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Stability analysis of fuzzy control systems simplified as a discrete system

by

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Abstract: Fuzzy controls can describe control rules using fuzzy if-then rules and it can incorporate experts' control rules. It is, however, difficult to guarantee the stability of fuzzy control system. This paper presents a new method for stability analysis of fuzzy control system using petri nets. The proposed method crisply divides all of the input and output variables of the fuzzy controller and the fuzzy model of the controlled object. This crisp division makes the fired rules of the fuzzy controller and the fuzzy model, which have truth values greater then 0, be single each. These simplified fuzzy rules can be considered as discrete description of the controller and the controlled object. By approximating the fuzzy control system as this discrete system, the system can be expressed by the petri nets. The proposed method describes the fuzzy control system using matrix based on a bipartite directed multigraph of the petri net, thereby enables to analyze the stability of the fuzzy control system. The analytic results using the petri nets have clear correspondence to the fired fuzzy rules. The dynamical behavior of the system are able to be understood easily. Simulation is done to verify the proposed stability analysis method.

Keywords: Stability analysis, Petri net, fuzzy control

1. Introduction

Fuzzy control can describe control rules using fuzzy if-then rules and it can incorporate experts' control knowledge. It is, however, difficult to guarantee the stability of fuzzy control system. Studies have been done to analyze the stability of fuzzy control System, Kitamura and Kurozumi (1991), Hojo, Terano, Masui (1992), Tanaka and Sugeno (1990, 1992). These studies are effective to analyze the stability of the fuzzy control system. However, the distinguishing feature of the fuzzy controls, i.e. easily understandable linguistic expressions, was not used in these analysis methods.

The authors have proposed a method to describe the dynamical behavior of the fuzzy control system Furuhashi, Horikawa and Uchikawa (1993), Hasegawa, Horikawa, Furuhashi and Uchikawa (1995), Adachi, Horikawa, Furuhashi and Uchikawa (1995). The authors call the method "Rule-to-Rule Mapping method". This method can describe the dynamical behavior of the fuzzy control system as transitions between the fuzzy rules of the controller and the controlled object. In Adachi, Horikawa, Furuhashi and Uchikawa (1995), a new design method of fuzzy controller from linguistic specifications which uses the linguistic rules of the fuzzy model of the controlled object was also proposed. However, the stability analysis using the Rule-to-Rule Mapping method was not able to be done.

The authors Hasegawa, Furuhashi and Uchikawa (1996a, b, c) have proposed a stability analysis method of fuzzy control system using petri nets. These methods simplify the fuzzy control system as a discrete system and describe the fuzzy control system using the petri nets, see Reisig (1985). The next-state function of the petri nets enables to analyze the stability of the fuzzy control system. In these proposed methods, all transitions of the petri net have clear relationships with the fuzzy rules of the controller and the controlled object. It is easy to grasp the relationships between the behavior of the control system and the fuzzy rules. This method made the best use of the distinguishing feature of fuzzy controls, i.e. easily understandable linguistic expressions. However, the proposed analysis method yet tests the firing vector of transitions experimentally to analyze the stability of the fuzzy control system.

This paper presents a new method for stability analysis of the fuzzy control system using a matrix based on a bipartite directed multigraph of the petri net. This method enables to analyze the stability of the fuzzy control system by calculation of matrices and the vectors. This paper presents a new theorem of stability of fuzzy control systems. Simulation is done to verify the stability analysis by the proposed method.

2. Description of the fuzzy rules using Petri nets

Fuzzy control is an effective tool to incorporate experts' control know-how into the controller. This paper deals with the design of fuzzy controller to automatize controls done by human experts. When the knowledge of the controlled object is available, the incorporation of experts' control know-how can be expedited. Fuzzy modeling of the controlled object is effective to acquire the knowledge of the object in the form of fuzzy rules Adachi, Horikawa, Furuhashi and Uchikawa (1995). These fuzzy rules can be utilized for the design of fuzzy controller. It is assumed that the i-th fuzzy rule of the fuzzy model of the controlled object is obtained as

$$\begin{aligned} R_P^i : &\text{ If } y_k \text{ is } A_{i1} \text{ and } y_{k-1} \text{ is } A_{i2} \text{ and } \cdots \\ &\text{ and } u_k \text{ is } B_{i1} \text{ and } u_{k-1} \text{ is } B_{i2} \text{ and } \cdots \\ &\text{ then } y_{k+1} = A_{i0}, \end{aligned}$$

where y is the output of the controlled object, u is the manipulated variable and subscript k means the sampling time. A_{ij}, B_{ij} $(j = 1, 2, \cdots)$ are fuzzy numbers. In this type of rule expressions, the inputs and outputs of the controlled object are bounded. These fuzzy rules of the model of the controlled object provide useful information for the design of the fuzzy controller. The following i-th fuzzy rule of the fuzzy controller can be designed by incorporating experts' know-how:

$$\begin{aligned} R_C^i : \text{If } & r_{k+1} \text{ is } A_{i0} \\ & \text{and } y_k \text{ is } A_{i1} \text{ and } y_{k-1} \text{ is } A_{i2} \text{ and } \cdots \\ & \text{and } u_{k-1} \text{ is } B_{i1} \text{ and } u_{k-2} \text{ is } B_{i2} \text{ and } \cdots \\ & \text{then } u_k = B_{i0}, \end{aligned}$$

where r denotes the command of the system.

Usually, the input and output variables of the fuzzy controller and the fuzzy model of the controlled object are divided fuzzily as shown in Fig. 1(a). For the rough evaluation of the designed fuzzy controller with the experts' control knowledge, this paper uses the following approximation. All of the input and output variables of the fuzzy controller and the fuzzy model of the controlled object are divided crisply as shown in Fig. 1(b). The divided points are set at the crossing points of the membership functions. This crisp division makes the fired rules of the fuzzy controller and the fuzzy model, which have truth values greater then 0, be single each. These simplified fuzzy rules can be considered as discrete description of the controller and the controlled object. This approximation sacrifices the rigorous description of the behavior of the fuzzy control system. But it is effective to grasp the overall behavior of the fuzzy control system easily. Fine tuning of membership functions is the next step after the guarantee of stability by this analysis.

3. Stability analysis

3.1. Matrix representation of network

In this paper, the fuzzy control system is described as a petri net. In this petri net, all the transitions have only one input each.

The fuzzy control system is expressed with a bipartite directed multigraph expression of the petri nets. Fig. 2 shows an example of the bipartite directed multigraph. ○ denotes the place P_i. The places describe a set of conditions. | denotes the transition t_j. The transitions ignite the transitions of tokens. • denotes the token and represents a condition of the system. → denotes the arc. Arcs indicate the directions of flow of tokens.

The transition occurs when the input place has a token. This transition removes the token from its input place and puts the token on its output place.

The proposed method describes the fuzzy control system as a bipartite directed multigraph according to the following steps:

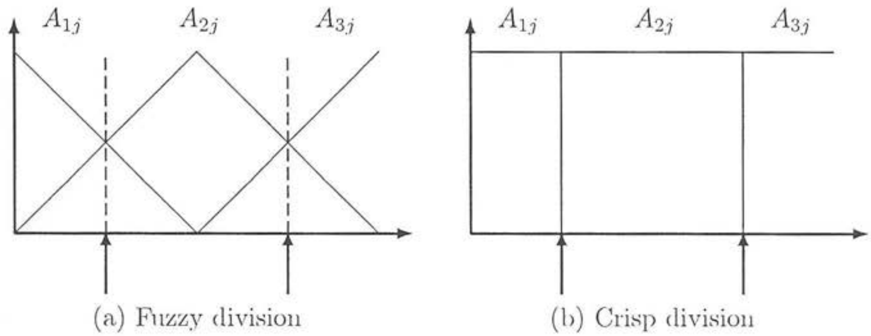


Figure 1. Approximation of membership function using crisp division

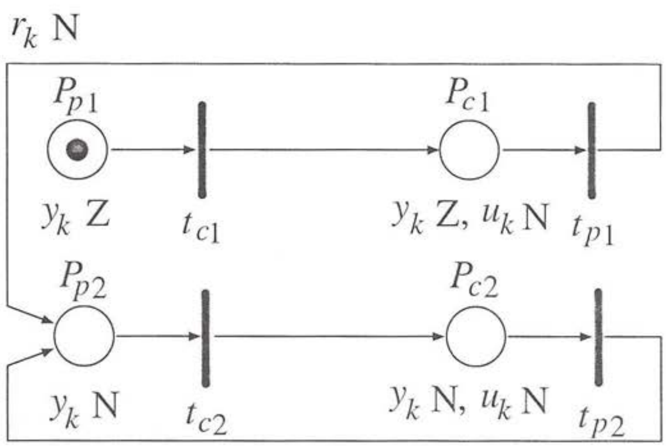


Figure 2. Example of petri net

1. The places P_{pi} are created. Each place corresponds to one of the states of the controlled object. The inner state of each place has the output of the controlled object y and the command value r.
2. The transitions t_{ci} are created. The number of the t_{ci} is the same as that of the fuzzy control rules. Each transition has a connection from one of the places P_{pi} with an arc, and to one of the places P_{ci} with another arc. The transition t_{ci} denotes the firing of the i-th rule of the fuzzy controller.
3. The places P_{ci} are created. Each place has a connection from one of the transition t_{ci} with an arc. P_{ci} denotes the state after the firing of the i-th fuzzy control rule. The inner state of each place has the manipulated value u, the output of the controlled object y, and the command value r.
4. The transitions t_{pi} are created. The number of t_{pi} is the same as that of the fuzzy control rules. Each transition t_{pi} does not have one-to-one correspondence to each rule of the fuzzy model of the controlled object. Each transition t_{pi} has a connection from one of the places P_{ci}, and to one of the places P_{pi}. The firing of the transition t_{pi} means the firing of the i-th rule of the fuzzy model of the controlled object.

By the above steps, the fuzzy control system can be expressed with bipartite directed multigraph. Each transition has only one input. Fig. 2 shows the bipartite directed multigraph of the following rules:

R_C^1 : If r_k is N and y_k is Z then u_k=N
R_C^2 : If r_k is N and y_k is N then u_k=N

R_P^1 : If y_k is Z and u_k is N then y_{k+1}=N
R_P^2 : If y_k is N and u_k is N then y_{k+1}=N.

The stability analysis of the fuzzy control system can be done using the obtained bipartite directed multigraph which can be transformed to a matrix.

Matrices $\boldsymbol{D}^-$, $\boldsymbol{D}^+$ are defined. $\boldsymbol{D}^-$ denotes the input function of the petri net and $\boldsymbol{D}^+$ means the output function of the petri net. Each matrix has n rows and m columns. n means the number of places and m means the number of transitions. ij-th elements of the matrices are defined as follows:

$$\begin{aligned} \boldsymbol{D}^-[i,j] &= \#(P_i, \boldsymbol{I}(t_j)) \\ \boldsymbol{D}^+[i,j] &= \#(P_i, \boldsymbol{O}(t_j)), \end{aligned} \tag{1}$$

where P_i denotes the state of i-th place and t_j denotes the j-th transition, $\boldsymbol{I}(t_j)$ describes a set of input places needed to fire the transition t_j, $\boldsymbol{O}(t_j)$ describes a set of output places of the transition t_j, $\#(P_i, \boldsymbol{I}(t_j))$ denotes the number of tokens on P_i which are needed to fire the transition t_j, $\#(P_i, \boldsymbol{O}(t_j))$ also denotes the number of tokens on P_i which are increased after the firing of the transition t_j.

From the above definitions, the input-output function of petri nets can be defined as

$$\boldsymbol{D} = \boldsymbol{D}^+ - \boldsymbol{D}^-. \tag{2}$$

The transition from the initial marking $\boldsymbol{\mu}_0$, which denotes the number of initial tokens on each place, to the marking $\boldsymbol{\mu}$ by the firing sequence of transitions $\sigma = t_{j_1} t_{j_2} \cdots t_{j_k}$ is expressed with

$$\boldsymbol{\mu} = \boldsymbol{\mu}_0 + \boldsymbol{D} \cdot \boldsymbol{f}(\sigma), \tag{3}$$

where $\boldsymbol{f}(\sigma)$ is the firing vector for the sequence $t_{j_1} \cdots t_{j_k}$. The i-th component $\boldsymbol{f}(\sigma)_i$ of $\boldsymbol{f}(\sigma)$ means the number of firings of t_i at sequence $t_{j_1} t_{j_2} \cdots t_{j_k}$. In this paper, all the transitions of the petri net have one input each. The transition possible to be fired at the marking $\boldsymbol{\mu}$ can be calculated as

$$\boldsymbol{D}^{-T} \cdot \boldsymbol{\mu}, \tag{4}$$

where $\boldsymbol{D}^{-T}$ means the transposed matrix of $\boldsymbol{D}^-$. From eqs. (3) and (4), the marking after the transition at the marking $\boldsymbol{\mu}_k$ is calculated as

$$\begin{aligned} \boldsymbol{\mu}'_k &= \boldsymbol{\mu}_k + \boldsymbol{D} \cdot \boldsymbol{D}^{-T} \cdot \boldsymbol{\mu}_k \\ &= (\boldsymbol{I} + \boldsymbol{D} \cdot \boldsymbol{D}^{-T}) \cdot \boldsymbol{\mu}_k, \end{aligned} \tag{5}$$

where $\boldsymbol{I}$ is unit matrix. Since this petri net describes the control system, the token moves from one of the places of the controller to one of the places of the controlled object and vice versa, alternately. Let $\boldsymbol{\mu}_k$ be the marking of the controller, then $\boldsymbol{\mu}'_k$ is the marking of the controlled object. At one sampling time, two transitions each in the controller and in the controlled object occur. The marking $\boldsymbol{\mu}_k$ at k samplings from the initial state $\boldsymbol{\mu}_0$ is described as follows:

$$\begin{aligned} \boldsymbol{\mu}_k &= (\boldsymbol{I} + \boldsymbol{D} \cdot \boldsymbol{D}^{-T}) \cdot \boldsymbol{\mu}'_{k-1} \\ &= (\boldsymbol{I} + \boldsymbol{D} \cdot \boldsymbol{D}^{-T}) \cdot (\boldsymbol{I} + \boldsymbol{D} \cdot \boldsymbol{D}^{-T}) \cdot \boldsymbol{\mu}_{k-1} \\ &= (\boldsymbol{I} + \boldsymbol{D} \cdot \boldsymbol{D}^{-T})^2 \cdot \boldsymbol{\mu}_{k-1} \\ &\ \ \vdots \\ &= (\boldsymbol{I} + \boldsymbol{D} \cdot \boldsymbol{D}^{-T})^{2k} \cdot \boldsymbol{\mu}_0, \end{aligned} \tag{6}$$

where a matrix $\boldsymbol{A}$ is defined as

$$\boldsymbol{A} = (\boldsymbol{I} + \boldsymbol{D} \cdot \boldsymbol{D}^{-T}).$$

Eq. (6) can be re-described as

$$\boldsymbol{\mu}_k = \boldsymbol{A}^{2k} \cdot \boldsymbol{\mu}_0. \tag{7}$$

In the case of Fig. 2, the marking $\boldsymbol{\mu}$ is described as

$$\boldsymbol{\mu} = \begin{pmatrix} P_{p1} \\ P_{p2} \\ P_{c1} \\ P_{c2} \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{8}$$

This marking shows that place P_{p1} has a token and the others have no token. The input function is

$$D^- = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{9}$$

$$= I. \tag{10}$$

The output function is

$$D^+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{11}$$

From the above functions, the input-output function is

$$D = D^+ - D^- \tag{12}$$

$$= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}. \tag{13}$$

As a result, the matrix A is

$$A = I + D \cdot D^{-T} \tag{14}$$

$$= D^- + D \cdot I^T \tag{15}$$

$$= D^+ \tag{16}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{17}$$

3.2. Stability

The stability of the fuzzy control system can be analyzed using the above matrices and vectors.

The behavior of the fuzzy control system simplified as the discrete system can be expressed with eq. (7). When the initial state of fuzzy control system has one token on an arbitrary place P_{pi}, this token moves onto one of the places P_{pj} at $2k(k \geq 1)$. As a result, the stability of the fuzzy control system can be defined as follows:

DEFINITION 3.1 (EQUILIBRIUM STATE) *The marking* $\boldsymbol{\mu}_k$ *at sampling time* k *is in the equilibrium state where*

$$\begin{aligned} \boldsymbol{\mu}'_k &\neq \boldsymbol{\mu}_k, & (18)\\ \boldsymbol{\mu}_{k+1} &= \boldsymbol{\mu}_k, & (19)\\ \boldsymbol{\mu}'_{k+1} &= \boldsymbol{\mu}'_k. & (20) \end{aligned}$$

This definition expresses that the markings for both the controller and the controlled object remain unchanged.

DEFINITION 3.2 (PERIODIC STATE) *The marking* $\boldsymbol{\mu}_k$ *at sampling time* k *is in the periodic state where there exists a positive integer* $\lambda \geq 2$, *and*

$$\begin{aligned} \boldsymbol{\mu}'_k &\neq \boldsymbol{\mu}_k, & (21)\\ \boldsymbol{\mu}_{k+\lambda} &= \boldsymbol{\mu}_k, & (22)\\ \boldsymbol{\mu}'_{k+\lambda} &= \boldsymbol{\mu}'_k. & (23) \end{aligned}$$

This definition expresses that the same markings appear periodically at more than 1 sampling time.

DEFINITION 3.3 (UNSTABLE STATE) *The marking* $\boldsymbol{\mu}_k$ *at sampling time* k *is in the unstable state where*

$$\boldsymbol{\mu}'_k = \boldsymbol{\mu}_k. \tag{24}$$

This method deals with bounded input-bounded output (BIBO) controlled object. The transitions occur in the bounded area. Unstable state of the system is defined as the one without any transitions to be fired next.

DEFINITION 3.4 (ASYMPTOTICALLY STABLE) *The fuzzy control system is asymptotically stable where there exist* $K \geq 0$ *for any initial marking* $\boldsymbol{\mu}_0$ *and*

$$\begin{aligned} \boldsymbol{\mu}'_K &\neq \boldsymbol{\mu}_K, & (25)\\ \boldsymbol{\mu}_{K+1} &= \boldsymbol{\mu}_K, & (26)\\ \boldsymbol{\mu}'_{K+1} &= \boldsymbol{\mu}'_K. & (27) \end{aligned}$$

A lemma is derived using the above definitions.

LEMMA 3.1 (UNSTABLE STATE) *Iff any one of ii-th elements of the matrix* $\boldsymbol{A}$ *in eq. (7) is 1, the fuzzy control system has unstable state(s).*

The theorem of stability is derived as follows:

THEOREM 3.1 (ASYMPTOTICALLY STABLE) *The necessary and sufficient condition of asymptotic stability of the fuzzy control system simplified as the discrete system for arbitrary initial state is that there exists no zero column vector and no unity ii-th element and*

$$rank\ \boldsymbol{A}^n = 2$$

for n-dimensional square matrix $\boldsymbol{A}$.

y_{k+1}		y_k		
		N	Z	P
u_k	N	N	N	Z
	Z	N	Z	Z
	P	Z	P	P

Table 1. Fuzzy rules of controlled object

[Proof]

(Sufficient condition)

The state of fuzzy control system $\boldsymbol{\mu}_k$ at $k(=n/2)$ sampling after the initial state $\boldsymbol{\mu}_0$ is described as

$$\boldsymbol{\mu}_k = \boldsymbol{A}^{2k} \cdot \boldsymbol{\mu}_0$$

from eq. (7). There exists no zero column vector in $\boldsymbol{A}$ and any ii-th element of $\boldsymbol{A}$ is not 1, so

$$\boldsymbol{\mu}'_k \neq \boldsymbol{\mu}_k.$$

Since,

$$\text{rank}\boldsymbol{A}^n = 2.$$

At $k = n/2$, the state of fuzzy control system $\boldsymbol{\mu}_k$ settles down to

$$\boldsymbol{\mu}_k = \boldsymbol{e}[i]$$

regardless of the initial state $\boldsymbol{\mu}_0$. $\boldsymbol{e}[i]$ denotes n-dimensional unit row vector. As a result, the marking will not change after k, and

$$\begin{aligned}\boldsymbol{\mu}_{k+1} &= \boldsymbol{\mu}_k \\ &= \boldsymbol{e}[i].\end{aligned}$$

(Necessary condition)

Self-evident by the definitions.

4. Simulation

In this section, the proposed method is applied to the simple control system to verify the proposed stability analysis method.

Table 1 and 2 show the fuzzy rules of a fuzzy model of a controlled object and a fuzzy controller used in this simulation.

The input function of the fuzzy control system defined in Table 1 and 2 is

$$\boldsymbol{D}^- = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (28)$$

u_k		y_k		
		N	Z	P
r_k	N	N	N	Z

Table 2. Fuzzy rules of controller

$$= \boldsymbol{I}.$$

The output function is

$$\boldsymbol{D}^{+} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}. \tag{29}$$

From the above functions, the input-output function is

$$\boldsymbol{D} = \begin{pmatrix} -1 & 0 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{pmatrix}. \tag{30}$$

As a result, the matrix $\boldsymbol{A}$ is

$$\boldsymbol{A} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}. \tag{31}$$

Here, this 6-dimensional square matrix satisfies

$$\boldsymbol{A}^{6} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{32}$$

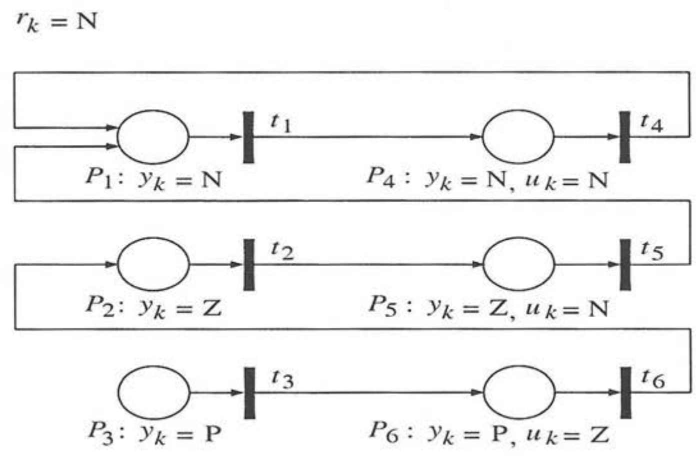


Figure 3. Bipartite directed multigraph of simulation system

and

$$\mathrm{rank}\boldsymbol{A}^6 = 2. \tag{33}$$

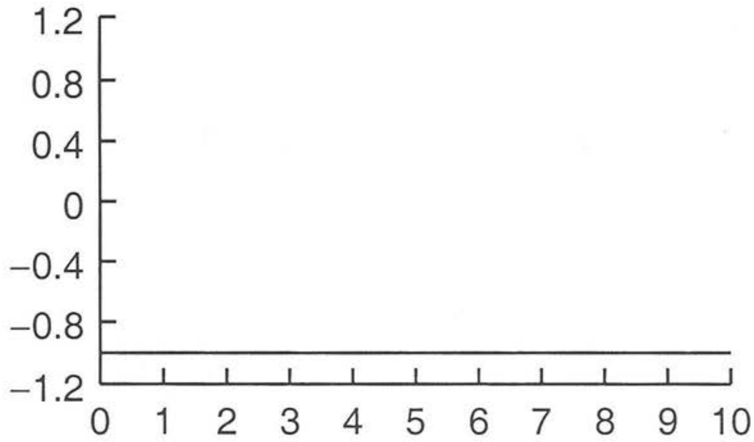
From Theorem 3.1, the behavior of this control system is asymptotically stable.

Fig. 3 shows the bipartite directed multigraph of the fuzzy control system. Since this controller is very simple, it is easy to see from this graph that this system is asymptotically stable. The stability analysis method has the merit to be able to examine the control system by matrix computation without exhausting all the paths in complex graphs.

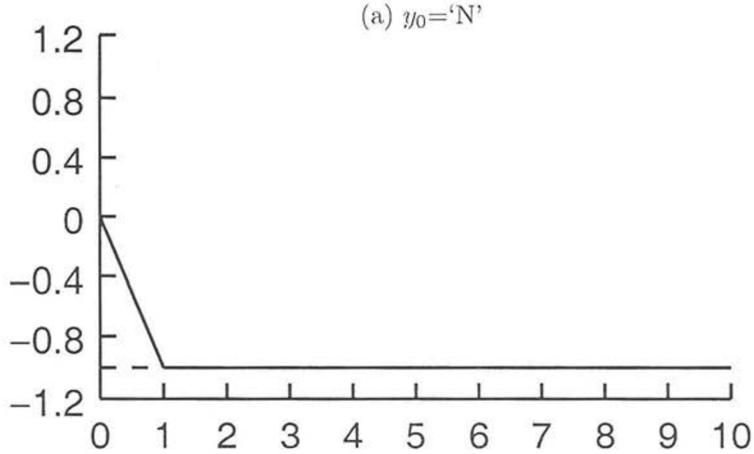
Fig. 4 shows the result of simulations whose initial states y_0 are 'N', 'Z' and 'P', respectively. The horizontal axes mean the sampling time and the vertical axes mean the output of the controlled object. The result of the simulations show that the outputs are settled down to the command value r_k = 'N'.

5. Conclusion

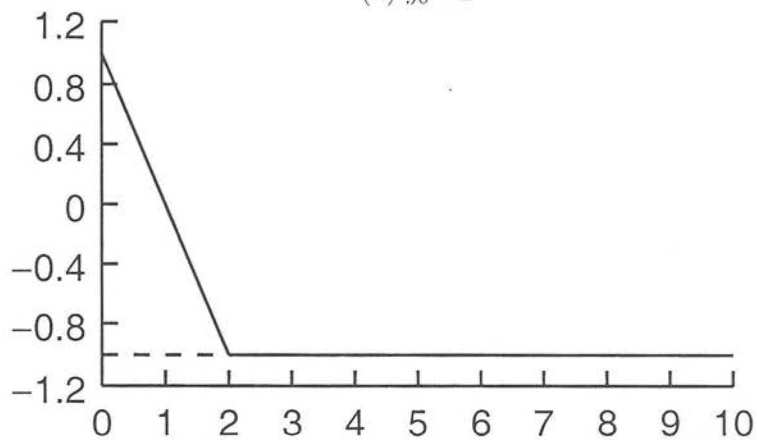
This paper presented a theory on the stability of the fuzzy control system using the petri nets. The fuzzy control system is described as a matrix based on the bipartite directed multigraph. The further work is to show the validity of the simplification of fuzzy control system into the discrete system. The condition of the fuzzy control system for this validity should be clarified, and the design of fuzzy controller will be guided for the valid evaluation of stability.



(a) y_0='N'



(b) y_0='Z'



(c) y_0='P'

Figure 4. Simulation result

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