Article citation info:
SU C, ZHANG YJ, CAO BX. Forecast model for real time reliability of storage system based on periodic inspection and maintenance data. Eksploatacja i Niezawodnosc – Maintenance and Reliability 2012; 14 (4): 342–348.

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FORECAST MODEL FOR REAL TIME RELIABILITY OF STORAGE SYSTEM BASED ON PERIODIC INSPECTION AND MAINTENANCE DATA

MODEL DO PROGNOZOWANIA NIEZAWODNOŚCI SYSTEMU MAGAZYNOWANIA W CZASIE RZECZYWISTYM W OPARCIU O DANE Z PRZEGLĄDÓW OKRESOWYCH ORAZ DANE EKSPLOATACYJNE

In recent years, storage reliability has attracted much attention for increasing reliability requirement. In this paper, forecast models for real-time reliability of storage system under periodic inspection and maintenance are presented, which is based on the theories of reliability physics and exponential distribution. The models are developed under two newly-defined imperfect repair modes, i.e., Improved As Bad As Old (I-ABAO) and Improved As Good As New (I-AGAN). A completion method for censored life data is also proposed by averaging the residual lifetime. According to the complete and censored lifetime data, parameters in the models are estimated by applying maximum likelihood estimation method and iterative method respectively. A numerical example of a storage system is given to verify the feasibility of the proposed completion method and the effectiveness of the two models.

Keywords: storage reliability, real-time reliability, periodic inspection and maintenance, censored data.

W ostatnich latach wiele uwagi poświęcono tematyce niezawodności magazynowania w odniesieniu do zwiększania wymogu niezawodności. W prezentowanym artykule, przedstawiono modele do prognozowania w czasie rzeczywistym niezawodności systemu magazynowania podlegającego przeglądom okresowym i obsłudze. Modele oparto na teoriach z zakresu fizyki niezawodności oraz na rozkładzie wykładniczym. Proponowane modele opracowano dla dwóch nowo zdefiniowanych opcji niepełnej odnowy, t.j. Improved-As Bad As Old (Jak Tuż Przed Uszkodzeniem – Wersja Udoskonalona) oraz Improved-As Good As New (Jak Fabrycznie Nowy – Wersja Udoskonalona). Zaproponowano także metodę uzupełniania danych cenzurowanych (uciętych) dotyczących trwałości polegającą na uśrednianiu trwałości resztkowej. Zgodnie z pełnymi i cenzurowanymi danymi trwałościowymi, parametry w proponowanych modelach ocenia się, odpowiednio, z zastosowaniem estymacji metodą największej wiarygodności oraz metody iteracyjnej. Poprawność przedstawionej metody uzupełniania oraz efektywność proponowanych dwóch modeli zweryfikowano na podstawie numerycznego przykładu systemu magazynowania.

***Słowa kluczowe**: niezawodność magazynowania, niezawodność w czasie rzeczywistym, okresowe przeglądy i obsługa, dane cenzurowane.*

1. Introduction

In engineering practice, there exit such kinds of systems. They are kept in storage state for most of their lifetime, but must keep high mission reliability and be ready for operation whenever needed. Strategic missile, spare parts for nuclear power plant, and fire protection system are typical examples of such systems. Periodic testing, replacement and other maintenance measures are necessary to avoid or reduce the occurrence of failure [6, 9, 12].

In the past decades, storage reliability has attracted much attention from both academia and engineering field. Merren studied the periodic test problem of electronic equipment in storage, and an algorithm for computing the reliability was developed based on test data [8]. Aneziris presented a method for calculating the dynamic reliability of safety systems, which was based on the theory of Markov chain [2]. Ref. [4] proposed an availability model for storage products under periodic inspection, but they ignored the initial failure. Zhao and Xie studied the problem of potential storage reliability estimation with initial storage failure [13]. Zhang and Zhao established a storage reliability model under periodic inspection [14]. Ref. [15] studied the storage reliability of missile-engine with Bayesian approach. Yang used stochastic process method to describe the degradation process of the components with multi-performance parameters, and proposed a storage reliability evaluation model [11]. Ref. [13] introduced an approach to evaluate the storage reliability of engine control circuit module.

Up to now, most existing research mainly concentrated on the data analysis methods for the components of a system. Real-time reliability estimation of products in storage on the system level has not received sufficient attention. Most studies assume the system is in perfect initial state and there is not much effort placed on empirical study based on actual storage reliability data. As for repair efficiency, two basic assumptions are known As Bad As Old (ABAO) and As Good As

New (AGAN). However, these two extreme cases seldom happen in most practical systems [7]. More reasonable repair models need to be developed to get a closer description of the real situation.

In this paper, real-time reliability evaluation models are established based on periodic inspection data, and two newly-defined imperfect repair modes are considered, i.e., Improved-ABAO and Improved-AGAN. A new completion method is also proposed to convert censored life data into complete data. Based on complete and censored lifetime data, the parameters in the models are estimated with maximum likelihood estimation method and iterative method, respectively. A numerical example is given to illustrate the models and method.

2. Reliability model for storage system

Due to different reasons, not all the storage systems are shipped out of the factory with reliability testing. Thus the initial use reliability of the system might not be 1.

Let A indicate the event "the storage system is qualified at t=0", denote T as the lifetime of the storage system. The real-time reliability at time $t(t\geq 0)$ can be expressed as:

$$R(t)=p(T>t,A)=p(T>t\mid A)p(A)=R_0R_s(t) \tag{1}$$

where R_0 is the inherent reliability of a storage system, $R_0=p(A)$; $R_s(t)=p(T>t\mid A)$ is the conditional reliability of the system, which changes with storage time and $R_s(0)=1$. Therefore, the system reliability can be represented as

$$R(t;\theta)=R_0R_S(t;\theta),\theta\in\Theta \tag{2}$$

where Θ is a measurable parameter set.

In order to keep high mission reliability, periodic inspection and maintenance are needed. Based on the difference of maintenance, two types of forecast models for real-time reliability are given as follows.

2.1. Reliability forecast model under I-AGAN

Assume that the storage system is put into use at t=0 and it is inspected at intervals of time $\tau(\tau\geq 0)$. If partial failure or aging occurs, the maintenance is carried out. After $x(0<x<\tau)$ time maintenance, the system is restored and keeps on storing. When the maintenance is perfect and leaves the system as if it were new, we call it repair under AGAN[7]. In engineering practice, however, maintenance can increase system reliability but the failure rate usually shows an increasing tendency owing to the degradation of material strength. Taking account of this, we propose a new repair mode called Improved-AGAN (I-AGAN) (See Fig. 1).

Under I-AGAN, three assumptions are made as below: (1) The system is stored under natural conditions, and the lifetime t obeys exponential distribution. (2) In the initial time, the system reliability is $R(0)=R_0\leq 1$ and lifetime t obeys exponential distribution with parameter θ_0. After the k[th] maintenance, lifetime obeys exponential distribution with parameter θ_k, and $\theta_0\geq\theta_1\geq\theta_2\geq\ldots\geq\theta_{k-1}\geq\theta_k$. (3) The inspection interval is τ. The maintenance time x_k (k = 1, 2, …) is a stochastic variable that varies with failure.

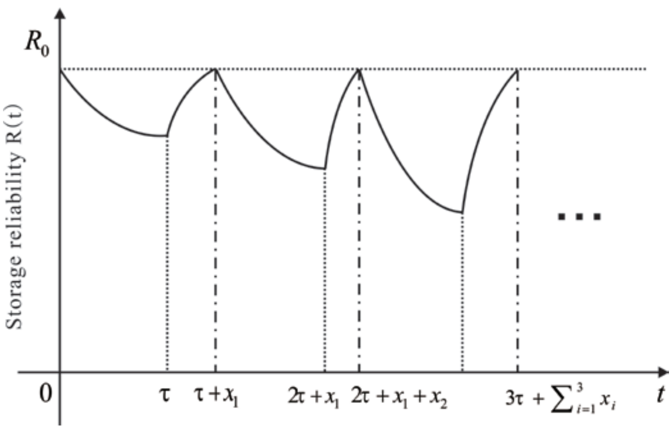

Fig. 1 Reliability change of storage system under I-AGAN

From Fig. 1, the reliability before the k[th] maintenance $k\tau+\sum_{i=1}^{k-1}x_i$ is $R_s(k\tau+\sum_{i=1}^{k-1}x_i)$. According to the definition under I-AGAN, after each maintenance the system is restored to its initial state, so the reliability at time $k\tau+\sum_{i=1}^{k-1}x_i$ is $R_s(k\tau+\sum_{i=1}^{k-1}x_i)\leq R_s(k\tau+\sum_{i=1}^{k}x_i)=R_0=R(0)$, where k=1, 2, 3, …, n; n is times of maintenance.

Denote $F(t)$ as the failure distribution function of the system at t, and $F(t)$=1-R(t). Hence, $F(t)$ obeys the following distribution:

$$F(t)=1-R_0\cdot R_s\left(t-r\tau-\sum_{i=1}^{r}x_i\right),r=\left[\left(t-\sum_{i=1}^{r}x_i\right)/\tau\right] \tag{3}$$

where [·] represents rounding down.

Based on the characteristic of exponential distribution, in non-maintenance period $R(t)$ can be calculated:

$$R(t)=R_0\exp\left(-\frac{t-r\tau-\sum_{i=1}^{r}x_i}{\theta_r}\right),r=\left[\left(t-\sum_{i=1}^{r}x_i\right)/\tau\right] \tag{4}$$

Suppose λ_i (i=1, 2, …, k, …) is the system failure rate after the i[th] maintenance. As $\lambda_i=1/\theta_i$, obviously, $\lambda_0\leq\lambda_1\leq\lambda_2\leq\ldots\leq\lambda_{k-1}\leq\lambda_k$. According to the reliability physics and engineering practice, after the k[th] maintenance the failure rate satisfies the following equation:

$$\lambda_k=\lambda_0\exp(k\beta),\ k=1,2,\cdots,n\ , \tag{5}$$

where $\beta(\beta>0)$ is the degradation factor. It can also be rewritten as $\theta_k=\theta_0\exp(-k\beta)$, k = 1, 2, …, n.

From Eqs. (4), (5), the real-time reliability at time t is:

$$R(t)=R_0\exp\left[-\lambda_0\exp(r\beta)(t-r\tau-\sum_{i=1}^{r}x_i)\right],r=\left[\left(t-\sum_{i=1}^{r}x_i\right)/\tau\right] \tag{6}$$

Based on the maintenance strategy, it can be obtained that:

$$\lim_{t\to(k\tau+\sum_{i=1}^{k-1}x_i)}\left[\left(t-\sum_{i=1}^{r}x_i\right)/\tau\right]=k-1,$$

$$\lim_{t\to(k\tau+\sum_{i=1}^{k}x_i)}\left[\left(t-\sum_{i=1}^{r}x_i\right)/\tau\right]=k,k=1,2,\cdots,n \tag{7}$$

From Eq. (6), it can be concluded that the system real-time reliability before the kth maintenance time $t=k\tau+\sum_{i=1}^{k-1}x_i$ is:

$$R_k^-(t)=R_0\exp(-\lambda_0e^{k\beta}\tau),k=1,2,\cdots,n \tag{8}$$

2.2. Reliability forecast model under I-ABAO

When the maintenance leaves the system in the same state as it was before failure, we call it repair under ABAO[7]. In practice, however, maintenance may leave the system reliability higher than it was before failure but lower than its initial state R_0. Additionally, material strength degradation causes failure rate to increase. Taking account of this, we

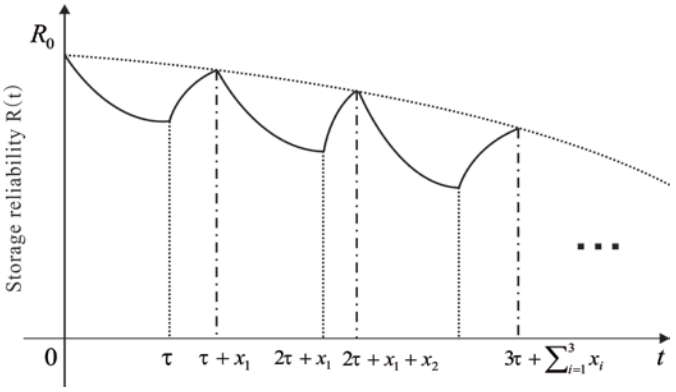

Fig. 2 Reliability change of storage system under I-ABAO

propose another imperfect repair mode called Improved-ABAO (I-ABAO), with the same assumptions in Section 2.1 (See Fig. 2).

According to reliability characteristics, we can divide a storage system into two subsystems. Subsystem 1 requires regular inspection and maintenance with an invariable or increasing failure rate. Subsystem 2 has a high reliability and does not require regular inspection and maintenance, but aging phenomenon may occur.

Under I-ABAO, after each maintenance the system reliability $R_s(t)$ cannot be restored to the initial reliability, that is: $R_s(k\tau + \sum_{i=1}^{k} x_i) < R_0 = R(0)$. During the storage, the real-time reliability of Subsystem 1 can be obtained by:

$$R_{s1}(t) = \exp\left[-\lambda_0 \exp(r\beta)(t - r\tau - \sum_{i=1}^{r} x_i)\right],\ r = \left[\left(t - \sum_{i=1}^{r} x_i\right)/\tau\right] \tag{9}$$

Subsystem 2 is composed of mechanical and electronic parts, which has usually undergone ageing screening tests, and the infancy failures have been eliminated. Basically, it is in the random failure period, so the system reliability is assumed to decrease exponentially, which is expressed as[1]:

$$R_{s2}(t) = \exp(-\delta t) \tag{10}$$

where δ is the degradation coefficient. From Eq. (2), it can be concluded that the system reliability at time t is:

$$R(t) = R_0 R_s(t) = R_0 R_{s1}(t) R_{s2}(t) = R_0 \exp\left[\lambda_0 \exp(r\beta)\left(r\tau + \sum_{i=1}^{r} x_i - t\right) - \delta t\right],$$

$$r = \left[\left(t - \sum_{i=1}^{r} x_i\right)/\tau\right] \tag{11}$$

Before the k^th^ inspection, which is at time $t = k\tau + \sum_{i=1}^{k-1} x_i$, the system reliability $R_k^-(t)$ is:

$$R_k^-(t) = R_0 \exp\left\{-\left[\lambda_0 \exp(r\beta) + \delta(k\tau + \sum_{i=1}^{k-1} x_i)\right]\right\},\ k = 1, 2, \cdots, n \tag{12}$$

3. Completion of censored data

Regular inspection is necessary for the storage system, but the failures are seldom found exactly at the inspection time. So it is difficult to obtain complete data, and the field lifetime data are mostly censored. If the censored data are simply regarded as complete data or treated with interpolation method, large errors will be brought out. The completion method of censored life data is discussed below.

3.1. Completion method of censored data

Let $T_1, T_2, \ldots, T_n$ be the lifetimes of storage system; $t_1, t_2, \ldots, t_n$ be the start times of inspection and $o(T_i)$ be the residual lifetime of the i^th^ system. If t_i is right censored or completely censored, $I_i = 0$; Otherwise, if t_i is left censored, I_i=1. It can be obtained that $o(T_i) = (T_i - t_i)(1 - I_i)$, $I_i = I\{t_i \le T_i\}, i = 1, 2, \cdots, n$. Consequently, the complete data of $t_i (i = 1, 2, \cdots, n)$ is:

$$\xi_i = t_i + o(T_i) \tag{13}$$

Accordingly, when the failure data of the i^th^ system is right censored, the average residual lifetime is:

$$E(T_i - t_i \mid T_i \ge t_i) = \frac{p\{T_i - t_i, T_i \ge t_i\}}{p\{T_i \ge t_i\}} = \frac{\int_{t_i}^{+\infty} (x - t_i) f(x; \theta_1, \theta_2, \cdots, \theta_m) dx}{1 - F_s(t_i; \theta_1, \theta_2, \cdots, \theta_m)} \tag{14}$$

Therefore, under periodic inspection the average residual lifetime of the system is:

$$\overline{o(T_i)} = (1 - I_i) \frac{\int_{t_i}^{+\infty} (x - t_i) f(x; \theta_1, \theta_2, \cdots, \theta_m) dx}{R_s(t_i; \theta_1, \theta_2, \cdots, \theta_m)} \tag{15}$$

If inspection data are discontinuous, the average residual lifetime is:

$$D\overline{o(T_i)} = \frac{(t_{i+1} - t_i) R_s(t_i) + \sum_{j \ge i+1} (t_{j+1} - t_j) R_s(t_j)}{R_s(t)} \tag{16}$$

where the measurable parameter set $\theta_1, \theta_2, \cdots, \theta_m \in \Theta$ and $\overline{o(T_i)}$ are the implicit functions of unknown parameters, which can be solved by iterative algorithm [10]. Assuming that the system lifetime T has density function $f(\xi; \Theta)$, the likelihood function of parameter set Θ within the complete sample data $\xi_1, \xi_2, \cdots, \xi_n$ is $L(\xi; \Theta) = \prod_{i=1}^{n} f(x_i; \Theta)$, so the likelihood equation is:

$$\left.\frac{\partial \ln L(\xi; \theta_1, \theta_2, \cdots, \theta_m)}{\partial \theta_i}\right|_{\theta_i = \hat{\theta}_i} = 0, i = 1, 2, \cdots, m \tag{17}$$

Suppose the initial value of parameter estimation is $\Theta_0 = \{(\theta_1)_0, (\theta_2)_0, \cdots, (\theta_m)_0\}$. Insert it into Eq. (17) and the first iterative parameter set $\Theta_1 = \{(\theta_1)_1, (\theta_2)_1, \cdots, (\theta_m)_1\}$ can be obtained. Circulating the iterative procedure, the kth iterative parameter set $\Theta_k = \{(\theta_1)_k, (\theta_2)_k, \cdots, (\theta_m)_k\}$ can also be obtained. Thereby, the maximum likelihood estimator set of Θ is the convergent parameter set $\hat{\Theta} = \lim_{k \to \infty} \Theta_k$.

3.2. Censored data completion of Weibull & exponential type system

For most storage system, the lifetime obeys Weibull distribution or memoryless exponential distribution. Based on cumulative damage effect, the method to convert censored data of Weibull storage system into complete ones is given below. Assume that the lifetime T has the density function:

$$f(t; \alpha, \lambda) = \lambda\alpha(\lambda t)^{\alpha - 1} \exp\left[-(\lambda t)^{\alpha}\right],\ t \ge 0 \tag{18}$$

where λ, α are parameters to be estimated. The likelihood function under the case of the complete sample is:

$$L(T_1,T_2,\cdots,T_n;\lambda,\alpha)=\prod_{i=1}^{n}\lambda\alpha(\lambda T)^{\alpha-1}\exp\left[-(\lambda T_i)^{\alpha}\right]. \quad (19)$$

Therefore, the logarithmic likelihood equations are:

$$\begin{cases}\dfrac{\sum_{i=1}^{n}T_i^{\alpha}\ln T_i}{\sum_{i=1}^{n}T_i^{\alpha}}-\dfrac{1}{\alpha}-\dfrac{1}{n}\sum_{i=1}^{n}\ln T_i=0\\ \lambda^{-\alpha}-\dfrac{1}{n}\sum_{i=1}^{n}T_i^{\alpha}=0\end{cases} \quad (20)$$

If x_i is left censored, it can be obtained from Eqs. (13), (15) that:

$$T_i=\frac{\lambda\int_{x_{i-1}}^{x_i}t^{\frac{1}{\alpha}}\exp(-t)dt}{\exp(-\pi_{i-1})-\exp(-\pi_i)},\ \pi_{i-1}=\left[\lambda\left((i-1)\tau+\sum_{j=1}^{i-1}x_j\right)\right]^{\alpha},\ \pi_i=\left[\lambda\left(i\tau+\sum_{j=1}^{i-1}x_j\right)\right]^{\alpha} \quad (21)$$

If x_i is right censored, then:

$$T_i=\frac{1}{\lambda}\exp\left[(\lambda x_i)^{\alpha}\right]\left[\Gamma\left(1+\frac{1}{\alpha}\right)-\int_0^{(\lambda x_i)^{\alpha}}t^{\frac{1}{\alpha}}\exp(-t)dt\right] \quad (22)$$

If x_i is completely censored, take T_i as:

$$T_i=x_i \quad (23)$$

For the exponential storage system, take $\alpha = 1$, then the completion of left censored data and right censored data are given as follows, respectively.

$$T_i=\frac{1}{\lambda}+\frac{\tau}{\exp(\lambda\tau)-1}+\left[(i-1)\tau+\sum_{j=1}^{i-1}x_j\right] \text{ and } T_i=x_i+\frac{1}{\lambda} \quad (24)$$

4. Parameter estimation of reliability model

When the censored data have been converted into complete data, the estimator $\hat{\Theta}$ of parameter set can be obtained by iteration of Eq. (17). Given the distribution function $F(t)=F(t;\hat{\Theta})$, the real-time reliability function of storage system can be obtained as $R(t)=\hat{R}_0\cdot[1-F(t;\hat{\Theta})]$.

The least square method was used to estimate the initial reliability in Ref. [13]. Here we take

$$\hat{R}_0=\min(R_0',\frac{R_0'+\hat{p}_0}{2}) \quad (25)$$

as the initial reliability of the system, where $\hat{p}_0=(N_0-f_0+1)/(N_0+2)$ is the Bayes estimation of use reliability, N_0 is the number of systems, and f_0 is the failure number. For Weibull storage system, the real-time reliability function is:

$$R(t)=\hat{R}_0\cdot\exp\left[-(\hat{\lambda}t)^{\hat{\alpha}}\right] \quad (26)$$

For the original censored data, $\{(N_i, S_i, t_i), i=1, 2, \ldots, n\}$ is the failure data set collected from periodic inspection, where S_i is the number of non-failed products within N_i of total stored products at time t_i. Supposing that the periodic inspection interval τ is a fixed time unit, and the maintenance time is negligible compared to the storage time, only parameters in Eqs. (8), (12) need to be estimated.

For real-time reliability model under I-AGAN:

$$R_k^-(t)=R_0\exp\left[-\lambda_0\exp(k\beta)\right],\ k=1,2,\cdots,n \quad (27)$$

The likelihood function is:

$$L_1(\lambda,\beta\mid\Theta)=\prod_{i=1}^{n}\binom{N_i}{S_i}\cdot\left[R_0\exp(-\lambda_0e^{i\beta})\right]^{S_i}\times\left[1-R_0\exp(-\lambda_0e^{i\beta})\right]^{N_i-S_i} \quad (28)$$

The maximum likelihood equations involving λ_0 and β are as follows:

$$\begin{cases}\dfrac{\sum_{i=1}^{n}S_ie^{i\beta}}{R_0}-\sum_{i=1}^{n}\dfrac{(N_i-S_i)e^{i\beta}\exp(-\lambda_0e^{i\beta})}{1-R_0\exp(-\lambda_0e^{i\beta})}=0\\ \dfrac{\sum_{i=1}^{n}iS_ie^{i\beta}}{R_0}-\sum_{i=1}^{n}\dfrac{ie^{i\beta}(N_i-S_i)\exp(-\lambda_0e^{i\beta})}{1-R_0\exp(-\lambda_0e^{i\beta})}=0\end{cases} \quad (29)$$

It can be concluded from Eq. (29) that there is no analytical solution for maximum likelihood estimation of λ_0 and β. However, the estimates $\hat{\lambda}$, $\hat{\beta}$ can be derived by numerical calculating method. The real-time system reliability is:

$$R(t)=\hat{R}_0\exp\left[-\hat{\lambda}_0\exp(r\hat{\beta})(t-r\tau-\sum_{i=1}^{r}x_i)\right],\ r=\left[\left(t-\sum_{i=1}^{r}x_i\right)/\tau\right] \quad (30)$$

For the real-time reliability model under I-ABAO:

$$R_k^-(t)=R_0\exp\left\{-\left[\lambda_0\exp(r\beta)+\delta(k\tau+\sum_{i=1}^{k-1}x_i)\right]\right\} \quad (31)$$

The corresponding maximum likelihood function is:

$$L_2(\lambda_0,\delta,\beta\mid\Theta)=\prod_{k=1}^{n}\binom{N_k}{S_k}\cdot\left\{R_0\exp\left[-\left(\lambda_0e^{k\beta}+\delta(k+\sum_{i=1}^{k-1}x_i)\right)\right]\right\}^{S_k}\times \left\{1-R_0\exp\left[-\left(\lambda_0e^{k\beta}+\delta(k+\sum_{i=1}^{k-1}x_i)\right)\right]\right\}^{N_k-S_k} \quad (32)$$

Then the maximum likelihood equations involving λ, δ, β are as follows:

$$\begin{cases}\dfrac{\sum_{k=1}^{n}S_ke^{k\beta}}{R_0}-\sum_{k=1}^{n}\dfrac{(N_k-S_k)e^{k\beta}\exp\left[-\left(\lambda_0e^{k\beta}+(k+\sum_{i=1}^{k-1}x_i)\delta\right)\right]}{1-R_0\exp\left[-\left(\lambda_0e^{k\beta}+(k+\sum_{i=1}^{k-1}x_i)\delta\right)\right]}=0\\ \dfrac{\sum_{k=1}^{n}kS_ke^{k\beta}}{R_0}-\sum_{k=1}^{n}\dfrac{ke^{k\beta}(N_k-S_k)\exp\left[-\left(\lambda_0e^{k\beta}+(k+\sum_{i=1}^{k-1}x_i)\delta\right)\right]}{1-R_0\exp\left[-\left(\lambda_0e^{k\beta}+(k+\sum_{i=1}^{k-1}x_i)\delta\right)\right]}=0\\ \dfrac{\sum_{k=1}^{n}kS_k}{R_0}-\sum_{k=1}^{n}\dfrac{(N_k-S_k)k\exp\left[-\left(\lambda_0e^{k\beta}+(k+\sum_{i=1}^{k-1}x_i)\delta\right)\right]}{1-R_0\exp\left[-\left(\lambda_0e^{k\beta}+(k+\sum_{i=1}^{k-1}x_i)\delta\right)\right]}=0\end{cases} \quad (33)$$

From Eq. (33), the estimates $\hat{\lambda}_0$, $\hat{\delta}$, $\hat{\beta}$ of λ_0, δ, β can be derived by Newton's iteration method. The real-time system reliability is:

$$R(t)=\hat{R}_0\exp\left[\hat{\lambda}_0\exp(r\hat{\beta})(r\tau+\sum_{i=1}^{r}x_i-t)-\hat{\delta}\,t\right],\ r=\left[\left(t-\sum_{i=1}^{r}x_i\right)/\tau\right] \quad (34)$$

5. Case study

In this section, a storage system composed of 18 subsystems is considered. It is maintained every other year to keep its availability. Compared with the storage time, the time consumed by maintenance is negligible. Those subsystems which have serious failure or have no value for maintenance will be withdrawn from storage, while others will be restored to storage state. Maintenance will continue until all the subsystems are withdrawn from storage. The detailed data are shown in Table 1.

Table 1. Recorded data of 18 subsystems under periodic inspection

Year (t_i)	Total storage (N_i)	Non-failures (S_i)	Cumulated withdrawals (F_i)
1	18	18	0
2	18	18	0
3	18	16	2
4	16	16	2
5	16	16	2
6	16	16	2
7	16	13	5
8	13	12	6
9	12	12	6
10	12	11	7
11	11	10	8
12	10	9	9
13	9	4	14
14	4	4	14
15	4	3	15
16	3	3	15
17	3	1	17
18	1	1	17
19	1	1	17
20	1	0	18

The reliability analysis of the system under I-AGAN and I-ABAO is showed as below.

5.1. Reliability function of Weibull & exponential type system

At the beginning of storage, all the 18 subsystems satisfy mission reliability. According to Eq. (25), the initial storage reliability R_0 is:

$$\hat{R}_0 = \min\left(1, \frac{1}{2}\left(1+\frac{S_0+1}{N_0+2}\right)\right) = 0.975$$

Rearrange the data in Table 1 on a monthly basis by left-censored data (denoted as x⁻), right-censored data (denoted as x⁺), and complete data (denoted as x): { 36+, 36+, 84+, 84+, 84+, 96+, 120+, 132-, 144+, 156+, 156+, 156+, 156+, 156, 180+, 204+, 204+, 228+ }.

If the storage lifetime obeys Weibull distribution, the complete lifetime data of the storage system T_i (month) are derived from Eqs. (21), (22), (23), as shown in the sequence: {248, 248, 262, 262, 262, 267, 278, 126, 291, 298, 298, 298, 298, 156, 312, 327, 327, 343}. Inserting it into Eq. (20), parameter estimators with a precision of 10-5 are obtained: $\hat{\alpha}$=2.08998, $\hat{\lambda}$=0.00363. The real-time reliability function with complete lifetime data is:

$$R(t) = 0.975\exp\left[-(0.00363t)^{2.08998}\right] \tag{35}$$

If the storage lifetime obeys exponential distribution, the complete lifetime data T_i (month) are derived from Eq. (24), as shown in the sequence: {1239, 1239, 1287, 1287, 1287, 1299, 1323, 126, 1347, 1359, 1359, 1359, 1359, 156, 1383, 1407, 1407, 1431}. The estimator of parameter λ is:

$$\hat{\lambda} = \left(\frac{1}{n}\sum_{i=1}^{n} T_i\right)^{-1} = 8.3126\times10^{-4}.$$

So the real-time reliability function with the complete lifetime data is:

$$R(t) = 0.975\cdot\exp(-0.00083126t) \tag{36}$$

Fig. 3 illustrates the reliability change of Weibull type system and exponential type system.

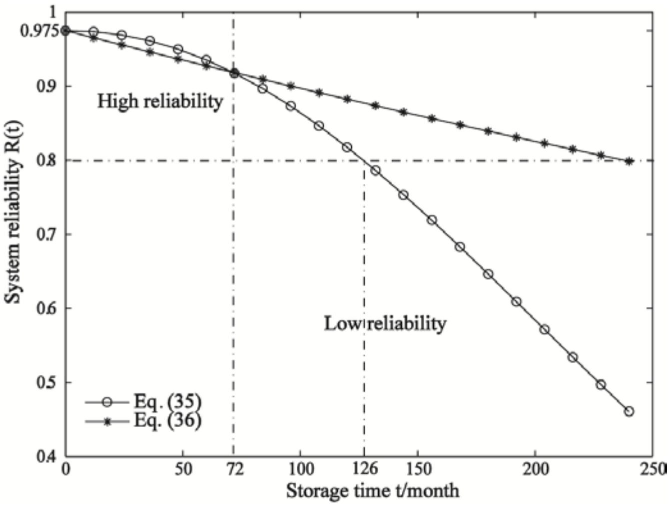

Fig. 3. Real-time reliability with complete lifetime data

From Fig. 3, it can be concluded that the degradation of Weibull type system is slower than that of exponential type, and exponential type appears more sensitive than Weibull type. However, as time goes by, exponential type exhibits higher physical strength and its mechanical behavior changes slower compared with Weibull type. Under a certain maintenance strategy, exponential type system keeps higher reliability within 20 years' storage period, while the reliability of Weibull type will be lower than 0.8 and enter into a low reliability area after 10 years.

In the next section, we choose exponential type system to verify the effectiveness of the proposed completion method and models under I-AGAN and I-ABAO.

5.2. Reliality analysis under I-AGAN

Based on the original exponential censored data (Table 1), when I-AGAN is adopted, the estimator of parameter λ_0 and β in Eq. (30) can be obtained using numerical analysis and iterative method: $\hat{\lambda}$=0.003865, $\hat{\beta}$=0.1102. Therefore, the real-time reliability under I-AGAN can be expressed as:

$$R(t) = 0.975\exp\left[-0.003865\exp\left(0.1102\cdot[\frac{t}{12}]\right)\right]\cdot\left(t-12\cdot[\frac{t}{12}]\right) \tag{37}$$

where $[\cdot]$ represents rounding down.

To validate the effectiveness of the estimation model, reliability curves drawn according to Eqs. (36), (37) are shown in Fig. 4.

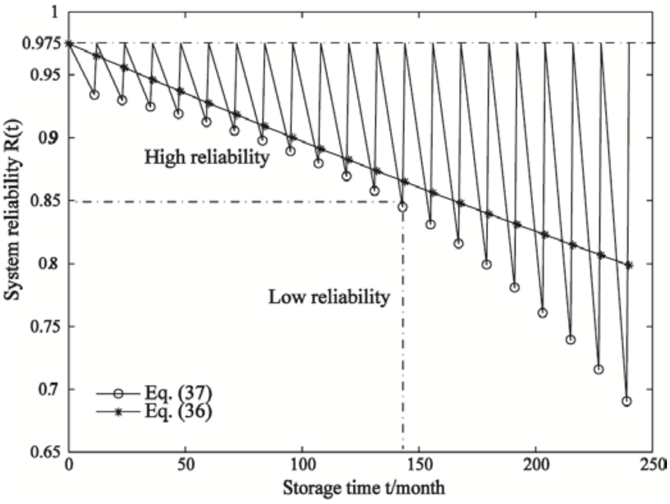

Fig. 4 Real-time reliability with censored data under I–AGAN

It can be concluded from Fig. 4 that during storage under I-AGAN, the reliability of exponential system decreases progressively faster over time, which accords with engineering practice. Within the first 10 years, the system keeps a reliability higher than 0.85, and within 19 years, the reliability still remains higher than 0.7.

5.3. Reliality analysis under I-ABAO

When I-ABAO is adopted, based on the data in Table 1 and Eq. (33), estimator of parameter λ_0, δ, β in Eq. (34) can be obtained using iterative method: $\hat{\lambda}_0$=0.001098, $\hat{\beta}$=0.2015, $\hat{\delta}$=0.000384.Therefore, the real-time reliability under I-ABAO is expressed as:

$$R(t) = 0.975\exp\left[-0.001098\exp\left(0.2015\cdot[\frac{t}{12}]\right)\cdot\left(t-12\cdot[\frac{t}{12}]\right)-0.000384t\right] \quad (38)$$

where $[\cdot]$ represents rounding down.

Reliability curves drawn according to Eqs. (36), (38) are shown in Fig. 5.

In Fig. 5, the reliability of exponential system under I-ABAO has similar downtrend with that under I-AGAN. Within the first 10 years of storage, the system keeps a reliability higher than 0.80. However, after 10 years, the reliability enters into a low reliability area or even fails. Compared with Fig. 4, I-AGAN can keep higher reliability.

Fig. 4 and Fig. 5 show the comparison between the reliability curves obtained from complete data and censored data under I-AGAN, I-ABAO respectively. The results show they share the similar downtrend and they are in accordance with engineering practice, which verifies the effectiveness of the proposed completion method and models.

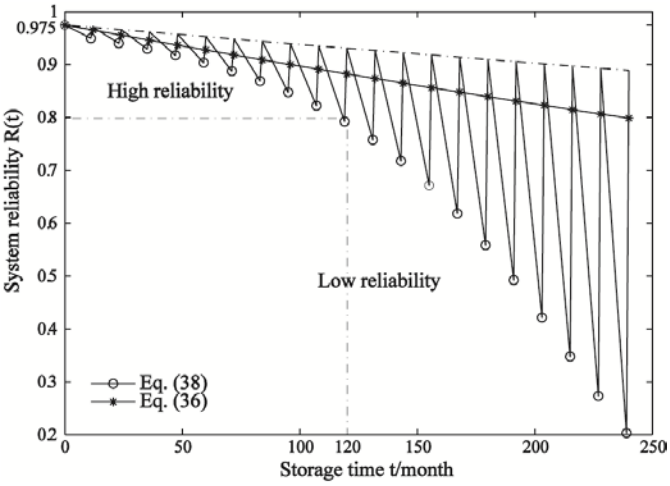

Fig. 5 Real-time reliability with censored data under I–ABAO

6. Conclusion

In this paper, we develop two forecast models for real-time reliability of storage system under two repair modes. A completion method is also proposed to convert censored data into complete data by averaging the residual lifetimes. The case study shows that the system reliability estimated using the complete data shares similar changing trend with that using directly recorded censored data. The result verifies the feasibility of the proposed completion method and the effectiveness of the two models. For systems in long-time storage state under periodic inspection, the real-time reliability can be effectively estimated by applying the forecast models proposed in this paper.

In this article, it is assumed that the repair cycle is a fixed value. It should be interesting to carry out further studies on the unfixed repair cycle. Furthermore, if it is difficult to identify a suitable theoretical distribution for an application, a nonparametric approach can be employed to estimate the probability distribution based on the periodic inspection data.

***Acknowledgement*:**

This project is supported by National Natural Science Foundation of China (Grant No. 50405021), the Key Program of Science Foundation of Higher Education Institutions of Anhui Province, China (Grant No. KJ2010A337).

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