varepsilon_0 \varepsilon_{ij}^{-1}$, where ε_{ij}^{-1} denotes the inverse of the matrix ε_{ij} and ε_0 is the dielectric permittivity of the vacuum. The relation between the electric field and the electric displacement is

$$E_i = \varepsilon_0^{-1} \eta_{ij} D_j\,. \tag{3}$$

The acousto-optic effect consists in changing the tensor η_{ij} due to strain of the medium. This is usually denoted by

$$\Delta\eta_{ij} = \eta_{ij}(1) - \eta_{ij}(0). \tag{4}$$

The argument 0 or 1 means that the strain is equal to zero or is different from zero. The tensor $\zeta_{ij} = \Delta\eta_{ij}$ is proportional to the strain tensor [2, 3], i.e.

$$\zeta_{ij} = p_{ijkl} S_{kl}\,, \tag{5}$$

for $k, l = 1, 2, 3$, where S_{kl} is the strain tensor and p_{ijkl} is the acousto-optic tensor.

Mutiplying Eq. (4) by $\varepsilon_0^{-1} D_j$ we get

$$E_i(1) - E_i(0) = \varepsilon_0^{-1} \zeta_{ij} D_j\,. \tag{6}$$

Denote by $|E_i|$ the length of the vector E_i. The ratio

$$\frac{|E_i(1) - E_i(0)|}{|E_i(0)|} = \frac{|\zeta_{ij} D_j|}{|\eta_{ij}(0) D_j|} \tag{7}$$

is the relative change of the length of E_i due to strain for a particular electric displacement D_j. It is obvious that this ratio is independent of the the length of D_j. If the vector D_j is parallel to the x axis then we may put $(D_j) = (1, 0, 0)$ in Eq. (7). This gives

$$\alpha_1 = |\zeta_{i1}|/|\eta_{i1}(0)|, \tag{8}$$

which will be called acousto-optic coupling coefficient in the x direction. In general,

$$\alpha_j = |\zeta_{i(j)}|/|\eta_{i(j)}(0)| \tag{9}$$

will be called acousto-optic coupling coeficients in the x, y, and z direction (for $j = 1, 2, 3$). The coefficient $\alpha = \alpha_1 + \alpha_2 + \alpha_3$ is an overall measure of acousto-optic coupling.

We assume that the acousto-optic coupling does not affect essentially the propagation of the surface wave. So, in the above formulae, by E_i and D_i we mean the fields other than those of the surface wave.

In order to calculate the tensor ζ_{ij} and then the acousto-optic coupling coefficients, we need the value of the strain tensor $S_{ij} = \dfrac{1}{2}(u_{i,j} + u_{j,i})$ at the conducting plane. It is seen that

$$u_{i,1} = -j\omega r_{\mathrm{p}} \tilde{u}_i\,, \tag{10}$$

and that $u_{i,2} = 0$. The remaining derivative can be found from Eq. (1). We have

$$u_{i,3} = H_{iL}(r_{\mathrm{p}}) \tilde{F}_L\,. \tag{11}$$

In Eqs. (10), (11), $\tilde{u}_i$ and $\tilde{F}_L$ are the complex amplitudes of u_i and F_L at the conducting plane, r_{p} is the slowness of PIW.

4. Numerical calculations

The complex amplitudes and other parameters of PIW are calculated for various orientations of the conducting plane with respect to the crystallographic axes of the medium and for different directions of propagation. This is done by solving the eigenvalue problem related to Eq. (2) with the use of EISPACK routines [4] for different crystal cuts or triplets of Euler angles. The three-dimensional space of Euler angles is scanned in steps of 2° in each of the three Euler angles (for details see Ref. [1]). Next, for each scanned crystal cut, acousto-optic coupling coefficients of PIW (if it exists) are calculated using Eq. (9).

Two piezoelectrics are investigated in this way: lithium niobate (trigonal *3m* symmetry class) and quartz (trigonal *32* symmetry class). They differ considerably in acousto-optic properties. For both piezoelectrics, the scanning is performed in the ranges of $0° - 30°$, $0° - 180°$, and $0° - 180°$ (first, second, and third Euler angle). In the calculations, $\omega = 10^6 \mathrm{s}^{-1}$. The material constants are taken from Ref. [5], and the acousto-optic tensors are taken from Ref. [3].

Table 1. PIW parameters and acousto-optic coupling coefficients for lithium niobate (physical units: A=10^{-12}m/P$^{1/2}$ and B=Nm^{-2}/P$^{1/2}$, where P=10^{-3}Wm^{-1}). Notes. Row 1: $\kappa > 3$. Rows 2–4: $\alpha > 6$. Row 5: $\alpha_1 > 3$. Row 6: $\alpha_2 > 3$. Row 7: $\alpha_3 > 3$.

Euler angles deg			v_p m/s	ψ deg	κ %	$\tilde{u}_1$ A	$\tilde{u}_2$ A	$\tilde{u}_3$ A	$\tilde{T}_1$ B	$\tilde{T}_2$ B	$\tilde{T}_3$ B	α_1 %	α_2 %	α_3 %
30	92	164	4075	−2.7	3.50	−2.07	−69.9	−0.11	$-j11.0$	$j0.52$	$-j128.$	0.33	2.99	1.77
10	106	128	3917	−8.6	0.65	−15.3	25.0	29.8	$-j542.$	$j10.6$	$-j286.$	4.28	1.24	1.69
20	96	132	3952	−8.2	0.65	−12.1	34.6	20.7	$-j379.$	$j14.0$	$-j244.$	2.07	2.27	2.14
30	46	62	3982	−3.0	0.45	−14.8	2.88	−39.0	$j739.$	$-j13.5$	$-j281.$	0.41	0.62	4.99
20	100	132	3971	−5.9	0.60	−11.0	38.2	14.8	$-j291.$	$j5.49$	$-j231.$	3.36	2.03	1.93
30	90	164	4074	−2.7	3.45	−2.11	−70.2	0.00	$-j0.01$	$j0.00$	$-j129.$	0.34	3.02	1.79
20	44	70	3980	−3.8	0.45	−14.3	7.75	−38.9	$j740.$	$-j14.0$	$-j274.$	0.42	0.71	3.98

Table 2. PIW parameters and acousto-optic coupling coefficients for quartz (physical units: A=10^{-12} m/P$^{1/2}$ and B=Nm^{-2}/P$^{1/2}$, where P=10^{-3} Wm^{-1}). Notes. Rows 1–2: $\kappa > 0.04$. Rows 3–4: $\alpha > 0.03$.

Euler angles deg			v_p m/s	ψ deg	κ %	$\tilde{u}_1$ A	$\tilde{u}_2$ A	$\tilde{u}_3$ A	$\tilde{T}_1$ B	$\tilde{T}_2$ B	$\tilde{T}_3$ B	α_1 %	α_2 %	α_3 %
20	134	154	3388	−10	0.04	−0.95	−44.0	21.5	$-j193.$	$j40.8$	$j75.1$	0.00	0.01	0.00
30	90	160	3410	−10	0.05	0.00	0.00	47.3	$-j421.$	$j83.4$	$j0.00$	0.00	0.01	0.00
30	84	156	3362	−10	0.02	0.59	5.28	29.5	$-j263.$	$j47.9$	$-j3.29$	0.00	0.05	0.00
30	90	154	3341	−8.4	0.01	0.00	0.00	23.2	$-j205.$	$j28.9$	$j0.00$	0.00	0.05	0.00

The results are presented in Tables 1 and 2. The tables give PIW parameters and acousto-optic coupling coefficients for several crystal cuts selected from tens of thousands of cuts where PIW exists. Each cut is representative of a group of cuts (see the notes to the tables).

The following parameters are given: phase velocity $v_{\mathrm{p}} = 1/r_{\mathrm{p}}$, beam steering angle ψ, piezoelectric coupling coefficient κ, normalized complex amplitudes of u_i and T_i at the conducting plane, acousto-optic coupling coefficients.

5. Conclusion

In the angle space, domains of high acousto-optic coupling are different from domains of high piezoelectric coupling, as can be seen from Table 2. Nevertheless, the two kinds of domains overlap partially so that there are domains where both the couplings are relatively high. (Domains of high piezoelectric coupling for quartz can be seen in Refs. [6] and [7] in the form of maps.)

Acousto-optic coupling coefficients for lithium niobate, which can be as large as 5%, are greater that those for quartz by two orders of magnitude. It is interesting to note that, for lithium niobate, cuts of high piezoelectric coupling (rows 1 and 6 of Table 1) are characteristic of large α_2 which is the greatest of the three coefficients α_i. This is even more conspicuous in the case of quartz: α_1 and α_3 are less than 0.01% for every crystal cut (Table 2 gives just four examples).

In the case of lithium niobate, there is a great freedom of choosing such a crystal cut that one of the coefficients α_i is greater than the other two (Table 1: row 2 for α_1, rows 1 and 6 for α_2, rows 4 and 7 for α_3). It is seen no correlation between the coefficients α_i and the amplitudes $\tilde{u}_i$ and $\tilde{T}_i$.

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