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ESTIMATION OF LOCAL MATERIAL AND STRUCTURE PARAMETERS OF A POLYCRYSTALLINE AGGREGATE FROM ULTRASONIC MEASUREMENTS

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The propagation velocities of four ultrasonic waves in a steel plate are measured in twelve equally spaced observation points lying in a rolling plane on a straight line perpendicular to the rolling direction. The plate material was rolled plastically and uniaxially in the situation where the edges parallel to the rolling direction were free. The plate is considered to be a bulk sample with orthorhombic symmetry of bulk mechanical properties made of cubic crystals of the highest symmetry. The local probability density function of the crystallite orientation and the local effective stiffness moduli of a single grain (crystallite) are found from four ultrasonic velocities and the rules of orthorhombic symmetry and Jaynes' principle of maximum Shannon entropy. These results, which have been obtained for twelve mesodomains centered at each of the twelve observation points, show the effect of the distance between an observation point and a free plate edge on the local effective stiffness moduli and on the local probability density function of the crystallite orientation.

1. Introduction

In an isotropic polycrystalline material the ultrasonic velocities are independent of the direction of the ultrasonics' propagation through a macroscopic sample of the material. Most polycrystalline materials (e.g. metals) were acted on by forming forces, which caused plastic deformation, subjected the body to a state of stress or deformation and left the crystallites (basic units, grains) in certain preferred orientations. Consequently, the forces of the forming process caused anisotropy of the overall (effective) mechanical (amongst them acoustical) properties of the material. The non-random distribution of the crystallite orientation, which is caused by plastic deformation, is called the texture. Therefore, most polycrystalline materials exhibit texture resulting from their forming processes. The acoustical anisotropy is revealed by the variations in speeds at which ultrasonic waves propagate through the sample, the variations being dependent on the directions of the wave propagation and polarization. In numerous situations, the texture considerably contributes to the mechanical and acoustical anisotropy of the material.

Among the problems of forming the texture in metals during plastic deformation, there are mainly two fields of interest for both a fundamental and an applied researcher. The first is directed to the effect of roll forces, their geometry as well as another rolling

parameters and conditions on the texture. The second is concerned with the influence of the texture on the mechanical properties of rolled metal (e.g. steel). For example, for the application of structural steels in various fields of engineering, high strength, sufficient ductility and a good weldability as well as formability are often highly desired. Therefore, the changes in these steel properties during a plastic forming process (e.g. rolling) may be more exactly predicted as our knowledge of the problems belonging to these two fields becomes better.

In the paper, the considerations are confined to ultrasonic plane and linearly polarized waves which propagate in macroscopic pollycrystalline aggregates with orthorhombic symmetry of the bulk (effective) elastic properties, the aggregates being composed of a large number of cubic crystallites of the highest symmetry. The macroscopic orthorhombic symmetry is of considerable practical interest since the rolling process in one direction only of a polycrystalline aggregate with virgin (before deformation) isotropic symmetry always results in the transformation of the isotropic symmetry (or another one) into the othorhombic symmetry.

In the paper, there is presented a nondestructive ultrasonic method of the estimation of some local material parameters and the local texture of a steel plate which was rolled uniaxially in the situation where the edges parallel to the rolling direction were free. Thus in this paper, only such orientation statistics of the crystallites is considered which contributes to the orthorhombic symmetry of the dynamic properties of a bulk specimen of a polycrystalline aggregate made of cubic crystals of the highest symmetry. Using the approach proposed by LEWANDOWSKI in [2], we do not neglect the effect of other causes on the effective mechanical and ultrasonic propagation properties of the polycrystalline aggregate as well as on the macroscopic symmetry of these properties, since this approach is based on taking into account the fact that the values of measured velocities of the ultrasonic waves are determined not only by the structure and physical properties of the acoustical medium under examination but also by all these physical phenomena occurring in the medium which influence on the propagation. In other words, the measured velocities of ultrasonics, which are the basis for all calculations presented in the subsequent text, contain information on the structure and properties of the material, among other on the values of the dynamic single-crystal material parameters influenced on by the defects and imperfections of crystal microstructure, residual stress, scattering and the phenomena of mechanical energy dissipation.

2. Formulation of the problem

Although numerous ultrasonic investigations have been carried out in connection with the rolling process, quantitative information on the complete distributions both of some local mechanical properties of the plate material and the local texture in a plate undergoing rolling seems to be little. Among others here arises the question how strongly are the local mechanical properties and texture influenced on by the distance x between an observation point determined by the position vector $\mathbf{r}$ and one of the two plate edges parallel to the rolling direction, in the situation where these edges are free.

It should perhaps be emphasized that in this paper the term *local* concerns the use of three measuring scales, the smallest of which refers to a structural grain or crystallite of the material. For simplification of the analysis, an assumption is used as the first approximation of the internal contribution to the material response on an external (ultrasonic transducer) loading. In accordance with this assumption, the constitutive internal relations of a microelement (grain or crystallite) are describable by continuum laws, whilst the stress and displacement fields are described either in terms of random variables or stochastic processes depending on the loading and the material under investigation. All parameters concerned with the smallest region are prefixed by *micro*. Next an intermediary scale is introduced referred to as a *mesodomain* that contains a statistical ensemble of crystallites. A mesodomain is interpreted to be much smaller than the *macroscopic domain* of the entire material body (macroscopic sample), but is much larger than the domain of a microelement as containing a statistical ensemble of crystallites. In the subsequent text, the term *local* texture (properties) does not mean the texture (properties) in a point in the plate under study, which is determined by a position vector $\mathbf{r}$, but means the texture (properties) of the plate material filling a mesodomain geometrically centered at the point determined by the position vector $\mathbf{r}$. This point is called in the subsequent text the observation point.

In the paper, we are interested in some mechanical properties and the local texture of a steel plate of dimensions of $0.38\,\mathrm{m} \times 0.26\,\mathrm{m} \times 0.02\,\mathrm{m}$ which was rolled uniaxially in the situation where the edges parallel to the rolling direction were free. We are interested first in showing that it is possible to estimate from ultrasonic measurements the distribution of the local propagation (mechanical) properties and texture of the steel plate. Secondly, we are going to reveal the influence of the distance x between an observation point and the reference free edge on the the local values of effective material parameters and the local texture. For this reason, we are interested in deducing from the ultrasonics measurements the local values of effective material parameters of a crystallite as well as the distributions of the local texture in the plate in twelve equally spaced points (0.02 m apart) lying in rolling plane on a straight line perpendicular to the rolling direction. In this way, the measurements of the propagation velocities, V_{ij}, where $i, j = 1, 2, 3$, of ultrasonic plane waves propagating and polarized in the directions of the Cartesian reference axes $0x_i$ and $0x_j$, respectively, are the only experimental tools for texture investigations discussed in this paper. The Cartesian coordinate system $0x_1x_2x_3$ with the axes $0x_1$, $0x_2$ and $0x_3$ is defined below.

Among the reasonable choices of the reference system for analysing this problem is the Cartesian coordinate system $0x_1x_2x_3$ with the axes $0x_1$, $0x_2$ and $0x_3$ chosen as the rolling R, transverse (perpendicular to R in the rolling plane) T and normal (to the rolling plane) N directions, respectively. Let the abbreviations $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$ denote the unit vectors along the directions of the axes $0x_1$, $0x_2$ and $0x_3$, respectively. The Cartesian coordinate system $0x_1x_2x_3$ will be called the macroscopic reference system. To define the texture precisely, we also make use of a local Cartesian coordinate system $0X_1X_2X_3$ called the microscopic reference system. This reference system is defined, similarly as in Refs. [1, 2], for each single cubic crystallite. Its reference $0X_1$ axes, $0X_2$ and $0X_3$ are chosen in the crystallographic directions [100], [010] and [001], respectively. Let the

abbreviations $\mathbf{E}_1$, $\mathbf{E}_2$ and $\mathbf{E}_3$ denote the unit vectors along the directions of the axes $0X_1$, $0X_2$ and $0X_3$, respectively. In the subsequent considerations, the orientation of a crystallite in the polycrystalline sample is described by giving the values of three Eulerian angles θ, φ and ϕ where θ, φ and ϕ denote the Eulerian angles defined in this paper in the same way as in Ref. [3], i.e. θ is the angle of nutation, φ is the angle of precession and ϕ is the angle of proper rotation. In the paper, the texture is described by using the probability density function of the crystallite orientation, $p(\xi, \varphi, \phi)$, defined in such a way that $p(\xi, \varphi, \phi)\, d\xi\, d\varphi\, d\phi$ expresses the probability of a crystallite having an orientation described by the Euler angles $\theta = \cos^{-1}\xi$, φ and ϕ, lying in the intervals $\langle \cos^{-1}\xi, \cos^{-1}(\xi + d\xi)\rangle$, $\langle \varphi, \varphi + d\varphi\rangle$ and $\langle \phi, \phi + d\phi\rangle$, respectively.

Using the approach proposed by LEWANDOWSKI [2], we utilize the fact that the propagation properties of the polycrystal under examination, which are revealed by the results of ultrasonic measurements, contain information on the structure and properties of the components of the medium as well as on the phenomena occurring in the polycrystal and influencing on the propagation (e.g., scattering). It means that the dynamic mechanical and propagation properties (the values of the components of the effective stiffness tensor) of a single grain in the approach proposed in Ref. [2] and applied in this paper are taken to be "as they are" by letting the experimental data (ultrasonics wave velocities and the rules of orthorhombic symmetry) to determine the single-grain *effective* elastic properties and texture of the bulk sample under consideration. The respective numerical calculations are performed by using the equations which are derived and listed in Ref. [2, Eqs. (5), (7), (9), (15), (21) - (23)]. Starting from these equations, we find the function $p(\xi, \varphi, \phi)$ and obtain values of the components of the so-called *effective* stiffness tensor, $c_{ij}^{(\mathrm{eff})}$, $(i, j = 1, 2, 3)$, of a single grain of the polycrystalline aggregate under examination. The reasoning leading us to such results may be presented shortly as follows:

We start from the hypothesis that the propagation properties of the bulk specimen under examination are defined by the macroscopic tensor of the *effective* elastic stiffness, $C_{i,j}^{(\mathrm{eff})}$, of the sample (or, equivalently, elastic compliance tensor) and the effective density, ρ, the last being assumed in this paper to be equal to the density averaged over the volume of a single bulk sample. Let us remind that the symmetry of the effective elastic stiffness tensor of the bulk sample of the polycrystalline aggregate under examination, which is called in the subsequent text the macroscopic symmetry of the bulk sample, is orthorhombic in the situation where the material was plastically rolled in one direction. More strictly speaking, the term *effective properties* of the bulk sample is used to describe the physical properties of the so-called equivalent homogeneous medium [4] that exhibits the same macroscopic symmetry as the bulk sample under study, and the displacement response of the equivalent medium to the transducer loading is the same as the averaged displacement response of the polycrystalline material to the same loading, the averaging being carried out over a statistical ensemble of bulk samples, i.e. over all crystallites through the function $p(\xi, \varphi, \phi)$. Similar to the effective density, the effective elastic moduli are also independent of the position vector (space coordinates), but they are dependent on the frequency of the loading transducer. In contrast, the average displacement field resulting from the dynamic load is dependent on the position vector, $\mathbf{r}$, time, t, and

load (angular) frequency, ω, and, consequently, is called the effective displacement field or effective wave — especially, if is harmonically dependent on the position vector and time.

Knowing the effective properties of the sample under examination, we are able to estimate the dynamic response of the sample being acted on by ultrasonic transducer. In the case, when the response is of the form of ultrasonic plane waves propagating and polarized in the directions of the macroscopic Cartesian reference axes $0x_i$ and $0x_j$ $(i,j = 1,2,3)$, respectively, the propagation velocities, V_{ij}, of these waves can be calculated from the Christoffel equation [5]

$$\det\left(\Gamma^{ik} - \rho V_{gh}^2 \delta_{ik}\right) = 0, \qquad g,h,i,k = 1,2,3 \tag{1}$$

in which

$$\Gamma^{ik} = C_{ijkl}^{(\mathrm{eff})} \eta_j \eta_l \tag{2}$$

is the so-called "Christoffel-Kelvin stiffness", ρ stands for the mass density, and δ_{ik} is the Kronecker delta. In Eq. (2) the definition of Γ^{ik}, the components $C_{ijkl}^{(\mathrm{eff})}$ (effective sample stiffness moduli) of the stiffness tensor of the macroscopic sample are related to $C_{ij}^{(\mathrm{eff})}$ by using the reduced subscript notation. The abbreviations η_j and η_l denote the components of the unit vector in the direction of the wave propagation.

On the other hand, by using a suitable averaging procedure the effective sample stiffness moduli, $C_{ij}^{(\mathrm{eff})}$, can be calculated from the values of the dynamic stiffness moduli c_{11}, c_{12}, c_{44} of a single cubic grain (crystal), its density ρ, and from the probability density function of the crystallite orientation, $p(\xi,\varphi,\phi)$. There are numerous procedures to approximating the effective elastic constants, proposed by such authors as VOIGT [6], REUS [7] and HILL [8]. The solutions of the Christoffel equations (1) for an orthorhombically textured solid, which are obtained with applying the Voigt approximation (averaging procedure) to the calculation of the effective sample stiffness moduli, $C_{ij}^{(\mathrm{eff})}$, are listed in Ref. [9] as formulae (10) – (21). It should perhaps be emphasized that the values of the moduli c_{11}, c_{12}, c_{44} of a single cubic grain (crystal) were considered in Refs. [9, 1] for a deformed and textured steel as being equal to the values of c_{11}, c_{12}, c_{44} and ρ, which had been determined for a single-crystal of pure BCC Fe with using a statical method. It is not to be expected that such an approximation, which can be called the long-wavelength and ideal Fe crystal approximation, would be always acceptable for rolled steel, which is a polycrystalline aggregate of Fe with impurities and structure defects. For this reason, herein is used a modified approach proposed by LEWANDOWSKI in Ref. [2], in which the values of c_{11}, c_{12}, c_{44} are replaced by the so-called effective dynamic stiffness moduli of a single grain in deformed steel, $c_{11}^{(\mathrm{eff})}$, $c_{12}^{(\mathrm{eff})}$ and $c_{44}^{(\mathrm{eff})}$, the last being determined also from measured values of ultrasonic velocities. Using the SAYERS' solutions [9, formulae (10) – (21)], LEWANDOWSKI [2] arrived at the following equations, after a little algebra and manipulation

$$\begin{aligned} \langle r_1(\xi,\varphi,\phi)\rangle &= \frac{1}{2\overline{c}}\left(\overline{c}_{11} - V_{11}^2\right), \qquad \overline{c}_{11} = \frac{c_{11}^{(\mathrm{eff})}}{\rho}, \\ \overline{c} &= \frac{1}{\rho}\left(c_{11}^{(\mathrm{eff})} - c_{12}^{(\mathrm{eff})} - 2c_{44}^{(\mathrm{eff})}\right) \doteq \overline{c}_{11} - \overline{c}_{12} - 2\overline{c}_{44}\,; \end{aligned} \tag{3}$$

$$\langle r_2(\xi,\varphi,\phi)\rangle = \frac{1}{2\overline{c}}\left(\overline{c}_{11} - V_{22}^2\right), \tag{4}$$

$$\langle r_3(\xi,\varphi,\phi)\rangle = \frac{1}{2\overline{c}}\left(\overline{c}_{11} - V_{33}^2\right), \tag{5}$$

$$\langle r_4(\xi,\varphi,\phi)\rangle = \frac{1}{\overline{c}}\left(V_{23}^2 - \overline{c}_{44}\right), \qquad \overline{c}_{44} = \frac{c_{44}^{(\mathrm{eff})}}{\rho}, \tag{6}$$

$$\langle r_5(\xi,\varphi,\phi)\rangle = \frac{1}{\overline{c}}\left(V_{13}^2 - \overline{c}_{44}\right), \tag{7}$$

$$\langle r_6(\xi,\varphi,\phi)\rangle = \frac{1}{\overline{c}}\left(V_{12}^2 - \overline{c}_{44}\right), \tag{8}$$

where

$$\begin{aligned} V_{ij} &= V_{ji}\,, \\ V_{i1}^2 + V_{i2}^2 + V_{i3}^3 &= V_{1j}^2 + V_{2j}^2 + V_{3j}^2 = \overline{c}_a\,, \\ \overline{c}_a &= \frac{1}{\rho}\left(c_{11}^{(\mathrm{eff})} + 2c_{44}^{(\mathrm{eff})}\right), \end{aligned} \tag{9}$$

$$r_4 = r_3 + r_2 - r_1\,, \qquad r_5 = 2(r_1 - r_2) + r_4\,, \qquad r_6 = 2r_1 - r_5\,, \tag{10}$$

$$\begin{aligned} r_1 &= l_1^2 l_2^2 + l_1^2 l_3^2 + l_2^2 l_3^2\,, \\ r_2 &= m_1^2 m_2^2 + m_1^2 m_3^2 + m_2^2 m_3^2\,, \\ r_3 &= n_1^2 n_2^2 + n_1^2 n_3^2 + n_2^2 n_3^2\,, \end{aligned} \tag{11}$$

$$l_i = E_i e_1\,, \qquad m_i = E_i e_2\,, \qquad n_i = E_i e_3. \tag{12}$$

The abbreviations $\langle r_q\rangle$, $q = 1, 2, ..., 6$, in Eqs. (3) – (8) denote averaging the above defined functions of a single-crystal orientation, $r_q(\theta,\varphi,\phi)$, over all the crystallites in the sample, i.e. $\langle r_q(\theta,\varphi,\phi)\rangle$ is $r_q(\theta,\varphi,\phi)$ weighted by $p(\theta,\varphi,\phi)$:

$$\langle r_q(\xi,\varphi,\phi)\rangle = \int_0^{2\pi}\int_0^{2\pi}\int_{-1}^{1} r_q(\xi,\varphi,\phi)p(\xi,\varphi,\phi)\,d\xi\,d\varphi\,d\phi. \tag{13}$$

Let us remind that $p(\xi,\varphi,\phi)d\xi\,d\varphi\,d\phi$ stands for the probability of a crystallite having an orientation described by the Euler angles θ $(= \cos^{-1}\xi)$, φ and ϕ, lying in the intervals $\langle \cos^{-1}\xi, \cos^{-1}(\xi + d\xi)\rangle$, $\langle \varphi, \varphi + d\varphi\rangle$ and $\langle \phi, \phi + d\phi\rangle$, respectively. The probability density function $p(\xi,\varphi,\phi)$ fulfils the normalization condition

$$\langle p(\xi,\varphi,\phi)\rangle = \int_0^{2\pi}\int_0^{2\pi}\int_{-1}^{1} p(\xi,\varphi,\phi)\,d\xi\,d\varphi\,d\phi = 1. \tag{14}$$

It should perhaps be emphasized that each left-hand side of the six equations (3) – (8) is of the form of an expectation value of one of known six functions, $r_q(\xi,\varphi,\phi)$, of a single-

crystal orientation. As it follows from Eqs. (9) – (12), only three functions $r_q(\xi, \varphi, \phi)$ are linearly independent of each other. Each right-hand side of the six equations (3) – (8) is of the form of a known function of an ultrasonic velocity, V_{ij}, the effective single-crystal stiffness moduli $c_{11}^{(\mathrm{eff})}$, $c_{12}^{(\mathrm{eff})}$, $c_{44}^{(\mathrm{eff})}$, and density ρ. In Ref. [1], where the polycrystalline aggregate was approximated by the respective ideal polycrystalline solid, the values of all the quantities appearing on the right-hand side of each of the six equations (5) – (10) are regarded to be known and are to be equal to the respective single-crystal stiffness moduli c_{11}, c_{12}, c_{44} and density ρ.

As it was shown in Ref. [1], in the situation, where the left-hand sides of Eqs. (3) – (8) are the expectation values of $r_q(\xi, \varphi, \phi)$ weighted with $p(\xi, \varphi, \phi)$ and the right-hand sides of these equations are functions of the observables V_{ij}, and when $\bar{c}_{11}$, $\bar{c}_{12}$, $\bar{c}_{44}$ are known, the information theory approach can be used successfully for determining the probability density function $p(\xi, \varphi, \phi)$ in the long-wavelength and ideal polycrystal approximation. It can be done for uniaxially rolled material from three of Eqs. (5) – (10) with such three functions $r_q(\xi, \varphi, \phi)$, which are linearly independent on each other. This requirement is fulfilled by three equations under testing, if each of the numbers 1, 2, and 3 appears as subscripts i or/and j at no more than two velocities involved in the equations [1]. Such a system of three equations was called in the Ref. [2] the *basic system of three equations*. From results of Refs. [1, 2] it follows that the measurements of three propagation velocities involved in the basic system of three equations, e.g. V_{11}, V_{33} and V_{13}, are sufficient for the probability function $p(\xi, \varphi, \phi)$ to be fully determined for aggregates with orthorhombic macroscopic symmetry and when $\bar{c}_{11}$, $\bar{c}_{12}$ $\bar{c}_{44}$ are known. Then the probability density function $p(\xi, \varphi, \phi)$ implied by the JAYNES' [10] principle of maximum Shannon entropy is given in terms used in [2, Appendix] by the following expression

$$p(\xi, \varphi, \phi) = \frac{1}{Z} \exp\left[-L_1 r_1(\xi, \varphi, \phi) - L_3 r_3(\xi, \varphi, \phi) - L_5 r_5(\xi, \varphi, \phi)\right], \tag{15}$$

where the partition function Z and the Lagrangian multipliers L_1, L_3 and L_5 may be determined from Eqs. (3), (5), (7) and the normalization condition (14).

The method proposed in Ref. [1] has been improved in Ref. [2] by avoiding the limiting assumptions concerning the length of ultrasonic waves (long-wavelength approximation) and the absence of imperfections (e.g. voids, imperfect adhesion of neighbouring grains, residual stress, impurities) of the material of the polycrystalline aggregate and its microstructure. In Ref. [2], a theoretical approach has been proposed utilizing the same as in Ref. [2] information theory method to determine the probability density function $p(\xi, \varphi, \phi)$ and the material parameters $\bar{c}_{11} \doteq c_{11}^{(\mathrm{eff})}/\rho$, $\bar{c}_{12} \doteq c_{12}^{(\mathrm{eff})}/\rho$, $\bar{c}_{44} \doteq c_{44}^{(\mathrm{eff})}/\rho$ from the rules of macroscopic orthorhombic symmetry and the results of the measurements of four respectively chosen ultrasonic velocities, V_{ij}, three of them being involved in the basic system of equations.

In the remainder of this paper, the procedure proposed in Ref. [2] will be utilized for solving the problem of determination the function $p(\xi, \varphi, \phi)$ and the single-crystal

material parameters $\overline{c}_{11}$, $\overline{c}_{12}$, and $\overline{c}_{44}$ in the case when the same three ultrasonic velocities Ve_{11}, Ve_{33}, Ve_{13} and additionally one of the velocities Ve_{22}, Ve_{33} and Ve_{12}, namely Ve_{23}, are known. Similarly as in Ref. [2], henceforth, a value of V_{ij}, which will be regarded to be obtained experimentally, will be denoted Ve_{ij}. The ultrasonic measurements were performed in the twelve observation points on the real material under examination. Hence, the values of measured velocities, Ve_{ij}, contain information on the structure and properties of the material as well as on the phenomena influencing on the propagation and occurring in the polycrystal under examination. Therefore, it may be expected that the values of Ve_{ij} together with the symmetry rules allow us to determine the function $p(\xi, \varphi, \phi)$ as well as the material parameters $\overline{c}_{11}$, $\overline{c}_{12}$, and $\overline{c}_{44}$. In this case, assuming that the probability density function $p(\xi, \varphi, \phi)$ is implied by the JAYNES' [10] principle of maximum Shannon entropy, $p(\xi, \varphi, \phi)$ is also of the form given by Eq. (15). Contrary to Ref. [1], now the partition function Z and the Lagrangian multipliers L_1, L_3 and L_5 are to be determined together with the material parameters $\overline{c}_{11}$, $\overline{c}_{12}$ and $\overline{c}_{44}$ from the seven Eqs. (3) – (8), (14), after setting the results of the measurements of the ultrasonic velocities Ve_{11}, Ve_{33}, Ve_{13}, and Ve_{23}. Therefore, from the four velocities Ve_{ij}, the maximum-entropy estimate of the probability density function, $p(\xi, \varphi, \phi)$, will be determined together with the material parameters $\overline{c}_{11}$, $\overline{c}_{12}$ and $\overline{c}_{44}$, after enlarging the approach presented in paper [1] by including a self-consistent computational procedure proposed in Ref. [2]. In using such a procedure the dynamic parameters of the ideal single-crystal material, $\overline{c}_{11}^{(0)} = c_{11}^{(0)}/\rho$, $\overline{c}_{12}^{(0)} = c_{12}^{(0)}/\rho$, $\overline{c}_{44}^{(0)} = c_{44}^{(0)}/\rho$, should be replaced by the effective ones, $\overline{c}_{11}$, $\overline{c}_{12}$, $\overline{c}_{44}$, of the crystallite in the real bulk specimen, the effective moduli being calculated also from the four ultrasonic velocities and symmetry rules given by Eqs. (9). Contrary to the problem defined Ref. [1], the problem of finding the maximum-entropy estimate of the function $p(\xi, \varphi, \phi)$ and the values of material parameters, $\overline{c}_{11}$, $\overline{c}_{12}$, $\overline{c}_{44}$, from the results of the measurements of four propagation velocities Ve_{ij} of ultrasonic waves and from the rules of macroscopic orthorhombic symmetry is not unambiguous. To make a choice between numerous solutions to the problem, LEWANDOWSKI [2] proposed the *criterion of the minimum relative difference* between the values of the material parameters, $\overline{c}_{11}$, $\overline{c}_{12}$, $\overline{c}_{44}$, obtained in the procedure of the maximum-entropy estimate, and their analogues, $\overline{c}_{11}^{(0)}$, $\overline{c}_{12}^{(0)}$, $\overline{c}_{44}^{(0)}$, referring to the same (or the most similar) ideal material in the virgin (before deformation) state. Obviously, a great difficulty of the analysis is encountered when one wishes to find the solution to such a problem of great complexity. In considering this problem, only the concepts and equations required in this study are reiterated herein after [2].

In the present paper, the method proposed in Ref. [2] is utilized for the estimation of the local texture and local basic material parameters of a steel plate which was rolled uniaxially in the situation where the edges parallel to the rolling direction were free. In this way, we seek the answer on the question how strongly are some local material parameters and the local texture influenced on by the distance x between an observation point $\mathbf{r}$ on a plate and one of the two plate edges, which are parallel to the rolling direction.

3. Measurements and numerical analysis

The measurements have been confined to measuring only the ultrasonic velocities Ve_{11}, Ve_{33}, Ve_{13}, which are involved in the so-called basic system of three equations, and additionally the velocity Ve_{32}. The twelve equally spaced *observation* (measurement) *points* have been chosen as lying on a straight line in the rolling plane, the straight line being perpendicular to the $0x_1$ (rolling) direction. The measurement points have been chosen at the distances $x = (0.02, 0.04, 0.06, ..., 0.24)$ m from the reference $0x_1$-edge of the plate.

The volume of a macroscopic sample is regarded as large enough to include a large number of crystallites with each of the occurring orientations. Then, it is reasonable to assume that the measured velocities of ultrasonic pulses propagating through such a macroscopic sample (the pulses being generated by a transducer oscillating normally or transversely to the coupling surface) are equal to the propagation velocities of the respective ultrasonic waves.

The propagation velocities of waves propagating normally to the rolling plane (Ve_{33}, Ve_{32}) have been measured by using commercially available wide-band ultrasonic transducers having a maximum middle frequency of 2.5 MHz. By making use of reflection, a single transducer served as both a transmitter (source) and a receiver (detector). The longitudinal and shear waves propagating parallel to the rolling plane, i.e. Ve_{11} and Ve_{13}, respectively, have been generated and detected by using transducers mounted on Plexiglas wedges and inclined at such angles to the plate surface that ensure the values of the refraction angles of the waves to be nearly critical (equal to 90°). These pieces of equipment allow the transit time of acoustic waves to be measured for all desired directions. The transit distance is to be regarded for the first pair of waves (Ve_{33}, Ve_{32}) as the double plate thickness at the region of the coupling of the transducer and plate, the plate thickness being equal to 0.02 m. Similarly, the transit distances are to be regarded for the second pair of waves, Ve_{11} and Ve_{13}, as the distances between the geometrical centres of the regions of coupling of the plate with the transmitter and receiver, the distances being equal to 0.182 m and 0.0985 m, respectively. The propagation velocities of the second pair of waves deduced from the transit times are regarded to be the velocities at the distances x from the reference $0x_1$-edge, x being determined by the middles of the respective distances between the geometrical centres of the regions of coupling of the plate with the transmitter and receiver. The transit time was measured with an error equal to $5 \cdot 10^{-10}$ s. The final results of the determination of the ultrasonic velocities for the twelve values of x from the transit time are presented in Figs. 1 – 2.

In the subsequent numerical analysis, we examine the texture in the twelve equally spaced observation points by utilizing for each observation point separately the results of the measurements of the ultrasonic velocities Ve_{11}, Ve_{33}, Ve_{13}, which are involved in the basic system of three equations, and additionally the velocity Ve_{32}. It means that first we find for each observation point separately the analytical form of the maximum-entropy probability density function $p(\xi, \varphi, \phi)$, which is that given by Eq. (15). On evaluating the material parameter $\overline{c}_a$ for each observation point from Eq. (9), after substituting $V_{ij} = Ve_{ij}$, $j = 1, 2, 3$, the second step was to calculate the two missing velocities Ve_{12}

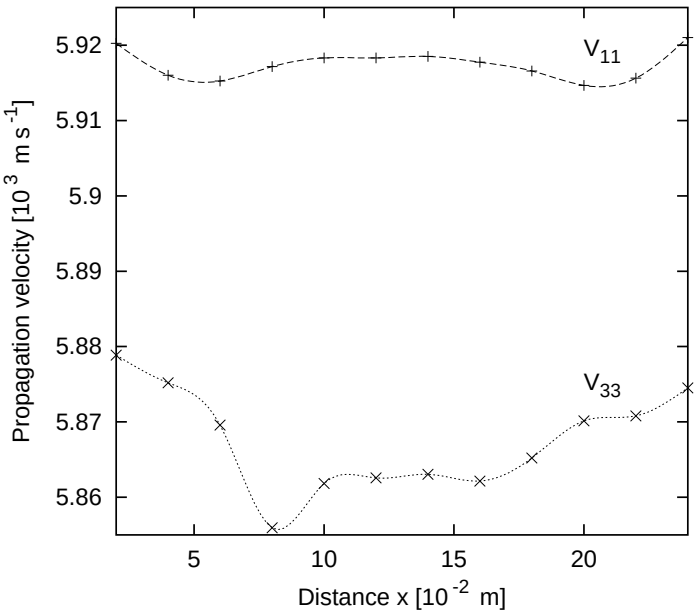

Fig. 1. Velocities Ve_{11} and Ve_{33} of longitudinal waves plotted against the distance x between an observation point on the steel plate and one of the two plate edges parallel to the rolling direction.

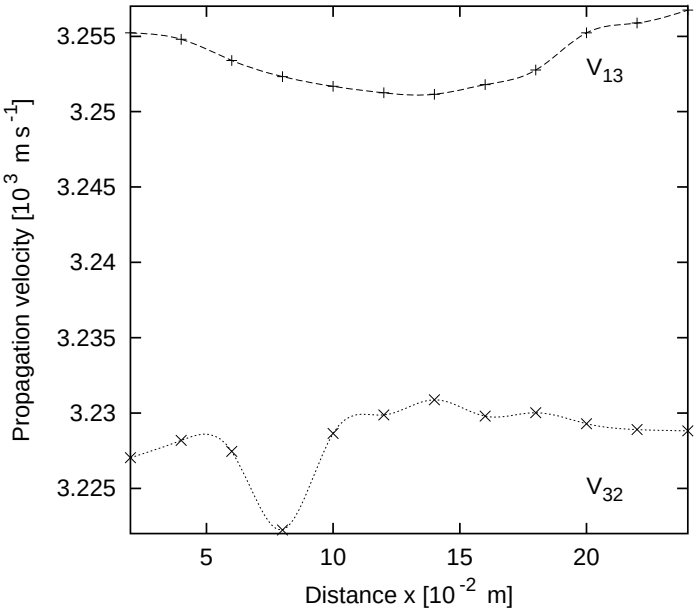

Fig. 2. Velocities Ve_{13} and Ve_{32} of shear waves plotted against the distance x between an observation point on the steel plate and one of the two plate edges parallel to the rolling direction.

and Ve_{22} (for each observation point as well) from the following equations:

$$\begin{aligned} Ve_{12} &= \sqrt{C_a - Ve_{11}^2 - Ve_{31}^2}\,, \\ Ve_{22} &= \sqrt{C_a - Ve_{12}^2 - Ve_{23}^2} \end{aligned} \tag{16}$$

which are deduced from the macroscopic othorhombic symmetry, i.e. are derived also from Eq. (9). In this way for each observation point we have arrived at the following set $\{Ve_{ij}\}$ of the values of the six velocities

$$\{Ve_{ij}\} = \{Ve_{11},\, Ve_{22},\, Ve_{33},\, Ve_{12},\, Ve_{23},\, Ve_{31}\}. \tag{17}$$

As it will be pointed out below, the knowledge of the set (17) enable us to make some check of the actual accuracy of calculating digitally the partition function Z, Lagrangian multipliers L_1, L_3, L_5 and material parameters $\bar{c}_{11}$, $\bar{c}_{12}$, $\bar{c}_{44}$. In the situation where the material parameters $\bar{c}_{11}$, $\bar{c}_{12}$ and $\bar{c}_{44}$ are unknown, the task consists of finding Z, L_1, L_3, L_5, $\bar{c}_{11}$, $\bar{c}_{12}$ and $\bar{c}_{44}$ for each observation point separately from Eqs. (3) – (8), (14). Seeking the texture for each of the twelve observation points, we utilize the numerical procedure proposed in Ref. [2] for each such a point separately. Any full description of the numerical method will be omitted from this paper for the sake of brevity.

As was mentioned above, the partition function Z, Lagrangian multipliers L_1, L_3, L_5 and single-crystallite (grain) material parameters $\bar{c}_{11}$, $\bar{c}_{12}$, $\bar{c}_{44}$ are to be determined from Eqs. (3) – (8), (14). However, these equations present so complicated nonlinear dependencies of the quantities Z, L_1, L_3, L_5, $\bar{c}_{11}$, $\bar{c}_{12}$ and $\bar{c}_{44}$ on each other that the problem of

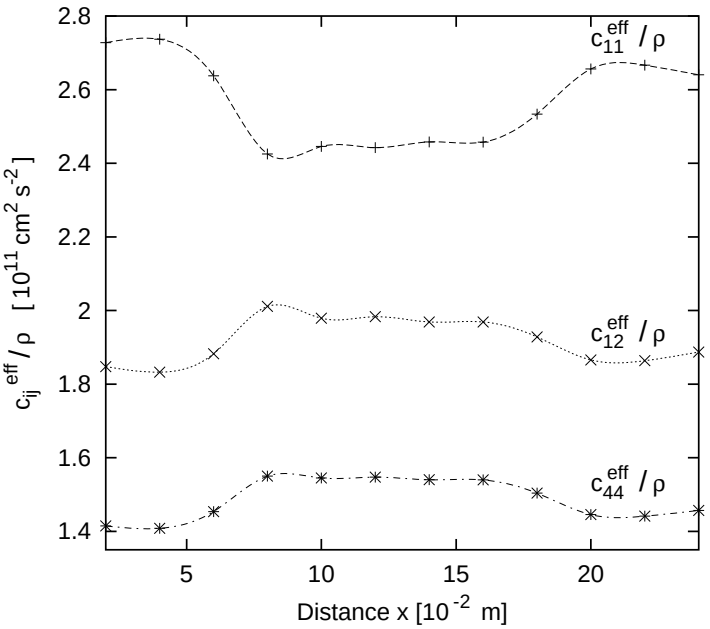

Fig. 3. Effective single-crystallite material parameters $\bar{c}_{11}$, $\bar{c}_{12}$, and $\bar{c}_{44}$ calculated from ultrasonic measurements plotted against the distance x between an observation point on the steel plate and one of the two plate edges parallel to the rolling direction.

evaluating these quantities from Eqs. (3) – (8), (14) is not unambiguous. This raises the need to make a choice between numerous solutions of the problem. In this way arises the need to provide a constructive criterion for choosing one set of Z, L_1, L_3, L_5, $\overline{c}_{11}$, $\overline{c}_{12}$ and $\overline{c}_{44}$ from all such sets satisfying Eqs. (3) – (8), (14). Following Ref. [2], in the subsequent text we confine ourselves only to present numerical results obtained in the situation where the criterion of the minimum value of a difference is used, the proposal of the criterion of the minimum difference being described after Ref. [2] below.

First we suppose that the rolled polycrystalline material (steel) we are dealing with herein is such that, on one hand, all the values of the effective material parameters, $\overline{c}_{11}$, $\overline{c}_{12}$ and $\overline{c}_{44}$, of a single cubic crystal in the bulk sample of the rolled material are unknown and, on the other hand, all the parameter values, $\overline{c}_{11}^{(0)}$, $\overline{c}_{12}^{(0)}$ and $\overline{c}_{44}^{(0)}$, of a single cubic crystal of the polycrystal material (or a material as similar to that as possible) in the virgin state (before deformation) are known from measurements. In accordance with the criterion of the minimum difference, $\overline{c}_{11}$, $\overline{c}_{12}$ and $\overline{c}_{44}$, are as close to $\overline{c}_{11}^{(0)}$, $\overline{c}_{12}^{(0)}$ and $\overline{c}_{44}^{(0)}$, respectively, as it is allowed by Eqs. (3) – (8), (14). Moreover, if we are interesting in rolled steel, similarly as in Refs. [1, 2], and its parameters' values $\overline{c}_{11}^{(0)}$, $\overline{c}_{12}^{(0)}$ and $\overline{c}_{44}^{(0)}$ are unknown, it is supposed that such a virgin material for the rolled steel may be approximated by BCC Fe, which is characterized by the following values of $C_{11}^{(0)}$, $C_{12}^{(0)}$, $C_{44}^{(0)}$:

$$\begin{aligned} \overline{c}_{11}^{(0)} &= 2.5982d + 07 \left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2, \\ \overline{c}_{12}^{(0)} &= 1.6857d + 07 \left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2, \\ \overline{c}_{44}^{(0)} &= 1.5843d + 07 \left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2. \end{aligned} \tag{18}$$

Now we define the difference parameter Qc by the following formula, using the FORTRAN 77 intrisinc functions DMAX1 and DABS

$$Qc = \mathrm{DMAX1}\,(Gc_{11},\, Gc_{12},\, Gc_{44}). \tag{19}$$

The nomenclature introduced in Eq. (17) is as follows:

$$Gc_{ij} = \mathrm{DABS}\left[(\overline{c}_{ij} - \overline{c}_{ij}^{(0)})/\overline{c}_{ij}^{(0)}\right]. \tag{20}$$

According to the choice rule applied herein after Ref. [2], we use this set of the values of Z, L_1, L_3, L_5, $\overline{c}_{11}$, $\overline{c}_{12}$ and $\overline{c}_{44}$ satisfying Eqs. (3) – (8), (14), *which contains such values of the material parameters* $\overline{c}_{11}$, $\overline{c}_{12}$ and $\overline{c}_{44}$ *that lead to the minimum value of the difference parameter* Qc *and simultaneously contains such values of* Z, L_1, L_3, L_5 *that lead to the probability density function* $p(\xi, \varphi, \phi)$ *achieving the maximum value of Shannon entropy*. In this way, we formulate the criterion of the minimum difference.

On finding for each of the twelve observation points the partition function Z and Lagrangian multipliers L_1, L_3, L_5, the maximum-entropy probability density functions, $p(\xi, \varphi, \phi)$, are known for all the observation points. Then the next step was to make the

use of the functions $p(\xi, \varphi, \phi)$, and the material parameters $\bar{c}_{11}$, $\bar{c}_{12}$ and $\bar{c}_{44}$ for calculating the ultrasonic velocities Vm_{11}, Vm_{22}, Vm_{33}, Vm_{12}, Vm_{23}, Vm_{31} for each of the twelve observation points. In the remainder of this paper, the value of a velocity V_{ij} will be denoted by Vm_{ij}, if it is calculated in the maximum-entropy approximation. Thus on the basis of the previously determined maximum-entropy estimate of the probability density function $p(\xi, \varphi, \phi)$ and the material parameters $\bar{c}_{11}$, $\bar{c}_{12}$ and $\bar{c}_{44}$, all the six ultrasonic velocities Vm_{11}, Vm_{22}, Vm_{33}, Vm_{12}, Vm_{23}, Vm_{31} were calculated from Eqs. (3) – (8) for each observation point. By comparing in pairs the values of velocities Vm_{11}, Vm_{22}, Vm_{33}, Vm_{12}, Vm_{23}, Vm_{31} with their analogues defined by Eqs. (17), it was possible to have some check of the actual accuracy of calculating digitally the partition function Z, Lagrangian multipliers L_1, L_3, L_5 and material parameters $\bar{c}_{11}$, $\bar{c}_{12}$ and $\bar{c}_{44}$ by employing a numerical method, which consists of a succession of iterations with increasing accuracy of calculation. To have some estimation of the actual accuracy of calculation, the error parameter Q_m has been used. Q_m had been defined in Ref. [2] using the FORTRAN 77 intrisinc function, DMAX1, which returns the maximum value in the argument list. Q_m had been defined by the following formula:

$$Q_m = \mathrm{DMAX1}(Gm_{11}, Gm_{22}, Gm_{33}, Gm_{12}, Gm_{23}, Gm_{31}). \tag{21}$$

The nomenclature introduced in Eq. (21) is as follows

$$Gm_{ij} = \mathrm{DABS}\left[(Vm_{ij} - Ve_{ij})/Ve_{ij}\right], \tag{22}$$

where the FORTRAN 77 intrisinc function DABS returns the absolute value of its argument.

In solving numerically the system of Eqs. (3) – (8), (14) with respect to Z, L_1, L_3, L_5, $\bar{c}_{11}$, $\bar{c}_{12}$ and $\bar{c}_{44}$ for each of the twelve observation points, the succession of iterations with increasing accuracy of calculation was continued as long as the error parameter Q_m became less than 1.0×10^{-6}. In this way we obtained the numerical results given in Table 1.

In each of the twelve columns of Table 1, there is presented a numerical solution of the system of Eqs. (3) – (8), (14) with respect to Z, L_1, L_3, L_5, $\bar{c}_{11}$, $\bar{c}_{12}$ and $\bar{c}_{44}$, which are calculated with an exactness characterized by the values of the parameters of error, Q_m, and difference, Q_c. The values of Q_m and Q_c are given in each column in the before last row and in the last one, respectively. In all columns, the error parameters Q_m are less than 1.1×10^{-8}. The x-th column ($x = 2, 4, 6, \ldots, 24$ cm) is the set of the values of Z, L_1, L_3, L_5, $\bar{c}_{11}$, $\bar{c}_{12}$ and $\bar{c}_{44}$ which satisfies Eqs. (3) – (8), (14), after inserting the results of the measurements the observables V_{11}, V_{33}, V_{13}, and V_{32} performed in the x-th observation point and employing the symmetry rules given by Eqs. (9). Moreover, one can say that the x-th column contains such values of the material parameters $\bar{c}_{11}$, $\bar{c}_{12}$ and $\bar{c}_{44}$ that lead to the minimum value of the difference parameter, Q_c, and simultaneously contains such values of Z, L_1, L_3, L_5 that lead to the maximum-entropy estimate of the function $p(\xi, \varphi, \phi)$ for the observables V_{11}, V_{33}, V_{13}, (and V_{32}). For these reasons, the values of Z, L_1, L_3, L_5, $\bar{c}_{11}$, $\bar{c}_{12}$ and $\bar{c}_{44}$, which are given in each column, should be regarded in the paper as the solution to the problem under consideration for the respective observation point.

Table 1. Results of numerical calculations of Lagrangian multipliers and material parameters from ultrasonic measurements for twelve observation points.

x [cm]	2	4	6	8
Z	6.98979	7.01548	7.12077	7.07534
L_1	−1.51127138407	−1.52080828560	−1.52376497056	−1.59628175935
L_3	0.839757785471	0.90380655508	0.89713214274	1.00258928214
L_5	1.30840170534	1.292058967	1.22100600410	1.2274019498
$\bar{c}_{11}$ [m^{2}s^{-2}]	2.72759×10^7	2.73678×10^7	2.63767×10^7	2.42525×10^7
$\bar{c}_{12}$ [m^{2}s^{-2}]	1.84753×10^7	1.83255×10^{-1}	1.88247×10^7	2.01129×10^7
$\bar{c}_{44}$ [m^{2}s^{-2}]	1.41477×10^7	1.40824×10^7	1.45381×10^7	1.55000×10^7
$\bar{c}$ [m^{2}s^{-2}]	-1.37794×10^7	-1.16776×10^7	-0.896665×10^7	-0.894442×10^7
$\bar{c}_a$ [m^{2}s^{-2}]	5.55712×10^7	5.55326×10^7	5.54530×10^7	5.52525×10^7
Q_m	9.85135×10^{-7}	9.22084×10^{-7}	8.68158×10^{-7}	1.54281×10^{-7}
Q_c	1.29988×10^{-1}	1.35226×10^{-1}	0.99639×10^{-1}	0.98409×10^{-1}

x [cm]	10	12	14	16
Z	7.41466	7.30817	7.43995	7.44211
L_1	−1.61829511744	−1.52896286792	−1.60507903061	−1.61990847820
L_3	0.85983673731	0.911152470345	0.87014804885	0.85225414589
L_5	1.14268717833	1.07185579723	1.09977675460	1.13241671318
$\bar{c}_{11}$ [m^{2}s^{-2}]	2.44562×10^7	2.44249×10^7	2.45795×10^7	2.45747×10^7
$\bar{c}_{12}$ [m^{2}s^{-2}]	1.97888×10^7	1.98309×10^7	1.96878×10^7	1.96895×10^7
$\bar{c}_{44}$ [m^{2}s^{-2}]	1.54512×10^7	1.54737×10^7	1.54021×10^7	1.53979×10^7
$\bar{c}$ [m^{2}s^{-2}]	-2.62351×10^7	-2.63534×10^7	-2.59126×10^7	-2.59106×10^7
$\bar{c}_a$ [m^{2}s^{-2}]	5.53586×10^7	5.53723×10^7	5.53838×10^7	5.53705×10^7
Q_m	9.69190×10^{-7}	9.77151×10^{-7}	8.52790×10^{-7}	9.45054×10^{-7}
Q_c	1.40431×10^{-1}	1.42255×10^{-1}	1.36023×10^{-1}	1.36097×10^{-1}

x [cm]	18	20	22	24
Z	7.38138	7.21346	7.18267	7.19703
L_1	−1.61571682028	−1.55860866956	−1.60577916205	−1.60124022736
L_3	0.85416318209	0.86426789517	0.85955105606	0.85111195804
L_5	1.16982116073	1.22217644441	1.30172316513	1.29450264631
$\bar{c}_{11}$ [m^{2}s^{-2}]	2.53361×10^7	2.65628×10^7	2.66659×10^7	2.64029×10^7
$\bar{c}_{12}$ [m^{2}s^{-2}]	1.92828×10^7	1.86581×10^7	1.86364×10^7	1.88732×10^7
$\bar{c}_{44}$ [m^{2}s^{-2}]	1.50390×10^7	1.44553×10^7	1.44134×10^7	1.45692×10^7
c [m^{2}s^{-2}]	-2.40246×10^7	-2.10060×10^7	-2.07972×10^7	-2.16058×10^7
$\bar{c}_a$ [m^{2}s^{-2}]	5.54140×10^7	5.54734×10^7	5.54927×10^7	5.55414×10^7
Q_m	4.79940×10^{-7}	9.42212×10^{-7}	3.86125×10^{-7}	9.66848×10^{-7}
Q_c	1.17873×10^{-1}	1.05939×10^{-1}	1.09158×10^{-1}	0.98728×10^{-1}

The degree and type of the texture in each of the twelve observation points can be determined in the fullest detail by making use of the function $p(\xi, \varphi, \phi)$, which gives the probability density of a given crystallite having a specified orientation with respect to the axes of the Cartesian coordinate system of the sample (plate), the crystallite being placed in the region of the considered observation point in the sample. Following Refs. [2], we use the quantities

$$n_\varphi(\varphi_2, \varphi_1) = \int_0^{2\pi} \int_{\varphi_1}^{\varphi_2} \int_{-1}^{1} p(\xi, \varphi, \phi)\, d\xi\, d\varphi\, d\phi, \tag{23}$$

$$n_\theta(\theta_2, \theta_1) = \int_0^{2\pi} \int_0^{2\pi} \int_{\xi_2}^{\xi_1} p(\xi, \varphi, \phi)\, d\xi\, d\varphi\, d\phi, \tag{24}$$

as examples of such specifications. Here

$$\theta_1 = \arccos \xi_1\,, \qquad \theta_2 = \arccos \xi_2\,, \qquad 0 \le \theta_2 \le \pi \tag{25}$$

and $n_\varphi(\varphi_1, \varphi_2)$, $n_\theta(\theta_1, \theta_2)$ denote the fractions of the total number of crystallites (in the region of the considered observation point in the sample) with the angle of precession, φ, lying in the interval $\varphi_1 \le \varphi \le \varphi_2$ and with the angle of nutation, θ, lying in the interval $\theta_1 \le \theta \le \theta_2$, respectively. In Fig. 4, examples of numerical calculations of $n_\varphi(\varphi_1, \varphi_2)$ are

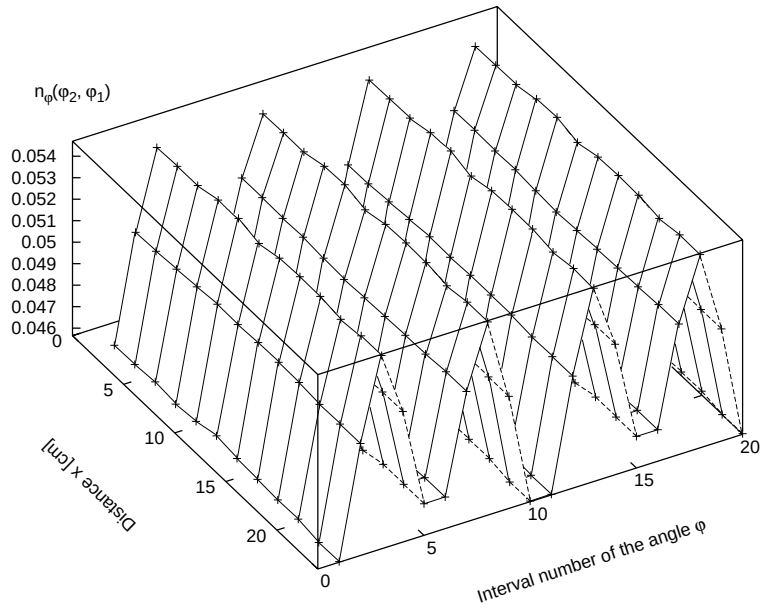

Fig. 4. $n_\varphi(\varphi_2, \varphi_1)$ defined by Eq. (23) and plotted against both the distance x between an observation point on the steel plate and one of the two plate edges parallel to the rolling direction as well as against the the number of each subdomain of the precession angle φ, the whole domain $[0°, 360°]$ of the precession angle φ being divided into parts (subdomains) of equal size, $18°$, numbered from 1 to 20, with centres at $\varphi_0 = (\varphi_1 + \varphi_2)/2 = 9°$, $27°$, $45°$, ..., $351°$.

presented (for each of the twelve observation points) with the whole domain $[0°, 360°]$ of the precession angle φ being divided into parts (subdomains) of equal size, $18°$, with centres at $\varphi_0 = (\varphi_1 + \varphi_2)/2 = 9°$, $27°$, $45°$, ..., $351°$. Similarly, in Fig. 5, examples of numerical calculations of $n_\theta(\theta_1, \theta_2)$ are presented (also for each of the twelve observation points) with the whole domain $[0°, 180°]$ of the nutation angle θ being divided into parts (subdomains) of equal size, $18°$, with centres at $\theta_0 = (\theta_1 + \theta_2)/2 = 9°$, $27°$, $36°$, ..., $171°$. For each region of the twelve observations points, the particle fractions $n_\varphi(\varphi_1, \varphi_2)$ and $n_\theta(\theta_1, \theta_2)$ were calculated separately for each subdomain and the results of these calculations are presented in Figs. 4 and 5.

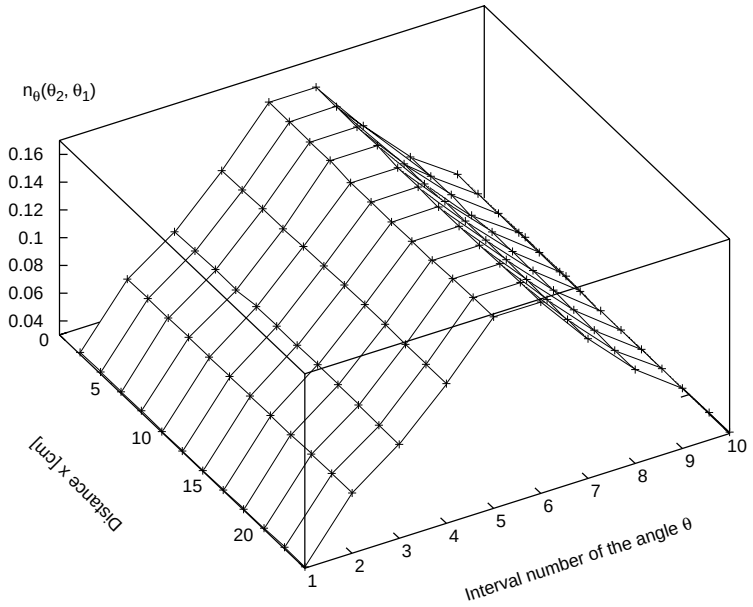

Fig. 5. $n_\theta(\theta_2, \theta_1)$ defined by Eq. (24) and plotted against both the distance x between an observation point on the steel plate and one of the two plate edges parallel to the rolling direction as well as against the number of each subdomain of the nutation angle θ, the whole domain $[0°, 180°]$ of the nutation angle θ being divided into parts (subdomains) of equal size, $18°$, numbered from 1 to 10, with centres at $\theta_0 = (\theta_1 + \theta_2)/2 = 9°$, $27°$, $36°$, ..., $171°$.

From Figs. 4 and 5 it can easily be seen that rolling, say, in the $0x_1$ direction, leaves the crystallites in some non-random orientations. The statistics of the forced non-random orientations leads to the occurrence of the most preferred intervals of the Euler angles, the preferred orientation being a periodic function of the Euler angles. Obviously, all the symmetry properties of $n_\varphi(\varphi_1, \varphi_2)$ and $n_\theta(\theta_1, \theta_2)$, which are shown in Figs. 4 and 5, result in the orthorhombic symmetry of the macroscopic mechanical properties of the polycrystalline aggregate (rolled steel). Similarly as in Ref. [2], the preference of the crystallite orientations, which is revealed by the numerical results presented in Figs. 4 and 5, can be defined in the crystallographic terms as follows: If a solid plate made of

cubic crystallites with the highest symmetry is uniaxially rolled, say, in the $0x_1$ direction, the rolling process leaves the crystallites in a non-random orientations with an orientation preference for the crystallographic plane $[1,1,0]$ to be parallel to the rolling plane x_1x_2 as well as for the crystallographic direction $\langle 1,1,0\rangle$ to be parallel to the rolling direction Ox_1.

4. Final remarks and conclusions

This paper is concerned with the nondestructive ultrasonic method of the estimation of the local texture and local basic material parameters of a steel plate which was rolled uniaxially in the situation where the edges parallel to the rolling direction were free. The aim of the paper was also to estimate how strongly are the local texture and mechanical properties influenced on by the distance x between an observation point determined by the position vector $\mathbf{r}$ and one of the two plate edges parallel to the rolling direction, in the situation where the edges parallel to the rolling direction were free during the rolling process. In the presented method, there is involved the inversion of the problem of calculating the ultrasonic velocities from texture with making use of the Voigt averaging procedure. The inversion, which has been performed with using the information theory approach, leads to the maximum-entropy estimate of the probability density function of the crystallite orientation, $p(\xi,\varphi,\phi)$. This function for each of the twelve observation points is of the form given by Eq. (15), this form being implied by Eqs. (3), (5), (7), (14) under the assumption that the ultrasonic velocities (observables) Ve_{11}, Ve_{33} and Ve_{13} in these regions are known from measurements. Next the same Eqs. (3), (5), (7), (14) together with Eqs. (4), (6), (8), (9) and the criterion of the minimum difference are used for determining, for each of the twelve observation points successively and separately, both the exact form of $p(\xi,\varphi,\phi)$ (by evaluating the three Lagrangian multipliers L_1, L_3, L_5 and normalization constant Z) and the values of three single-crystal material parameters ($\overline{c}_{11} \doteq c_{11}^{(\mathrm{eff})}/\rho$, $\overline{c}_{12} \doteq c_{12}^{(\mathrm{eff})}/\rho$, $\overline{c}_{44} \doteq c_{44}^{(\mathrm{eff})}/\rho$). These quantities and parameters are to be calculated from the values of four observables (Ve_{11}, Ve_{33}, Ve_{13} and Ve_{32}) and orthorhombic symmetry rules given by Eqs. (9).

The analysis presented in Ref. [2] leads to the relations between the probability density function of the crystallite orientation, $p(\xi,\varphi,\phi)$, as well as the values of single-crystal material parameters $\overline{c}_{ij}$, $ij = 11,\ 12,\ 44$, of the rolled material and ultrasonic velocities (Ve_{11}, Ve_{33}, Ve_{13} and Ve_{32}). The present study, moreover, leads to showing that the quantities and parameters involved in the analysis performed by using the method of Ref. [2] are considerably influenced on by the distance $x = (0.02,\ 0.04\,0.06, ...,\ 0.24)$ m between an observation point on the steel plate and one of the two plate edges parallel to the rolling direction, the edges being free during the rolling process. If along a straight line of distance x the rolling load and other forces acting in the rolling process were constant and the microstructure of the material in the virgin (before deformation) state had been statistically homogeneous, then the changes in the values of all the quantities and parameters (Ve_{11}, Ve_{33}, Ve_{13}, Ve_{32}; $p(\xi,\varphi,\phi)$, L_1, L_3, L_5, Z; $\overline{c}_{ij}$, $ij = 11,\ 12,\ 44$), which are observed in the material in a plane parallel to the rolling plane, would be both

constant along the straight line and symmetrical with respect to the axis of geometrical symmetry of the plate, the symmetry axis being parallel to the rolling direction. From Table 1 and Figs. 1 – 3 it can immediately be seen that the deviations of the changes from the axial symmetry are considerable in the case of the material under examination. These deviations contain information on the local inhomogeneity of the material under investigation.

In the presented method, there is involved the inversion of the problem of the relations between the probability density function of the crystallite orientation, $p(\xi, \varphi, \phi)$, and the values of the effective single-crystal material parameters, $\overline{c}_{ij}$, $ij = 11, 12, 44$, of the rolled material as well as of the measured ultrasonic velocities (Ve_{11}, Ve_{33}, Ve_{13} and Ve_{32}). This analysis has been performed for the polycrystalline aggregate with orthorhombic macroscopic symmetry, the aggregate being composed of cubic crystals. In every heterogeneous elastic body, the ultrasonic velocities depend on the effective density and components of the so-called effective dynamic tensor of stiffness of the bulk sample as well as on the frequency of the ultrasonic waves. In turn, the components of the effective dynamic tensor of stiffness of the bulk sample are determined by the probability density function of the crystallite orientation, $p(\xi, \varphi, \phi)$, as well as by the values of the effective single-crystal material parameters, $\overline{c}_{ij}$, $ij = 11, 12, 44$. In the limit, as the wavelength increases to infinity (or the frequency diminishes to zero), the dynamic effective moduli in these relations may be replaced by the static effective moduli, if the polycrystal under consideration may be regarded as an ideal polycrystalline aggregate. In accordance with the long-wavelength approximation and in view of the common opinion that plastic deformation does not induce any considerable changes in the static values of single-crystal material parameters, c_{ij}/ρ, $ij = 11, 12, 44$, it seems to be reasonable to replace the effective dynamic values of these material parameters, which are involved in problems of ultrasonic testing of plastically deformed materials, by their static values measured before deformation. However, from the the twelve examples, which has been considered above and in which $9.84\% < Q_c < 14.23\%$, it can easily be seen that such assumptions may lead to considerable errors in analysing problems concerning the application of ultrasonic methods in material science, because observed changes in acoustic (ultrasonic) anisotropy may be accompanied by considerable changes in the values of c_{ij}/ρ, $ij = 11, 12, 44$.

In this context, it can easily be seen the advantage of the approach applied above and proposed by LEWANDOWSKI in [2] over that which are based on the long-wavelength and ideal polycrystal approximations. By contrast, one might claim that the approach proposed in this paper is more free because it allows us to determine completely the effective single-grain dynamic properties and the texture of the bulk sample under examination from experimentally observed data (velocities Ve_{11}, Ve_{33}, Ve_{13}, Ve_{32}) and Eqs. (3) – (8), (9), (14). In this way we avoid neglecting the effects of the changes in the grain shape (morphological texture) as well as the influence of the distributions of such material defects as voids, impurities, residual stress, etc. on the mechanical and propagation properties of the polycrystalline aggregate made of steel. In this approach we also avoid neglecting the influence of the scattering and mechanical energy dissipation on the propagation of ultrasonic waves.

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