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# SOME NOISE CANCELLATION AND PREDICTION METHODS ON THE RESPONSE PROBABILITY DISTRIBUTION FORM FOR COMPLICATED SOUND WALL SYSTEMS WITH BACKGROUND NOISE

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In the actual situation of measuring an environmental noise, it is very often that only the resultant stochastic fluctuation contaminated by an additional noise of arbitrary distribution type can be observed. In this paper at first a noise cancellation for reasonably removing the effect of the above additional noise, especially in a whole probability distribution form, is derived theoretically in order to estimate only the undisturbed objective output response. Next, for the purpose of predicting a whole expression form of the output response probability of an acoustic system excited by an arbitrary stochastic input with the additional noise, a new stochastic signal processing method, reflecting the effect of the additional noise fluctuation, is proposed in a whole probability distribution form. The effectiveness of the proposed theoretical methods is experimentally confirmed too by applying them to the actual data measured in the complicated sound wall systems.

## 1. Introduction

In the actual measurement of environmental noise, the desired signal is usually contaminated by an additional noise of an arbitrary distribution type and it is only the resultant signal that can be observed [1].

In this paper, at first, a new practical trial of estimating (especially in a whole probability distribution form) the uncontaminated output response probability of sound wall systems with background noise is derived without using any artificial error criterion like the least-squares method. More concretely, a mathematical model of arbitrary sound environmental systems is introduced by using a physical law of additive principle on the energy scale [2] in a form of a linear system on the intensity scale. At first, after introducing a probability expression form of the resultant output response contaminated by the background noise, a noise cancellation method in a whole probability distribution form is developed by which only the uncontaminated output response probability function form for the above sound environmental systems can be detected from the data contaminated by the background noise. Next, for the purpose of predicting the output response probability excited by an arbitrary stochastic input with background noise, a new signal processing method of probabilistically reflecting the effect of the background noise is proposed. More specifically, a relationship between two kinds of the probability density function (abbr. p.d.f.) and the cumulative distribution function (abbr. c.d.f.) forms on the system output excited by a specific stochastic input of reference type and an arbitrary random input without the additional noise for an arbitrary environmental systems is discussed in the form on an intensity scale. Then, a relationship between two kinds of p.d.f.s of the system outputs excited by an arbitrary stochastic input in the absence and in the presence of background noise is also derived in the form on an intensity scale. Based on these relationships, a new prediction method on a whole p.d.f. and/or c.d.f. forms of the system output for the arbitrary environmental systems with the background noise is proposed especially by the use of the observed data excited by the specific stochastic input of reference type with the background noise. Finally, the effectiveness of the proposed methods is confirmed experimentally too by applying them to the actual type sound wall systems.

## 2. Theoretical consideration

### *2.1. Noise cancellation on a whole probability form*

The observed data are usually given in a sound level form (dB scale) based on the logarithmic type non-linear transformation of the sound pressure. Therefore, for the purpose of determining the uncontaminated output response, it is necessary to find a method of reasonably removing the effect of the background noise and that of the observation mechanism based on the above non-linear transformation.

Based on the additive principle of sound energy, the arbitrary sound environmental systems on an intensity scale can be described in a simplified form of the following linear system:

$$\xi = \sum_{i=0}^{N} a_i \cdot x_i \,, \tag{1}$$

where $\xi$ and $x_i$ are the system output and input, respectively. Here, the acoustic system order $N$ and the system parameters $a_i$ ($a_{N+1} = 1$) have been found in advance in the previous paper [3]. Let us consider the observation mechanism based on the linear and/or non-linear transformations as following equations:

$$y = f(\xi), \tag{2}$$

$$z = f\left(\xi + \sum_{i=0}^{N+1} a_i \cdot v_i\right). \tag{3}$$

Hereupon, $f(\cdot)$ denotes the mechanism of the linear and/or non-linear transformation measurement. Also, $z$ and $y$ are two kinds of observed data with and without the background noise. And, $v_i$ $(i = 1, 2, ..., N)$ and $v_i$ $(i = N + 1)$ show the sound intensities of background noises added on the input and output sides, respectively.

Let us derive a synthetic probability density function of the stochastic sound environmental system with background noise, after the linear and/or non-linear transformations in Eqs. (2) and (3). If employing this synthetic probability expression into an inverse direction of analysis, it becomes possible to estimate reasonably a p.d.f. of the output response uncontaminated by the background noise without introducing any artificial error criterion like the well-known least-squares method. More concretely, we introduce an arbitrary function $\psi(z)$ which plays the role of a certain kind of the catalytic like operation in the decomposition of the above synthetic expression for the p.d.f. Here, let us write the expectation value of this arbitrary function under consideration, as a certain catalytic function of analysis, as follows:

$$I \equiv \langle \psi(z) \rangle = \int_{-\infty}^{\infty} \psi(z) p_z(z)\, dz, \tag{4}$$

where $p_z(z)$ is a p.d.f. of $z$ and $\langle * \rangle$ denotes an expectation operation with respect to the variable $*$. Here, it seems to be natural to assume that the $i$-th $(i = 0, 1, 2, ...)$ successive derivatives of $\psi(z)$ and/or $p_z(z)$ tend to zero at the boundary region $z \to \pm\infty$. After substituting Eq. (3) into $\psi(z)$ and expanding it in a Taylor's expansion series form under the above natural boundary condition, $\psi(z)$ can be rewritten as follows:

$$\psi(z) = \sum_{n=0}^{\infty} \left[ \left( \sum_{i=0}^{N+1} a_i \cdot v_i \right)^n \Big/ n! \right] \cdot (d/d\xi)^n \cdot \psi(f(\xi)). \tag{5}$$

Accordingly, after substituting Eq. (5) into Eq. (4) and successively integrating by parts, the expectation $I$ of the arbitrary function $\psi(z)$ can be concretely expanded under the above natural boundary condition as follows:

$$I = \int_{-\infty}^{\infty} \psi(y) \cdot \left\{ \sum_{n=0}^{\infty} (-1)^n \Big/ n! \cdot A^n \cdot \left[ \langle B^n \mid f^{-1}(y) \rangle \cdot p_y(y) \right] \right\} dy, \tag{6}$$

where

$$A = \left( \frac{1}{\dfrac{df^{-1}(y)}{dy}} \frac{d}{dy} \right), \qquad B = \left( \sum_{i=0}^{N+1} a_i \cdot v_i \right).$$

After replacing $y$ with $z$ owing to the property of the definite integral operation in Eq. (6) and comparing the definition of the expectation of the arbitrary function in Eq. (4) with Eq. (6), the above p.d.f. $p_z(z)$ of $z$ can be derived as the following equations:

$$p_z(z) = p_y(z) + \sum_{n=1}^{\infty} (-1)^n \Big/ n! \cdot A^n \cdot \left[ \langle B^n \mid f^{-1}(z) \rangle \cdot p_y(z) \right], \tag{7}$$

or

$$p_y(z) = p_z(z) - \sum_{n=1}^{\infty}(-1)^n\Big/n! \cdot A^n \cdot \left[\langle B^n \,|\, f^{-1}(z)\rangle \cdot p_y(z)\right], \tag{8}$$

Here, we must notice the fact that $p_y(z)$ means to replace only a stochastic variable $y$ with $z$ in the p.d.f. expression $p_y(y)$ of $y$ itself. Based on the above synthetic probability expression Eq. (8), it is possible to estimate reasonably only the undisturbed p.d.f. $p_y(y)$ of the objective output $y$ without the background noise for arbitrary sound environmental systems. That is, after substituting $p_y(z)$ in the expansion series expression on the right hand side of Eq. (8) by the whole right side of this equation and successively repeating the same procedure, the following expression of $p_y(y)$ can be derived:

$$\begin{aligned} p_y(y) = p_z(y) &- \sum_{n_1=1}^{\infty} A_{n_1} \cdot A^{n_1} \cdot \left[\langle B^{n_1} \,|\, f^{-1}(y)\rangle\right] \cdot p_z(y) + \cdots \\ &+ (-1)^s \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty} \dots \sum_{n_s=1}^{\infty}\prod_{k=1}^{s} A_{n_s} \cdot A^{n_k} \cdot \left[\langle B^{n_k} \,|\, f^{-1}(y)\rangle\right] \cdot p_z(y) + \cdots, \end{aligned} \tag{9}$$

where

$$A_{n_k} = (-1)^{n_k}/n_k!.$$

Therefore, the p.d.f. expression for the output response of sound environmental systems after noise cancellation can be explicitly estimated from the observed actual data obtained by the logarithmic type non-linear transformation of the data including background noise.

### *2.2. Prediction of the system response probability with background noise and arbitrary input*

First, we derive, both on the intensity scale and in the parameter differential form, the relationship between two p.d.f.s of the system outputs excited by a reference stochastic input and an arbitrary input without additional noise. Let a system output without additional noise change from $y_0$ to $y$:

$$y = y_0(1 + \gamma/s_0), \tag{10}$$

where $y_0$ and $y$ denote two system outputs emitted by a specific stochastic input of reference type and an arbitrary random input without additional noise in the form of the intensity scale, respectively. The $\gamma/s_0$ shows some ratio of a dimensionless deviation from a standard distribution type and is statistically independent of $y_0$. We can express a relationship between two p.d.f.s of acoustic system responses excited by a specific input of reference type and an arbitrary input without additional noise in the expression form of p.d.f. as follows [4]:

$$p_y(y) = \sum_{l=0}^{\infty}(-1)^l/l! \cdot (d/dy)^l \left[\left\langle(\gamma \cdot y/s_0)^l \,|\, y\right\rangle \cdot p_{y_0}(y)\right]. \tag{11}$$

Here, we must notice the fact that only a random variable is changed from the original $y_0$ to $y$ in the proper p.d.f. expression $p_{y_0}(*)$ of $y_0$. Also, the conditional moment can be directly obtained as:

$$\langle (\gamma \cdot y/s_0)^l \mid y \rangle = y^l/s_0^l \cdot \langle \gamma^l \rangle. \tag{12}$$

Accordingly, after substituting Eq. (12) into Eq. (11), the latter can be easily rewritten as follows:

$$\begin{aligned} p_{y(y)} &= \sum_{l=0}^{\infty} (-1)^l/l! \cdot (d/dy)^l \left[ y^l/s_0^l \cdot \langle \gamma^l \rangle \cdot p_{y_0}(y) \right] \\ &= \sum_{l=0}^{\infty} (-1)^l/l! \cdot \langle \gamma^l \rangle (d/dy)^l \left[ p_{y_0}(y) \cdot y^l/s_0^l \right]. \end{aligned} \tag{13}$$

Paying our attention to the fact that the system output on an intensity scale, $y$ always fluctuates in a non-negative region. The probability density function for the system output can be expressed in advance especially in the general form of a statistical Laguerre expansion series [5] as:

$$p_{y_0}(y) = \left\{ 1 + \sum_{n=1}^{\infty} C_n \cdot L_n^{(m_0-1)}(y/s_0^l) \right\} p_\Gamma(y; m_0, s_0), \tag{14}$$

where

$$m_0 = \langle y \rangle^2 \Big/ \left\langle (y - \langle y \rangle)^2 \right\rangle, \qquad s_0 = \left\langle (y - \langle y \rangle)^2 \right\rangle \Big/ \langle y \rangle, \tag{15}$$

$$p_\Gamma(y; m_0, s_0) = y^{m_0-1} \cdot e^{-y/s_0} / \left( \Gamma(m_0) \cdot s_0^{m_0} \right) \tag{16}$$

and

$$C_n = \Gamma(m_0) \cdot n!/\Gamma(m_0 + n) \cdot \left\langle L_n^{(m_0-1)}(y/s_0) \right\rangle, \tag{17}$$

where $L_n^{(m_0-1)}(*)$ is a Laguerre polynomial of the $n$-th order, and $C_n$ is the expansion coefficient reflecting hierarchically the lower and higher order statistics of the output intensity fluctuation. Furthermore, Eq. (14) can be transformed into a parameter differential type series expansion expression taking a gamma distribution function as the first expansion term:

$$p_{y_0}(y) = \left\{ 1 + \sum_{n=1}^{\infty} C'_n (\partial/\partial s_0)^n \right\} p_\Gamma(y; m_0, s_0), \tag{18}$$

where

$$C'_n = \Gamma(m_0) \cdot (-s_0)^n \Big/ \Gamma(m_0 + n) \cdot \left\langle L_n^{(m_0-1)}(y_0/s_0) \right\rangle. \tag{19}$$

After some complicated calculation procedures, the following relationship between the variable differential and the parameter differential can be derived as:

$$(\partial/\partial y)^l \left[ p_{y_0}(y) \cdot y^l/s_0^l \right] = (-1)^l (\partial/\partial s_0)^l p_{y_0}(y). \tag{20}$$

Consequently, by employing Eq. (20), Eq. (13) can be rewritten as follows:

$$p_y(y) = \sum_{l=0}^{\infty} 1/l! \cdot \langle \gamma^l \rangle \cdot (\partial/\partial s_0)^l p_{y_0}(y). \tag{21}$$

Next, the relationship between the two kind p.d.f.s of the system output emitted by the arbitrary stochastic input with and without additional noise is also introduced by the expression of the parameter differential form on an intensity scale. Based on these relationships, we derive a new prediction method being able of evaluating the acoustic system output excited by the arbitrary stochastic input in the presence of additional noise. The output fluctuation on the intensity scale for the arbitrary sound environmental system can be written in the following linear form:

$$z = y + \sum_{i=0}^{N+1} a_i v_i \,, \tag{22}$$

where $z$ and $y$ denote the system outputs with and without additional noise in the form of an intensity scale. Here, $N$ and $a_i$ are the system order and system parameter. Also, $v_i$ $(i = 1, 2, ..., N)$ and $v_i$ $(i = N + 1)$ denote the intensities of additional noises on the input and output sides, respectively. The following expression can be simply deduced from Eq. (7) taking into consideration that $df^{-1}(z)/dz = 1$ for the system output given by Eq. (22) and $\langle B^n | f^{-1}(z) \rangle = \langle B^n \rangle$ when the system output $y$ and the additional noise are statistically independent:

$$p_z(z) = \sum_{n=0}^{\infty} (-1)^n/n! \cdot \left\langle \left( \sum_{i=0}^{N+1} a_i v_i \right)^n \right\rangle \cdot (d/dz)^n p_y(z). \tag{23}$$

After substituting Eq. (21) into Eq. (23) under the above condition, it is possible to rewrite Eq. (23) as:

$$\begin{aligned} p_z(z) &= \sum_{n=0}^{\infty} (-1)^n/n! \cdot \left\langle \left( \sum_{i=0}^{N+1} a_i v_i \right)^n \right\rangle \cdot (d/dz)^n \left\{ \sum_{l=0}^{\infty} 1/l! \cdot \langle \gamma^l \rangle \cdot (\partial/\partial s_0)^l p_{y_0}(z) \right\} \\ &= \sum_{n=0}^{\infty} (-1)^n/l! \cdot (\partial/\partial s_0)^l \left\{ \sum_{n=0}^{\infty} (-1)^n \Big/ n! \cdot \left\langle \left( \sum_{i=0}^{N+1} a_i v_i \right)^n \right\rangle \cdot (d/dz)^n p_{y_0}(z) \right\}. \end{aligned} \tag{24}$$

Consequently, after taking into consideration a p.d.f. $p_{y_0}(z)$ of $z$ corresponding only to $y_0$ (instead of $y$) expressed in the same form as Eq. (23), we directly have:

$$p_z(z) = \sum_{l=0}^{\infty} 1/l! \cdot \langle \gamma^l \rangle \cdot (\partial/\partial s_0)^l p_{z_0}(z). \tag{25}$$

Thus, we can predict theoretically the response p.d.f. for an actual sound environmental system with an arbitrary stochastic input in the presence of additional noise, especially by employing the information on the system output p.d.f. for the same system with a specific reference input in the presence of an additional noise knowing its statistics.

## 3. Experimental consideration

### *3.1. Experimental arrangement*

Figure 1 shows a block diagram of the experimental arrangement in two reverberation rooms. The speaker excites the transmission room and two microphones receive the input and output intensity fluctuations of the sound insulation system respectively. Table 1 shows values of the system parameters for the sound insulating structures considered in the experiment (the system order $N = 2$). We have employed the actual road traffic noise measured in Hiroshima City and the white noise as the stochastic input and the background noise, respectively. The aperture of the wall between the transmission and the reception has an area of $1.74\,\mathrm{m} \times 0.84\,\mathrm{m}$.

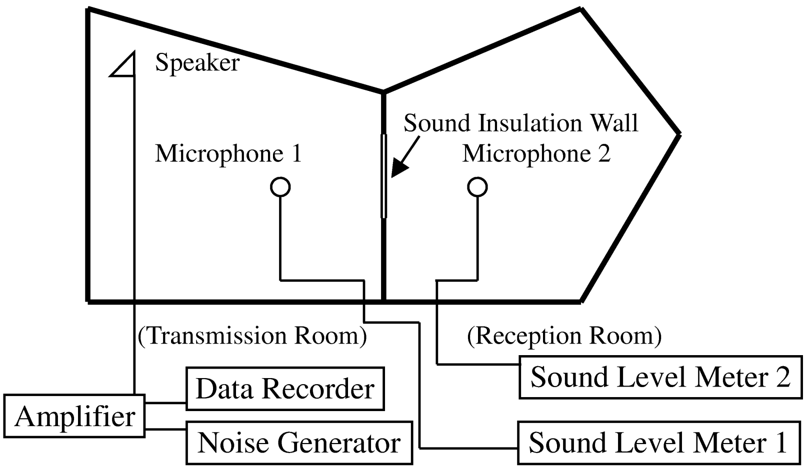


Fig. 1. Block diagram of experimental arrangement.

**Table 1.** Values of the system parameters.

| | $a_0$ | $a_1$ | $a_2$ |
|---|---|---|---|
| Single wall | $2.43 \times 10^{-3}$ | $2.10 \times 10^{-3}$ | $1.95 \times 10^{-3}$ |
| Non-parallel double wall | $9.28 \times 10^{-3}$ | $6.08 \times 10^{-3}$ | $5.53 \times 10^{-3}$ |
| Double wall with sound bridge | $5.52 \times 10^{-3}$ | $4.18 \times 10^{-3}$ | $3.37 \times 10^{-3}$ |

The proposed methods are applied to three types of the sound insulation wall systems, a) a single wall — an aluminum panel (surface density : $3.22\,\mathrm{kg/m^2}$, thickness : 1.2 mm), b) a non-parallel wall — composed of aluminum (at an angle 9 degrees each other), and c) a double wall with sound bridge — composed of aluminum with a sound bridge (air gap thickness : 50 mm).

### *3.2. Experimental results*

*3.2.1. Noise cancellation on a whole probability form.* The results of the c.d.f. for the estimation of the output response probability after the background noise cancellation in cases of the double wall with sound bridge and the non-parallel double wall are shown in Fig. 2. The good agreement between the theoretically calculated values and experi-

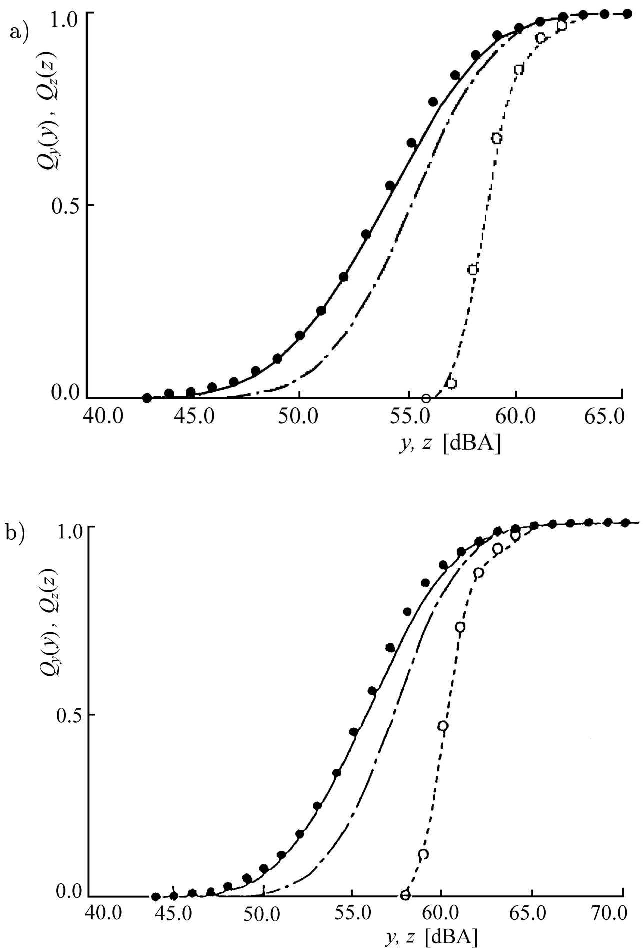


Fig. 2. Comparison between theoretically estimated curves and experimentally sampled values for cumulative distribution function; a) a double wall with sound bridge, b) a non-parallel double wall. The observed and fitted curve for $Q_z(z)$ are shown as (○) and (· · ·). The true and estimated curves for $Q_y(y)$ are shown as (•), (– · –): 1st approx. and (—): 2nd approx.

mentally observed data is recognized in Fig. 2 by employing only the first few expansion terms in the proposed theoretically expansion expression.

*3.2.2. Prediction of the system response probability with background noise and an arbitrary input.* The results of the c.d.f. for the prediction of the system output are shown in Fig. 3 in cases of the single wall and the non-parallel double wall, respectively. Here, the 1st, 2nd or 3rd approximations correspond to the cases of employing the 1st, 2nd or 3rd terms in the above theoretical expansion expression, respectively. In the inverse problem of infinite series expression, there is generally some risk of series divergency even if its original series expression is convergent. So, some reasonable countermeasure of divergent error seem fairly important. For the purpose of reasonably minimizing this

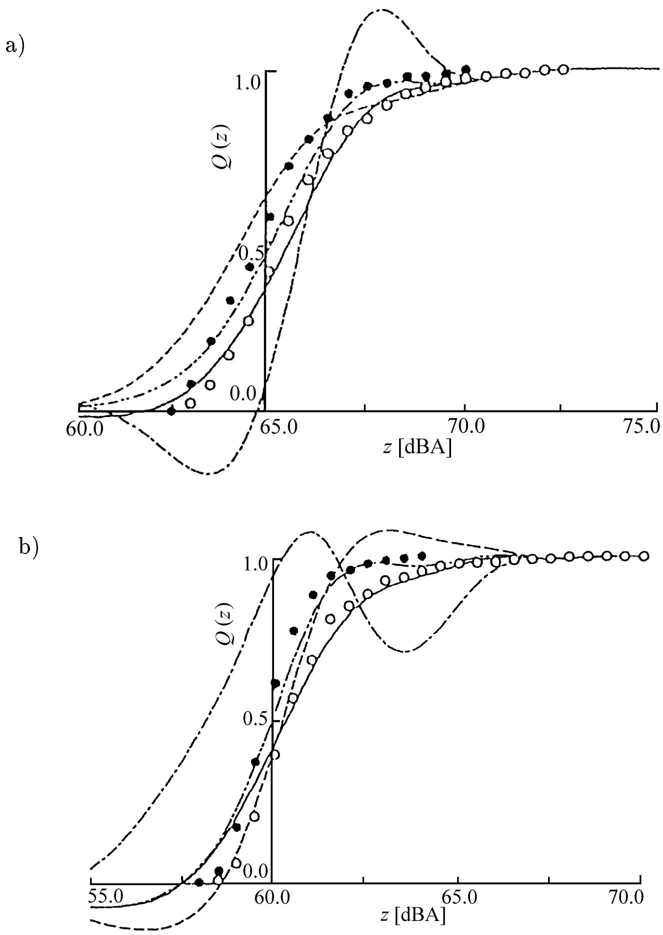


Fig. 3. Comparison between theoretically predicted curves and experimentally sampled values for cumulative distribution function; a) a single wall, b) a non-parallel double wall. Experimentally sampled values in cases of the arbitrary input and the reference input are marked by (•) and (○), respectively. Theoretically predicted curves are shown by (—): 1st approx., (···): 2nd approx., (– · –): 3rd approx. and (– ·· –): averaging method.

divergency error caused by employing only the first finite terms in the above infinite series expansion expression, some averaging evaluation procedure can be derived theoretically as follows:

$$Q(z) = Q_0(z) + (b+c)/(a+b+c) \cdot Q_1(z) + c/(a+b+c) \cdot Q_2(z) \\ +1/(a+b+c) \cdot (a \cdot \varepsilon_0 + b \cdot \varepsilon_1 + c_2 \cdot \varepsilon_2), \qquad (26)$$

where $Q_i(z)$'s $(i = 0, 1, 2)$ are respectively the c.d.f. in the special cases taking the 1st, 2nd or 3rd terms in the above infinite series type theoretical p.d.f. expansion expression, and $a$, $b$ and $c$ are the arbitrary constants. Also, the $\varepsilon_i$'s $(i = 0, 1, 2)$ denote the errors caused by use of the finite expansion terms in the cases of $Q_i(z)$ $(i = 0, 1, 2)$, respectively.

From Fig. 3, it seems that the 1st, 2nd and 3rd approximation curves do not show any agreement with the experimentally sampled points owing to the above error. The averaging method in Eq. (26), however, shows a better agreement with the experimentally sampled points compared with the other curves.

## 4. Conclusion

In this paper, we have proposed two stochastic signal processing methods on a whole probability distribution form without introducing in advance any artificial error criterion. That is, for the arbitrary sound environmental system under the existence of background noise, we have developed the method of estimating the output response probability after noise cancellation and that of predicting the system output emitted by an arbitrary input with background noise by employing the information on the system output p.d.f. for the same system excited only by a specific reference input in the presence of the background noise knowing its statistics.

Finally, the practical effectiveness of the proposed methods have been experimentally confirmed too by applying them to the actually observed response data in the reverberation room.

Since the present methods are at an earlier stage of study, there still remain some kinds of future problems, for example, to apply them to many other actual systems and to find more simplified methods for practical use through some approximation of the proposed methods.

## Appendix A

### *A.1. Simplified determination method of the order for an arbitrary sound insulation system based on time series model*

An arbitrary sound insulation system can be described by the following discrete-time type:

$$z_k = f(X_k; A) \qquad X_k \equiv (x_k, x_{k-1}, ..., x_{k-l}), \tag{A.1}$$

where $x_k$ and $z_k$ are the system input and output at the discrete-time $k$, and $f(\;)$ denotes the linear and/or non-linear mechanisms of the system. Furthermore, a vector $A \equiv (a_1, a_2, ..., a_N)^T$ show system parameters.

We introduce a somewhat more simplified method rather than such methods as the well-known AIC method or the FPE method for determining the system order on the time series model. When the white noise is adopted on trial as a test input of the system described by Eq. (A.1), the relationship between the test input ($= u_k$) and the system output ($= y_k$) can be written in the following form:

$$y_k = f(U_k; A) \qquad U_k \equiv (u_k, u_{k-1}, ..., u_{k-l}). \tag{A.2}$$

Because the statistical independence property originally does not change, even in an arbitrary nonlinear transformation of the systems, the statistical independence for the arbitrary random signal $y_k$ can be evaluated in terms of the following measure $\varepsilon(y_k, y_{k+j})$:

$$\varepsilon(y_k, y_{k+j}) = \frac{\langle y_k y_{k+j} \rangle}{\langle y_k \rangle \langle y_{k+j} \rangle} - 1, \tag{A.3}$$

where $\langle * \rangle$ denotes the expected value of $*$ and $\varepsilon(y_k, y_{k+j}) = 0$ when $y_k$ and $y_{k+j}$ are statistically independent. Then, it is surely reasonable that we adapt the system order as $l = j - 1$, in the case when the value of $\varepsilon(y_k, y_{k+j})$ is saturated downward the neighborhood of zero at $j$.

Figure 4 shows the result of the system order determined by Eq. (A.3). From this figure, it can be found that the system order is approximately 2 because the value of $\varepsilon(y_k, y_{k+j})$ is close to zero at $j = 3$.

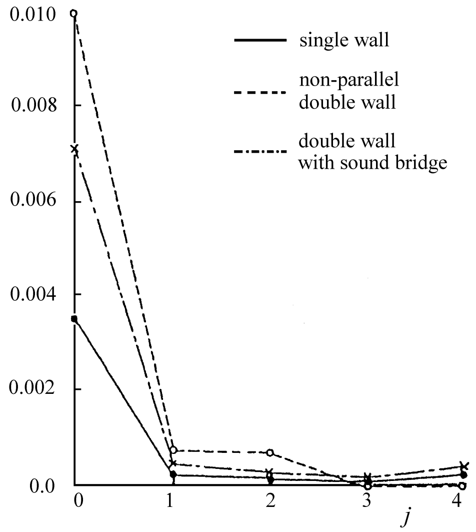


Fig. 4. Identification results for the order of time series model based on the criterion function of independency; $\varepsilon(j) = \langle y_k, y_{k+j} \rangle / \langle y_k \rangle \cdot \langle y_{k+j} \rangle - 1$.

### *A.2. Prediction of output probability distribution*

Equation (A.1) can be rewritten as the following linear system on an intensity scale supported by the well-known statistical energy analysis method:

$$z_k = \sum_{i=0}^{2} a_i x_{k-i} \,. \tag{A.4}$$

By use of Eq. (A.4), the prediction of the output probability distribution can be obtained for the sound insulation system excited by random input contaminated by background noise.

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