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RELIABILITY ALLOCATION USING PROBABILISTIC ANALYTICAL TARGET CASCADING WITH EFFICIENT UNCERTAINTY PROPAGATION

ALOKACJA NIEZAWODNOŚCI Z WYKORZYSTANIEM PROBABILISTYCZNEJ METODY ANALITYCZNEGO KASKADOWANIA CELÓW ZAPEWNIAJĄCA WYDAJNĄ PROPAGACJĘ NIEPEWNOŚCI

Analytical target cascading (ATC) provides a systematic approach in solving reliability allocation problems for large scale system consisting of a large number of subsystems, modules and components. However, variability and uncertainty in design variables (e.g., component reliability) are usually inevitable, and when they are taken into consideration, the multi-level optimization will be very complicated. The impacts of uncertainty on system reliability are considered in this paper within the context of probabilistic ATC (PATC) formulation. The challenge is to reformulate constraints probabilistically and estimate uncertainty propagation throughout the hierarchy since outputs of subsystems at lower levels constitute inputs of subsystems at higher levels. The performance measure approach (PMA) and the performance moment integration (PMI) method are used to deal with the two objectives respectively. To accelerate the probabilistic optimization in each subsystem, a unified framework for integrating reliability analysis and moment estimation is proposed by incorporating PATC with single-loop method. It converts the probabilistic optimization problem into an equivalent deterministic optimization problem. The computational efficiency is remarkably improved as the lack of iterative process during uncertainty analysis. A nonlinear geometric programming example and a reliability allocation example are used to demonstrate the efficiency and accuracy of the proposed method.

Keywords: optimal reliability allocation, hierarchical decomposition, probabilistic analytical target cascading, uncertainty propagation.

Analityczne kaskadowanie celów (ATC) stanowi systematyczne podejście do rozwiązywania zagadnień alokacji niezawodności dotyczących systemów wielkoskalowych składających się z dużej liczby podsystemów, modułów i elementów składowych. Jednakże zmienność i niepewność zmiennych projektowych (np. niezawodności elementów składowych) są zazwyczaj nieuniknione, a gdy weźmie się je pod uwagę, optymalizacja wielopoziomowa staje się bardzo skomplikowana. W prezentowanym artykule, wpływ niepewności na niezawodność systemu rozważano w kontekście formuły probabilistycznego ATC (PATC). Wyzwanie polegało na probabilistycznym przeformułowaniu ograniczeń oraz ocenie propagacji niepewności w całej hierarchii, jako że wyjścia podsystemów na niższych poziomach stanowią wejścia podsystemów na poziomach wyższych. Cele te realizowano, odpowiednio, przy użyciu metody minimum funkcji granicznej (performance measure approach, PMA) oraz metody całkowania momentów statystycznych funkcji granicznej (performance moment integration, PMI). W celu przyspieszenia probabilistycznej optymalizacji w każdym podsystemie, zaproponowano ujednolicone ramy pozwalające na integrację analizy niezawodności z oceną momentów statystycznych poprzez połączenie PATC z metodą jednopoziomową (pojedynczej pętli, single-loop method). Zaproponowana metoda polega na przekształceniu probabilistycznego zagadnienia optymalizacyjnego na deterministyczne zagadnienie optymalizacyjne. Zwiększa to znacznie wydajność obliczeniową w związku z brakiem procesu iteratywnego podczas analizy niepewności. Wydajność i trafność proponowanej metody wykazano na podstawie przykładów dotyczących programowania nieliniowego geometrycznego oraz alokacji niezawodności.

***Słowa kluczowe**: optymalna alokacja niezawodności, dekompozycja hierarchiczna, probabilistyczna metoda analitycznego kaskadowania celów, propagacja niepewności.*

1. Introduction

Optimal reliability design (reliability allocation) aims to determine the reliability of constituent subsystems and components so as to obtain targeted overall system reliability. It should be performed early in the design cycle to guide later tradeoff and improvement studies of more detailed designs. However, reliability allocation for designing complex system, such as structural, aerospace or automotive systems, is a complicated large-scale problem. Decomposition can result in improved computational efficiency because the formulation of each element typically has fewer degrees of freedom and fewer constraints than the all-in-once (AIO) formulation. Since the subsystems are coupled, their interactions need to be taken into consideration to achieve consistent designs. Zhang [20] proposed the collaborative allocation (CA) to deal with optimum allocation problem in aircraft conceptual design, which is of similar solution procedure with collaborative optimization (CO). However, in CA the auxiliary constraints are equality constraints, and the convergence has not been demonstrated yet. Recently, analytical target cascading (ATC) has been applied successfully to a variety of reliability allocation problems [4, 11, 21]. ATC is a methodology for cascading upper level design targets to lower level while the element at the lower level tries to provide responses as close to these targets as possible [8]. It has a few features which are applicable to optimum allocation problem. Firstly, upper level providing lower level with targets of variables is similar to allocation of design requirements. Secondly, the hierarchic multilevel optimization of ATC is similar to system structure composed of subsystems, components and parts. Finally, by forcing the consistency between each subsystem, ATC has proven to be convergent to the original undecomposed problem.

However, there exists uncertainty in design variables or parameters in the early development stage. For example, component reliability estimates are often uncertain, particular for new products with few failure data [2]. Thus, accurate estimates of system risk should be sought and used in system design and trade studies. In response to these new requirements, the ATC formulation has been extended to solve probabilistic design optimization problems using random variables to represent uncertainty [10], and generalized with general probabilistic characteristics by Liu [14]. In the previously published probabilistic ATC (PATC) formulations, the first few moments are usually used as targets and responses since matching two random variables is not practically doable in most cases. Even with the first few moments, however, computing the solution is very expensive due to computational difficulty in estimating propagated uncertainty. An efficient and accurate mechanism is required for propagating probabilistic information throughout the hierarchy.

$$\begin{aligned}
&\text{Given } \boldsymbol{\mu}^{\mathrm{U}}_{\mathbf{R}_{ij}}, \boldsymbol{\sigma}^{\mathrm{U}}_{\mathbf{R}_{ij}}, \boldsymbol{\mu}^{\mathrm{U}}_{\mathbf{Y}_{ij}}, \boldsymbol{\sigma}^{\mathrm{U}}_{\mathbf{Y}_{ij}}, \boldsymbol{\mu}^{\mathrm{L}}_{\mathbf{R}_{(i+1)k}}, \boldsymbol{\sigma}^{\mathrm{L}}_{\mathbf{R}_{(i+1)k}}, \\
&\qquad \boldsymbol{\mu}^{\mathrm{L}}_{\mathbf{Y}_{(i+1)k}}, \boldsymbol{\sigma}^{\mathrm{L}}_{\mathbf{Y}_{(i+1)k}}, \quad k = 1,\ldots,n_{ij} \\
&\text{find } \boldsymbol{\mu}_{\mathbf{R}_{(i+1)k}}, \boldsymbol{\sigma}_{\mathbf{R}_{(i+1)k}}, \mathbf{X}_{ij}, \boldsymbol{\mu}_{\mathbf{Y}_{ij}}, \boldsymbol{\sigma}_{\mathbf{Y}_{ij}}, \boldsymbol{\mu}_{\mathbf{Y}_{(i+1)k}}, \\
&\qquad \boldsymbol{\sigma}_{\mathbf{Y}_{(i+1)k}}, \varepsilon^{\boldsymbol{\mu}\mathbf{R}}_{ij}, \varepsilon^{\boldsymbol{\sigma}\mathbf{R}}_{ij}, \varepsilon^{\boldsymbol{\mu}\mathbf{Y}}_{ij}, \varepsilon^{\boldsymbol{\sigma}\mathbf{Y}}_{ij}, k = 1,\ldots,n_{ij} \\
&\min \left\| \boldsymbol{\mu}_{\mathbf{R}_{ij}} - \boldsymbol{\mu}^{\mathrm{U}}_{\mathbf{R}_{ij}} \right\| + \left\| \boldsymbol{\sigma}_{\mathbf{R}_{ij}} - \boldsymbol{\sigma}^{\mathrm{U}}_{\mathbf{R}_{ij}} \right\| + \left\| \boldsymbol{\mu}_{\mathbf{Y}_{ij}} - \boldsymbol{\mu}^{\mathrm{U}}_{\mathbf{Y}_{ij}} \right\| \\
&\qquad + \left\| \boldsymbol{\sigma}_{\mathbf{Y}_{ij}} - \boldsymbol{\sigma}^{\mathrm{U}}_{\mathbf{Y}_{ij}} \right\| + \varepsilon^{\boldsymbol{\mu}\mathbf{R}}_{ij} + \varepsilon^{\boldsymbol{\sigma}\mathbf{R}}_{ij} + \varepsilon^{\boldsymbol{\mu}\mathbf{Y}}_{ij} + \varepsilon^{\boldsymbol{\sigma}\mathbf{Y}}_{ij} \\
&\text{s.t. } \sum_{k=1}^{n_{ij}} \left\| \boldsymbol{\mu}_{\mathbf{R}_{(i+1)k}} - \boldsymbol{\mu}^{\mathrm{L}}_{\mathbf{R}_{(i+1)k}} \right\| \le \varepsilon^{\boldsymbol{\mu}\mathbf{R}}_{ij} \\
&\qquad \sum_{k=1}^{n_{ij}} \left\| \boldsymbol{\sigma}_{\mathbf{R}_{(i+1)k}} - \boldsymbol{\sigma}^{\mathrm{L}}_{\mathbf{R}_{(i+1)k}} \right\| \le \varepsilon^{\boldsymbol{\sigma}\mathbf{R}}_{ij} \\
&\qquad \sum_{k=1}^{n_{ij}} \left\| \boldsymbol{\mu}_{\mathbf{Y}_{(i+1)k}} - \boldsymbol{\mu}^{\mathrm{L}}_{\mathbf{Y}_{(i+1)k}} \right\| \le \varepsilon^{\boldsymbol{\mu}\mathbf{Y}}_{ij} \\
&\qquad \sum_{k=1}^{n_{ij}} \left\| \boldsymbol{\sigma}_{\mathbf{Y}_{(i+1)k}} - \boldsymbol{\sigma}^{\mathrm{L}}_{\mathbf{Y}_{(i+1)k}} \right\| \le \varepsilon^{\boldsymbol{\sigma}\mathbf{Y}}_{ij} \\
&\qquad \Pr\left(G_{ij,m}\left(\mathbf{R}_{ij}, \mathbf{X}_{ij}, \mathbf{Y}_{ij} \right) \ge 0 \right) \ge P_{ij,m}, m = 1,\ldots,M \\
&\text{where } \mathbf{R}_{ij} = \mathbf{f}_{ij}\left(\mathbf{R}_{(i+1)1},\ldots,\mathbf{R}_{(i+1)n_{ij}}, \mathbf{X}_{ij}, \mathbf{Y}_{ij} \right)
\end{aligned} \tag{1}$$

The paper proceeds as follows. First, the general PATC formulation is revisited in Section 2. Section 3 provides an introduction of existing methods for uncertainty propagation. Section 4 develops an efficient methodology integrating single loop method to deal with the issue of modeling uncertainty in multilevel hierarchies. The efficiency and accuracy of the proposed algorithm is demonstrated on two examples in Section 5. Finally, conclusions are presented in Section 6.

2. PATC formulation based on matching mean and variance

The choice of probabilistic characteristic is an important issue in decomposition based system design optimization under uncertainty because it is not practical to match two distributions exactly. For distributions with negligible higher-order moments, matching only the first two moments (mean value and variance) should be sufficient. According to the general PATC formulation provided by Liu [14], the design optimization problem for element j at level i (element O_{ij}) is shown in equation (1).

For a subsystem at certain level, its neighboring lower-level subsystems are called its children, while the neighboring upper-level subsystem is called its parent. In equation (1), $\mathbf{R}_{ij}$ and $\mathbf{Y}_{ij}$ are vectors of random responses and linking variables, respectively. $\mathbf{R}_{ij}$ are evaluated using analysis or simulation models $\mathbf{R}_{ij} = \mathbf{f}_{ij}\left(\mathbf{R}_{(i+1)1},\ldots,\mathbf{R}_{(i+1)n_{ij}}, \mathbf{X}_{ij}, \mathbf{Y}_{ij}\right)$. Targets for mean and standard deviation of $\mathbf{R}_{ij}$ and $\mathbf{Y}_{ij}$ are assigned by the parent element as $\boldsymbol{\mu}^{\mathrm{U}}_{\mathbf{R}_{ij}}$, $\boldsymbol{\sigma}^{\mathrm{U}}_{\mathbf{R}_{ij}}$ and $\boldsymbol{\mu}^{\mathrm{U}}_{\mathbf{Y}_{ij}}$, $\boldsymbol{\sigma}^{\mathrm{U}}_{\mathbf{Y}_{ij}}$. Achievable mean and standard deviation of $\mathbf{R}_{ij}$ and $\mathbf{Y}_{ij}$ are feed back to its parent element as $\boldsymbol{\mu}^{\mathrm{L}}_{\mathbf{R}_{ij}}$, $\boldsymbol{\sigma}^{\mathrm{L}}_{\mathbf{R}_{ij}}$ and $\boldsymbol{\mu}^{\mathrm{L}}_{\mathbf{Y}_{ij}}$, $\boldsymbol{\sigma}^{\mathrm{L}}_{\mathbf{Y}_{ij}}$. Similarly, achievable values of its children element responses and linking variables are passed to O_{ij} as $\boldsymbol{\mu}^{\mathrm{L}}_{\mathbf{R}_{(i+1)k}}$, $\boldsymbol{\sigma}^{\mathrm{L}}_{\mathbf{R}_{(i+1)k}}$ and $\boldsymbol{\mu}^{\mathrm{L}}_{\mathbf{Y}_{(i+1)k}}$, $\boldsymbol{\sigma}^{\mathrm{L}}_{\mathbf{Y}_{(i+1)k}}$. The design consistency is formulated as the first four constrains in equation (1). The optimization problem for O_{ij} is to find the optimum values for local design variables $\mathbf{X}_{ij}$, linking variables $\mathbf{Y}_{ij}$ and the target values for responses $\boldsymbol{\mu}_{\mathbf{R}_{(i+1)k}}$, $\boldsymbol{\sigma}_{\mathbf{R}_{(i+1)k}}$ and linking variables $\boldsymbol{\mu}_{\mathbf{Y}_{(i+1)k}}$, $\boldsymbol{\sigma}_{\mathbf{Y}_{(i+1)k}}$ of its children. Generally, the optimization problem in equation (1) can be formulated as

$$\begin{aligned}
&\min_{\mathbf{d},\mathbf{X}} f\left(\boldsymbol{\mu}_{\mathbf{R}_{ij}}(\mathbf{d},\mathbf{X},\mathbf{P}), \boldsymbol{\sigma}_{\mathbf{R}_{ij}}(\mathbf{d},\mathbf{X},\mathbf{P}), \mathbf{d}, \mathbf{X} \right) \\
&\text{s.t. } \Pr\left(G_{ij,m}(\mathbf{d},\mathbf{X},\mathbf{P}) \ge 0 \right) \ge P_{ij,m} \quad m = 1,\ldots,M \\
&\qquad \mathbf{R}_{ij} = \mathbf{f}_{ij}\left(\mathbf{d},\mathbf{X},\mathbf{P}\right)
\end{aligned} \tag{2}$$

where **d** is the vector of deterministic design variables, **X** is the vector of random design variables and **P** is the vector of random parameters. The optimization problem contains probabilistic constraints and the probability of success should be calculated. Besides, in a multilevel hierarchy, outputs of subsystems at lower levels constitute inputs of subsystems at higher levels. It is thus necessary to estimate the statistical moments of these outputs with adequate accuracy. This needs to be done for all problems at all levels of the hierarchy, and the high computational cost is a great challenge.

In previous work, the Monte Carlo simulation (MCS) is used to calculate all the probabilistic characteristics of the responses, and all probabilistic constraints are simplified into the moment-matching formulations [14]. However, computational time becomes a significant challenge. MCS may not be a practical approach for design optimization problems that require a significant number of iterations. An effective way to improve efficiency is based on the Taylor series expansions, which may introduce large approximation errors of expected values for the nonlinear responses [10]. Therefore, an appropriate uncertainty propagation method needs to be selected to achieve an appropriate balance between accuracy and efficiency.

3. Uncertainty propagation methods

One of the key components of uncertainty analysis is the quantification of uncertainties in the system output performances propagated from uncertain inputs, named as uncertainty propagation (UP) [12]. For the optimization problem in equation (2), it should be point out that the emphases of the two kinds of uncertainty calculation problems are different. One emphasizes on assessing the performance reliability. And the other focuses on evaluating the low-order moments (mean and variance) of a performance. Thus, they are discussed separately.

3.1. Reliability analysis

Reliability analysis is a tool to compute the reliability index or the probability of failure corresponding to a given failure mode or for the entire system [5]. To deal with the reliability constraints in equation (2), the reliability index is statistically defined by a cumulative distribution function $F_{G_{ij,m}}(0)$ as

$$P\left(G_{ij,m}(\mathbf{d},\mathbf{X},\mathbf{P}) \geq 0\right) = F_{G_{ij,m}}(0) = \int_{G_{ij,m}(\mathbf{d},\mathbf{X},\mathbf{P})\geq 0} \cdots \int f_{\mathbf{X},\mathbf{P}}\left(\mathbf{X},\mathbf{P}\right) d\mathbf{X} d\mathbf{P} \geq \Phi\left(\beta_{t,m}\right) = P_{ij,m} \tag{3}$$

where Φ is the cumulative distribution function for standard normal distribution and $\beta_{t,m}$ is the target reliability index. $f_{\mathbf{X},\mathbf{P}}\left(\mathbf{X},\mathbf{P}\right)$ is a joint probability density function (PDF), which needs to be integrated. There are two different methods for the reliability assessment: the reliability index approach (RIA) [16] and the performance measure approach (PMA) [18]. RIA uses the reliability index (equation (4)) to describe the probabilistic constraint in equation (3).

$$\beta_{s,m} = \left(\Phi^{-1}\left(F_{G_{ij,m}}\left(0\right)\right)\right) \geq \beta_{t,m} \tag{4}$$

where $\beta_{s,m}$ is the safety reliability index for the m[th] probabilistic constraint. In RIA, the first-order safety reliability index is obtained using first-order reliability method (FORM). It is formulated as an optimization problem, with an implicit equality constraint in a standard U space defined as the limit state function.

$$\begin{aligned} &\min \|\mathbf{U}\| \\ &\text{s.t.} \quad G_{ij,m}\left(\mathbf{U}\right) = 0 \end{aligned} \tag{5}$$

where the vector **U** represents the random variables in the standard normal space. However, RIA may yield singularity in many reliability based design optimization (RBDO) applications. Moreover, it is more efficient in evaluating the violated probabilistic constraint. If the probabilistic constraint is inactive, RIA often yields a low rate of convergence [18]. To overcome these difficulties, PMA was developed to solve the RBDO problem. In this method, the reliability constraints are stated by an R-percentile formulation as

$$\Pr\left(G_{ij,m}\left(\mathbf{d},\mathbf{X},\mathbf{P}\right) \geq G_{ij,m}^{R}\right) = R \tag{6}$$

Equation (6) indicates that the probability of $G_{ij,m}$(**d, X, P**) greater than or equal to the R-percentile $G_{ij,m}^{R}$ is exactly equal to the desired reliability *R*. Instead of calculating the probability of failure directly, PMA judges whether or not a given design satisfies the probabilistic constraint with a given target reliability index R. Therefore, the original constraints that require the reliability assessment are now converted to constraints that evaluate the R-percentile. The percentile $G_{ij,m}^{R}$ can be evaluated by the inverse reliability analysis

$$\begin{aligned} &\min G_{ij,m}\left(\mathbf{U}\right) \\ &\text{s.t.} \quad \|\mathbf{U}\| = R \end{aligned} \tag{7}$$

3.2. Moment estimation

One purpose of statistical moment estimation stems from the robust design optimization, which attempts to minimize the quality loss, which is a function of the statistical mean and standard deviation [3]. The first two statistical moments of linking variables are estimated here to solve the higher-level problems and the overall multilevel design problem. Several methods are proposed to estimate the statistical moments of the output response. Monte Carlo simulation could be accurate for the moment estimation, however it requires a very large number of function evaluations. The first order Taylor series expansion has been widely used to estimate the first and second statistical moments in robust design. Nevertheless, the first order Taylor series expansion results in a large error especially when the input random variables have large variations. To overcome the shortcomings explained above, numerical integrations method have been recently proposed. The numerical integration methods rely on the principle that the first few moments of a random variable will adequately describe the complete PDF of the variable. The random variables are assumed to be statistically independent. Analytically, the statistical moments of the performance function H(**X**) can be expressed in an integration form as

$$\begin{aligned} E\left[H\right]^{1} &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} H(\mathbf{X}) f_{\mathbf{X}}\left(\mathbf{X}\right) d\mathbf{X} = \mu_{H} \\ E\left[\left(H(\mathbf{X}) - \mu_{H}\right)\right]^{k} &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} \left(H\left(\mathbf{X}\right) - \mu_{H}\right)^{k} f_{\mathbf{X}}\left(\mathbf{X}\right) d\mathbf{X} \end{aligned} \tag{8}$$

where $f_{\mathbf{X}}(\mathbf{X})$ is a joint PDF of the random parameters **X**. The numerical integration can be used either on the input domain or on the output domain. Since the computation of the moment could be very expensive through numerical integration on the input domain, a new formulation called performance moment integration (PMI) method is proposed for statistical moment calculation, which using numerical integration on the output domain [19]. The statistical moment calculation can be rewritten as

$$\begin{aligned} E\left[H\right]^{1} &= \int_{-\infty}^{\infty} h f_{H}\left(h\right) dh = \mu_{H} \\ E\left[\left(H - \mu_{H}\right)\right]^{k} &= \int_{-\infty}^{\infty} \left(h - \mu_{H}\right)^{k} f_{H}\left(h\right) dh \end{aligned} \tag{9}$$

where $f_H(h)$ is a probability density function of H. To approximate the statistical moments of H accurately, N-point numerical quadrature technique can be used as

$$\begin{aligned} E\left[H\right]^{1} &= \mu_{H} \cong \sum_{i=1}^{N} w_{i} h_{i} \\ E\left[H - \mu_{H}\right]^{k} &\cong \sum_{i=1}^{N} w_{i} (h_{i} - \mu_{H})^{k} \text{ for } 2 \leq k \leq 5 \end{aligned} \tag{10}$$

At minimum, the three-point integration is required to maintain a good accuracy in estimating first two statistical moments. By solving equation (10), three levels and weights on the output domain are obtained as $\{h_1, h_2, h_3\} = \left\{h_{\beta=-\sqrt{3}}, h(\mu_{\mathbf{X}}), h_{\beta=+\sqrt{3}}\right\}$ and $\{w_1, w_2, w_3\} = \left\{\frac{1}{6}, \frac{4}{6}, \frac{1}{6}\right\}$, respectively. Then, the mean and standard variation of the output response are approximated to be

$$\begin{aligned} E\left[H\right]^{1} &= \mu_{H} \cong \frac{1}{6} h_{\beta=-\sqrt{3}} + \frac{4}{6} h\left(\boldsymbol{\mu}_{\mathbf{X}}\right) + \frac{1}{6} h_{\beta=+\sqrt{3}} \\ E\left[H - \mu_{H}\right]^{2} &= \sigma_{H}^{2} = \int_{-\infty}^{\infty} \left(h - \mu_{H}\right)^{2} f_{H}\left(h\right) dh \\ &\cong \frac{1}{6}\left(h_{\beta=-\sqrt{3}} - \mu_{H}\right)^{2} + \frac{1}{6}\left(h_{\beta=+\sqrt{3}} - \mu_{H}\right)^{2} \end{aligned} \tag{11}$$

In equations (11), $h_{\beta=-\sqrt{3}}$ and $h_{\beta=+\sqrt{3}}$ can be obtained through inverse reliability analysis. The optimization problem used to approximate $h_{\beta=-\sqrt{3}}$ can be denoted as

$$\begin{aligned} &\min h(\mathbf{U}) \\ &\text{s.t.} \quad \|\mathbf{U}\| = \sqrt{3} \end{aligned} \tag{12}$$

The term $h_{\beta=+\sqrt{3}}$ in equations (11) can be approximated as the optimal cost obtained by maximizing $h(\mathbf{U})$ in equation (12). The term $h(\boldsymbol{\mu}_{\mathbf{X}})$ is the performance function value at the design point.

4. Single-loop method based probabilistic analytical target cascading

As mentioned above, since both the reliability analysis and the moment estimation make use of inverse reliability analysis to get percentile performance, it is very natural for the two different methodologies to be treated in a unified manner. In addition, each inverse reliability analysis is a separate optimization loop in the standard normal space. Then each subsystem optimization will be a nested, double-loop approach, which can drastically increase the computational cost. To accelerate the subsystem optimization, we employ the single loop method that has been developed for single-disciplinary systems [13]. It eliminates the need for inner reliability loops without increasing the number of design variables by using a relation representing the Karush-Kuhn-Tucker (KKT) optimality conditions instead of solving a nonlinear constrained optimization problem. The single loop method is used to efficiently evaluate percentile performances for both moment estimation and reliability assessments in PATC. The proposed strategy is named PATC-SL. For the optimization problem of equation (7), letting $R = \beta_{\mathrm{t,m}}$, the following KKT optimality condition is satisfied at the optimal point.

$$\nabla G_{ij.m}(\mathbf{U}) + \lambda \nabla H(\mathbf{U}) = 0 \tag{13}$$

where $H(\mathbf{U}) = \|\mathbf{U}\| - \beta_{t,m}$ is an equality constraint and λ is the corresponding Lagrange multiplier. According to the geometric explanation in reference [13], equation (13) states that the gradients $\nabla G_{ij,m}(\mathbf{U})$ and $\nabla H(\mathbf{U})$ are collinear and point in opposite directions. This condition yields

$$\begin{aligned} &\mathbf{U} = -\beta_{t,m} * \boldsymbol{\alpha} \\ &\boldsymbol{\alpha} = \nabla G_{\mathbf{U}}(\mathbf{d},\mathbf{X},\mathbf{P}) / \|\nabla G_{\mathbf{U}}(\mathbf{d},\mathbf{X},\mathbf{P})\| \end{aligned} \tag{14}$$

where $\boldsymbol{\alpha}$ is the constraint normalized gradient in U-space. Under the assumption for the PATC that the random variables are normally or can be approximated to be normally distributed [6], Equations (14) yield the following relationship between the most probable point (MPP) $\mathbf{X}_{mpp}$, $\mathbf{P}_{mpp}$ and the mean $\boldsymbol{\mu}_{\mathbf{X}}$, $\boldsymbol{\mu}_{\mathbf{P}}$.

$$\begin{aligned} &\mathbf{X}_{mpp} = \boldsymbol{\mu}_{\mathbf{X}} - \boldsymbol{\sigma} * \beta_{t,m} * \boldsymbol{\alpha}_{ij,m}, \quad \mathbf{P}_{mpp} = \boldsymbol{\mu}_{\mathbf{P}} - \boldsymbol{\sigma} * \beta_{t,m} * \boldsymbol{\alpha}_{ij,m} \\ &\boldsymbol{\alpha}_{ij,m} = \boldsymbol{\sigma} * \nabla G_{ij,m_{\mathbf{X},\mathbf{P}}}(\mathbf{d},\mathbf{X}_{mpp},\mathbf{P}_{mpp}) / \left\|\boldsymbol{\sigma} * \nabla G_{ij,m_{\mathbf{X},\mathbf{P}}}(\mathbf{d},\mathbf{X}_{mpp},\mathbf{P}_{mpp})\right\| \end{aligned} \tag{15}$$

where $\boldsymbol{\sigma}$ is the standard deviation vector of random variables $\mathbf{X}$ and random parameters $\mathbf{P}$. Equations (15) hold for each constraint $G_{ij,m}$ of equation (2). Similarly, according to equation (12), $h_{\beta=\pm\sqrt{3}}$ are obtained through reliability analysis at $\beta = \pm\sqrt{3}$ confidence levels. The approximate MPP can be denoted by

$$\begin{aligned} &\mathbf{X}_{\mathbf{R}_{ij},\beta=\pm\sqrt{3}} = \boldsymbol{\mu}_{\mathbf{X}} - \left(\pm\sqrt{3}\boldsymbol{\sigma}\right) * \boldsymbol{\alpha}_{\mathbf{R}_{ij},\beta=\pm\sqrt{3}} \\ &\mathbf{P}_{\mathbf{R}_{ij},\beta=\pm\sqrt{3}} = \boldsymbol{\mu}_{\mathbf{P}} - \left(\pm\sqrt{3}\boldsymbol{\sigma}\right) * \boldsymbol{\alpha}_{\mathbf{R}_{ij},\beta=\pm\sqrt{3}} \\ &\boldsymbol{\alpha}_{\mathbf{R}_{ij},\beta=\pm\sqrt{3}} = \frac{\boldsymbol{\sigma} * \nabla \mathbf{f}_{ij_{\mathbf{X},\mathbf{P}}}\left(\mathbf{d}, \mathbf{X}_{\mathbf{R}_{ij},\beta=\pm\sqrt{3}}, \mathbf{P}_{\mathbf{R}_{ij},\beta=\pm\sqrt{3}}\right)}{\left\|\boldsymbol{\sigma} * \nabla \mathbf{f}_{ij_{\mathbf{X},\mathbf{P}}}\left(\mathbf{d}, \mathbf{X}_{\mathbf{R}_{ij},\beta=\pm\sqrt{3}}, \mathbf{P}_{\mathbf{R}_{ij},\beta=\pm\sqrt{3}}\right)\right\|} \end{aligned} \tag{16}$$

Using equations (15) and equations (16), the double-loop optimization problem in equation (2) is transformed to the following single-loop, equivalent deterministic optimization problem.

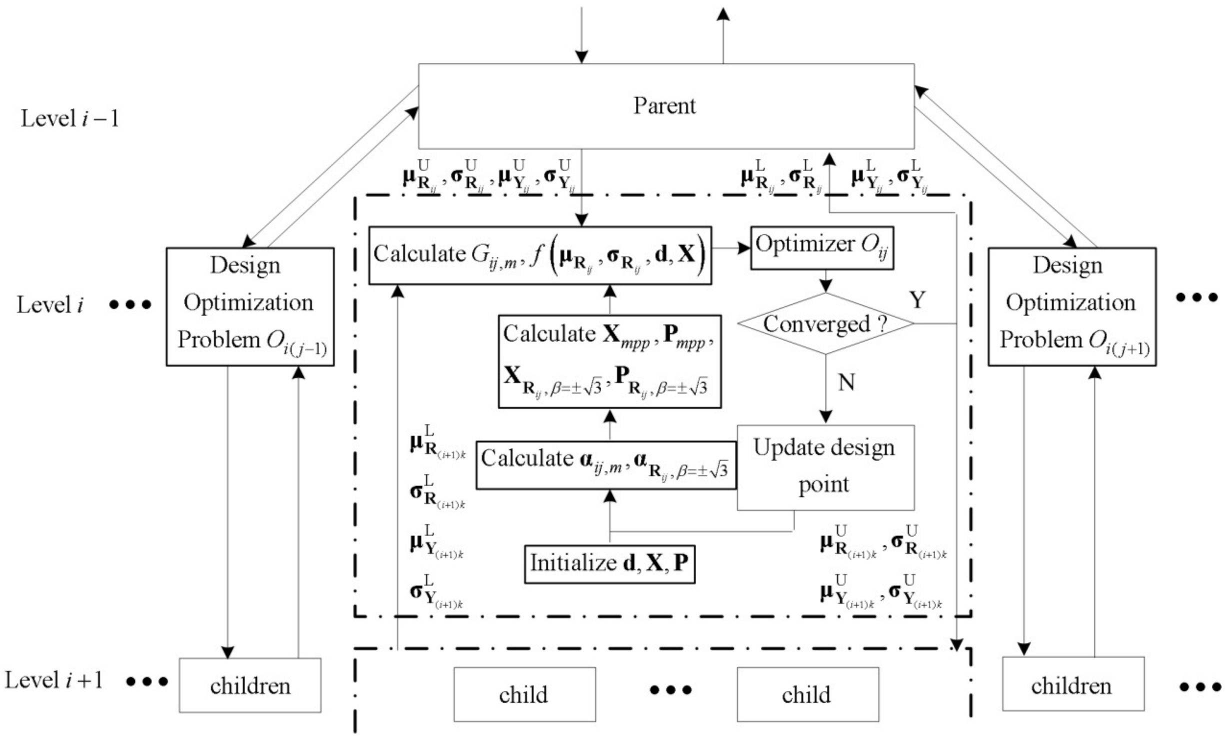

Fig. 1. Numerical process of single-loop method based PATC

$$
\begin{aligned}
&\min_{\mathbf{d},\mathbf{X}} f\left(\boldsymbol{\mu}_{\mathbf{R}_{ij}}, \boldsymbol{\sigma}_{\mathbf{R}_{ij}}, \mathbf{d}, \mathbf{X}\right) \\
&\text{s.t. } G_{ij,m}(\mathbf{d}, \mathbf{X}_{mpp}, \mathbf{P}_{mpp}) \ge 0 \quad m = 1,\ldots,M \\
&\boldsymbol{\mu}_{\mathbf{R}_{ij}} \cong \frac{1}{6}\mathbf{f}_{ij}(\mathbf{d}, \mathbf{X}_{\mathbf{R}_{ij},\beta=-\sqrt{3}}, \mathbf{P}_{\mathbf{R}_{ij},\beta=-\sqrt{3}}) + \frac{4}{6}\mathbf{f}_{ij}(\mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}, \boldsymbol{\mu}_{\mathbf{P}}) \\
&\qquad + \frac{1}{6}\mathbf{f}_{ij}(\mathbf{d}, \mathbf{X}_{\mathbf{R}_{ij},\beta=+\sqrt{3}}, \mathbf{P}_{\mathbf{R}_{ij},\beta=+\sqrt{3}}) \\
&\boldsymbol{\sigma}^2_{\mathbf{R}_{ij}} \cong \frac{1}{6}\left(\mathbf{f}_{ij}(\mathbf{d}, \mathbf{X}_{\mathbf{R}_{ij},\beta=-\sqrt{3}}, \mathbf{P}_{\mathbf{R}_{ij},\beta=-\sqrt{3}}) - \boldsymbol{\mu}_{\mathbf{R}_{ij}}\right)^2 \\
&\qquad + \frac{1}{6}\left(\mathbf{f}_{ij}(\mathbf{d}, \mathbf{X}_{\mathbf{R}_{ij},\beta=+\sqrt{3}}, \mathbf{P}_{\mathbf{R}_{ij},\beta=+\sqrt{3}}) - \boldsymbol{\mu}_{\mathbf{R}_{ij}}\right)^2
\end{aligned}
\tag{17}
$$

The single-loop method does not search for the MPP at each iteration. This dramatically improves the efficiency of the single-loop method without compromising the accuracy. Since $\boldsymbol{\alpha}_{ij,m}$ is a function of $\mathbf{X}_{mpp}$, equations (15) must be solved iteratively. That is, an iterative solution is obtained, where the normalized gradient from the previous iteration is used for constraint evaluation in the current iteration. The vectors $\boldsymbol{\alpha}_{ij,m}$ and $\mathbf{X}_{mpp}$ are alternately updated until the computations converge to a final probabilistic design. The same strategy is used for the calculation of $\boldsymbol{\mu}_{\mathbf{R}_{ij}}$ and $\boldsymbol{\sigma}_{\mathbf{R}_{ij}}$. Propagating uncertainty information during the PATC process should start at the bottom level of the hierarchy, where probability distribution on the input random variables and parameters are assumed as known. If such information is not available at the bottom level, start at the lowest level possible where it is available [10]. The process of PATC-SL is shown in Fig. 1.

To improve the convergence, formal methods for setting proper weights for element responses and linking variables can be found in Kim [9], Michalek [15], and Tosserams [17]. The augmented Lagrangian approach which shows stable convergence properties is used in this paper.

5. Numerical Examples

In this section, two examples are solved by the single-loop method based PATC. Comparing to other approaches, Performance of the proposed method is validated with respect to two criteria: accuracy of the solution and efficiency of the coordination process. For the accuracy comparison, the method is compared with the probabilistic all-in-one (PAIO) formulation using MCS technique (with 10000 samples), denoted as PAIO-MCS [14]. For the efficiency comparison, the current process is compared to probabilistic ATC employing linearization techniques (FORM and Taylor expansion), denoted as PATC-L [1].

5.1. Geometric programming problem

Geometric programming problem with polynomials is usually used to test the effectiveness of ATC formulations. The deterministic AIO and ATC formulations are provided by Kim [8]. Then it is formulated in a probabilistic form to demonstrate whether the PATC is capale of reaching the same optimal solution[6, 14]. The PAIO problem is formulated as

$$
\begin{aligned}
&\min E\left[f\right] = \mu^2_{X_1} + \mu^2_{X_2} \\
&\text{with respect to} \\
&x = \left\{x_4, x_5, x_7, \mu_{X_8}, x_9, x_{10}, \mu_{X_{11}}, x_{12}, x_{13}, x_{14}\right\}^T \\
&\text{subject to} \Pr\left[g_i \le 0\right] \ge \alpha_i, \quad i = 1,\ldots,6
\end{aligned}
\tag{18}
$$

where
$$
\begin{aligned}
g_1 &= \left(X_3^{-2} + x_4^2\right)x_5^{-2} - 1 \\
g_2 &= \left(x_5^2 + X_6^{-2}\right)x_7^{-2} - 1 \\
g_3 &= \left(x_8^2 + x_9^2\right)X_{11}^{-2} - 1 \\
g_4 &= \left(x_8^{-2} + x_{10}^2\right)X_{11}^{-2} - 1 \\
g_5 &= \left(X_{11}^2 + x_{12}^{-2}\right)x_{13}^{-2} - 1 \\
g_6 &= \left(X_{11}^2 + x_{12}^2\right)x_{14}^{-2} - 1 \\
X_1 &= \left(X_3^2 + x_4^{-2} + x_5^2\right)^{1/2} \\
X_2 &= \left(x_5^2 + X_6^2 + x_7^2\right)^{1/2} \\
X_3 &= \left(X_8^2 + x_9^{-2} + x_{10}^{-2} + X_{11}^2\right)^{1/2} \\
X_6 &= \left(X_{11}^2 + x_{12}^2 + x_{13}^2 + x_{14}^2\right)^{1/2}
\end{aligned}
$$

Design variables X_8 and X_{11} are assumed to be independent and normally distributed with constant standard deviations $\sigma_{X_8} = \sigma_{X_{11}} = 0.1$. The required reliability level is 99.865% for all probabilistic constraints.

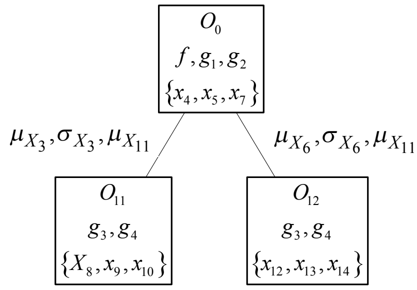

Fig. 2. Hierarchical structure of example 1

Table. 1. Optimal solutions and number of function evaluations for example 1

	PAIO-MCS	PATC-L	PATC-SL
X_4	0.76	0.754	0.76
X_5	0.86	0.855	0.86
X_7	0.91	0.905	0.906
μ_{X_8}	1.03	1.04	1.046
X_9	0.76	0.7	0.69
X_{10}	0.81	0.76	0.78
$\mu_{X_{11}}$	1.68	1.645	1.651
X_{12}	0.84	0.923	0.824
X_{13}	2.31	2.24	2.3
X_{14}	2.15	2.17	2.13
E[*f*]	24.67	24.9	24.7
Relative error of σ_{X_3}	0.272%	1.05%	0.397%
Relative error of σ_{X_6}	0.0437%	0.177%	0.081%
Number of function evaluations	5243×10000	40599	3305

The structure of the decomposed problem is illustrated in Fig. 2. The randomness in X_8 and X_{11} results in uncertainties in all computed response X_3, X_6, each described by its mean and standard deviation. Note that since the standard deviation of the random design variable X_{11} is assumed constant, it is not included as a linking variable. The initial point is set to the deterministic optimal point, {0.76, 0.87, 0.94, 0.97, 0.87, 0.80, 1.30, 0.84, 1.76, 1.55}.

Table 1 summarizes the results obtained from the three algorithms. The optimization algorithm used is sequential quadratic programming. The specified tolerance of consistency is 1.0×10^{-4}. σ_{X_3} and σ_{X_6} are validated via MCS with 100000 samples, and the relative errors are also displayed in Table 1. It is found that PATC-SL is more accuracy in estimating σ_{X_3} and σ_{X_6}, and achieves better solutions than PATC-L. More importantly, with no nested optimization loops, the number of function evaluations for PATC-SL is significantly smaller than those for others. Furthermore, the average number of iteration cycles of each system-level optimization using PATC-SL method is about 25, and the PATC-L method is around 70. Higher rate of convergence further improves the algorithmic efficiency. Compared to PATC-L, the results show that the PATC-SL improves the computational efficiency by more than 12 times.

5.2. Reliability optimum allocation problem

In this section, we demonstrate the methodology for reliability allocation using a two-level example. The deterministic formulation of decomposed optimization problem is presented in reference [20]. Through Fig. 3 it is apparent that the system is composed of five subsystems and each subsystem encompasses two components.

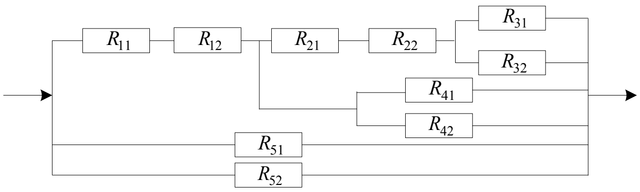

Fig. 3. The topology structure of system in the example 2

$$
\begin{aligned}
&\min \sum_{i=1}^{5}\sum_{j=1}^{2} C_{ij}\\
&\text{s.t. } R_s \ge 0.999\\
&\quad R_s = R_5 + R_1(1-R_5)(R_2R_3 + R_4 - R_2R_3R_4)\\
&\quad 0.5 \le R_{ij} \le 0.98, i=1,2; j=1,2\\
&\quad 0.2 \le R_{ij} \le 0.99, i=3,4,5; j=1,2\\
&\quad R_i = R_{i1}R_{i2} \ge 0.5, i=1,2\\
&\quad 0.5 \le R_i \le 0.998, i=3,4,5\\
&\quad C_{i1} = R_{i1}^2/3, C_{i2} = R_{i2}^2/2, i=1,2\\
&\quad C_{i1} = [\ln(1-R_{i1})]^2/100, i=3,4,5\\
&\quad C_{i2} = [\ln(1-R_{i2})]^2/60, i=3,4,5
\end{aligned}
\tag{19}
$$

where R is the reliability requirement and C is the cost. Subscript 's', 'i' and 'ij' indicate corresponding value of main system, subsystem i and component j in subsystem i, respectively. Treating component reliability as uncertain parameters is necessary as they are usually empirically determined [7]. Then all the reliability constraints should be transformed into confidence-level formulation to ensure that system reliability requirements are met with high probability. Here, the component reliability R_{ij} follows a normal distribution. The standard deviation is assumed to be 0.005. A 95% confidence level is used for every system and subsystem reliability constraint. The corresponding PATC decomposition is shown in Fig. 4.

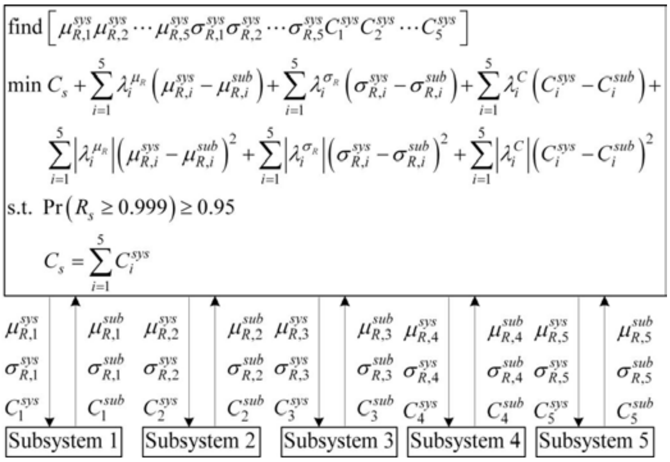

Fig. 4. The PATC-decomposed formulation of the reliability allocation problem

According to PATC, the mean and standard deviation of R_i is defined as linking variables, denoted as $\mu_{R,i}$ and $\sigma_{R,i}$ correspondingly. Auxiliary variables C_i is also introduced to calculate the total cost. Superscript '*sys*' or '*sub*' indicates the value allocated by main system or subsystem, respectively. Under the augmented Lagrangian ATC formulation [9], the consistency constraints can be incorporated into the objective function. $\lambda_i^{\mu_R}$, $\lambda_i^{\sigma_R}$ and λ_i^{C} denote the Lagrange multipliers associated with the deviations of $\mu_{R,i}$, $\sigma_{R,i}$ and C_i. The subsystem optimization can be formulated as

$$
\begin{aligned}
&\text{find } \mu_{R_{i1}}, \mu_{R_{i2}}\\
&\min \lambda_i^{\mu_R}\left(\mu_{R,i}^{sys}-\mu_{R,i}^{sub}\right) + \lambda_i^{\sigma_R}\left(\sigma_{R,i}^{sys}-\sigma_{R,i}^{sub}\right) + \lambda_i^{C}\left(C_i^{sys}-C_i^{sub1}\right) +\\
&\quad \left|\lambda_i^{\mu_R}\right|\left(\mu_{R,i}^{sys}-\mu_{R,i}^{sub}\right)^2 + \left|\lambda_i^{\sigma_R}\right|\left(\sigma_{R,i}^{sys}-\sigma_{R,i}^{sub}\right)^2 + \left|\lambda_i^{C}\right|\left(C_i^{sys}-C_i^{sub1}\right)^2\\
&\text{s.t. } \Pr\left(0.5 \le R_1^{sub1} \le 1\right) \ge 0.95\\
&\quad 0.5 \le \mu_{R_{i1}} \le 0.98; 0.5 \le \mu_{R_{i2}} \le 0.98, i=1,2
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
&\text{find } \mu_{R_{i1}}, \mu_{R_{i2}}\\
&\min \lambda_i^{\mu_R}\left(\mu_{R,i}^{sys}-\mu_{R,i}^{sub}\right) + \lambda_i^{\sigma_R}\left(\sigma_{R,i}^{sys}-\sigma_{R,i}^{sub}\right) + \lambda_i^{C}\left(C_i^{sys}-C_i^{sub1}\right) +\\
&\quad \left|\lambda_i^{\mu_R}\right|\left(\mu_{R,i}^{sys}-\mu_{R,i}^{sub}\right)^2 + \left|\lambda_i^{\sigma_R}\right|\left(\sigma_{R,i}^{sys}-\sigma_{R,i}^{sub}\right)^2 + \left|\lambda_i^{C}\right|\left(C_i^{sys}-C_i^{sub1}\right)^2\\
&\text{s.t. } \Pr\left(0.5 \le R_i^{sub1} \le 0.998\right) \ge 0.95\\
&\quad 0.2 \le \mu_{R_{i1}} \le 0.99; 0.2 \le \mu_{R_{i2}} \le 0.99, i=3,4,5
\end{aligned}
\tag{21}
$$

The means of component reliability $\mu_{R_{i1}}$ and $\mu_{R_{i2}}$ are treated as design variables. ATC is implemented first to find the deterministic optimal point, which is chosen as the initial design point of PAIO-MCS to prevent unnecessary and expensive reliability analyses for infeasible and otherwise undesirable design points.

The results are listed in Table 2 for comparison, where S_{ij} (i=1,2,…, 5,j=1,2) represents the component j in subsystem i. It shows that, the deterministic results have low confidence level once the uncertainty of the input variables is considered. With confidence-level constraints, both PAIO-MCS and PATC-SL improve the probability of meeting the reliability requirements. The accuracy of PATC-SL is excellent for this example as well. Fig. 5 shows the iteration histories for main optimization of ATC and PATC-SL. The efficiency of PATC-SL is comparable to the deterministic optimization.

Table. 2. Reliability optimum allocation results for example 2

		Subsystem(S_1)		Subsystem(S_2)		Subsystem(S_3)		Subsystem(S_4)		Subsystem(S_5)	
		S_{11}	S_{12}	S_{21}	S_{22}	S_{31}	S_{32}	S_{41}	S_{42}	S_{51}	S_{52}
ATC	R_{ij}	0.804	0.631	0.788	0.643	0.381	0.250	0.544	0.375	0.978	0.908
	C_{ij}	0.215	0.199	0.207	0.207	0.0023	0.0014	0.006	0.003	0.146	0.095
	R_i	0.507		0.51		0.535		0.715		0.998	
		Pr($Rs \geq$ 0.999)=0.249, Cs=1.083									
PAIO-MCS	$\mu_{R_{ij}}$	0.947	0.774	0.789	0.644	0.366	0.276	0.726	0.628	0.961	0.933
	C_{ij}	0.299	0.299	0.208	0.207	0.0021	0.0017	0.017	0.016	0.105	0.122
	μ_{R_i}	0.733		0.508		0.56		0.928		0.997	
		Pr($Rs \geq$ 0.999)=0.95, Cs=1.280									
PATC-SL	$\mu_{R_{ij}}$	0.929	0.788	0.741	0.686	0.341	0.274	0.881	0.385	0.950	0.948
	C_{ij}	0.288	0.310	0.183	0.235	0.0017	0.0017	0.045	0.004	0.090	0.146
	μ_{R_i}	0.732		0.516		0.521		0.927		0.997	
		Pr($Rs \geq$ 0.999)=0.952, Cs=1.304									

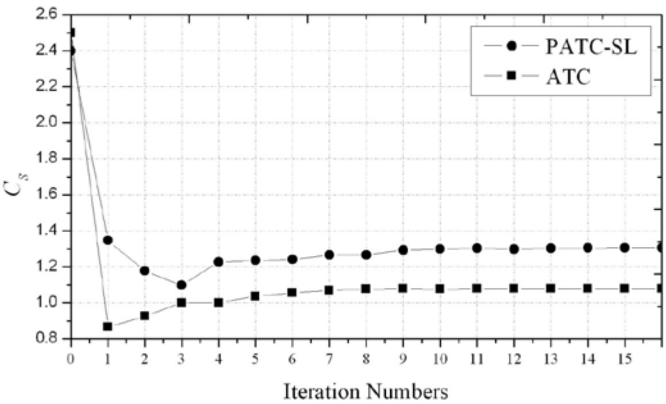

Fig. 5. Optimization history for the reliability allocation problem

6. Conclusions

The estimation uncertainty of component reliability is considered in this paper. To deal with the issue of modeling uncertainty propagation in multilevel hierarchies, PMA and PMI are investigated in the paper. A new PATC framework is developed by combining the single loop method with the uncertainty propagation techniques to solve the reliability allocation problem under uncertainty. Compared with the previous methods, the new approach requires no nested optimization loop. This makes it extremely efficient. Through the present study, it is shown that:

1) Compared to the all-in-one (AIO) method with MCS, The accuracy of the proposed PATC-SL formulation is demonstrated. The single-loop method based PATC can be useful for many nonlinear engineering systems.
2) Compared to PATC-L and ATC, the efficiency of PATC-SL is validated. Its efficiency is almost equivalent to deterministic optimization.
3) Evaluating system reliability in a probabilistic approach is meant to aid system architects make informed risk-based decisions rather than the traditional safety factor approaches. The proposed PATC-SL method is more practical for engineering application with an acceptable accuracy and better computational. Higher efficiency can be achieved by improving the convergence speed, which needs to be further studied.

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