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Professor Jakub Gutenbaum
on his 70th birthday

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Optimization of survival strategy by application of safety dependent utility model

by

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Abstract: This paper deals with life-saving decisions. The decision maker can buy a number of death-averting devices or services which increase his safety S at the expense of the discounted future consumption R. The safety and consumption depend parametrically on the death probability p and the probability reducing strategy x, i.e. $S(p/x); R(p/x)$. The two-factors utility function $U(S, R)$ is used to find the optimum strategy $x = \hat{x}$, which maximizes $U(x)$. One can show that the unique strategy $\hat{x}$ exists and can be effectively derived. Many applications of proposed methodology are indicated.
Keywords: optimization, decision support, life saving, death-averting, risk, safety, utility models, expcted return, optimum life-saving strategy, probability of death and survival.

1. Introduction

The survival model introduced in the present paper is supposed to support life saving decisions, e.g. when one decides to increase the probability of survival by buying the death-averting devices or services, which decrease his consumption. A typical decision of that type concerns the buyer of a car, which can be optionally equipped with a number of death-averting devices, such as: air bags, ABS, ESP, winter tyres, tyre chains, etc. The more devices one buys, the less is left for the remaining consumption. In other words, the driver's safety increases at the expense of consumption. A similar problem is obtained when trying to find the rational number of medical prophylactic and other death averting services, or - buying anti assassination devices or services. In order to find the best relation between increase of safety and loss of consumption, one should apply a utility function depending explicitly on two factors: safety and expected future consumption (discounted future return). In the present paper the two factors model of utility, proposed by Kulikowski (1998A, B, C) has been used. The notion of safety S is opposite to the notion of risk, expressed by standard deviation. Besides safety the model uses (as the second factor) the expected future return, i.e. consumption R. Both factors depend parametrically on the death probability p over the probability reducing factor x, i.e. p/x. Then, one can try to find the best utility $(U[S(x), R(x)])$ maximizing strategy $x = \hat{x}$. It is shown that the unique $\hat{x}$ exists and it can be derived effectively.

Several extensions of the basic problem are possible. One of them is concerned with discrete $(x_k, k = 1, 2, \ldots, M)$ strategies. In such a case one should choose the best number $k = \hat{k}$ of independent death averting devices or services.

2. The survival model

Assume that an investor expects to get a return R_a, out of the capital P_0, which is used for consumption next year. When the investor, in addition, receives a salary w (consumed within the year) his total consumption becomes $P_0 R_a + w$. Assume also that such an investment-consumption pattern is repeated in the future years, so in case the investor faces life expectancy T_e his discounted rate of consumption (denoted R^u) becomes

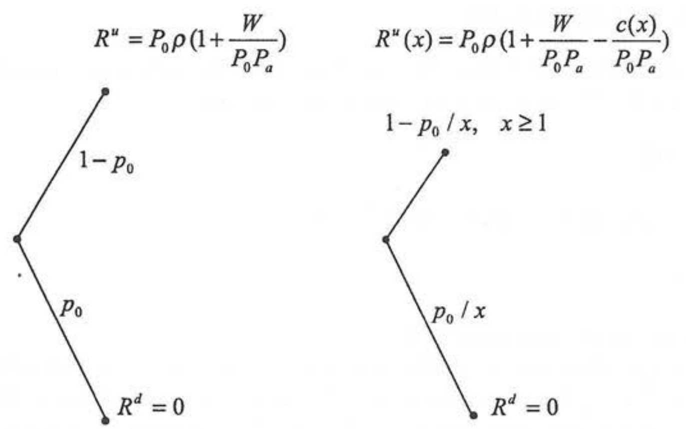
$$R^u = P_0 R_a (1 + \frac{w}{P_0 P_a}) \rho; \qquad \rho = \Sigma_{j=1}^{T_e} (1 + r)^{-j}, \tag{1}$$

where r = discount factor.

The investor is, however, exposed to a death accident with the given probability p_0 and his consumption rate, in such a case, $R^d = 0$. In other words one gets a binary model, shown in Fig. 1, where the state of being dead is characterized by $R^d = 0$, and p_0; while the state of survival is characterized by R^u and the probability of survival $1 - p_0$. The expected rate of consumption for the present model is obviously

$$R = (1 - p_0) R^u \tag{2}$$

Assume also that the investor can reduce the death probability by a factor of x at the expense of cost $c(x)$. For example, a car driver is exposed to death in a road accident. He can, however, reduce the value of p_0 by buying a car equipped with airbags, winter tyres, ABS, the Bosch ESP system, etc. The dealers promote the safety increasing systems by specifying the reduction of death casualties. For example, ESP is claimed to reduce the number of accidents by 25 % (or the probability of death casualties by $x = 3/4$). The cost of each safety oriented system increases along with the x factor. A similar situation exists in the area of health services. One can reduce the death probability by prophylactic actions, vaccination, screening etc., at the expense of additional



a. b.

Figure 1. Modelling the death-averting system

costs $c(x)$. For additional references see Jones-Lee (1974), Linnerooth (1975), Reynolds (1956).

One can also decrease the probability of assassination by employing the bodyguards or buying an armoured car etc. Since generally the cost function increases along with x, with an increasing rate, it is possible to approximate $c(x)$ by a convex function increasing at the rate λ

$$c(x) = \overline{c}[e^{\lambda(x-1)} - 1], \tag{3}$$

where $\overline{c}, \lambda$ - given positive numbers, and $c(1) = 0$.

As a result the expected return

$$R(x) = R^u(x)(1 - p_0/x), \tag{4}$$

where

$$R^u(x) = R^u - b[e^{\lambda(x-1)} - 1], \qquad b = \frac{\overline{c}}{P_0}p;$$

is a strictly concave, decreasing function of x which attains maximum value $R(1) = R^u$.

The application of death-averting systems transforms the model of Fig. 1a into the model of Fig. 1b, where the reduced death probability $p = p_0/x$ is achieved at the expense of the cost $c(x)$ reducing original consumption R^u. In order to compare the lotteries a and b in Fig. 1, and find the best survival strategy $x = \hat{x}$, the investor should apply a properly chosen utility model. The utility function should be sensitive to the death probability, survival strategy x, and consumption $R(x)$.

3. The utility model

The most popular utility function, used in economic sciences to evaluate consumption (called Z), is a concave increasing function f, i.e.

$$U = f(Z)$$

A typical example of such a function is

$$\overline{U} = aZ^{\beta}, \tag{5}$$

where a, β are given positive numbers, $\beta < 1$.

Since in the investment model one uses the notion of future consumption (denoted by $\widetilde{Z} = R_a P_0$), which is generally unknown, it is natural to regard $\widetilde{Z}$ as a random variable with expected value $Z = E\{\widetilde{Z}\}$. The expected value, however, does not characterize the random variable completely. When constructing a subjective utility model, which is supposed to account for decision under risk, it is desirable to have a complete description of random variable, e.g. in the form of probability distribution function (p.d.f.). In many situations, such a knowledge is missing and one should be satisfied with knowing certain p.d.f. parameters, such as expected value and variance (σ^2) only. It should be mentioned that many economists, e.g. Irving Fisher (1906), Allais (1953), argued that people base their choices among gambles on the variance of the gambles as well as expectations. The expectation alone is insufficient to explain risky decisions.

In the model proposed by Kulikowski (1998A, B, C) one assumes that utility depends on two factors: **1.** the expected return $Z = P_0 R$, and **2.** the net expected return $Y = P_0(R - \kappa\sigma)$.

The standard deviation σ is a measure of risk. It is multiplied by the subjective price of the risk (κ) the investor attaches to σ. Then $\kappa\sigma$ can be regarded as the subjective cost of risk characterizing the investor.

When κ (and consequently Y) is fixed, the random variable $\widetilde{Y}$ can belong to two separate sets of: **a.** worse (unprofitable) returns: $\widetilde{Y} \leq Y$ **b.** better (profitable) returns): $\widetilde{Y} > Y$.

In the case when the probability density function of $\widetilde{R}$, i.e. $F(\widetilde{R})$, is known, one can easily derive the probabilities of worse (p) and better ($1 - p$) returns:

$$p = \int_{-\infty}^{Y} F(R)dR, \qquad 1 - p = \int_{Y}^{\infty} F(R)dR$$

If, for instance, $F(R)$ is normal one gets for $\kappa = 1, p \approx 1/6$ and $1 - p = 5/6$. When the investor feels more averse to the risk he will set $\kappa > 1$, getting smaller probability of worse outcomes, i.e. $p < 1/6$, and vice versa - assuming $\kappa < 1$, i.e. when he becomes risk fond, he gets $p > 1/6$. In other words the investor should decide what probability for worse cases (when $\widetilde{Y} \leq Y(p)$) he can tolerate or accept. He can also regard Y as the lower bound for better returns i.e. $\widetilde{Y} > Y$ (which, e.g. will not induce bankruptcy). Knowing R, σ and Y the investor can derive easily κ and p.

It should be observed that fixing κ one can also formulate a simple acceptance rule: the investment is acceptable when the return R is at least equal $R_F + \kappa\sigma$, i.e.

$$R \geq R_F + \kappa\sigma, \tag{6}$$

where R_F is the return of risk free investment, such as buying the government bonds. Obviously R_F and R are positive.

The two-factors utility can be written in the form

$$U = F[Z, Y]. \tag{7}$$

Since Z and Y are expressed in monetary terms it is natural to assume that F is "constant returns to scale function" (otherwise one could increase utility by converting each \$ to 100 cents). As a consequence of that assumption one can express U in an equivalent form, Kulikowski (1998A, B):

$$U = Yf(\frac{Z}{Y}) = P_0RSf(\frac{1}{S}), \tag{8}$$

where

$$S = \frac{Y}{Z} = 1 - \kappa\frac{\sigma}{R} \text{ can be called the safety index.}$$

Observe that the maximum of safety, $S_m = 1$, one gets for $\sigma = 0$, and due to (6), $S \geq R_F \setminus R > 0$.

In the similar way (as in the case of (5)) one can approximate the function (8) by $f(\cdot) = (\cdot)^\beta$, which yields

$$\overline{U} = P_0RS^{1-\beta}, \quad 0 < \beta < 1 \tag{9}$$

The function (9) is expressed in monetary units, in the same form as the expected monetary return $Z = P_0R$. However, the real value of that return is decreased by the factor $S^{1-\beta}$, which for risky investments is less than 1.

One should observe that the parameter $1 - \beta$ can be regarded as the sensitivity of the investor ($\frac{dU}{U}$) to small change of safety ($\frac{dS}{S}$). Indeed

$$\frac{dU}{U} : \frac{dS}{S} = 1 - \beta \, .$$

When one uses the death averting system, shown in Fig. 1b, the expected return $R(x)$, given by (4), should be introduced in formula (8). One can also find the corresponding safety index $S(x) = 1 - \kappa\frac{\sigma(x)}{R(x)}$. Since

$$\begin{aligned}\sigma^2(x) &= p_0/x[R(x)]^2 + (1-p_0/x)[R^u(x) - R(x)]^2 = \\ &= [p_0/x(1-p_0/x)^2 + (1-p_0/x)(p_0/x)^2][R^u(x)]^2 = \\ &= p_0/x(1-p_0/x)[R^U(x)]^2.\end{aligned}$$

one gets

$$\sigma(x) = \sqrt{\frac{p_0}{x}(1-\frac{p_0}{x})R^u(x)}.$$

Then

$$S(x) = 1 - \kappa\frac{\sigma(x)}{(1-\frac{p_0}{x})R^u(x)} = 1 - \kappa\sqrt{\frac{p_0}{x-p_0}}. \tag{10}$$

Since

$$S(1) = 1 - \kappa\sqrt{\frac{p_0}{1-p_0}}$$

is the tolerable safety index, denoted by S_0, one can write

$$S(x) = 1 - \kappa\sqrt{\frac{p_0}{1-p_0}\cdot\frac{1-p_0}{x-p_0}} = 1 - (1-S_0)\sqrt{\frac{1-p_0}{x-p_0}}. \tag{11}$$

It is necessary to mention that the notion of maximum tolerable probability (p_0) as well as the minimum tolerable safety (S_0) is subjective. Some individuals can regard the level of safety offered by the social security system as tolerable. Others, mostly richer people, will regard S_0 as untolerable and spend a lot to increase their safety.

Observe that $R(x)$ is a strictly concave, decreasing function of x and $S(x)$ is strictly concave, increasing. Then $R(x) = \varphi[S(x)]$ for growing x ($x \geq 1$) is strictly concave, decreasing.

As an example, in Fig. 2, the graph of $R = \varphi[S]$ is plotted for $R^u = 1.1$; $b = 0.05$; $\lambda = 0.4$; $p_0 = 0.001$; $S_0 = 0.1$. A constant utility curve $R(x)[S(x)]^{1-\beta} = \overline{U} = const.$ for $\beta = 0.5$ is also plotted in Fig. 2. There is a unique point $x = \hat{x}$ where both curves are tangent. Regarding x as a control parameter one can say that a unique control strategy $x = \hat{x}$ such that $max_x U(x) = U(\hat{x})$ subject to the constraint $R(x) = \varphi[S(x)]$ exists and is unique.

One can derive the optimum strategy analytically. Indeed, the necessary condition of optimality requires that

$$\begin{aligned}\frac{dU}{dx} &= \frac{\delta U}{\delta R}\frac{dR}{dx} + \frac{\delta U}{\delta S}\frac{ds}{dx} = P_0[S^{1-\beta}\frac{dR}{dx} + (1-\beta)RS^{-\beta}\frac{ds}{dx}] = \\ &= U[\frac{1}{R}\frac{dR}{dx} + (1-\beta)\frac{1}{S}\frac{dS}{dx}] = 0\end{aligned} \tag{12}$$

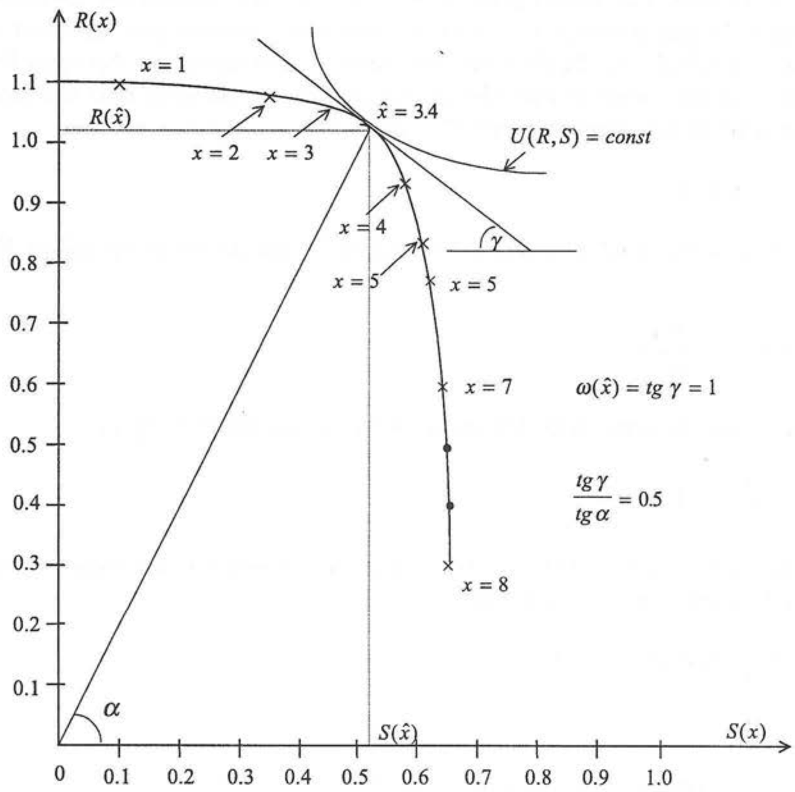


Figure 2. Interpretation of optimum strategy

Since $U > 0$ equation (12) can be written in an equivalent form, Kulikowski (1998C):

$$\frac{S}{R} = \frac{1-\beta}{\omega}, \tag{13}$$

where $\omega = -\frac{dR}{dx} : \frac{dS}{dx}$ can be called the marginal price of substitution (between $-dR$ and dS).

The relation (13) called optimum S/R relation, which specifies the necessary condition for the strategy $x = \hat{x}$ to be optimum, becomes also sufficient when the function $R(x) = \varphi[S(x)]$ is strictly concave, decreasing, as shown in Fig. 2.

One can also observe that the optimum strategy requires that the slope of the tangent to the admissible set $R = \varphi(S)$, i.e.

$$\omega(\hat{x}) = \tan\gamma,$$

derived from the slope of straight line, which intersects the point $\{S(\hat{x}), R(\hat{x})\}$, i.e.

$$\tan\alpha = \frac{R(\hat{x})}{S(\hat{x})},$$

should equal the sensitivity (of the investor) to safety, $(1-\beta)$, i.e.

$$\frac{\tan\gamma}{\tan\alpha} = 1 - \beta. \tag{14}$$

The rules of (13) or (14) can be used to find effectively the unique optimum survival strategy $x = \hat{x}$, such that:

$$\max_{x\in\Omega} U(x) = U(\hat{x}); \tag{15}$$

where

$$\Omega = \{x | R(x) - \varphi[S(x)] = 0, \quad x \geq 1\}.$$

In the numerical example, illustrated by Fig. 2,

$$\begin{aligned} R(x) &= 1.1 - 0,05[e^{0.4(x-1)} - 1](1 - \frac{0.001}{x}), \\ S(x) &= 1 - 0.9\sqrt{\frac{0.999}{x - 0.001}}. \end{aligned}$$

The optimum strategy $\hat{x} \approx 3.4$; $R(\hat{x}) = 1.04$; $S(\hat{x}) = 0,52$; $\tan\alpha = 2$; $\omega = \tan\gamma = 1$, and the condition (14) for $\beta = 0.5$ holds.

4. Extensions

In Section 3 we dealt with the continuous survival strategy x. However, in many situations the survival strategy is discrete. For example, people are confronted with a set Ω of death involving accidents $(A_1, A_2, \ldots, A_M)$, each characterized by the given probability of occurrence $p_1, \ldots, p_M$. They have also a possibility to reduce each probability by a given factor $x_k > 1, k = 1, \ldots, M$, at the expense of given cost $(\Sigma_k c_k)$, of the death-averting devices applied. To each A_k one can assign an x_k-factor and the corresponding values of R_k, S_k. The problem is to find a subset $\Omega_0 \subset \Omega$ consisting of accidents with reduced probabilities, i.e. with $x_k > 1$.

In order to solve the present problem it is helpful to assume:

a. The accidents $A_k \in \Omega$ are independent in the set Ω, so the probability of death following at least one accident becomes

$$p^d = p\{A_1 \cup A_2, \ldots, \cup A_M\} = 1 - \Pi_{k=1}^{M}[1 - p(A_k)] = 1 - \Pi_{k=1}^{M}(1 - \frac{p_k}{x_k}) \quad (16)$$

For the survival probability one gets

$$p^u = 1 - p^d = \Pi_{k=1}^{M}(1 - \frac{p_k}{x_k}). \quad (17)$$

b. The indices k in the set Ω are arranged according to the increasing marginal prices of substitution $\omega_k = -\frac{\Delta R_k}{\Delta S_k}$, $\Delta R_k = R_k - R_{k-1}$; $\Delta S_k = S_k - S_{k-1}$; $k = 1, \ldots, M$, so the most effective device (i.e. one with the smallest value of c_1/x_1 or ω_1) is the first and the device having the largest value of ω_M is the last in the sequence.

In order to find $\omega_k, k = 1, \ldots, M$ one should apply the multistage (i.e. for $k = 1, 2, \ldots$) model shown in Fig. 1, where the R_k^u value is attained with probability

$$p_k^u = \pi_{k+1}\Pi_{j=1}^{k}(1 - p_j/x_j), \qquad \pi_{k+1} = \Pi_{j=k+1}^{M}(1 - p_j).$$

The value $R_k^d = 0$ is attained with probability $P_k^d = 1 - p_k^u$. Then the expected return (R_k) at the stage k becomes

$$R_k = R_k^u p_k^u, \quad R_k^u = R_a(1 + \frac{w}{P_0 R_a} - \frac{\Sigma^k c_j}{P_0 R_a})p \quad (18)$$

and the risk

$$\delta_k = \sqrt{p_k^u p_k^d} R_k^u. \quad (19)$$

Using (18) (19) one can derive the safety at each stage:

$$S_k = 1 - \frac{\kappa \sigma_k}{R_k} = 1 - \kappa \sqrt{\frac{p_k^d}{p_k^u}}.$$

Since κ is unknown, one can assume that it is derived for the worst tolerable case, i.e. $k = 0$; and $p^u = \pi_1$, which corresponds to the minimum tolerance safety S_0. For example, the minimum tolerable safety can be set equal to 0.1. Then the value of κ becomes

$$\kappa = (1 - S_0)\sqrt{\frac{p^u}{p^d}} = (1 - S_0)\sqrt{\frac{\pi_1}{1 - \pi_1}},$$

and S_k can be written as

$$S_k = 1 - (1 - S_0)\sqrt{\frac{\pi_1 p_k^d}{(1 - \pi_1)p_k^u}}. \tag{20}$$

One can also derive the values of the marginal price of safety ω_k:

$$\omega_k = \frac{-\Delta R_k}{\Delta S_k} = \frac{R_{k-1} - R_k}{S_k - S_{k-1}}, \tag{21}$$

as well as the increasing (by assumption b) sequences of

$$\tan\gamma_k \text{ and decreasing } \tan\alpha_k = \frac{R_k}{S_k}, k = 1, 2, \ldots$$

One should observe that upon constructing the discrete process (18), (20) the continuous function $R = \varphi(S)$, introduced in Section 3, can be piecewise approximated by the linear segments with the slopes $-\omega_k, k = 1, 2, , \ldots$. As a result the optimum strategy $\hat{x} = \{x_1, x_2, \ldots, x_k\}$ should be formulated in terms of the number k of death-averting devices, which maximes utility, i.e. choosing the segment tangent to constant utility curve.

Taking into account the geometrical interpretation of γ_k, α_k (see Fig. 2), one can try to reach the optimum relation between γ_k, α_k, by approximation. For that purpose one can derive the sequence of errors

$$\delta_k = \left| \frac{\tan\gamma_k}{\tan\alpha_k} - (1 - \beta) \right|, \quad k = 1, 2, 3, \ldots \tag{22}$$

The optimum value of $k = \hat{k}$ will correspond to

$$\min_k \delta_k = \delta_{\hat{k}}.$$

When $\hat{k}$ is derived one finds easily the optimum life-saving strategy, i.e. the strategy which maximizes the utility function:

$$\max_k U[R_k; S_k] = U[R_{\hat{k}}; S_{\hat{k}}].$$

5. Numerical example

In order to facilitate the understanding of the proposed model consider a numerical example. One has to choose a survival programme within $M = 3$, statistically independent survival areas, characterized by probabilities $p_1 = p_2 = p_3 = p$. Let the probability reducing factors (x_i) and costs of death averting devices $\Delta R_i)$ be given, i.e.

$$x_i = 4;\ x_2 = 3;\ x_3 = 2;$$

$$-\Delta R_1 = 0.1,\ -\Delta R_2 = 0.11,\ -\Delta R_3 = 0.12.$$

Assume also $R_0 = 1.1,\ S_0 = 0.1,\ p = 0.001,\ \beta = 0.5.$

Using (16) and (17) one gets

$$\begin{array}{ll} p_1^u = (1-p/x_1)(1-p)(1-p) = 0.998; & p_1^d = 1 - p_1^u = 0.002, \\ p_2^u = (1-p/x_1)(1-p/x_2)(1-p) = 0.9985 & p_2^d = 1 - p_2^u = 0.0015 \\ p_3^u = (1-p/x_1)(1-p/x_2)(1-p/x_3) = 0.999 & p_3^d = 1 - p_3^u = 0.001 \end{array}$$

Then, by (20), (21) and (22) one obtains

$$S_1 = 1 - 0.9\sqrt{\frac{0.997}{0.003} \cdot \frac{0.002}{0.998}} = 0.266; \omega = -\frac{\Delta R_1}{\Delta S_1} = \frac{0.1}{0.266 - 0.1} = 0.602;$$

$$\delta_1 = \left|0.602\frac{S_1}{R_1} - 0.5\right| = \left|0.602\frac{0.266}{1.1 - 0.1} - 0.5\right| = |-0.340| = 0.34$$

In the similar way one finds also

$$S_2 = 0.364,\ \omega_2 = 1.122;\ \delta_2 = |-0.041| = 0.041,$$

$$S_3 = 0.481,\ \omega_3 = 1.026;\ \delta_3 = |0.158| = 0.158$$

It is possible to observe that $\min_k(\delta_k), k = 1, 2, 3$, is attained for $k = 2$. Then, the optimum death aversion programme should consist of two devices (with $x_1 = 4$ and $x_2 = 3$). The device with $x_3 = 2$ is too expensive and does not contribute to an increase of utility.

It should be observed that according to the model proposed in order to increase the probability of survival one should decrease the lifetime consumption as the cost of buying a number of death-averting devices or services.

It is well known that the lifetime consumption can be also increased by dropping life-shortening activities, such as smoking cigarettes or taking drugs. In such a case the life expectancy T_e increases by the factor $1 + y$, and the discount factor increases by

$$\Delta\rho = \Sigma_{j=1}^{T_e(i+y)}(1+r)^{-j} - \Sigma_{j=1}^{T_e}(1+r)^{-j} = \Sigma_{j=T_e}^{T_e(i+y)}(1+r)^{-j}$$

At the same time the consumption R_1^u increases to

$$R_1^* = R_a(1 + \frac{w+c}{P_0 R_0})(\rho + \Delta\rho),$$

where c = savings on cigarettes & drugs. As a result one gets an increase of p_1^u and an increase of safety, say to $S_1^* > S_1$, and of consumption, to $R_1^* > R_1$. If the decision was taken at the starting point ($k = 1$) one gets an improved new starting position $\{S_1^*, R_1^*\}$, as compared to the starting position $\{S_1, R_1\}$.

The last observation indicates that using the proposed methodology one can find the best survival strategy, enabling one to buy a proper number of death-averting devices and to drop the activities which endanger life expectancy.

One can also observe that $-\Delta R_k^u = R_{k-1}^u - R_k^u = c_k \frac{\rho}{P_0 R_a}$ and the rich people (with $P_{0r} > P_0$, and other parameters unchanged) face a smaller price of safety ($\omega_{kr} < \omega_k$). Then, they can afford a richer death-averting program (i.e. $\hat{k}_r > \hat{k}$).

The present methodology can be also applied to study the impact of social expenditures on the social safety. If, e.g., the government considers to increase safety (i.e. by reducing p_0/x) by increasing taxes for expenditures in health services etc., according to the cost function (3) the best strategy ($\hat{x}$) to make the individuals happy (i.e. to maximize individual utility) can be derived by formulae (13) and (14).

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