

## ANALYSIS OF BLOOD PRESSURE WAVE IN THE HUMAN COMMON CAROTID ARTERY ON THE BASIS OF NON-INVASIVE ULTRASONIC EXAMINATIONS

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The aim of the study was the examination of the forward and reflected blood pressure waves in the common carotid artery on the basis of non-invasive ultrasonic examinations. The study concerned the effect of stenosis of the internal carotid artery caused by atherosclerosis on the mean reflection coefficient modulus and the time delay between the reflected blood pressure wave and the forward blood pressure wave. The investigations were carried out on a group of healthy persons (30 cases) and on a group of sick persons (17 cases) with stenosis or occlusion of the internal carotid artery.

**Keywords:** forward and reflected blood pressure waves, vascular input impedance, common carotid artery, ultrasound.

### 1. Introduction

The blood pressure wave propagating along the vascular tree is reflected, the main points of reflection being the places of stenosis, or bifurcations of arteries [5]. In the case when behind the point of measurements there is a series of blood pressure wave reflections at various points spaced from one another, we cannot practically separate from one another the individual reflected waves reaching the measurement point. Under such circumstances, the wave propagating in the direction opposite to the forward wave and being superposition of the reflected waves is considered reflected wave. The shape of the blood pressure wave observed in particular points along the vascular tree can be reconstructed by a sum of the cosine waves which amplitudes and phases are determined in frequency domain as the results from the Discrete Fourier Transform of the blood pressure wave. The relationship between the spectrum components of the total  $P$ , forward  $P_f$  and reflected  $P_r$  blood pressure waves for the successive harmonics  $n$  of the heartbeat frequency in frequency domain is as follows:

$$P_n = P_{fn} + P_{rn} = P_{fn}(1 + \Gamma_n), \quad (1)$$

where

$$P_n = |P_n|e^{j\theta_n}, \quad P_{fn} = |P_{fn}|e^{j\theta_{fn}}, \quad P_{rn} = |P_{rn}|e^{j\theta_{rn}}, \quad \Gamma_n = |\Gamma_n|e^{j\gamma_n}.$$

The reflection coefficient  $\Gamma_n$  present in formula (1) is calculated on the basis of the vascular input impedance  $Z_n$  and characteristic impedance  $Z_{on}$  measured in the chosen cross-section of the artery:

$$\Gamma_n = \frac{P_{rn}}{P_{fn}} = \frac{Z_n - Z_{on}}{Z_n + Z_{on}} = |\Gamma_n|e^{j\gamma_n}. \quad (2)$$

The input and characteristic impedances are defined in the frequency as:

$$Z_n = \frac{P_n}{Q_n} = |Z_n|e^{j\varphi_n}, \quad Z_{on} = \frac{P_{fn}}{Q_{fn}} = |Z_{on}|e^{j\phi_n}, \quad (3)$$

where  $P_n, P_{fn}$ , are spectrum components of the total and forward blood pressure waves,  $Q_n, Q_{fn}$  are spectrum components of the total and forward blood flow waves.

According to formula (1) the spectrum components  $P_{fn}$  and  $P_{rn}$  of the forward and reflected blood pressure waves are calculated for the successive harmonics  $n$  of the heart-beat frequency as follows:

$$P_{fn} = \frac{P_n}{(1 + \Gamma_n)} = |P_{fn}|e^{j\theta_{fn}}, \quad P_{rn} = P_n - P_{fn} = |P_{rn}|e^{j\theta_{rn}}. \quad (4)$$

The components  $P_{fn}$  and  $P_{rn}$  are the basis for determination of the course of the forward and reflected blood pressure waves in time domain.

The phenomenon of blood pressure wave reflection has been considered up to now only on the basis of the invasive measurements performed mainly on animals [17, 20]. In this study the forward and reflected blood pressure waves in the human common carotid artery was determined on the basis of ultrasonic non-invasive measurements of the blood pressure and volumetric blood flow. The aim of the study was to estimate the effect of the internal carotid artery stenosis caused by atherosclerosis on the mean reflection coefficient modulus and delay of the reflected wave relative to the forward wave. The time delay  $\Delta t$  was determined by the zero-crossing method between the rising slopes of the forward and reflected blood pressure waves at the mean blood pressure level  $P_a$  (Fig. 5 and 7). Moreover the time delay was also determined by the correlation method. The values of the mean reflection coefficient modulus  $|\Gamma|_a$  was calculated on the basis of the values of the moduli  $|P_{fn}|$  and  $|P_{rn}|$  of the spectrum components of the forward and reflected blood pressure waves for the first ten harmonics of the heartbeat frequency according to the formula:

$$|\Gamma|_a = \frac{\frac{1}{10} \sum_{n=1}^{10} |P_{rn}|}{\frac{1}{10} \sum_{n=1}^{10} |P_{fn}|}. \quad (5)$$

The mean reflection coefficient modulus thus determined is mainly dependent upon those harmonic components, which have major influence on the shape of the forward and reflected blood pressure waves.

## 2. Method and equipment

In the developed method the instantaneous values of blood pressure  $P(t)$  in the common carotid artery was determined on the basis of ultrasonic measurements of the instantaneous artery diameter  $D(t)$ . The function  $D(P)$  presented by POWAŁOWSKI and PEŃSKO [10] having the following form is assumed as the basis of calculations:

$$D(P) = D_{\min} \sqrt{1 + \frac{1}{\alpha} \ln \left( \frac{P}{P_d} \right)} \quad \text{for } P \geq \frac{P_d}{\exp(\alpha)} > 0, \quad (6)$$

where  $\alpha$  has been defined as a logarithmic artery wall rigidity coefficient and has the form expressed by the following formula:

$$\alpha = \frac{D_{\min}^2}{(D_{\max}^2 - D_{\min}^2)} \ln \left( \frac{P_s}{P_d} \right), \quad (7)$$

where  $D_{\min}$  and  $D_{\max}$  are minimum and maximum artery diameters corresponding to the diastolic blood pressure  $P_d$  and systolic blood pressure  $P_s$ , respectively. Upon transformation of formula (6) the instantaneous blood pressure  $P(t)$  is given as:

$$P(t) = P_d \exp \left[ \frac{D^2(t) - D_{\min}^2}{D_{\max}^2 - D_{\min}^2} \ln \left( \frac{P_s}{P_d} \right) \right]. \quad (8)$$

In the non-invasive examinations the blood pressure  $P(t)$  thus determined was calibrated by means of the systolic blood pressure and diastolic blood pressures, measured by means of a sphygmomanometer. In the case of the blood pressure  $P(t)$  determination in the carotid arteries, the blood pressure  $P_s$  and  $P_d$  measurements were performed on the brachial artery, when the patient was lying down. A right to use the function described by formula (6) has been confirmed for a group consisting of 20 persons by POWAŁOWSKI *et al.* [13, 14] for the common carotid artery for different values of the systolic and diastolic blood pressures. The results of the investigations of the relationship between the diameter of the common carotid artery and the variations of the systolic and diastolic blood pressure in the brachial artery obtained by the above mentioned authors are presented for the three chosen persons in Fig. 1.

The relationship between the common carotid artery diameter and the blood pressure was also investigated in the conditions of the dynamic blood pressure variations. Investigations were carried out in the common carotid artery of a dog<sup>(1)</sup>. The instantaneous blood pressure and the instantaneous artery diameter were measured simultaneously in the same artery cross-section. The blood pressure was measured invasively by means of 1 mm pressure catheter developed by Sentron. The artery diameter was examined non-invasively using VED ultrasonic equipment described in the further part of this paper. The results of the examinations in the common carotid artery of a dog (Fig. 2) have shown that relationship (8) describes perfectly well the blood pressure variations. The coefficient

<sup>(1)</sup> Investigations were performed at the University in Maastricht (the Netherlands) within research project PL 92.0907 financed by the European Commission.

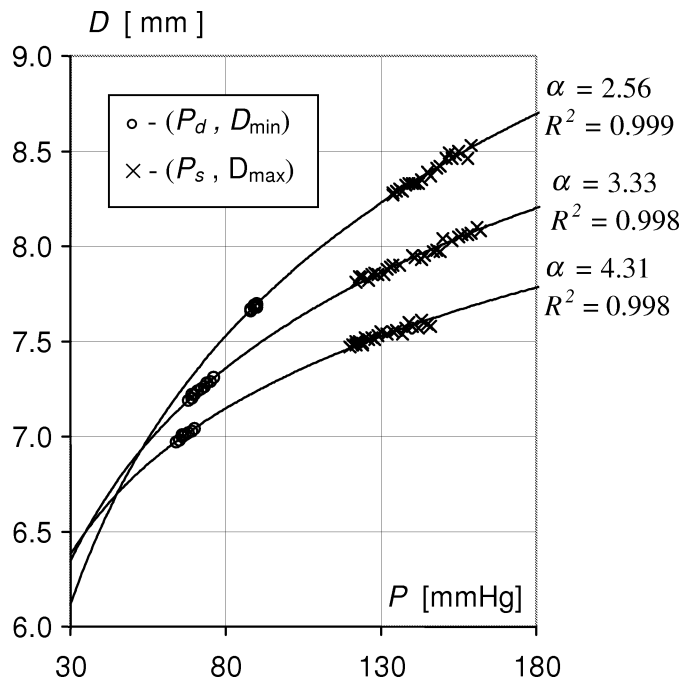


Fig. 1. Results of the measurements [13] of the relationship between diameter  $D$  (minimum  $D_{\min}$  and maximum  $D_{\max}$  diameter) of the common carotid artery and the blood pressure  $P$  (diastolic  $P_d$  and systolic  $P_s$  pressure) on the brachial artery for three patients aged 36, 38, and 42 years, for whom the mean value of  $\alpha$  coefficient as calculated from formula (7), was 3.33, 2.56 and 4.31, respectively. Solid line presents the function  $D(P)$  determined from formula (6). The function  $D(P)$  was plotted by means of the least squares method. Conformity of description of the experimental points by the assumed function  $D(P)$  has been expressed by the coefficient of determination  $R^2$ .

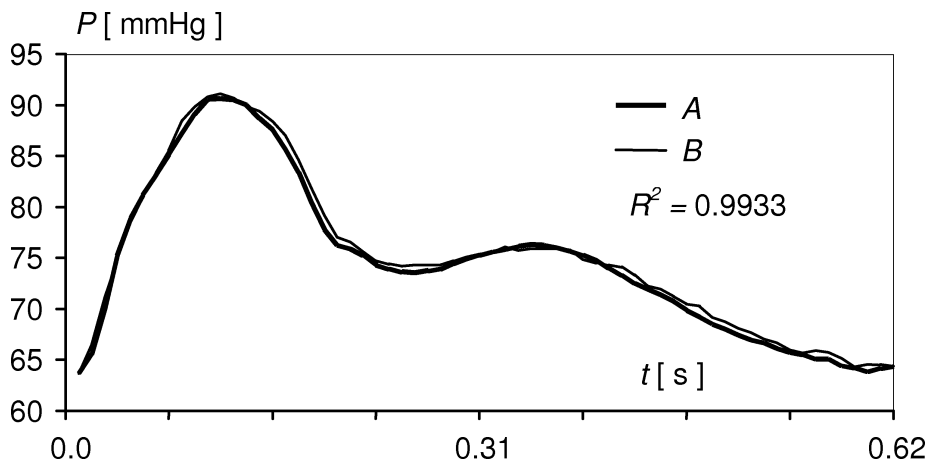


Fig. 2. Blood pressure  $P$  averaged from the consecutive ten cardiac cycles in the common carotid artery of a dog: (A) — measured, (B) — calculated on the basis of artery diameter variations, according to relationship (8).

of determination  $R^2$  between the blood pressure measured in the invasive way and that calculated from formula (8) was 0.9933. Moreover, the obtained results of investigations have shown that the effect connected with the viscous properties of the artery wall in the relationship between the artery diameter variations and blood pressure variations may be neglected. The phase shift in the relationship  $D(P)$  did not exceed  $9^\circ$  for the first five harmonic components of the instantaneous blood pressure  $P(t)$ . This has confirmed the earlier works done by BERGEL [1, 2], LEAROYD *et al.* [8], GOW *et al.* [6, 7] and WESTERHOF *et al.* [19]. The above mentioned authors agree that the phase shift between the artery diameter variations and blood pressure variations does not exceed  $10^0$ .

Non-invasive blood pressure measurements, together with non-invasive ultrasonic measurements of the volumetric blood flow were basis for determination of the vascular input impedance [12].

Besides the input impedance, subsequent magnitude necessary for determination of the blood pressure wave reflection coefficient is the characteristic impedance. In this study the characteristic impedance was determined on the basis of the formula given by WOMERSLEY [21]. Womersley has presented a relationship, which describes characteristic impedance of the artery, based on the assumptions that artery with ideally elastic wall is contracted and does not move in the longitudinal direction and that blood can be treated as viscous Newtonian liquid:

$$Z_{on} = \frac{\rho c}{\pi R^2 \sqrt{(1 - \sigma^2) M'_{10n}}} e^{-j \frac{\varepsilon_{10n}}{2}}, \quad (9)$$

where  $\rho$  is the blood density,  $R$  is the artery radius,  $\sigma$  is the Poisson constant,  $M'_{10n}$ , and  $\varepsilon_{10n}$  are values, which are functions of the artery radius, blood viscosity and harmonics  $n$  of the heartbeat frequency,  $c$  is the pulse wave velocity.

Values  $M'_{10n}$  and  $\varepsilon_{10n}$  are given in tables [9]. Pulse wave velocity  $c$  given in formula (9) has been defined on the basis of the Moens–Korteweg equation:

$$c = \sqrt{\frac{Eh}{2R\rho}}, \quad (10)$$

where  $E$  is the Young's modulus of the artery wall,  $h$  is the artery wall thickness,  $\rho$  is the blood density,  $R$  is the mean artery radius.

Moens–Korteweg formula has been derived for an extremely thin-walled tube for which the condition  $h/R \ll 1$  is satisfied. In order to eliminate difficulties connected with the measurement of the artery wall thickness and Young's modulus, BRAMWELL and HILL [3] in 1922 proposed the following relationship describing the value of the pulse wave velocity  $c$ :

$$c = \sqrt{\frac{V dP}{\rho dV}}, \quad (11)$$

where  $\rho$  is the blood density,  $dP$  is the blood pressure variation producing relative variation of the artery volume  $dV/V$ .

Having assumed that the length of the artery has not changed due to blood pressure variations, we can obtain formula (11) in the following form:

$$c = \sqrt{\frac{S dP}{\rho dS}}, \tag{12}$$

where  $dP$  is the blood pressure variation, which causes relative variation of the cross-section area of the artery  $dS/S$ .

The formula given above has been confirmed theoretically in the paper published by TEDGUI *et al.* [18] for an incompressible Newtonian liquid flowing in an elastic tube. In the paper cited above it has been proved that the pulse wave velocity described by formula (12) corresponds to the pulse wave velocity as calculated on the basis of the Moens–Korteweg formula. According to formula (12), in the non-invasive ultrasonic investigations, the pulse wave velocity has been calculated from the following relationship:

$$c = \sqrt{\frac{S \Delta P}{\rho \Delta S}} = \sqrt{\frac{(P_s - P_d) D_{\min}^2}{\rho (D_{\max}^2 - D_{\min}^2)}}, \tag{13}$$

where  $D_{\max}$  and  $D_{\min}$  are maximum and minimum internal artery diameter with the systolic  $P_s$  and diastolic  $P_d$  blood pressures being subordinated to these values, respectively.

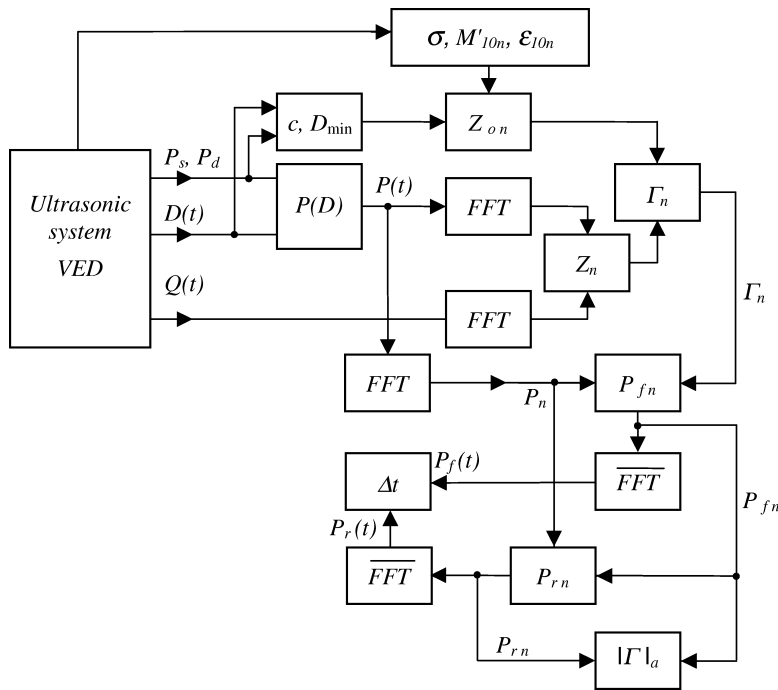


Fig. 3. Flow chart of determination and analysis of the forward  $P_f$  and reflected  $P_r$  blood pressure waves according to the formulae: (8), (4), (2), (3), (9), (13) and (5); FFT — Fast Fourier Transform,  $\overline{FFT}$  — inverse Fast Fourier Transform,  $|\Gamma|_a$  — mean reflection coefficient modulus,  $\Delta t$  — time delay of the reflected blood pressure wave relative to the forward blood pressure wave.

The forward and reflected blood pressure waves were determined in conformity with formulae: (8), (4), (2), (3), (9) and (13) on the basis of the simultaneous measurements of the instantaneous values of the volumetric blood flow  $Q(t)$  and artery diameter  $D(t)$  in the same cross-section area of the common carotid artery and on the basis of the measurement of the systolic blood pressure  $P_s$  and diastolic blood pressure  $P_d$  on the brachial artery by means of sphygmomanometer. The flow chart of determination and analysis of the forward and reflected pressure waves is given in Fig. 3. The investigations were carried out by means of VED ultrasonic equipment developed at the Institute of Fundamental Technological Research of the Polish Academy of Sciences. This equipment consists of a continuous wave Doppler flowmeter with a two-channel 128 point FFT Doppler signal analyser and a pulse wall tracking system [11]. The frequency of the ultrasonic wave transmitted in the Doppler flowmeter was 4.5 MHz, and in the tracking system — 6.75 MHz. The longitudinal resolution at the artery diameter measurements as determined on the basis of the model investigations was  $< 0.33$  mm. Measuring accuracy of artery wall displacements was  $7\ \mu\text{m}$ . Measuring data were presented during the investigations on the screen of an IBM PC connected on line with an ultrasonic equipment and were stored in the computer memory (Fig. 4). Apart from the data obtained from ultrasonic measurements, also the values of the systolic and the diastolic blood pressures were transmitted to the computer memory. The forward and reflected blood pressure waves were determined using 128 point Fast Fourier Transform (FFT).

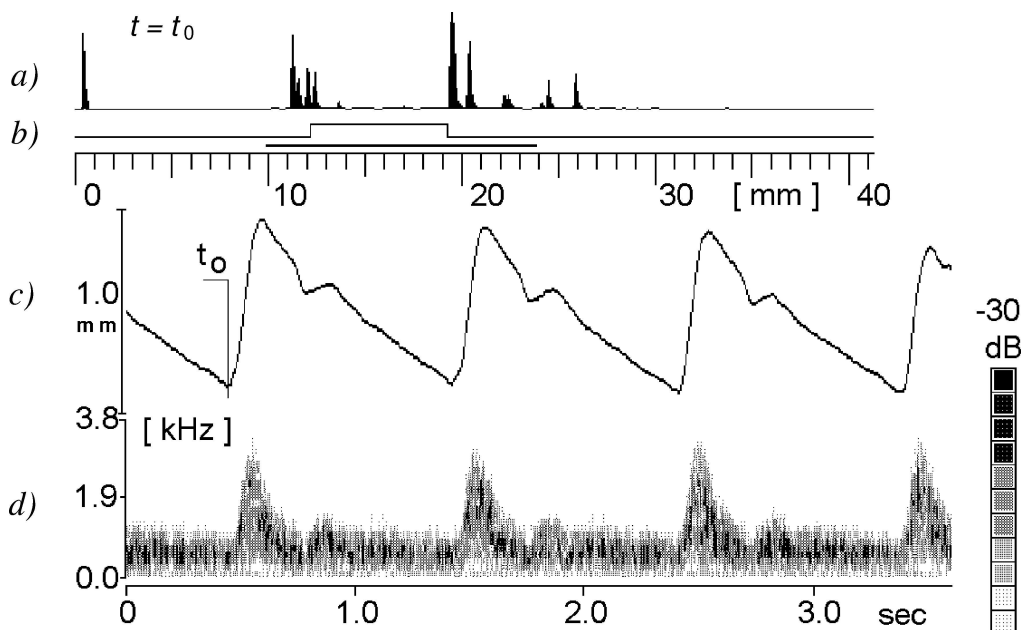


Fig. 4. Data presented in the course of the measurements in the human common carotid artery: a) echoes from artery wall surface, b) gate presenting internal artery diameter, c) relative artery diameter variation, d) power density spectrum of the Doppler signal;  $t_0$  is a moment of taking of the picture of the recorded echoes.

### 3. Results

The examinations performed in the common carotid artery were preceded by an estimate of influence of stenosis of the brachial artery caused by compression on the values of the mean reflection coefficient modulus and the time delay of the reflected blood pressure wave with respect to the forward blood pressure wave. (POWAŁOWSKI *et al.* [15]). The measurements were performed on a male 22 years old. The results of examinations are presented in Fig. 5. The measuring point was located 56 cm from fingertips. An apparent blood pressure wave reflection point as determined on the basis of the pulse wave velocity (formula (13)) and the time delay  $\Delta t$  between the reflected and forward waves was spaced by 60 cm from the measuring point. After compressing the artery at a distance of 12 cm from the measuring point, the time delay  $\Delta t$  has been reduced from 132 ms to 35 ms and the value of the mean reflection coefficient modulus  $|\Gamma|_a$  has increased from 0.4398 to 0.7983. Compressing of the brachial artery brought a 60% reduction in the volumetric blood flow.

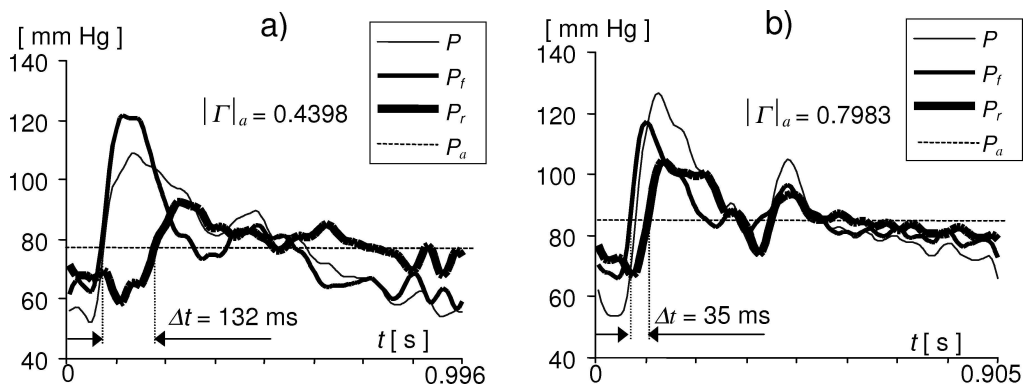


Fig. 5. Blood pressure waves: total  $P$ , forward  $P_f$  and reflected  $P_r$ , determined on the basis of the brachial artery measurements: a) without artery compression, b) with artery compression at a point lying 12 cm distal the measurement point;  $P_a$  is the mean blood pressure.

The clinical examinations (POWAŁOWSKI *et al.* [16]) were performed at the Department of the General and Thoracic Surgery of the Medical Academy in Warsaw on a control group of healthy persons without any atherosclerotic lesions in the carotid arteries and on a group of sick persons with atherosclerotic lesions in the initial segment of the internal carotid artery (Fig. 6). The examinations were carried out in the common carotid artery at a distance of 3–4 cm proximal to the bifurcation of the artery. The ultrasonic examinations (B-mode+Doppler) did not show any atherosclerotic plaque in the common carotid artery where the measurements were taken. The measurements were done while the subjects were lying down, following 15 minute rest periods. The stenosis range of the internal carotid artery was classified on the basis of the combined ultrasonographic and Doppler measurements. Generally accepted criteria were used (DE BRAY and GLATT [4]). The results of the blood flow and blood pressure measurements have been averaged for four cardiac cycles. In Fig. 7 are presented the results of the chosen blood



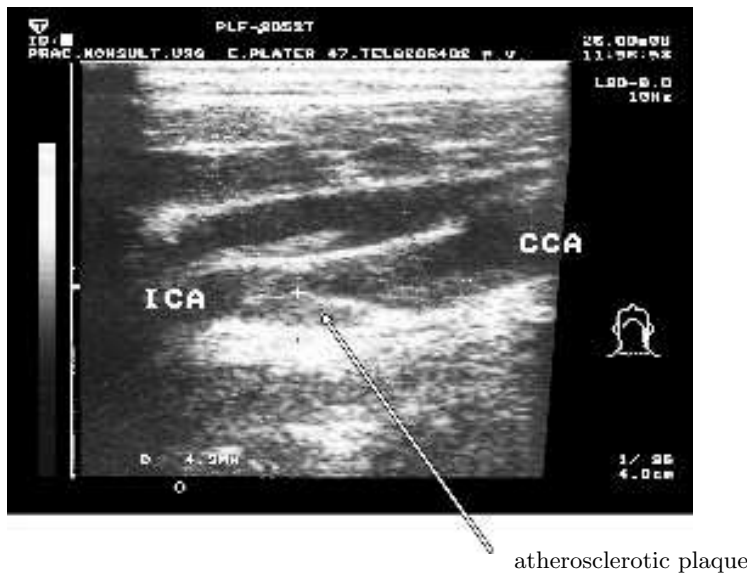


Fig. 6. Ultrasonic B-mode image of the common carotid artery (CCA) bifurcation in the case of a person with atherosclerosis of the internal carotid artery (ICA).

pressure  $P$  and blood flow  $Q$  measurements, the input and characteristic impedance, as well as the forward and reflected blood pressure wave for the healthy person and for the patients with 50% and 70% stenosis of the internal carotid artery. It may be seen that in the case of persons with stenosis of the internal carotid artery there is a visible increase in the input impedance modulus  $|Z_n|$  of with respect to the characteristic impedance modulus  $|Z_{0n}|$ , coupled with an increase of the reflected wave amplitude and shortening of the time delay  $\Delta t$  of the reflected blood pressure wave  $P_r$  with respect to the forward blood pressure wave  $P_f$ .

The total results of examinations have been given in Table 1. Amplitude and time delay of the reflected blood pressure wave measured in the common carotid artery were obtained as a sum of the waves reflected from various points of vascular system fed by the common carotid artery. The results of the examinations have shown that an increase of the degree of stenosis of the internal carotid artery is also accompanied by an increase in the value of the mean reflection coefficient modulus and decrease in the time delay of the reflected blood pressure wave relative to the forward blood pressure wave. In the case of the persons with a critical stenosis, or occlusion of the internal carotid artery, the mean reflection coefficient modulus was greater by about 48% and the apparent reflection point as determined on the basis of the time delay and the pulse wave velocity (formula (13)) was situated at distance  $\Delta L$  relative to the measuring point about 4.4 times shorter than in the case of the healthy persons.

The time delay  $\Delta t$  of the reflected blood pressure wave relative to the forward blood pressure wave obtained by the zero crossing method was compared with the time delay obtained by the correlation method. In order to enhance resolution of the correlation

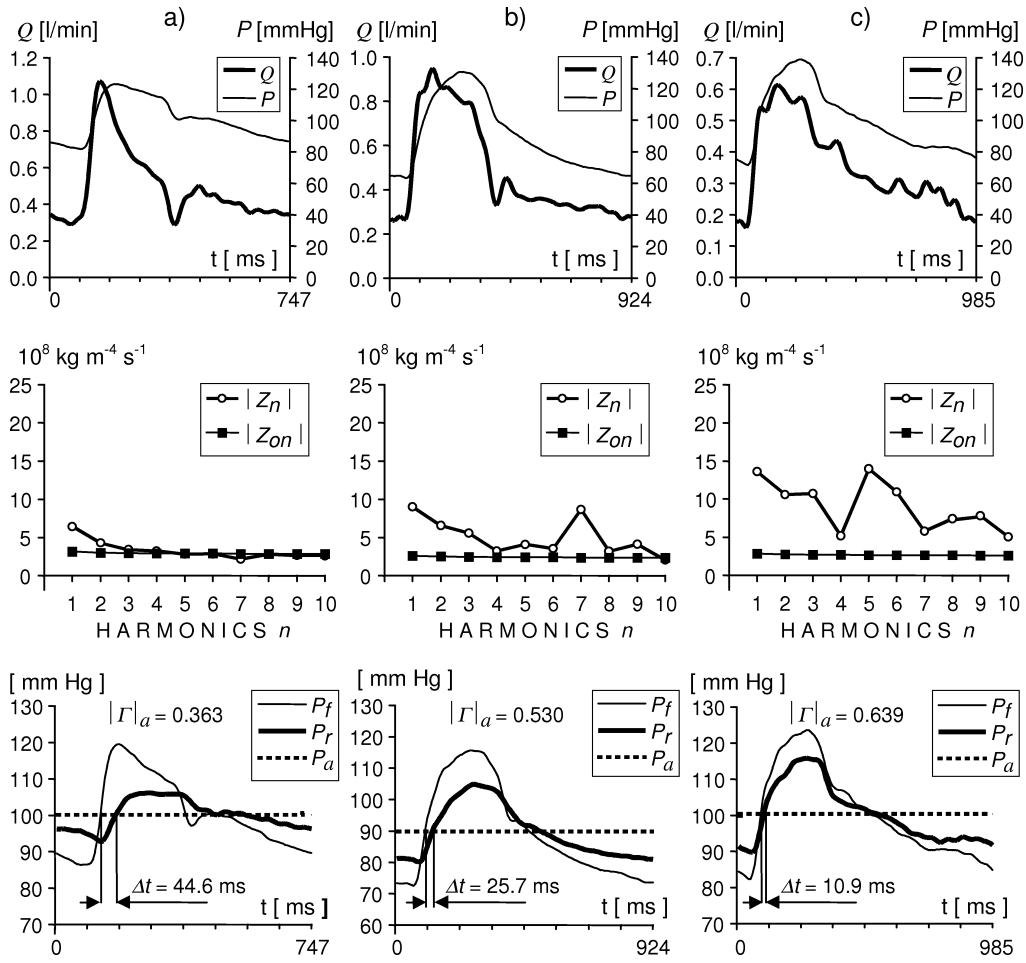


Fig. 7. Results of examination in the common carotid artery in the case of the healthy person (a), the person with 50% stenosis of the internal carotid artery (b) and the person with 70% stenosis of the internal carotid artery (c).

method, the time course of the forward and reflected blood pressure waves for the given cardiac cycle was divided into  $2^{12}$  samples having assumed linear approximation between the primary samples ( $2^7$  samples) of both waves. Average difference of the values of the time delay  $\Delta t$  following from the above methods of  $\Delta t$  determination was 2%, for A, C and D groups of the persons being examined given in Table 1, and 10% — for group B.

In this paper the propagation of the forward wave and the wave reflected between two measuring points situated at a known distance from each other was also considered. The distance between the measuring points was determined on the basis of the time delay difference at both measuring points and on the basis of pulse wave velocity, according to the following relationship:

$$\Delta L^* = \frac{c(\Delta t_1 - \Delta t_2)}{2}, \quad (14)$$

where  $\Delta t_1$  and  $\Delta t_2$  are time delays between the reflected wave and the forward wave at a measuring point situated at a greater and smaller distance from the wave reflection point respectively,  $c$  is the pulse wave velocity in the distance between the measuring points.

**Table 1.** Results of measurements in the common carotid artery for a control group of healthy persons (A) and the persons with stenosis (B – D) of the internal carotid artery (ICA).  $P_s$ ,  $P_d$  are systolic and diastolic blood pressures on the brachial artery,  $D_{\min}$  is minimum internal diameter,  $Q_{\text{med}}$  is the mean volumetric blood flow,  $c$  is the pulse wave velocity,  $|I|_a$  is the mean reflection coefficient modulus,  $\Delta t$  is the time delay of the reflected blood pressure wave relative to the forward blood pressure wave determined by zero-crossing method,  $\Delta L$  is the distance between the measuring point and the apparent blood pressure wave reflection point,  $\Delta t^*$  is the time delay of the reflected blood pressure wave relative to the forward blood pressure wave determined by the correlation method.

Examined group	(A) Control	(B) ICA stenosis 50%	(C) ICA stenosis 50 – 70%	(D) ICA stenosis > 90% or occlusion
Number of cases	30	5	6	6
Age [years]	48.3 ± 14.2	59.0 ± 12.5	62.3 ± 10.0	64.8 ± 7.4
$P_s$ [mmHg]	119.7 ± 11.2	138.4 ± 7.4	158.7 ± 24.9	142.5 ± 19.4
$P_d$ [mmHg]	74.1 ± 8.6	79.6 ± 9.5	83.5 ± 11.1	75.8 ± 9.4
$D_{\min}$ [mm]	7.182 ± 1.015	8.294 ± 0.733	8.728 ± 0.750	9.528 ± 0.780
$Q_{\text{med}}$ [l/min]	0.605 ± 0.113	0.589 ± 0.082	0.595 ± 0.153	0.416 ± 0.094
$c$ [m/s]	6.80 ± 1.54	8.70 ± 1.96	8.65 ± 2.43	8.94 ± 1.58
$ I _a$	0.448 ± 0.048	0.527 ± 0.024	0.603 ± 0.083	0.661 ± 0.060
$\Delta t$ [ms]	52.7 ± 13.4	27.0 ± 6.1	22.1 ± 10.6	9.1 ± 4.8
$\Delta L$ [cm]	17.6 ± 5.5	11.4 ± 1.2	9.0 ± 3.9	4.0 ± 2.0
$\Delta t^*$ [ms]	53.8 ± 13.9	24.6 ± 5.7	21.7 ± 9.3	9.0 ± 4.6

The examinations were carried out in the human common carotid artery and on a model<sup>(2)</sup> in elastic tube ( $\sigma = 0.5$ ) of inside diameter 19.8 mm, through which liquid of viscosity  $\eta$  and density  $\rho$  similar to those of blood ( $\eta = 3.3 \cdot 10^{-2}$  P,  $\rho = 1100$  kg/m<sup>3</sup>) flowed. The obtained results are shown in Table 2. It may be seen that the distance between the measuring points as determined from the time delay difference and the pulse wave velocity differs only slightly from the existing distance between the measuring points. This difference was equal to 5% for a segment of the common carotid artery 3 cm long and 1% for a segment of elastic tube 25 cm long.

**Table 2.** Distance between the measuring points: measured ( $\Delta L$ ) and determined on the basis of the pulse wave velocity and on the basis of the time delay difference at two measuring points ( $\Delta L^*$ ) (formula (14)).

	$\Delta L$	$\Delta L^*$
Elastic tube	25.0 cm	24.75 cm
Common carotid artery	3.0 cm	2.86 cm

The influence of atherosclerosis in the internal carotid artery upon the phenomenon of blood pressure wave reflection has also been considered theoretically, by calculation

<sup>(2)</sup> The model investigations were carried out at the Department of Mechanical Engineering, Technical University of Eindhoven [15].

of the value of the reflection coefficient in the common carotid artery for the case of the normal internal and external carotid arteries, as well as for the case of an occlusion of the internal carotid artery. In the calculations the results of non-invasive blood pressure and volumetric blood flow measurements in the internal and external carotid arteries for a healthy male aged 38 were applied. The measurements were carried out close to the bifurcation of the carotid artery. Neglecting the distance of the measuring points from the bifurcation of the common carotid artery, the time course of the blood flow velocity in the common carotid artery at the point of bifurcation was calculated as a sum of the time courses of the blood flows in the external and the internal carotid arteries. Moreover, blood pressure has been determined as an arithmetic mean of the blood pressures in the external and the internal carotid arteries. The obtained results of measurements were basis for the determination of the reflection coefficients in the three above mentioned carotid arteries at the point of bifurcation of the common carotid artery. Reflection coefficient in the common carotid artery was determined from the successive harmonic  $n$  of the heartbeat frequency from the following relationship:

a) for the case of the normal (without atherosclerotic plaques) internal and external carotid arteries:

$$\Gamma_{cn} = \frac{Z_{cn} - Z_{con}}{Z_{cn} + Z_{con}} = \frac{\frac{Z_{en}Z_{in}}{Z_{en} + Z_{in}} - Z_{con}}{\frac{Z_{en}Z_{in}}{Z_{en} + Z_{in}} + Z_{con}}$$

$$= \frac{\frac{1}{Z_{con}} - \left[ \frac{1}{Z_{eon}} \left( \frac{1 - \Gamma_{en}}{1 + \Gamma_{en}} \right) + \frac{1}{Z_{ion}} \left( \frac{1 - \Gamma_{in}}{1 + \Gamma_{in}} \right) \right]}{\frac{1}{Z_{con}} + \left[ \frac{1}{Z_{eon}} \left( \frac{1 - \Gamma_{en}}{1 + \Gamma_{en}} \right) + \frac{1}{Z_{ion}} \left( \frac{1 - \Gamma_{in}}{1 + \Gamma_{in}} \right) \right]}. \quad (15)$$

b) for the case of the internal carotid artery occlusion:

$$\Gamma_{cn} = \frac{Z_{cn} - Z_{con}}{Z_{cn} + Z_{con}} = \frac{Z_{en} - Z_{con}}{Z_{en} + Z_{con}} = \frac{\frac{1}{Z_{con}} - \frac{1}{Z_{eon}} \left( \frac{1 - \Gamma_{en}}{1 + \Gamma_{en}} \right)}{\frac{1}{Z_{con}} + \frac{1}{Z_{eon}} \left( \frac{1 - \Gamma_{en}}{1 + \Gamma_{en}} \right)}, \quad (16)$$

where  $Z_{cn}$ ,  $Z_{en}$ ,  $Z_{in}$  are the input impedances,  $Z_{con}$ ,  $Z_{eon}$ ,  $Z_{ion}$  are the characteristic impedances,  $\Gamma_{cn}$ ,  $\Gamma_{en}$ ,  $\Gamma_{in}$  are the reflection coefficients in carotid arteries: common, external and internal respectively.

In the case of the normal carotid arteries it has been assumed at the determination of the reflection coefficient  $\Gamma_{cn}$  in the common carotid artery that the characteristic admittance in the common carotid artery at the point of bifurcation is equal to the sum of the characteristic admittances in the external and internal carotid arteries. The reflection coefficient  $\Gamma_{cn}$  calculated from formulae (15) and (16) was basis for determination of the forward and reflected blood pressure waves in the common carotid artery. The mean modulus  $|\Gamma|_a$  of the reflection coefficient was calculated on the basis of forward wave modulus and reflected wave modulus according to the formula (5). The results of calculations of the mean modulus  $|\Gamma|_a$  of the reflection coefficient were presented in Table 3.

The performed calculations have confirmed the phenomenon of an increase of the value of mean modulus  $|I|_a$  for the case of the occluded internal carotid artery (Table 1).

**Table 3.** Mean modulus  $|I|_a$  of the reflection coefficient calculated on the basis of the non-invasive blood pressure and blood flow velocity measurements in the common carotid artery (CCA), external carotid artery (ECA) and internal carotid artery (ICA) and on the basis of a model of bifurcation of the common carotid artery (\*) described by the formulae (15) and (16).

Artery	$ I _a$
ECA	0.444
ICA	0.540
CCA (for normal ECA and ICA)	0.471, (0.490*)
CCA (for occluded ICA+ normal ECA)	0.663*

#### 4. Conclusions

The results of examinations obtained in the human common carotid artery indicate that the stenosis of the internal carotid artery caused by atherosclerosis was the source of the reflected blood pressure wave. This is expressed by an increase in value of the mean reflection coefficient and a decrease in the time delay between the reflected and forward blood pressure wave accompanying the degree of stenosis of the common carotid artery. These observations indicate that the proposed method of investigation of the forward and reflected blood pressure waves may be in the future a new diagnostic tool useful for the detection of atherosclerotic lesions in the arteries.

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