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Selection of lease contracts in an asset-backed securitization: a real case analysis

by

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Abstract: Asset-backed securities represent one of the most attractive financial novelties for institutional investors. More precisely, asset-backed securities are the output of a complex financial operation, so-called securitization, in which bonds are issued to finance a collection of assets in such a way that the bonds issue design is integrated with the funded assets. We consider a real case of securitization, in which the assets are lease contracts. Through the market issuance of tranches of notes a bank (seller) receives funds from a factor (purchaser) and pay them back in terms of pools of credits associated with lease contracts. In this paper we analyze the problem of selecting the most convenient collection of lease contracts (assets) for the seller to hand over to the purchaser.

A 0-1 linear programming model is presented which is shown to be equivalent to a 0-1 Multidimensional Knapsack Problem. Heuristic solution procedures are proposed and computational results based on data from the discussed real case are presented.

Keywords: lease contracts, securitization, multidimensional knapsack problem.

1. Introduction

Nowadays, quantitative methodologies for selecting optimal portfolios of bonds or stocks are well known and are widely adopted in practice. Recently, new problems which require the application of quantitative methods have arisen in complex securitization process in which bonds are issued to finance a collection of assets in such a way that the bonds issue design is integrated with the funded assets. *Financial assets*, which cannot be directly traded, such as *lease contracts* or mortgage contracts, are grouped and transformed, through securitization, into securities which are easily negotiated on the market (Szegö, 1993). A high rating (Aaa or AAA) is usually assigned by the rating agencies to the issued securities. This makes the financial product more attractive for institutional investors. While the securities are the output of the securitization, the underlying assets represent the input. To be eligible for sale the assets must be characterized by a cash flow generated by interest payments (*securitization of receivables*) and the presence of a real security (*asset-backed securitization*).

Simulation and optimization models for a portfolio of mortgage-backed securities have been formulated by Zenios and Kang (1993), while a wide literature considered the evaluation and pricing of mortgage-backed securities and the prepayment of the underlying assets (see, for example, Schwartz and Torous, 1989; Kang and Zenios, 1992). For a careful investigation on the topic we refer to Barham and Letebvre (1990), Donaldson (1989), Henderson and Scott (1988), Norton and Spellman (1991). Thus, while tools exist to assist investors and traders in asset-backed securities, the problem of selecting the assets to associate with the securities issuance has, to the best of our knowledge, never been treated. In the present paper we analyze such a problem in relation to a real case. The institution which is in charge of the selection is an Italian bank for leasing, Banca per il Leasing-Italease S.p.A. (from now on simply Italease), and the assets associated with the securities issuance are lease contracts.

Herein we analyze the kernel of a securitization of flows generated by rental payments (installments) under lease contracts. It is noteworthy that the securitization has no effect on the original lease agreement conditions. In other words, the handing over of the contracts to a specialized organization (*the factor*) does not break off the original agreement between *the leaser*, Italease and the leasees. In the real case we refer to the technique used for the securitization as *pay-through model*: the leaser hands over the lease contracts but keeps on drawing the financial flows for the factor. At the same time an external organization is created, the so-called *Special Purpose Vehicle* (SPV), which is in charge of the securities (from now on *notes*) issuance (see Fig. 1). Within this process Italease lends a collection of lease contracts to the factor in such a way that the outstanding principals of the lease contracts handed over by Italease must, over time, not exceed a given profile. The problem for Italease is to select the collection of lease contracts in such a way that the "distance" between the lease contracts and the profile is as small as possible. This is done in order to benefit as much as possible from the notes issuance.

We formalize the selection problem by means of a mixed integer binary linear programming model and show that the model is strongly related with the multidimensional knapsack problem. Then, given the complexity of the model

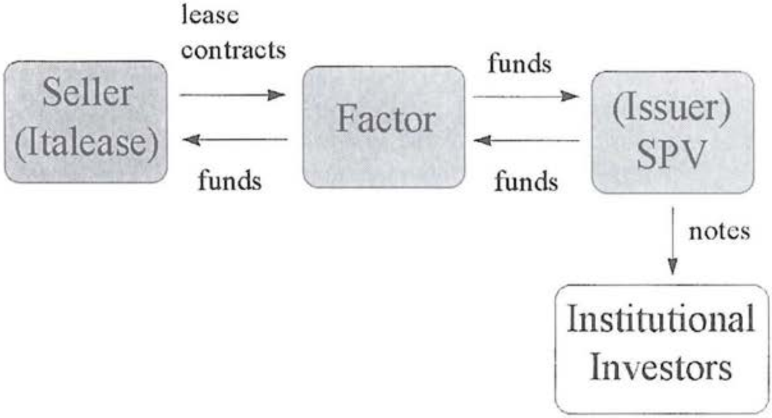

Figure 1.

procedures and tested them against the optimum of the continuous relaxation of the problem. The computational results obtained on data taken from the real case show that the problem can be effectively solved.

The paper is organized as follows. After discussing the different phases which characterize the analyzed securitization, Section 2 focuses on the description of the selection problem. In Section 3 the formulation of a 0-1 linear programming model is introduced and the model is analyzed. The heuristic solution procedures are described in Section 4 while the computational results on the real case data are given in Section 5. Finally, in Section 6 conclusions are drawn.

2. Problem description

Although the framework is common to other securitization processes, the details described herein are specific for the real case under examination. In particular, we will describe the kernel of a securitization of lease contracts recalling that such a process can be defined as a complex framework within which different agreements take place. It starts with the issuance of tranches of notes and the assignment of a long term loan by the SPV to a factor which uses the funds to acquire lease contracts from Italease. The proceeds of the issuance of the notes are used by the SPV to make limited recourse loans to the factor. Such limited recourse loans are made pursuant to a Loan Agreement between the factor, as borrower, and the issuer, as lender. The factor (from now on *the purchaser*) uses the funds obtained from the issuer to make revolving purchases from Italease (from now on *the seller*) of lease receivables (*the receivables*) under floating or fixed rate lease contracts for the leasing of personal property (equipment) or real property (real estate). The seller reimburses the loan obtained by the purchaser seller according to the terms and conditions set forth, starting at a *closing date* (initial date for the loan) and on a quarterly basis thereafter, during the *revolving period* (the first 30 months since the closing date), and, to the extent permitted, during the *accumulation period* (the period starting with the end of the revolving period and expiring at the end of the loan). Each date of the revolving and accumulation periods where a purchase takes place is called *settlement date*. The first receivables handed over by the seller at the closing date are called the *initial portfolio*. Sales of additional receivables, called *subsequent portfolios*, will take place thereafter on the settlement dates. Subsequent portfolios are purchased with the proceeds of a *seller advance* made on each settlement date. Any seller advance is equal to the aggregate amount of all receivables expected during the following *settlement period* (the period between two consecutive settlement dates). The initial and subsequent portfolios are collectively referred to as the *portfolios*.

The problem for the seller consists in selecting, at the closing date and successively at each settlement date, which lease contracts will be a part of the portfolio handed over to the purchaser (composition of the initial and subsequent portfolios). Such a choice has to be made guaranteeing a minimum gap between the outstanding principal of the loan received by the purchaser and the amounts transferred under the lease contracts. Since such a gap has to be filled in by cash, the seller has to hold a money deposit large enough to cover it. As it is known, money deposits earn lower return than any other investment alternatives. This represents the missing of higher gains for the seller.

The objective of the seller is to benefit as much as possible from the securitization, i.e. to minimize the gap and as a consequence the size of the deposit to hold.

In the real case upon which this description is based, each portfolio has to satisfy some necessary conditions. Some of these conditions simply exclude from consideration sets of lease contracts (for example, because of their dimension). Moreover, as each lease contract belongs to one among a set of pools (motor vehicles, plant and machinery, real estate, etc.). the other conditions establish that the lease contracts of a certain pool cannot exceed a fixed maximal percentage of the portfolio handed over.

We briefly introduce the required notation. Let $T = \{1, 2, ..., n\}$ be the discretized time period between the closing date (time 1) and the end of the loan received by the seller (time n), with $|T| = n$. For example, if the day is taken as a time unit and the duration of the loan is theree years, then $n = 3*365$. Since the issued notes have different maturity dates, for each tranche of notes payable at a certain date there is a corresponding date of reimbursement for the loan. Each reimbursement installment, paid in correspondence of notes deadline, reduces the amount of the outstanding principal. The outstanding principal of the loan at time $t \in T$ is denoted by d_t, while the set $T^L = \{t_i | i = 1, \ldots, m_L\}$, $T^L \subset T$, denotes the set of reimbursement dates of the loan. The last date

installment of the loan at the reimbursement date t_i, $i = 2, \ldots, m_L$, is $d_{t_{i-1}} - d_{t_i}$. We conventionally assume that the first date of reimbursement is $t_1 = 1$ and that the installment of the loan at time 1 is equal to 0.
Let r_{jt} be the outstanding principal at time t for lease contract $j \in C$, where C is the set of the lease contracts available. We set $r_{jt} = 0$ for any t before the starting date and after the expiring date of contract j. We denote by $T^S = \{\tau_i | i = 1, \ldots, l\}$, $T^S \subset T$, the set of the settlement dates, where the first date $\tau_1 = 1$ is the closing date. Let $k_t(\tau_i)$, $\tau_i > 1$, be the sum at time t of the outstanding principals of the lease contracts handed over to the purchaser before time τ_i. We set $k_t(\tau_1) = 0$, $\forall t$. Finally, we denote by $d'_t = d_t - k_t(\tau_i)$ the gap at time t between the outstanding principal of the loan and the outstanding principals of the lease contracts handed over up to τ_i, excluding time τ_i. The sum of the outstanding principals of the contracts which are handed over at time τ_i cannot overcome at time t the gaps d'_t.

3. A Multidimensional Knapsack formulation

We formulate in this section a model for the selection of the portfolio of lease contracts to hand over to the purchaser at time τ_i. The model defines the original portfolio when $\tau_i = 1$ and a subsequent portfolio when $\tau_i > 1$. We define the following binary variables

$$x_j = \begin{cases} 1 & \text{if contract } j \text{ is included in the portfolio} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

while δ_t is the gap between the outstanding principal of the loan at time t and the sum, at time t, of the outstanding principals of the lease contracts in the portfolios handed over up to time τ_i, including time τ_i.

We formulate an optimization model for the selection of the portfolio of lease contracts at the settlement date $\tau_i \in T^S$:

Problem $A(\tau_i)$

$$\min \frac{\sum_{t=\tau_i}^{n} \delta_t}{n - \tau_i + 1} \tag{2}$$

$$\sum_{j \in C} r_{jt} x_j + \delta_t = d'_t \qquad t = \tau_i, \ldots, n \tag{3}$$

$$x_j \in \{0, 1\} \qquad j \in C \tag{4}$$

$$\delta_t \geq 0 \qquad t = \tau_i, \ldots, n. \tag{5}$$

The objective function is the average gap between the outstanding principal of the loan and the sum of the outstanding principals of the lease contracts and, together with (5), states that the outstanding principals of the contracts cannot overcome the outstanding principal of the loan at any time t greater than or equal to τ_i.

The above model can be transformed into a 0-1 Multidimensional Knapsack Problem, simply by observing that

$$\sum_{t=\tau_i}^{n} \delta_t = \sum_{t=\tau_i}^{n} (d'_t - \sum_{j \in C} r_{jt} x_j) = \sum_{t=\tau_i}^{n} d'_t - \sum_{j \in C} \sum_{t=\tau_i}^{n} r_{jt} x_j.$$

Therefore, the objective function of Problem $A(\tau_i)$ is

$$\min \frac{\sum_{t=\tau_i}^{n} \delta_t}{n - \tau_i + 1} = \frac{\sum_{t=\tau_i}^{n} d'_t}{n - \tau_i + 1} - \max \frac{\sum_j \sum_{t=\tau_i}^{n} r_{jt} x_j}{n - \tau_i + 1}.$$

The constraints of Problem $A(\tau_i)$ state that the sum of the outstanding principals of the contracts cannot overcome the outstanding principal of the loan at any time. Noting that the outstanding principal of each contract is nonincreasing over time and that the outstanding principal of the loan has a stepwise shape over time, we can conclude that only the constraints corresponding to time τ_i and to the dates of reimbursement of the loan succeeding τ_i are necessary. Moreover, there is no need for the constraint at time n, the last date of reimbursement, as this is automatically satisfied when only the contracts which expire before the end of the loan are considered as candidate for the portfolio selection. Therefore, by defining $v_j = \frac{\sum_{t=\tau_i}^{n} r_{jt}}{n-\tau_i+1}, \forall j$, Problem $A(\tau_i)$ becomes equivalent to the following

Problem $B(\tau_i)$

$$\max \sum_{j \in C} v_j x_j \tag{6}$$

$$\sum_{j \in C} r_{jt} x_j \le d'_t \qquad t \ge \tau_i, t \in T^L \setminus \{n\} \tag{7}$$

$$x_j \in \{0, 1\} \qquad j \in C. \tag{8}$$

Problem $B(\tau_i)$ is a 0-1 Multidimensional Knapsack Problem with the number of constraints depending on the number of the dates of reimbursement of the loan which succeed τ_i. In the case where the only date of reimbursement after τ_i is the end of the loan, only the constraint at time τ_i is necessary and Problem $B(\tau_i)$ simply becomes a 0-1 Knapsack Problem.

In the case where the additional condition on maximum percentage per pool has to be taken into account, for any such pool s the following constraint should be added to the model

$$\sum r_{j\tau_i} x_j \le p_s(\sum r_{j\tau_i} x_j), \tag{9}$$

where p_s and P_s represent the maximal percentage of contracts of pool s which is possible to hand over and the set of contracts available for pool s at time τ_i, respectively.

4. Heuristic procedures

In this Section we describe some heuristic solution procedures which can be used to select the portfolio of contracts to hand over at a given date $\tau_i \in T^S$. Other simple heuristic procedures for the same problem have been proposed by Mansini and Speranza (1997). The computational complexity of each procedure is given.

Procedure A: Greedy

Let S_A be the subset of C selected by the procedure. Let $S_j = \frac{\sum_{t=\tau_i}^{n} r_{jt}}{r_{j\tau_i}}$ be the sum of the outstanding principal values for contract j on the time interval $[\tau_i, n]$ divided by the outstanding principal of contract j at time τ_i (the date τ_i is taken as evaluation time).

1. Sort the contracts in the non-increasing order of S_j.
2. Choose the first contract k and cancel it from C.
3. If contract k satisfies the pools constraints and its outstanding principal does not exceed the outstanding principal of the loan $\forall t \in T$, $t \geq \tau_i$, then assign the contract to S_A and compute $d'_t = d'_t - r_{kt}$, $\forall t \geq \tau_i$.
4. If $C = \emptyset$, then stop; otherwise go to step 2.

This procedure corresponds to the most popular approximate algorithm for the Knapsack Problem, usually called the *Greedy Algorithm*, where the items are ordered according to the non-increasing values of the profit per unit of weight. The time complexity is $O(|C|log|C|)$.

Procedure B: Present Value

Let S_B be the subset of C selected by the procedure B and

$$VA_j = \sum_{t=\tau_i}^{n} \frac{r_{jt-1} - r_{jt}}{(1+\alpha)^t}$$

be the actual value of the differences $(r_{jt-1} - r_{jt})$ for contract j at times $t \geq \tau_i$, $t \in T$, with α, $0 < \alpha < 1$, a suitable discount rate.

1. Sort the contracts according to the non-decreasing order of $VA_j/r_{j\tau_i}$, where $r_{j\tau_i}$ is the outstanding principal of contract j at time τ_i.
2. - 4. As in Procedure A, replacing S_A with S_B.

The higher the actual value of contract j at time τ_i the sharper the profile of is with respect to the loan outstanding principal. The computational complexity of the procedure is $O(|C| log|C|)$.

Procedure Big

This procedure is inspired by the behaviour of the decision maker observed in the real case.

Let $\tilde{S}$ be the subset of the contracts C selected by the procedure.

1. Sort the contracts in non-increasing order of their outstanding principal at time τ_i.
2. – 4. As in Procedure A, replacing S_A with $\tilde{S}$.

Such procedure has been taken into account to verify the effectiveness of a natural behaviour with respect to the other implemented heuristics. The computational complexity of the procedure is $O(|C| log|C|)$.

Finally, among the other procedures we also implemented Toyoda's algorithm (Toyoda, 1975) which is a known method for obtaining approximate solutions to large scale zero-one programming problems. This algorithm, which has been initially devised for the problem of selecting projects under limited resources, fits a Multidimensional Knapsack Problem. Toyoda's procedure assigns to each project (in our case to each contract), a measure of its relative value (its effective gradient).

5. Computational results

Real case data involving up to 2400 contracts have been used for computational experiments. We have tested the effectiveness of the heuristic methods by comparing the value of their solutions with the corresponding optimal solution found using one of the most efficient packages for integer programming (CPLEX). In all cases when no optimal solution was available within a fixed treememory space of 50 Mb, which was set up as an upper limit for the branch and bound procedure (i.e. within a given computational time), the errors have been computed with respect to the LP Relaxation. The computations have been carried out using a PC COMPAQ microprocessor Intel Pentium with 16Mb of RAM.

The computational results refer to three different securitizations for all of which the main contracts selection at the closing date (initial portfolio) has already taken place. More precisely, we have considered, for each securitization, the selection of the subsequent portfolio handed over at the date of the 1st of September 1996. For a detailed description of these and other results see Mansini (1997), while Mansini and Speranza (1997) describe the results obtained for the same problem but using different heuristic methods tested on simulated outlines of the outstanding principal.

Both the cases of portfolio selection with and without constraints on its straints deal with the selection of contracts satisfying a maximum percentage for each pool. The data available are divided into five different pools, for each of which the maximum percentage allowed in a portfolio is shown in Table 1.

Pools	Underlying Asset	Average Term (months)	Max Percentage
P1	vehicles(1)	35	20%
P2	plant/machinery(1)	54	35%
P3	real estate	95	32%
P4	vehicles(2)	48	32%
P5	plant/machinery(2)	47	15%

Table 1. Pool composition.

The table reports the main characteristics for each pool. The first column refers to the underlying asset of the lease contract, where (1) means that the lease contract has a purchase price lower than 150 million Italian Liras, while (2) means the price is over 150 million. The second column gives the average term for each pool, while the third one shows the maximum percentage allowed for a given pool in the composition of the portfolio. For example, the first pool (referred to as P1) corresponds to lease contracts for commercial vans and other motor vehicles having a purchase price lower than 150 million. Its average original term is of 35 months, while the maximum percentage allowed in the portfolio for this type of contract is 20%. Since we analyze a subsequent portfolio selection, all the contracts belonging to pool 3 (see Table 1) have already been handed over: in the analyzed real case all the real estate contracts must be a part of the initial portfolio selected at the closing date.

The following tables show the solutions found for all the algorithms and for the LP relaxation of Problem $B(\tau_i)$ in the three securitizations. In each of them the first column gives the number of contracts selected; the second column shows the objective function value of the Problem $B(\tau_i)$. The third column gives the percentage errors computed for all the solution algorithms with respect to the optimal solution of the LP relaxation of Problem $B(\tau_i)$. Since no integer solution has been found within the time limits involved by a branch and bound tree memory limits of 50 Mb, all the computed errors are overestimations of the real ones. Finally, the last column is associated with a financial evaluation of the results.

Since the sum of the gaps between the outstanding principal of the loan and the total sum of the outstanding principals of the contracts handed over, computed for all $t \geq \tau_i$, $t \in T$ (objective function of the Problem $A(\tau_i)$), has the meaning of forced liquidity for Italease, we normalize such figure by the number of days in a year and apply a spread rate. Such rate represents the opportunity cost for all the alternative and possibly more profitable investments.

The computation is as follows:

$$\text{profit loss} = \frac{\sum_{t=\tau_i}^{n} \delta_t}{\ldots}\beta$$

where δ_t is the difference between the outstanding principal of the loan and the sum of the outstanding principals of the contracts handed over at time t; h is equal to 365 if the time unit is the day and β is the corresponding spread rate which allows one to measure the profit loss as the missing from a more profitable investment of the liquidity. In the experiments the spread rate has been set to 0.4% per year, according to the situation of Italease. All the values are expressed in millions of Italian Liras.

All the described methods have been applied to six different instances arising in the three different securitizations where the cases with and without composition constraints are taken into account. The value of α in procedure B (Present Value) is set equal to 0.06.

The first securitization has 571 contracts available to be handed over on the 1st of September 1996. This process, which will expire at the 5th of May 1999, has the following outline: the outstanding principal will decrease from 100 billion (this is also the value at the settlement date of the 1st September 1996) to 80 billion at the 5th of May 1997 (first principal reimbursement date) and, finally, to 60 billion at the same date in 1998.

Securitization	P1	P2	P3	P4	P5	total
Sec. 1:	77	220	0	271	3	**571**
Sec. 2:	239	1480	0	613	53	**2385**
Sec. 3:	239	1488	0	620	53	**2400**

Table 2. Number of contracts for each pool.

In the first securitization the partition among pools of the contracts selected from the different procedures is shown in the graph of Fig. 2, while in Table 2 the first line reports the composition with respect to the 5 pools of the 571 contracts that were available for the selection of the subsequent portfolio.

It is easy to see that the restricted number of contracts belonging to pool 5 (only 3) will influence the effectiveness of the solutions found when the composition constraints are taken into account. Tables 3 and 4 show the number of contracts selected, the objective function value with respect to problem $B(\tau_i)$ formulation, the percentage errors computed with respect to the LP relaxation of the same problem, and the value of the profit loss found for all the methods using data from the first securitization for both cases, the ones with composition constraints and the ones without.

The comparison of the results shown in Tables 3 and 4 allows us to evaluate the effect on the solutions produced by the introduction of the composition constraints. The presence of pools increases, on average, by more than 30%, the number of contracts selected by the procedures. For the procedure Big which hands over the biggest contracts, the increase is up to 90% (from 195 contracts selected without pools to 372 selected for the case with pools). Table 5 shows the composition of the 195 contracts selected when no composition constraints

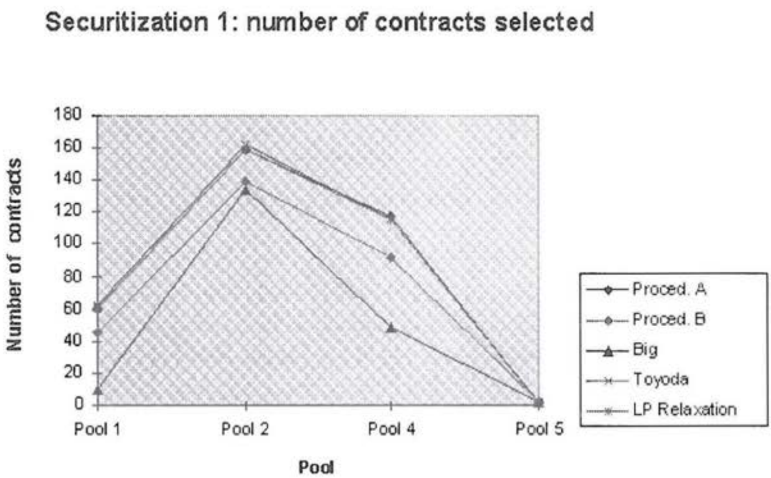

Figure 2.

Procedure	Number of Contracts	Obj. Func. (Max)	Perc. Errors	Profit Loss
A-Greedy	339	10371	0.009%*	276.0
B-Present Value	278	9940	4.16%*	280.2
Big	195	9843	5.10%*	281.2
Toyoda	340	10371	0.01%*	276.0
LP Relaxation	337	10372		275.8

Table 3. Securitization 1 (571 contracts) - Case without pools.

Procedure	Number of Contracts	Obj. Func. (Max)	Perc. Errors	Profit Loss
A-Greedy	443	7490	0.009%*	297.6
B-Present value	394	7359	1.77%*	299.0
Big	372	6992	6.67%*	302.6
Toyoda	438	7491	0.0089%*	297.6
LP Relaxation	440	7492		297.5

Table 4. Securitization 1 (571 contracts) - Case with pools.

into account.

Pools	Without Pool	With Pool
P1	10	77
P2	134	22
P3	0	0
P4	49	270
P5	2	3

Table 5. Portfolio composition. Procedure Big with and without pools - Securitization 1.

We notice that, without pools, there is less than one third of the contracts available for pool 4, only 2 contracts out of 3 for pool 5, and 134 contracts belonging to pool 2 are handed over. When composition constraints are taken into account pool 2 cannot exceed the 35% of the total portfolio value, the consistent reduction of the contracts of such a pool is compensated by the complete handover of the contracts available for pools 4 and 5. These pools do not have enough contracts to exceed the corresponding maximum percentage. Comparing the errors with and without pools found by the different heuristic procedure in the first securitization, we notice that while procedure A and Toyoda do not modify their errors, procedure B (Present Value) slightly improves with the introduction of the pool constraints while procedure Big, even if the number of contracts selected has increased, shows an increase of the percentage error from 5.10% (case without pools) to 6.67% (case with pools).

In the second securitization the outstanding principal reduces from 420 billion on the 1st September 1996 to 110 billion on the 1st November 1999 and finally to 45 billion, and remains the same from the 1st of November 2000 until the end of the securitization. The number of contracts available is 2385. The gap between the sum of outstanding principals of the contracts already handed over before this settlement date and the outstanding principal of the loan is equal to 62,734 million Italian Liras on the 1st October 1996, worth 76,277 and 39,749 million Liras in the two following reimbursement dates, respectively. The results corresponding to the different methods are shown in the Tables 6 and 7.

Procedure	Number of Contracts	Obj. Func. (Max)	Perc. Errors	Profit Loss
A-Greedy	397	32072	0.0024%*	2548.1
B-Present Value	204	31325	2.33%*	2557.9
Big	55	27804	13.31%*	2596.5
Toyoda	398	32066	0.0026%*	2548.2
Relaxation PL	393	32073		2547.1

Table 6. Securitization 2 (2385 contracts) - Case without pools.

The third securitization is characterized by a total number of 2400 contracts available at the settlement date of September 1st, 1996. The level of the out-

Procedure	Number of Contracts	Obj. Func. (Max)	Perc. Errors	Profit Loss
A-Greedy	763	24608	0.0036%*	2610.4
B-Present Value	727	24076	2.17%*	2615.5
Big	593	22677	7.85%*	2634.0
Toyoda	771	24584	0.004%*	2610.7
Relaxation PL	762	24610		2609.3

Table 7. Securitization 2 (2385 contracts) - Case with pools.

dates (November 1st, 2000 and 2001) is equal to 450, 230 and 105 billion Italian Liras, respectively. The difference between the oustanding principal of the loan and the sum of the outstanding principals of contracts handed over before the 1st of September 1996 was equal to 45,861 million, at the settlement date, and equal to 194,546 and 88,684 million at the two following reimbursement dates.

Pools	Without Pools	With Pools
P1	3	239
P2	75	55
P3	0	0
P4	42	225
P5	3	53

Table 8. Portfolio composition. Procedure Big with and without pools - Securitization 3.

As for the previous securitization pool 2 is the largest one having a number of contracts greater than the other pools (Table 2). This implies that in the instances without composition constraints, the most part of the contracts selected in the solutions comes from the pool with the largest availability of contracts. This is not the case when, due to the introduction of pools constraints, the contracts from pool 2 cannot exceed 35% of the total value of the portfolio. The difference between the contracts selected by the procedure Big in the case with and without pools is shown in Table 8. Tables 9 and 10 show the behaviour of the different methods.

Procedure	Number of Contracts	Obj. Func. (Max)	Perc. Errors	Profit Loss
A-Greedy	360	21872	0.0012%*	5135.5
B-Present Value	233	21008	3.96%*	5142.2
Big	123	20006	8.26%*	5150.5
Toyoda	360	21872	0.0012%*	5135.5
Relaxation PL	356	21873		5135.3

Table 9. Securitization 3 (2400 contracts) - Case without pools.

Comparing at the same time the three securitizations we notice that Procedure A and Toyoda, in all the tested instances, with and without composition

Procedure	Number of Contracts	Obj. Func. (Max)	Perc. Errors	Profit Loss
A-Greedy	684	18825	0.006%*	5162.0
B-Present Value	639	18537	1.53%*	5164.6
Big	572	17705	5.95%*	5172.1
Toyoda	684	18825	0.006%*	5162.0
Relaxation PL	679	18826		5160.0

Table 10. Securitization 3 (2400 contracts) - Case with pools.

procedure Big finds, on average, errors which are much greater than those of the other procedures, it has a better behaviour in the case with pools rather than without them. The introduction of composition constraints allows the procedure Big to select contracts otherwise discarded. From a maximum error of 13.31% in the second securitization and a minimum one of 5.10% in the first securitization for the instances without pools the procedure Big shows a maximum error of 7.85% in the second securitization and a minimum error of 5.95% in the third securitization for the instances with pools. Similarly, if we analyze the profit loss with all the financial interpretations which are connected with, we can draw the same conclusions. In the first securitization, assuming a spread rate equal to 0.40% per year, the profit loss for the procedures A and Toyoda is some hundred thousands Liras more than the profit loss found by the LP relaxation; such a result is true both for the case with pools and for that without them. With respect to the same instances the profit loss rises up to 5.2 and 5 million Italian Liras when the procedures Big and Toyoda are compared. In the second securitization the gap of the profit loss between the procedures Big and Toyoda increases up to 48.3 million Italian Liras in the instance without pools and up to 23.3 million in that with pools. Once more it is shown that the low performance of the procedure Big improves with the introduction of the composition constraints. Finally, in the third securitization, the procedure Toyoda allows a lower profit loss with respect to the procedure Big by 15 and 10.1 million Italian Liras, in the instance without and with pools, respectively, and with respect to the Present Value procedure - by 6.7 and 2.6 million Italian Liras.

It is important to notice that, although the percentage errors of the heuristic procedures reduce on average in the instances with pools, the introduction of the portfolio composition constraints increases the profit loss. In fact, in the first securitization the profit loss (column 4) rises, as an effect of pool introduction, from 275.8 to 297.5 million Italian Liras (an increase of about 22 million); in the second securitization the profit loss value is pushed up from about 2,547 to 2,609 million with an increase of about 62 million Italian Liras; finally, in the third securitization - from 5,135 to 5,160 with an increase of 25 million. This suggests that the optimal solution tends to move far away from the ideal one (where the value of the portfolio handed over equals the outstanding principal of the loan).

more on a noncorrespondence between the outline of the outstanding principal of the loan and the outlines of the outstanding principals of the contracts than to a non optimization of the contracts handover. However, its value is in any case very high in the instances with pools: greater than 297 million for the first securitization, than 2,609 million in the second securitization, and than 5,160 million in the third one. It seems evident that in order to shrink such values in any securitization it would be necessary to modify the corresponding outline of the outstanding principal and, thus, the value of the tranches of notes issued on the market as well as the dates of their reimbursement.

6. Conclusions

The computational complexity of the problem has prevented from finding the optimal solution for all the instances. However, in spite of their simplicity, the proposed heuristics find solutions quite close to the optimal one. On the basis of what has been found with the computational results the following conclusions can be drawn.

- The experiments made on real data show how the method A and the method of Toyoda outperform all the other methods described. Toyoda is on average the most effective method, finding solutions closer to the optimal one and thus it seems to be the most reliable one.
- With the optimization of the contracts handover, the profit loss reduces by a value of some tens of millions, for the spread rate set equal to 0.40%. This value will be proportionally greater (lower) if we assume a greater (lower) level for the spread rate.
- The comparison between the results found for the case with pools and those found without them, puts evidence on how the introduction of these constraints increase the profit loss by some tens of millions.
- The profit loss remains very high even after the optimization of the contracts handover. This is due to the noncorrespondence between the outline of the outstanding principal of the loan and that of the contracts available.

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