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DIRECTIONAL CHARACTERISTICS OF A PLANAR ANNULAR PLATE FOR AXIALLY–SYMMETRIC FREE VIBRATIONS

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There is solved a coastal problem of the acoustic wave radiation at Fraunhofer zone for a planar annular plate vibrating harmonically. It is assumed that a plate is clamped with both its banks, inner and outer, into a planar rigid baffle. There is an analysis of axially-symmetric free vibrations. There are directional-frequency characteristics for both kinds of sources — annular plate and annular membrane.

1. Introduction

Many of substantial factors should be taken into consideration during theoretical analysis of complex surface vibrating systems generation and propagation phenomena. These factors are acoustic influences of source surface particular elements, individual sources of a system and of vibration form upon vibrating system radiation resultant field. There are many scientific works considering these problems. E.g. works of Levin and Leppington [6] in case of axially-symmetric sources. In works [9, 10] authors considered analytic active and reactive radiation power of a planar annular membrane for axially-symmetric vibrations forms. In work [8] there are presented directional characteristics of vibrating annular plate. Natural frequencies of transversely vibrating plates analysis is contained in work [11]. Energetic radiation aspect of an annular plate is transformed to integral form with Hankel transform in work [1]. Authors analyzed in detail non-dimensionalized added virtual mass incremental (NAVMI) factors. A considerable part of works contain problems of radiation conditions optimization of vibrating systems (comp. Engel [3], Fuller [4], Nelson and Deffayet [2]).

More complex, in respect of analytic research, is acoustic wave radiation of vibrating annular plate. There is a theoretical analysis of linear and sinusoidal in time phenomena. As a result acoustic pressure at Fraunhofer zone formula of elementary form of is produced. The source of pressure is a planar annular plate clamped with both its banks into a planar rigid baffle. Frequency-directional characteristics of annular plate radiation are presented in respect of source sizes. These characteristics are also compared with corresponding characteristics of annular membrane.

2. Free vibrations of an annular plate

In the plane $z = 0$, which is perfectly rigid, there is a planar thin annular plate. It is clamped with its banks, i.e. for $r = r_1$ and $r = r_2$, and $s = r_2/r_1 > 0$. There are considered axially-symmetric free vibrations, sinusoidal in time. Transverse deflection of plate surface points $\eta(r,t) = \eta(r)\exp(i\omega t)$ with boundary conditions

$$\eta(r_2,t) = \eta(r_1,t) = 0, \qquad \left.\frac{\partial\eta(r,t)}{\partial r}\right|_{r=r_2} = \left.\frac{\partial\eta(r,t)}{\partial r}\right|_{r=r_1} = 0, \tag{2.1}$$

there is

$$\frac{\eta_n(r)}{A_n} = J_0\left(x_n\frac{r}{r_1}\right) + B_n I_0\left(x_n\frac{r}{r_1}\right) - C_n N_0\left(x_n\frac{r}{r_1}\right) - D_n K_0\left(x_n\frac{r}{r_1}\right). \tag{2.2}$$

There are here functions of null order: J_0 — Bessel, N_0 — Neumann, I_0 – modified Bessel and K_0 — McDonald. A value $x_n = k_n r_1$ ($k_n^4 = \omega_n^2\varrho h/B$) is n–th root of frequency equation

$$[sN(sx_n) - N(x_n)][sT(sx_n) - T(x_n)] = [sS(sx_n) - S(x_n)][sR(sx_n) - R(x_n)], \tag{2.3}$$

where

$$\begin{aligned}
S(x) &= J_1(x)I_0(x) + J_0(x)I_1(x),\\
T(x) &= N_1(x)I_0(x) + N_0(x)I_1(x),\\
N(x) &= J_1(x)K_0(x) - J_0(x)K_1(x),\\
R(x) &= N_1(x)K_0(x) - N_0(x)K_1(x).
\end{aligned} \tag{2.4}$$

There are constants in the vibration equation (2.2)

$$\begin{aligned}
B_n &= sx_n\frac{R(sx_n)N(x_n) - R(x_n)N(sx_n)}{sR(sx_n) - R(x_n)}\,, \qquad C_n = \frac{\lambda S(sx_n) - S(x_n)}{sT(sx_n) - T(x_n)}\,,\\
D_n &= sx_n\frac{T(sx_n)S(x_n) - T(x_n)S(sx_n)}{sT(sx_n) - T(x_n)}\,.
\end{aligned} \tag{2.5}$$

The constant A_n is calculated from normalization condition (comp. [9])

$$\int\limits_{r_1}^{r_2} \eta_n^2(r)\,r\,dr = \frac{1}{2}(r_2^2 - r_1^2). \tag{2.6}$$

There is obtained

$$A_n^2 = \frac{1}{2}(s^2-1)\left\{s^2[J_0(sx_n) - C_nN_0(sx_n)]^2 - [J_0(x_n) - C_nN_0(x_n)]^2\right\}^{-1}. \tag{2.7}$$

In Table 1 there are given values of frequency equation several roots (2.3).

Table 1. Roots x_n of the frequency equation (2.3).

	s					
n	1.1	1.2	1.5	2	3	5
1	47.299	23.648	9.4554	4.7236	2.3579	1.1766
2	78.531	39.264	15.703	7.8477	3.9200	1.9569
3	109.96	54.976	21.988	10.991	5.4923	2.7433
4	141.37	70.685	28.272	14.134	7.0640	3.5295
5	172.79	86.393	34.555	17.276	8.6354	4.3154
6	204.20	102.10	40.839	20.418	10.207	5.1013

3. Acoustic pressure at the Fraunhofer zone

An annular plate, which transverse vibration distribution is calculated from the formula (2.2), radiates a wave of acoustic pressure into a half space $z \geq 0$ which is filled with a perfect gas medium of rest density ϱ_0 and wave propagation velocity c_0.

Acoustic pressure distribution of vibrating source clamped into a planar rigid baffle at the Fraunhofer zone is [7]:

$$p(R,\vartheta,\varphi) = \frac{i\varrho_0\omega}{2\pi}\,\frac{\exp(-ik_0R)}{R}\int\limits_{\sigma_0} v(r_0,\varphi_0)\exp[ik_0r_0\sin\vartheta\cos(\varphi-\varphi_0)]\,d\sigma_0 \tag{3.1}$$

and $\frac{1}{2}k_0r_0\{r_0/R\} \ll 1$, R, ϑ, φ are spherical coordinates of field point and r_0, φ_0 are polar coordinates of source point, $k_0 = 2\pi/\lambda$, λ is radiated wavelength, $v(r_0,\varphi_0)$ is a normal component of surface source vibration velocity.

In case of axially-symmetric vibrations $v(r_0) = i\omega\eta(r_0)$ the formula (3.1) is of form

$$p(R,\vartheta) = -\varrho_0\omega^2\frac{\exp(-ik_0R)}{R}\int\limits_{r_1}^{r_2}\eta(r_0)J_0(k_0r_0\sin\vartheta)r_0\,dr_0\,. \tag{3.2}$$

As a result of calculation of this integral, with specified vibration distribution (2.2), there is

$$\begin{aligned} p_n(R,\vartheta) = {} & -\varrho_0\omega_n^2A_n\frac{\exp(-ik_0R)}{R}\,\frac{2(r_1x_n)^2}{x_n^4-u^4} \\ & \times\{x_n[sC_1'(sx_n)J_0(su) - C_1'(x_n)J_0(u)] - u[sC_0'(sx_n)J_1(su) - C_0'(x_n)J_1(u)]\}\,, \end{aligned} \tag{3.3}$$

where $u = k_0r_1\sin\vartheta$, $C_0'(x) = J_0(x) - C_nN_0(x)$ and $C_1'(x) = J_1(x) - C_nN_1(x)$.

Result (3.3) presents an elementary form of an acoustic pressure expression at the Fraunhofer zone of an annular plate, which is activated to axially-symmetric form of vibrations $(0, n)$.

At the main direction there is

$$p_n(R, 0) = -2\varrho_0 r_1^2 A_n \frac{\omega_n^2}{x_n} \left[sC_1'(sx_n) - C_1'(x_n)\right] \frac{\exp(-ik_0 R)}{R}, \tag{3.4}$$

where A_n is the normalization constant defined by formula (2.7).

Directional characteristic of an annular clamped plate is in the form

$$\begin{aligned} K_n(\vartheta) &= \left|\frac{p_n(R,\vartheta)}{p_n(R,0)}\right| \\ &= \frac{sC_1'(sx_n)J_0(su) - C_1'(x_n)J_0(u) - (u/x_n)[sC_0'(sx_n)J_1(u) - C_0'(x_n)J_1(u)]}{[1-(u/x_n)^4][sC_1'(sx_n) - C_1'(x_n)]}, \end{aligned} \tag{3.5}$$

where $u = \beta \sin\vartheta$.

If an annular membrane is stimulated to axially-symmetric form of vibrations $(0, n)$ then pressure at the Fraunhofer zone is (comp. [9])

$$p_n(R, \vartheta) = -i\varrho_0\, \omega_n W_n(\vartheta) \frac{\exp(-ik_0 R)}{R}, \tag{3.6}$$

where

$$\begin{aligned} W_n(\vartheta) = \sqrt{s^2-1}\, \frac{i\omega_n x_n}{N_0(x_n)} \left\{ \frac{1}{N_0^2(sx_n)} - \frac{1}{N_0^2(x_n)} \right\}^{-1/2} & \\ \times \frac{\alpha_n J_0(s\beta\sin\vartheta) - J_0(\beta\sin\vartheta)}{x_n^2 - \beta^2 \sin^2\vartheta}, & \end{aligned} \tag{3.7}$$

where $\alpha_n = J_0(x_n)\big/J_0(sx_n)$, x_n is a root of characteristic equation (comp. [5] and [9, 10])

$$\frac{J_0(x_n)}{J_0(sx_n)} = \frac{N_0(x_n)}{N_0(sx_n)}. \tag{3.8}$$

Annular membrane directional characteristic for mode $(0, n)$ is in the form

$$K_n(\vartheta) = \left[1 - (\beta/x_n \sin\vartheta)^2\right]^{-1} \frac{\alpha_n J_0(s\beta\sin\vartheta) - J_0(\beta\sin\vartheta)}{\alpha_n - 1}. \tag{3.9}$$

4. Numerical analysis and concluding remarks

Diagrams of annular plate radiation direction indicator of axially-symmetric vibration forms are presented in Figs. 1–4. In case of odd vibrations forms ($n = 1, 3, 5, \ldots$) the direction indicator is calculated from the formula [7]

$$K_n(\vartheta) = \left|\frac{p_n(R,\vartheta)}{p_n(R,\vartheta_0)}\right| \qquad \text{where} \quad \vartheta_0 = \begin{cases} 0 & \text{for} \ \ n = 1, 3, 5, \ldots, \\ \max\limits_{p_n(R,\vartheta)} \vartheta & \text{for} \ \ n = 2, 4, 6, \ldots. \end{cases} \tag{4.1}$$

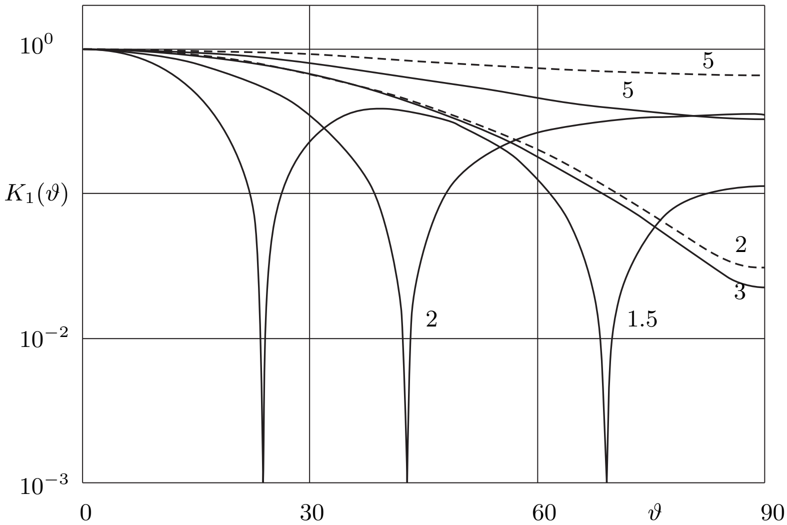

Fig. 1. Directional characteristics of annular source radiation for $\beta/x_1 = 0.5$, $s = 5, 3, 2$ and 1.5. Attention for Figs. 1–4, 7: Curves in figures are: solid — vibrating plate is a source, dashed — vibrating membrane is a source.

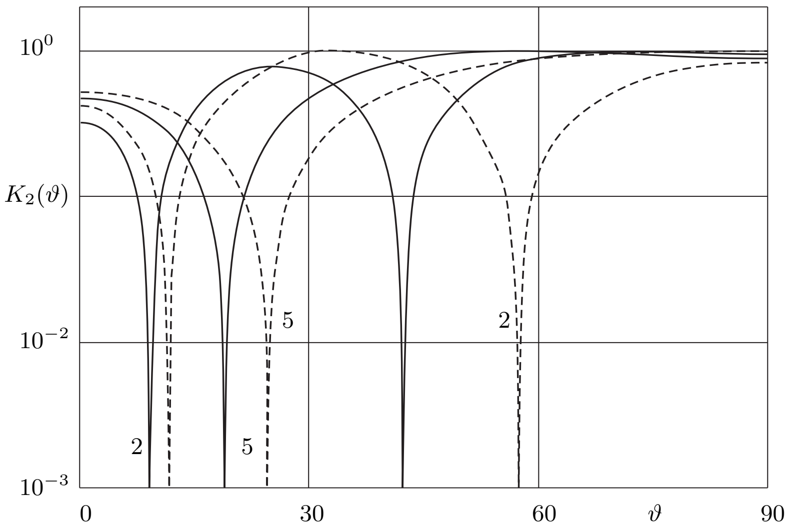

Fig. 2. Directional characteristics of annular source radiation for $\beta/x_2 = 0.5$, $s = 5$ and 2.

If an annular plate is stimulated to one of the even forms of vibrations $(n = 2, 4, 6, ...)$, then pressure $p_n(R, \vartheta)$ is related to pressure $p_n(R, \vartheta_0)$. Angle ϑ_0 determines the direction of the maximal radiation.

Directional characteristics are also presented graphically in case of vibrating annular membrane. It allows comparing directional characteristics of two different annular sources

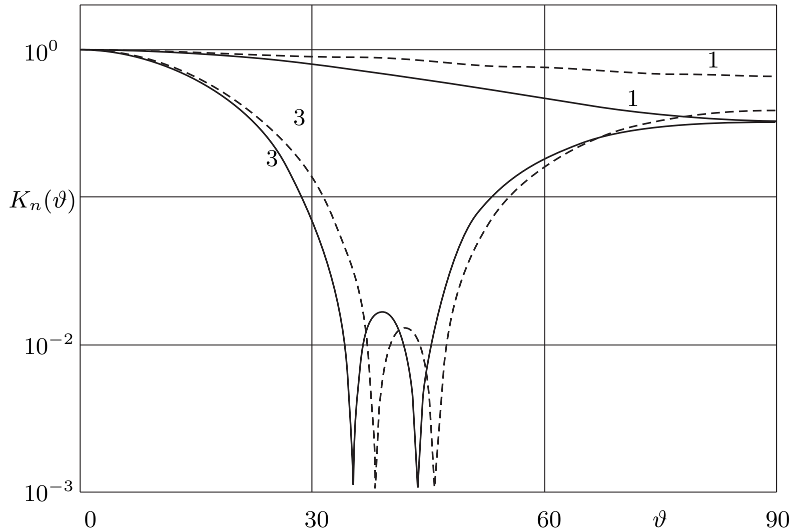

Fig. 3. Directional characteristics of annular source for $s = 5$, $\beta/x_n = 0.5$, $n = 1$ and 3.

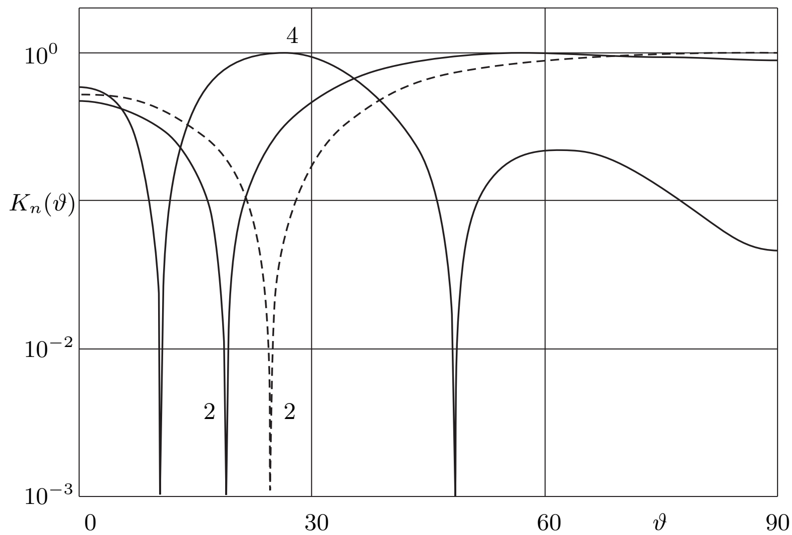

Fig. 4. Directional characteristics of annular source for $s = 5$, $\beta/x_n = 0.5$, $n = 2$ and 4.

— the plate and the membrane. There are diagrams of $K_n(u)$ versus variable $u = \beta \sin\vartheta$ in Figs. 5, 6 and versus variable $u = \beta/x_n \sin\vartheta$ in Fig. 7.

In boundary case, when $r_1 \to 0$ and $r_2 = a$, there are formulas describing a circular source (membrane or plate) instead of formulas describing an annular source (membrane or plate). Instead of characteristic equation (2.3) there is characteristic equation for a

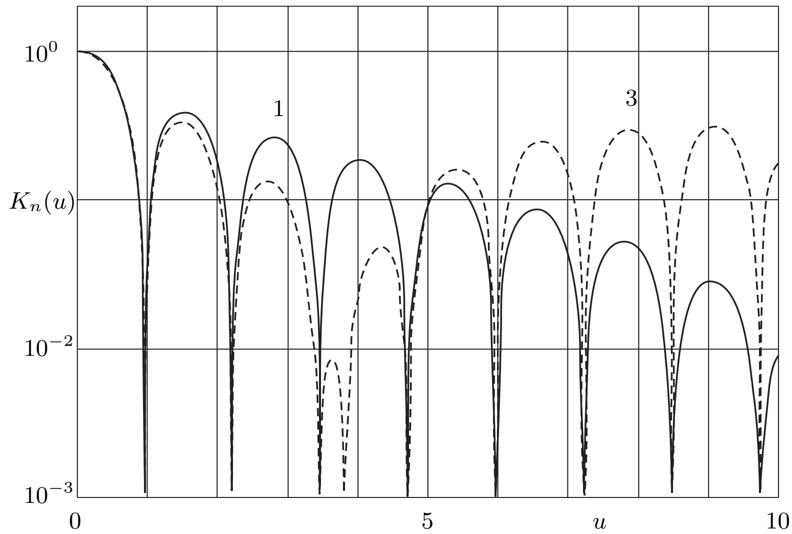

Fig. 5. Diagrams of $K_n(u)$ function versus variable $u = \beta \sin\vartheta$ for annular plate. It is assumed that $s = 1.5$, $n = 1$ and 3.

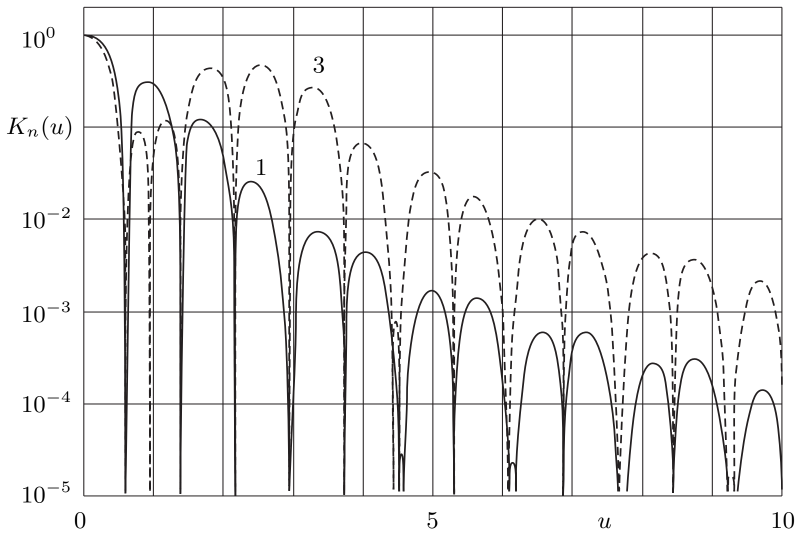

Fig. 6. Diagrams of $K_n(u)$ function versus variable $u = \beta \sin\vartheta$ for annular plate. It is assumed that $s = 3$, $n = 1$ and 3.

circular plate of radius $r = a$, i.e. when $r_1 \to 0$, $C_n \to 0$,

$$S(sx_n) = S(k_n a) = J_1(k_n a)\, I_0(k_n a) - J_0(k_n a)\, I_1(k_n a) = 0, \tag{4.2}$$

where $k_n^2 = \omega_n \sqrt{\varrho h/\beta}$. There is also that $\lim\limits_{r_1 \to 0} A_n(r_1) = 2^{-1/2} J_0^{-1}(k_n a)$.

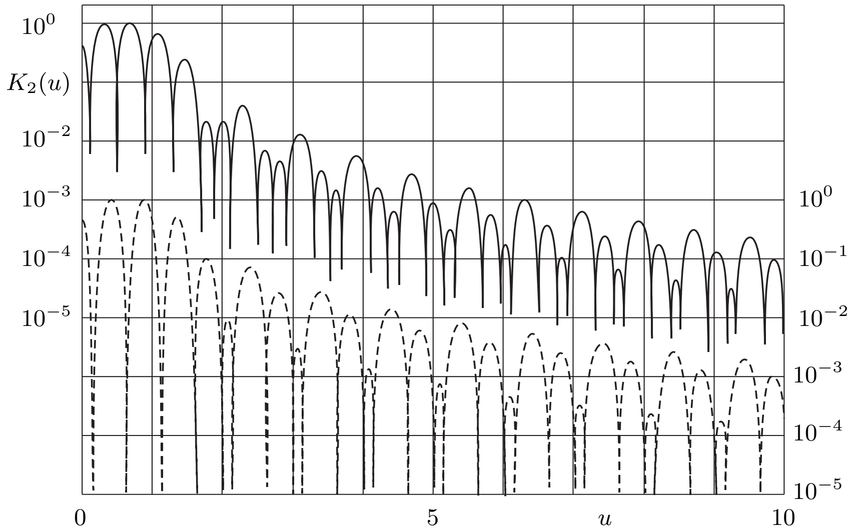

Fig. 7. Diagrams of $K_2(u)$ function versus variable $u = \beta/x_2 \sin\vartheta$ for annular source. It is assumed that $s = 3$.

From the formula (3.5) there is

$$\lim_{r_1\to 0} K_n(r_1,\vartheta) = \frac{J_0(k_0 a\sin\vartheta) - \dfrac{k_0}{k_n}\dfrac{J_0(k_n a)}{J_1(k_n a)}\sin\vartheta\, J_1(k_0 a\sin\vartheta)}{1-(k_0/k_n)^4\sin^4\vartheta}\,. \tag{4.3}$$

Instead of characteristic equation (3.8) there is a characteristic equation for a circular membrane of radius $r = a$, i.e.

$$J_0(k_n a) = 0, \tag{4.4}$$

where $k_n = \omega_n\sqrt{\sigma/T}$, σ is the membrane surface density, T is the stretching force. From the formula (3.9) there is

$$\lim_{r_1\to 0} K_n(r_1,\vartheta) = \frac{J_0(k_0 a\sin\vartheta)}{1-(k_0/k_n)^2\sin^2\vartheta}\,. \tag{4.5}$$

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