ARCHIVES OF ACOUSTICS
25, 2, 157–174 (2000)

ANALYSIS OF A RESONATOR WITH A DIRECTIONAL ULTRASONIC VIBRATION CONVERTER OF R–L TYPE USING THE FINITE ELEMENTS METHOD

T. GUDRA

Wrocław University of Technology
Institute of Telecommunication and Acoustics
(50-370 Wrocław, Wybrzeże Wyspiańskiego 27, Poland)
e-mail: gudra@zakus.ita.pwr.wroc.pl

Some results of the analysis of the R–L type resonator with directional converter using the finite elements method (FEM) are presented in the paper. Three types of resonator tuning result from the analysis and they can be used for converters of other type. The modal patterns in anti-phase and in-phase vibration modes for all types of tuning are presented. The choice of particular tuning depends on the way of using the converter. The modulus of the relation between the surface displacement amplitude for a longitudinal vibrating resonator and the side displacement amplitude for a radial vibrating resonator can be interpreted as the gain coefficient of converter's vibration amplitude. Moreover the influence of rode and disk resonator dimensions on proper frequencies of the converter vibration is described.

Keywords: resonator, directional converter, ultrasonic vibration.

1. Introduction

In ultrasonic technology piezomagnetic and piezoelectric transducers are generally applied to obtain high amplitude vibrations. The maximum value of vibration amplitude are limited by the magnetostrictic effect of nonlinearity for a piezomagnetic transducer and by the low mechanical strength of piezoceramic material for a piezoelectric transducer. One method of increasing the vibration amplitude upper limit consists in the use of a radiating source of special design, proposed by K. Itoh and E. Mori [3–6, 8, 9]. Four resonator types with a directional converter have been proposed by those authors: L–L type, L–L–L type, R–L type and L–R type (L — longitudinal, R — radial vibration). In these resonators the ultrasonic energy can be transmitted from the direction of driver to the other and can be concentrated or divided into plural loads from one vibration source.

The goal of this work was an investigation of the manner of the resonator with a directional converter of the R–L type. The finite elements method (FEM) was applied to analyse vibrations. This method enables the application of any dividing density for elements in the real elastic continuum, which allows us to observe local disturbance effects in any chosen fragments of the area; it is not easy to obtain by using the classical method. The latest literature concerning the usage of this type of resonators (including [12–15]) points to the fact that despite the time that has passed since their invention, they are still technologically attractive.

The goal of this work was also to verify the usefulness of the FEM for the design and optimization of this type of resonators.

2. Structure of the R–L type converter

The resonator with directional R–L conversion consists of a radial vibration disk and a longitudinal vibrating rod. The rod and the disk are coupled together at the velocity node of radial vibration for the disk and that of longitudinal vibration for the rod as shown in Fig. 1. With the suitable choice of dimensions, such a system permits comparatively large amplitudes of rod surface vibrations to be obtained. An energy transmission from the disk to the rod occurs due to the Poisson effect in the mechanically coupled common part of the converter. When the free vibration frequencies of individual resonators are different from each other, each of them vibrates as a free system, whereas when these frequencies are close to one another, there is an interaction between the two resonators.

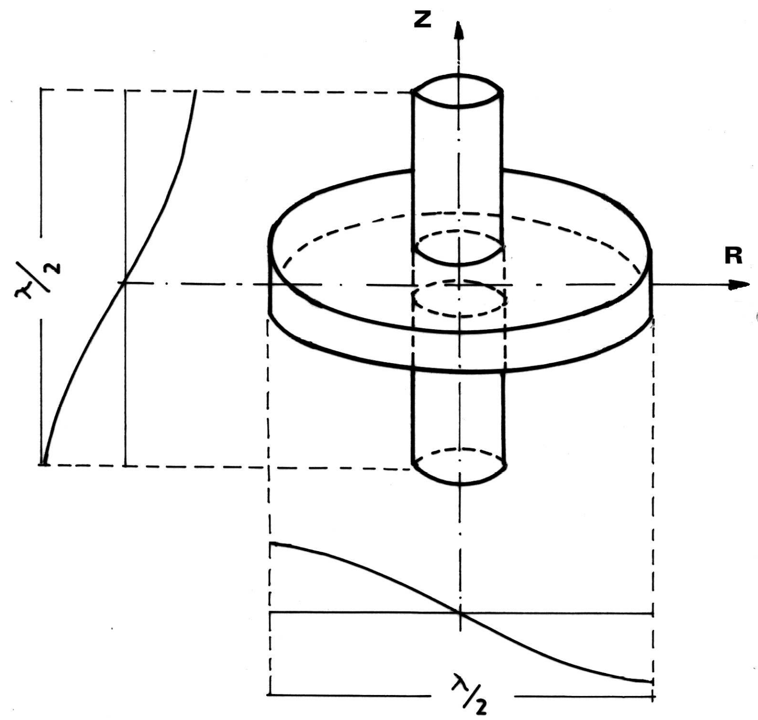

Fig. 1. Resonator with directional converter of the R–L type.

This system has two resonance frequencies depending on the vibration phase of the rod and the disk. In-phase vibrations occur when the ends of the rod and the side surface of the disk vibrate in the phase, whereas for anti-phase vibrations end surfaces of the rod vibrate in the opposite phase in relation to the side surface of the disk.

The resonators with directional converters can be constructed as homogeneous and heterogeneous systems. The homogeneous converters may be excited to vibration by external ultrasonic transducers [11 – 15], in heterogeneous converters the resonator of longitudinal or radial vibration can be a properly vibrating piezoelectric [1, 2] or piezomagnetic [7] ultrasonic transducer.

3. Geometry of the system

The resonator with directional conversion of vibration of the R – L type is a system with an axial symmetric stresses distribution. From the mathematical point of view the problem is similar to the two-dimensional problem. Because of the symmetry of the system in each point of the cross-section led along the symmetry axis the displacement is defined by two components. If we mark the radial co-ordinate of the point with R and the axial coordinate of the point with Z, u and v will be displacements related to these coordinates then we obtain the two-dimensional case. A chosen element, turning around the axis determines the volume with reference to which all calculations should be done.

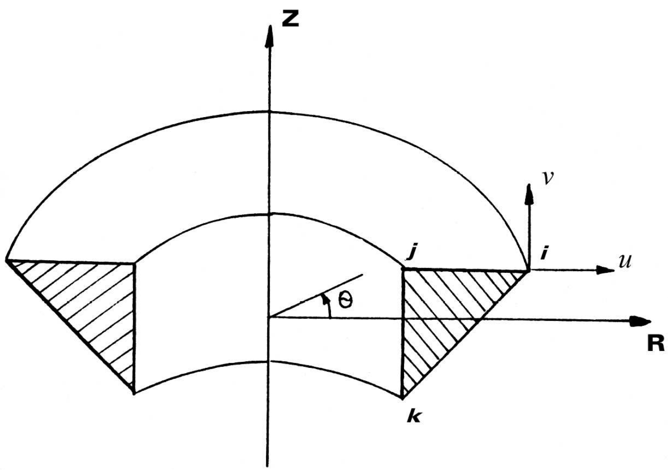

Fig. 2. Triangular ring shaped element.

For a triangular element, as shown in Fig. 2, the nodes are defined as i, j, k. The displacement δ of the i-node is defined by its two components:

$$\{\delta_i\} = \left\{ \begin{array}{c} u_i \\ v_i \end{array} \right\}, \tag{1}$$

so the displacement of an element is defined by the vector

$$\{\delta\}^e = \begin{Bmatrix} \delta_i \\ \delta_j \\ \delta_k \end{Bmatrix}. \tag{2}$$

Taking into account the relations characteristic of the FEM and solving the move equation for an elastic system [16] we can calculate free vibration frequencies of the system and node displacements of triangular elements the investigated resonator was divided into.

4. R–L type homogeneous converter

The R – L type homogeneous converter made of steel presented in Fig. 3 was subject to an analysis. Dimensions of this converter were such so as to be matched with experimental results obtained by other authors [3, 6].

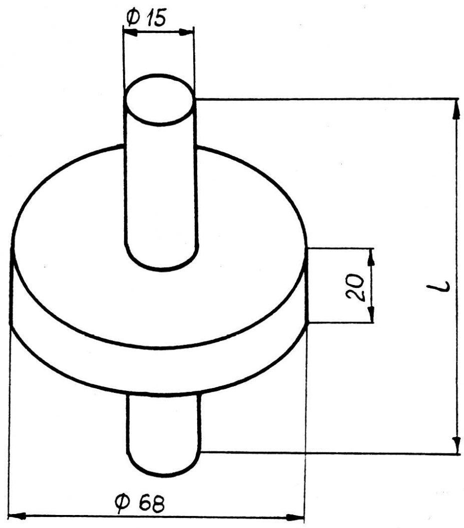

Fig. 3. Dimensions of homogeneous resonator with the R – L type conversion which are the grounds for analysis.

For calculations it has been assumed that the converter is made of steel whose material constants are:

$$\begin{aligned} E &= 0.206 \cdot 10^{12}\,[\mathrm{Pa}], \\ \nu &= 0.283, \\ \rho &= 7700\,[\mathrm{kg/m^3}], \end{aligned}$$

where E — Young modulus, ν — Poisson's ratio, ρ — density.

The division of the converter into elements is represented in Fig. 4. In consideration of the symmetry of the system, one fourth of the cross-sectional area was divided into 28 elements. The dependence of the converter's free vibration frequency on the length of a rod vibrating in the longitudinal mode is presented in Fig. 5. This dependence is illustrated by two curves corresponding to the frequencies of two modes of vibration: f_s — frequency for the in-phase (synphase) mode, f_a — frequency for the anti-phase mode.

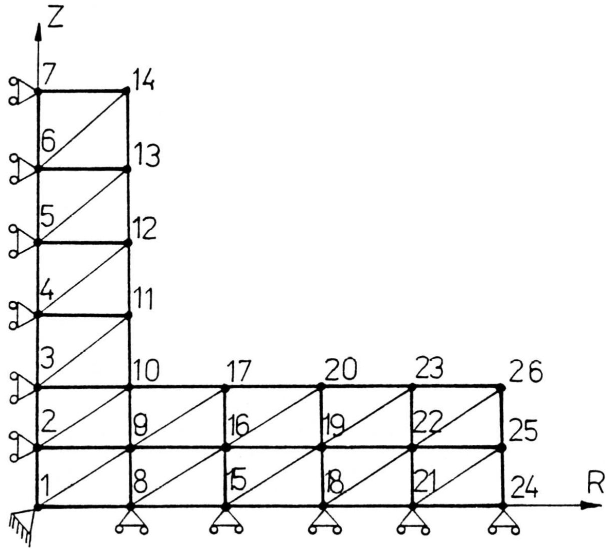

Fig. 4. Division of the R–L type converter into elements. Number of the elements – 28; Number of the nodes – 26; Number of movable degrees of freedom for direction R – 5; Number of movable degrees of freedom for direction Z – 6; Number of movable degrees of freedom for direction R and Z – 14; Number of immovable degrees of freedom – 1.

The distance between the curves for particular lengths of the rod resonator is related to the frequency difference $\Delta f = f_s - f_a$. The characteristic curve for $\Delta f = f(l_{\mathrm{rod}})$ is presented in Fig. 6.

Numbers on particular points on the curve in Fig. 5 are the calculated modules of the relation between the surface displacement amplitude for a longitudinal vibration resonator and the side horizontal displacement amplitude for a radial vibration resonator. This relation can be interpreted as the gain coefficient of converter's vibration amplitude. This is of a great practical value since it allows the application of this type of resonator to increase the amplitude of vibrations. For the minimum value of $(f_s - f_a)$ the value of this relation stays the same for both in-phase mode and anti-phase mode vibrations.

For $\min(f_s - f_a)$ we get

$$\left|\frac{\delta_{Z_7}}{\delta_{R_{24}}}\right|\Bigg|_{f_a} = \left|\frac{\delta_{Z_7}}{\delta_{R_{24}}}\right|\Bigg|_{f_s}, \tag{3}$$

where δ_{Z_7} — displacement of the node No 7 towards Z, $\delta_{R_{24}}$ — displacement of the node No 24 towards R.

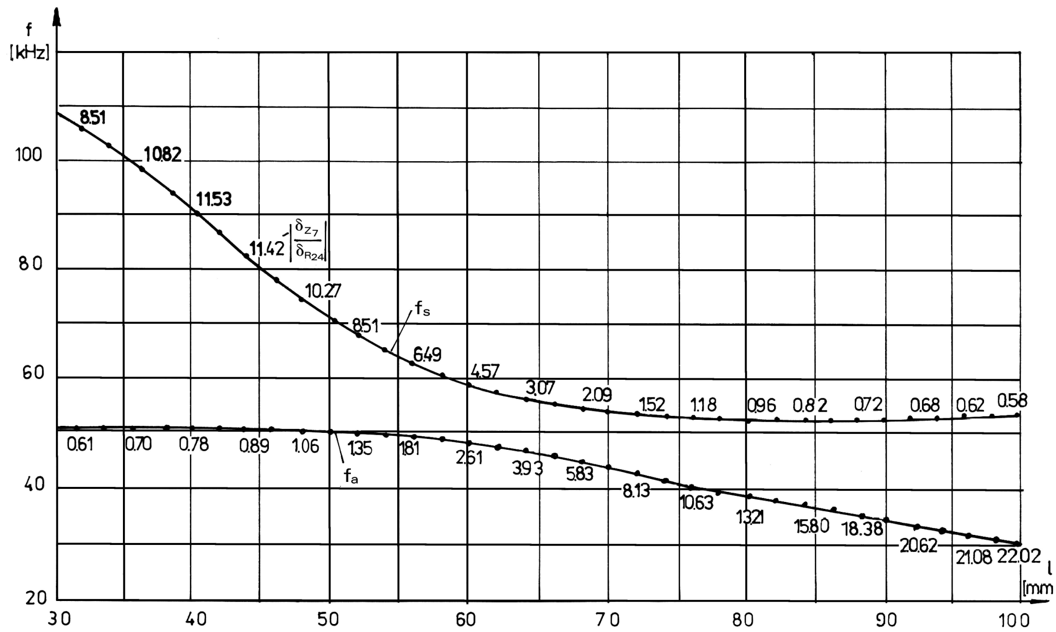

Fig. 5. The dependence of the R – L type converter's free vibration frequency on the length of the longitudinal resonator.

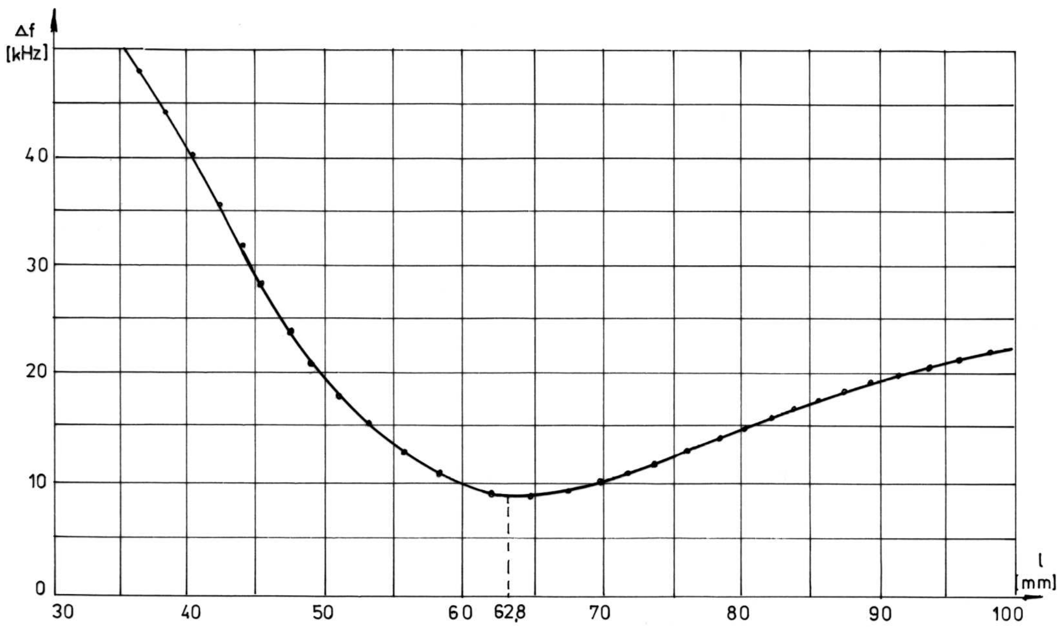

Fig. 6. The dependence of $\Delta f = f_s - f_a$ for the R – L type converter on the length of the rod resonator.

This is explained in a more detailed way in Fig. 7. where the relation $|\delta_{Z_7}/\delta_{R_{24}}| = f(l_{\text{rod}})$ is presented for both modes of vibrations. For the same value of this relation (the point of intersection of both curves) the length of the rod resonator was determined with the assumption that this is one of the possible criteria of "tuning" the R – L type converter. This criterion can be written in a generalised form:

$$\left|\left|\frac{\delta_{Z_{\text{rod}}}}{\delta_{R_d}}\right|\right|_{f_a} = \left|\left|\frac{\delta_{Z_{\text{rod}}}}{\delta_{R_d}}\right|\right|_{f_s}, \tag{4}$$

where δ_{rod} — the rod surface displacement towards Z, δ_{R_d} — the disk side horizontal displacement towards R.

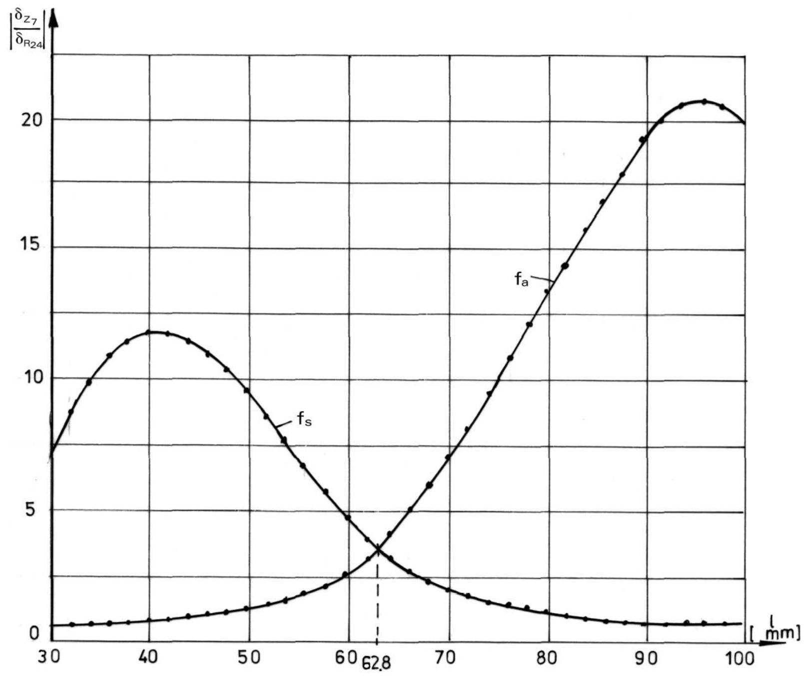

Fig. 7. Characteristic curve for the in-phase and the anti-phase vibration modes.

For such a "tuned" converter the displacements of particular nodes were determined which allowed us to obtain the form of vibrations both for anti-phase and in-phase vibration modes (Fig. 8).

In both figures the direction of vibrations of particular converter surfaces is marked by arrows. The calculated results obtained by means of FEM allow us to follow the way in which particular fragments of the resonator behave during vibrations. For the in-phase and the anti-phase vibration mode there exists a certain optimum length of the rod resonator at which the vibration amplitude gain coefficient $\delta_{Z_{\text{rod}}}/\delta_{R_d}$ reaches its maximum. This is of great importance when designing such resonators since it allows us to define the optimum dimensions of the resonator at the preset frequency and the preset vibration

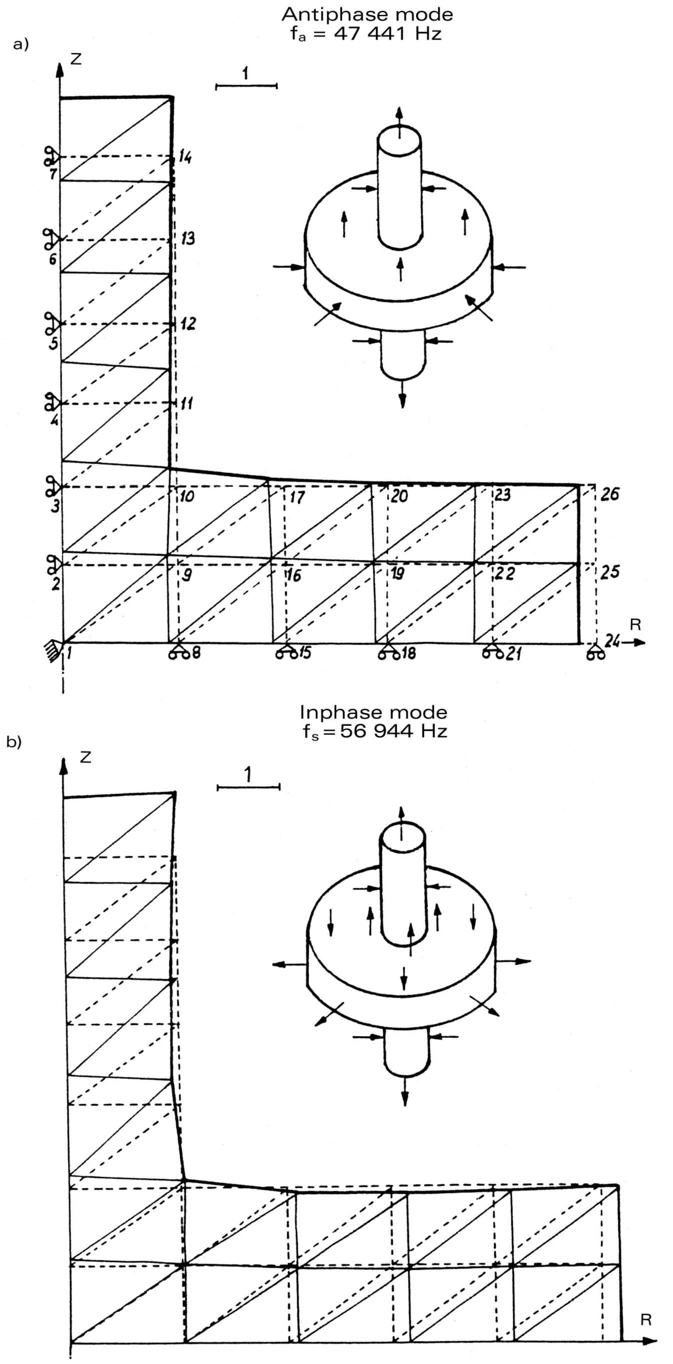

Fig. 8. Vibrations modes of homogeneous the R – L type converter: a) for the anti-phase mode of vibration, b) for the in-phase mode of vibration presented for the aligned converter in accordance with Eq. (3).

mode. Besides the criterion of tuning the R–L type converter (shown as (4)) another criterion can also be considered, i.e., the same value of the modulus representing the relation of surface displacement amplitude in a given direction to maximum displacement amplitude for particular vibration modes which can be written as

$$\left|\left|\frac{\delta_{Z_{\text{rod}}}}{\delta_{R_{\max}}}\right|\right|_{f_a} = \left|\left|\frac{\delta_{Z_d}}{\delta_{R_{\max}}}\right|\right|_{f_a}, \tag{5}$$

and

$$\left|\left|\frac{\delta_{Z_{\text{rod}}}}{\delta_{R_{\max}}}\right|\right|_{f_s} = \left|\left|\frac{\delta_{Z_d}}{\delta_{R_{\max}}}\right|\right|_{f_s}. \tag{6}$$

Figure 9 and 10 show vibration modes of the R–L type converter for such criteria of tuning. When comparing vibration modes in Figs. 9a, b and 10a, b one can easily notice

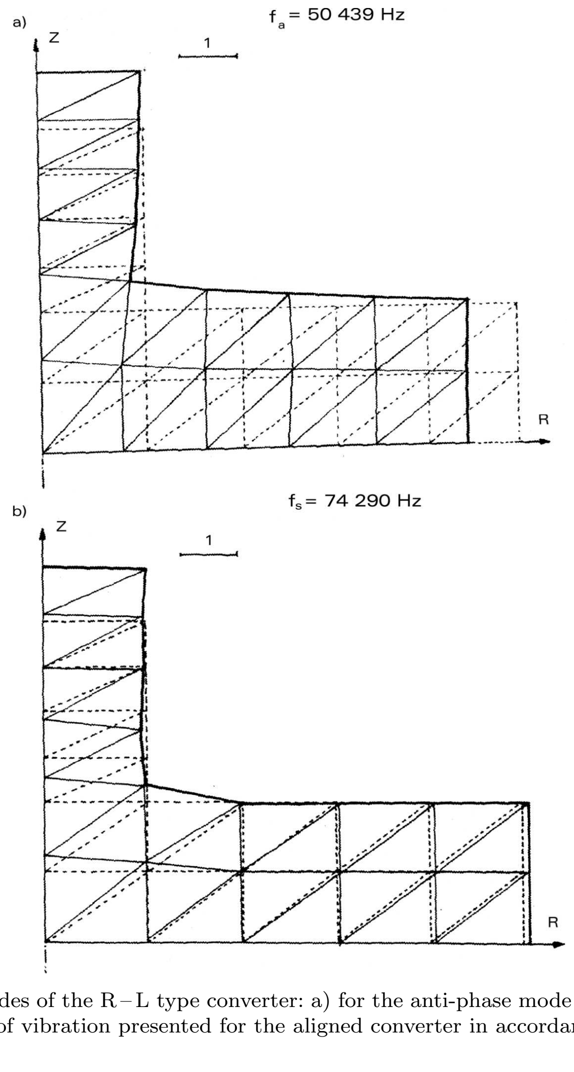

Fig. 9. Vibrations modes of the R–L type converter: a) for the anti-phase mode of vibration, b) for the in-phase mode of vibration presented for the aligned converter in accordance with Eq. (5).

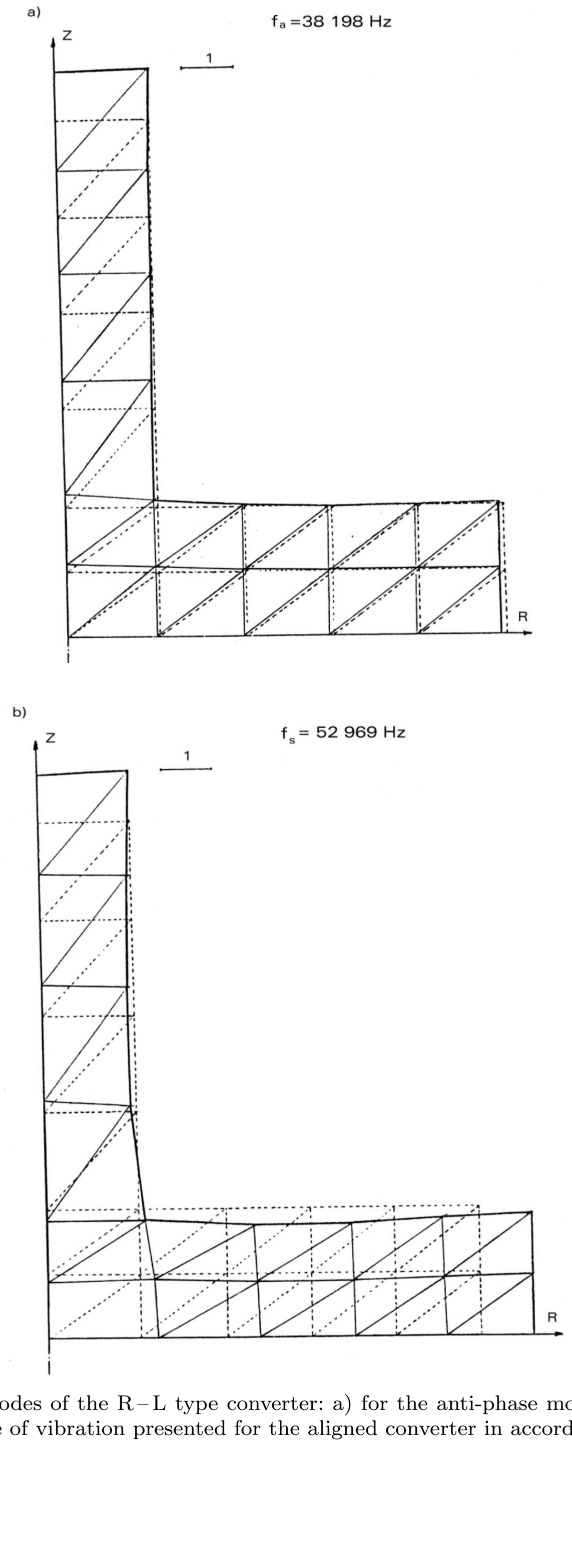

Fig. 10. Vibrations modes of the R – L type converter: a) for the anti-phase mode of vibration, b) for the in-phase mode of vibration presented for the aligned converter in accordance with Eq. (6).

that there is a considerable difference between the magnitude of displacement amplitudes of particular surfaces for both vibration modes at the pre-determined tuning criterion. Whereas the comparison of the obtained free vibration frequencies and the length of the rod resonator for vibration modes determined on the basis of all "tuning" criteria (Eqs. (4), (5), (6)) points to considerable discrepancies between these parameters (which is important in cascade-coupling of several converters).

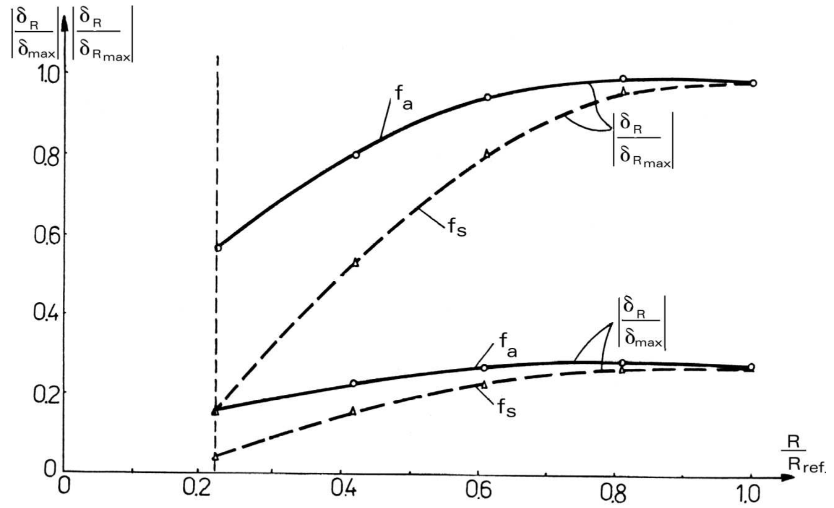

Fig. 11. Displacements distribution along the radius of the disk resonator presented for the aligned converter in accordance with Eq. (3).

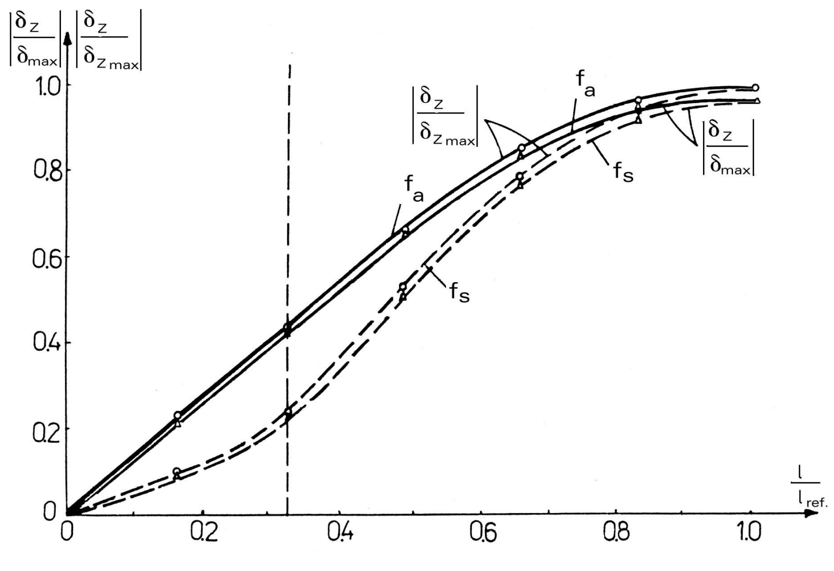

Fig. 12. Displacements distribution along the axis of the rod resonator presented for the aligned converter in accordance with Eq. (3).

Figures 11 and 12 present displacement distributions along the radius of the disk resonator and along the axis of the rod resonator for both vibration modes. For the anti-phase vibration mode the displacements are in each case bigger than for the in-phase mode.

Figures 13 and 14 show the distribution of displacements over the surface of resonators. What follows from both figures is that the displacements along the axis are bigger than those on the surface of particular resonators (for both vibration modes).

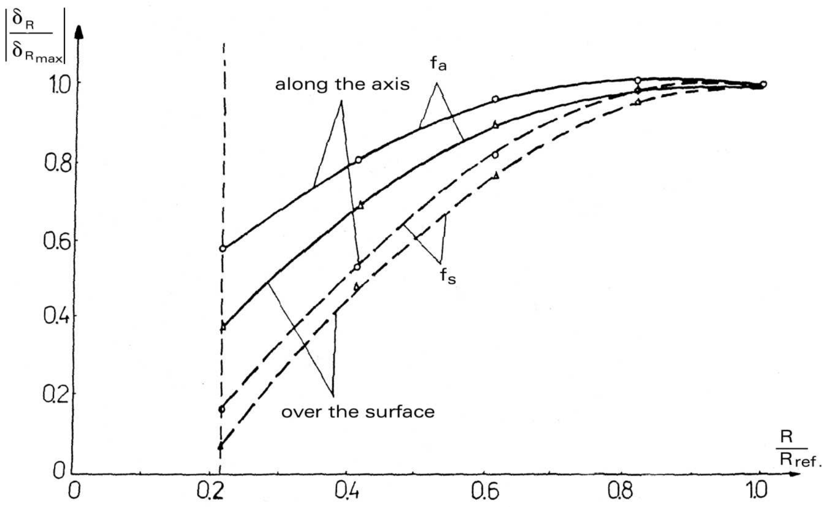

Fig. 13. Displacements distribution along the radius of the disk resonator over its surface and along its symmetry axis presented for the aligned converter in accordance with Eq. (3).

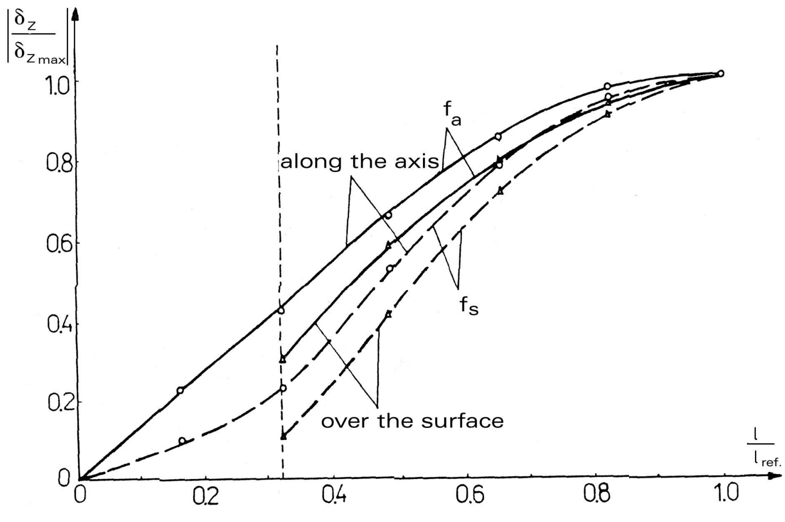

Fig. 14. Displacements distribution along the axis and over the surface of the rod resonator presented for an aligned converter in accordance Eq. (3).

5. R – L type heterogeneous converter

Finite elements methods also makes the analysis of resonators made from different materials possible. Figure 15 shows such an R – L type heterogeneous converter which was subject to analysis. The resonator of radial vibrations is a piezoelectric disk made of piezoceramic PXE-4 (PHILIPS). The resonator of longitudinal vibration is a titanium rod fixed in piezoceramic. The piezoelectric disk and the metal rod have been divided here into elements in the same way as the homogeneous converter. The following material constants were assumed for the calculation:

Titanium:

$$\begin{aligned} E &= 0.1157 \cdot 10^{12} [\mathrm{Pa}], \\ \nu &= 0.21, \\ \rho &= 4580\,[\mathrm{kg/m^3}]. \end{aligned}$$

Piezoceramic:

$$\begin{aligned} E &= 0.85 \cdot 10^{11}\,[\mathrm{Pa}], \\ \nu &= 0.3 \\ \delta &= 7500\,[\mathrm{kg/m^3}]. \end{aligned}$$

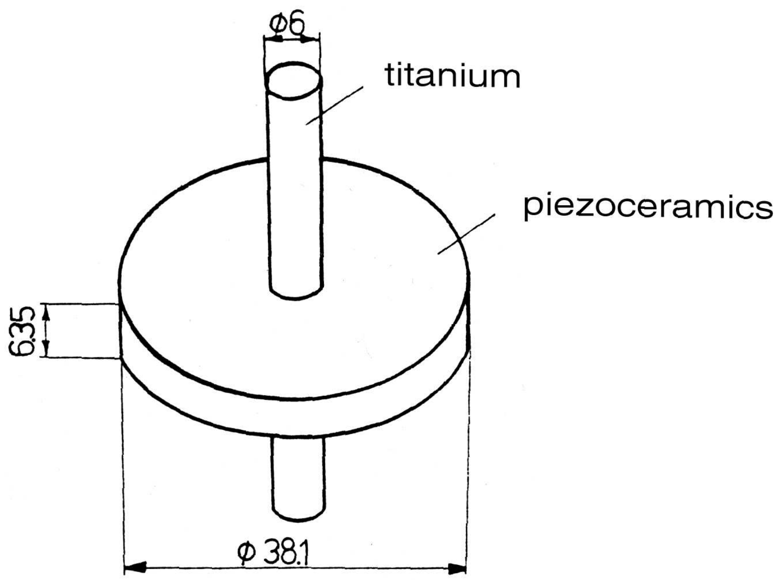

Fig. 15. The R – L type heterogeneous converter.

The analysis of this type of converter with the help of FEM yields characteristic curves and vibration modes similar those in a homogeneous converter. In view of similar character of the curves and vibration modes only some of the analysis results are presented here in Figs. 16, 17 and 18.

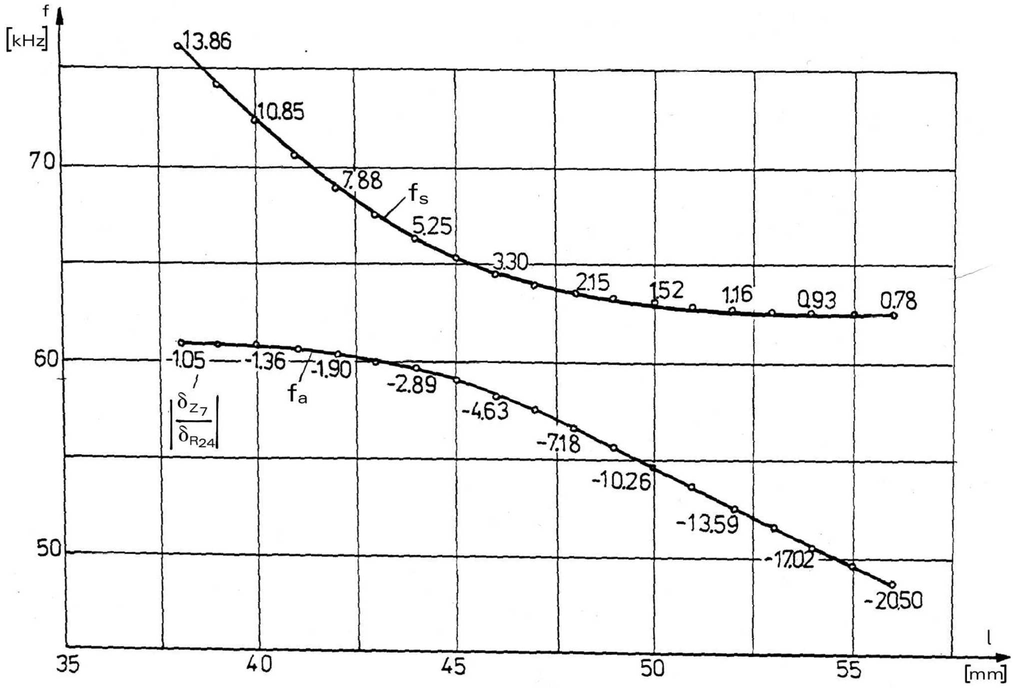

Fig. 16. The dependence of the R – L type heterogeneous converter's free vibration frequency on the length of the longitudinal resonator.

This resonator was experimentally tested by the author and described in [1, 2]. One characteristic of such a converter is its electrical input admittance. An example of a graph of the admittance modulus is presented in Fig. 19. The maxima of this graph correspond to free vibration frequencies of the converter presented in Fig. 17. The frequency spacing depends on the dimensions of the part common for both resonators and is longer the bigger the dimensions are [10]. The maximal values of the modulus of the electrical admittance of the converter depend on the resonator length and are equal for an aligned converter. Other results of investigations of this converter are described in [1, 2]. This construction has already been put into practice.

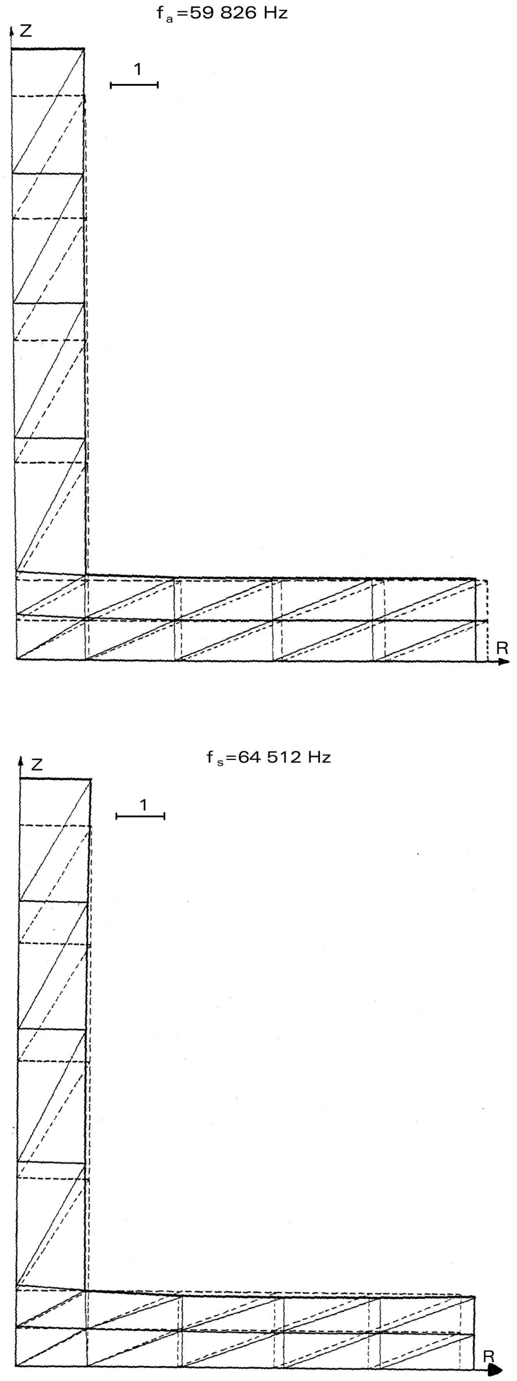

Fig. 17. Vibrations modes of the R–L type heteerogeneous converter: a) for the anti-phase mode of vibration, b) for the in-phase mode of vibration.

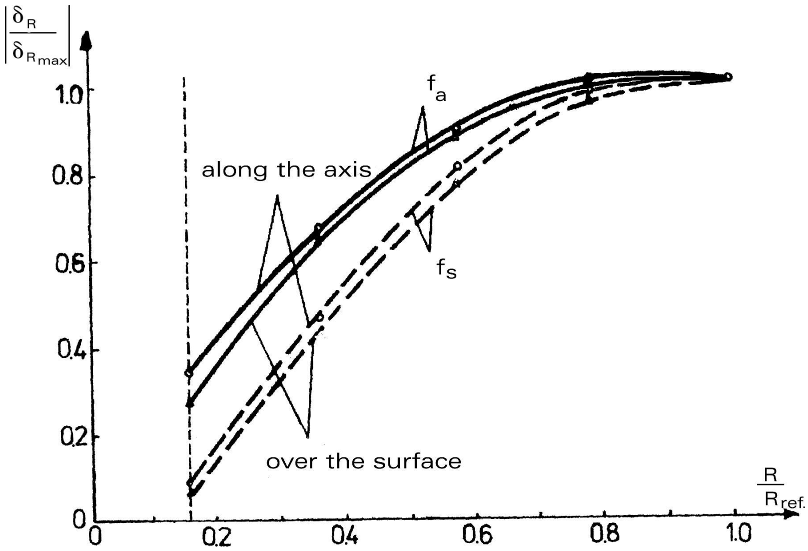

Fig. 18. Displacements distribution along the radius of the disk resonator, over its surface and along its symmetry axis for the aligned converter in accordance with Eq. (3).

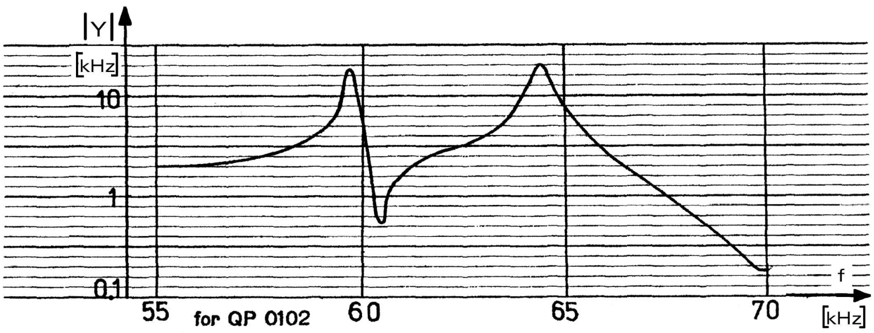

Fig. 19. The modulus of electrical admittance for the heterogeneous converter presented in Fig. 15.

6. Conclusions

The obtained results are, in part, compatible with those cited in works of K. Itoh and E. Mori [3, 6]. The authors, when experimenting on a homogeneous converter made of steel S45c (Japanese designation) with dimensions:

- Ø of the rod = 15 mm,
- length of the rod = 52.4 mm,
- Ø of the disk = 68 mm,
- thickness of the disk = 20 mm.

Obtained high free vibration frequencies corresponding to the anti-phase vibration mode $f_a = 50.235$ Hz and to the in-phase vibration mode $f_s = 60.4873$ Hz, result in $\Delta f = f_s - f_a = 10.252$ Hz.

The results obtained with the help of FEM point to the fact that the length of the rod was not optimum and did not allow us to obtain a converter which would be "tuned" according to any of the criteria suggested here. The most approximate result corresponds to the type of tuning in accordance with Eq. (4), for which:

- Ø of the rod = 15 mm,
- length of the rod = 62.8 mm,
- Ø of the disk = 68 mm,
- thickness of the disk = 20 mm,
- $f_a = 47.441$ Hz,
- $f_s = 56.944$ Hz,
- $\Delta = 9.505$ Hz.

Unconformity material constants may have a certain effect on the discrepancy between results presented in [3] and those obtained by means of FEM.

The three types of tuning (Eqs. (4)–(6)) postulated in this paper allow us to apply this type of resonators on large scale. Special attention should be paid to the possibility of using the converters for increasing the vibration amplitude. With the help of FEM it is possible to find the magnitude of this gain with preset input parameters as well as to determine the converter free vibration frequency at which the maximum gain for given resonator dimensions occurs.

The results obtained with the help of the FEM for the R–L type heterogeneous converter correspond to the results obtained from an experimental investigation of this converter.

References

[1] J. Golanowski and T. Gudra, *Experimental study on resonators with directional converter* [in Polish], Proc. of XXIII Open Seminar on Acoustics, Wisła 1975, 116–117.

[2] J. Golanowski and T. Gudra, *Ultrasonic transducers using radial vibrations of a piezoelectric disc*, Archives of Acoustics, **4**, 3, 245–255 (1979).

[3] K. Itoh and E. Mori, *Experimental study on directional converters of ultrasonic vibration*, Seventh International Congress on Acoustics, Budapest 1971, 641–644.

[4] K. Itoh and E. Mori, *Study on resonator with directional converter (L–L type converter)*, J. Acoustic. Soc. Japan, **28**, 3, 127–135 (1972).

[5] K. Itoh and E. Mori, *Studies on resonator with directional converter (L–L–L type converter)*, J. Acoustic. Soc. Japan, **29**, 1, 28–36 (1973).

[6] K. Itoh and E. Mori, *Study on resonator with directional converter (R–L type converter)*, J. Acoustic. Soc. Japan, **29**, 5, 307–314 (1973).

[7] G.F. Lvovskaya, *Teoreticheskoe issledovaniye characteristik diskosterzhnevogo ultrazvukovogo instrumenta*, Akusticheskii Zhurnal, **22**, 4, 552–557 (1976).

[8] E. Mori and K. Itoh, *The investigation of resonator with directional converter*, The 6-th International Congress on Acoustics, Tokyo 1968, K 89–92.

[9] Y. Morihiro and E. Mori, *Elastic vibration mode converter for ultrasonic power application*, Seventh International Congress on Acoustics, Budapest 1971, 25 U8, 645–648.

[10] G.F. Povolockaya, *Teoreticheskoe issledovaniye instrumentov s preobrazovaniyem napravleniya kolebanii*, Akusticheskii Zhurnal, 20, 1, 87–94 (1974).

[11] G.F. Povolockaya, *Razchet dvuhstierzhnevogo ultrazvukovogo instrumenta rabotayushchego s maloy nagruskoy*, Akusticheskii Zhurnal, **21**, 4, 598–604 (1975).

[12] J. Tsujino and K. Hasegawa, *Ultrasonic wire bonding using high frequency 330, 600 kHz and complex vibration 190 kHz welding system*, Ultrasonics, **34**, 223–228 (1996).

[13] J. Tsujino, T. Ueoka, K. Hasegawa, Y. Fujita, T. Shiraki, T. Okada and T. Tamura, *New methods of ultrasonic welding of metal and plastics materials*, Ultrasonics, **34**, 177–185 (1996).

[14] J. Tsujino, H. Yoshihara, K. Kamimoto and Y. Osada, *Welding characteristics of ultrasonic wire bonding using high frequency and complex vibration locus*, Proc. of World Congress on Acoustics, Yokohama 1997, pp. 156–157.

[15] J. Tsujino, H. Yoshihara, K. Kamimoto and Y. Osada, *Welding characteristics and temperature rise of high frequency and complex vibration ultrasonic wire bonding*, Ultrasonics, **36**, 56–59 (1998).

[16] O.C. Zienkiewicz, *Finite elements method*, Mc Graw Hill Company Limited, UK 1977.