

Multiple criteria decision making
with deductive approximate
representation of efficient outcomes

by

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Abstract: We propose an approach, which we believe, can be pivotal for wider applications of Multiple Criteria Decision Making methods in practical problems. The idea is to represent decisions by approximate rather than by exact values of criteria. By this it is possible to eliminate the need of solving optimization problems from decision making processes. This in turn has far reaching consequences for versatility of decision making methods when modified accordingly to absorb the proposed approach.

Keywords: decision theory, multiple criteria decision making, deductive representation, approximate representation.

1. Introduction

Multiple Criteria Decision Making (MCDM) problems are nowadays solved most often via interactive methods, which consist of man (decision maker) - machine (the underlying mathematical computer based model) interactions. During each interaction:

- in the man phase - the decision maker reveals his partial preferences,
- in the machine phase - the underlying model is solved to derive decisions (or a representation of such), which satisfy the partial preferences at best.

In the course of interactions the decision maker's preferences are revealed gradually. This process is amplified by the learning effect: the decision maker learns (from the model) the possible relations between values of criteria.

Interactive MCDM methods draw from a wide spectrum of sciences: from applied mathematics, artificial intelligence, decision theory, to psychology. In this framework it is more and more common to make no or little assumptions about decision maker's decisional behavior. If such assumptions are made a priori, they have no practical significance for in the field applications there is no operational procedure to verify if they are valid. The a posteriori decision maker's

behavior identification, seldom complete, is also of no practical use unless the decision making process is to be repeated under the same circumstances.

With this in mind, one can view interactive methods as free walks among feasible decisions, where the walk map is decided via the decision maker – model interactions.

The purpose of this paper is to show how in the context of MCDM deductive approximate decision representation can be used instead of exact values. Here, the notion of approximation is used to refer to a representation of a value by its lower and upper bound, and not to inexact values one gets as a result of digital computations and round-off errors. We make this idea operational by demonstrating how such bounds can be derived with a tremendous computation savings as compared to deriving exact values. Such an approach drastically decreases technical complexity of MCDM methods.

The plan of the paper is as follows. The next section brings preliminaries and notation. We discuss the motivation behind this paper in detail in Section 3. In Section 4 we present the main idea on how decision outcomes can be represented by deductive bounds. In the same section we discuss how the assumed approximate representation accuracy can be ensured. Section 5 brings discussion on possible applications of outcome deductive approximate representation to MCDM methods. To make the argument illustrative we present a rough taxonomy of MCDM methods and show how our idea applies to each taxonomy class. In this section we discuss also how our idea impacts upon the fundamental issues related to MCDM methods: the issue of optimization, and the issue of method efficiency. Section 6 concludes.

2. Preliminaries and notation

The underlying model of MCDM problems we shall refer to in the paper is as follows:

$$\text{“max” } f(x) \text{ s.t. } x \in X_0 \subseteq X, \quad (1)$$

where $f : X \rightarrow R^k$, $k \geq 2$, $f = (f_1, f_2, \dots, f_k)$, is a vector of criteria $f_i : X \rightarrow R$, X_0 is the set of feasible decisions, and “max” stands for what we without loss of generality assume: all the criteria are of the type “the more the better”.

Below we shall be dealing mostly with elements $f(x)$ of the set $f(X_0)$ and for the sake of notational simplicity we shall use notation $y = f(x)$ and $f(X_0) = Z$. Elements of the set Z shall be called *outcomes*. Under this convention, for a given feasible decision x , $y_i = f_i(x)$ is the value of the i -th component of the outcome $y = f(x)$.

We shall use the commonly accepted definitions of *efficiency*, *proper efficiency*, *weak efficiency*, *dominating outcome*, and *dominated outcome* (see, e.g. Miettinen 1999). A set of all efficient elements of Z is called *Pareto set*.

3. Motivation

The motivation for this work has been a generally shared conviction that what we witness over time is a failure of the utility function paradigm. This paradigm, which originates from the seminal works of von Neuman and Morgenstern (von Neuman, Morgenstern, 1953, see also Edwards, 1992), has it that it is possible to elicit a complete decision maker preference structure in a functional form. A quest for methods to construct DM's utility function and to reduce decision making to utility function maximization over a feasible set shaped research in the field of decision theory until around the decade of years 1970 - 1980. It is still a lively stream of academic research. In this framework many interesting and far reaching decision models have been obtained (Keeney, Raiffa, 1976, Fishburn, 1986).

However, the utility function paradigm turned out to be non-operational and impractical. No work, except a few experiments performed in academic settings, reports on successful real application of this paradigm. There has been and very probably there will be ever no practical method to elicit from a DM his utility function, even if the DM has one, is conscious of, and willing to disclose it. To reveal one's utility function amounts to a consent to encapsulate ones knowledge and preferences into a model, and by this token to become replaceable by the model. Indeed, with a utility function identified the optimal decision is identifiable automatically and the DM is "reduced to a formula". A human being will never accept this.

Another paradigm, namely the interactive framework of decision making, which follows the hybrid DM - mathematical model scheme mentioned before, is therefore believed to be an appropriate mechanism for DM partial preference formulation and, at the same time, DM knowledge acquisition resulting from the learning effect. The combined effect of preference formulation and knowledge acquisition can be used to support DM in his free walk around the set of feasible decisions with the ultimate goal of identifying the decision which he regards as best.

In a more detailed perspective a motivation for this work has been the author's attempt to eliminate from the MCDM interactive framework the following paradox. The paradox is a clear but seemingly generally accepted asymmetry between the DM and the model with respect to accuracy of information provided. Though it is accepted that the DM provides his preferences incomplete and in general terms, the existing interactive methods require the model to provide an exact representation of decisions. With imprecise information (partial preferences) being provided by the DM one would rather expect, and be satisfied with, some margin of imprecision in representation of decisions provided by the model. This, in principle, should neither distort decision making processes nor impact the DM ability to arrive at the most preferred decision. Below, we shall propose to represent imprecise information in the form of bounds, lower and upper, on decision outcome components (in other words, on criteria values)

The paradox we mention has a grave consequence for the practical applicability of the interactive framework paradigm. Namely, the exact representation of decisions is provided by solving optimization problems. Optimization is not a barrier, technical or perceptual, only for those who have acquired at least some knowledge in that field, but they certainly constitute a negligible minority of all potential decision makers. Thus, optimization is a barrier for a widespread acceptability and popularity of MCDM methods. The main and direct objective of this work is to present a method to eliminate the need for optimization from interactive decision processes. This, we believe, is a way to make MCDM methods a more versatile tool for both professional and lay applications.

4. Bounds

Throughout the paper we assume that there exist two vectors $\bar{L} = \{\bar{L}_i\}$ and $\bar{U} = \{\bar{U}_i\}$, $i = 1, \dots, k$, such that

$$\bar{L} \leq y \leq \bar{U}, \quad y - \text{any efficient outcome of } Z.$$

The fact that a weakly efficient element of Z can be derived by solving

$$\min_{y \in Z} \max_i \lambda_i (y_i^* - y_i) \quad \text{for some } \lambda > 0, \quad (2)$$

where $y_i^* = \bar{U}_i + \epsilon$, $i = 1, \dots, k$, ϵ - any positive number, belongs to the folklore of MCDM (see, e.g. Benayoun et al., 1971, Bowman, 1973).

4.1. Bounds

Below we show how to derive deductive approximate representation of efficient outcomes in the form of pairs of lower and upper bounds.

4.1.1. Lower bound

Let $y^w(\lambda)$ denote a weakly efficient outcome which solves (2) for some $\lambda > 0$.

With a subset of weakly efficient outcomes S , $S \subseteq Z$, $|S| \geq 1$, called *shell*, we have (Kaliszewski 2001, 2002, 2003)

$$y^w(\lambda)_i \geq L^{GEN}(S, \lambda)_i = \max\{y_i^* - \lambda_i^{-1} \max_{y \in S} \max_l \lambda_l (y_l^* - y_l)\}; \bar{L}_i\},$$

$$i = 1, \dots, k. \quad (3)$$

The idea of deriving a lower bound is illustrated in Fig. 1 (⊙ - location of the sought point $L^{GEN}(S, \lambda)$, black bullets - shell elements).

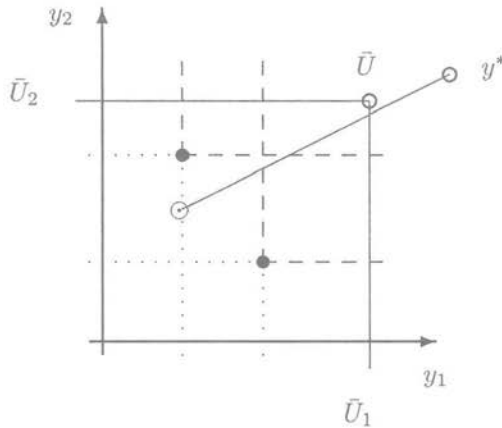


Figure 1.

4.1.2. Upper bound

Let S be a shell. Let us take any $\lambda > 0$ such that $\lambda_i \neq (y_i^* - y_i)^{-1}$ for at least one $i = 1, \dots, k$, for all $y \in S$. An upper bound for $y^{w(\lambda)}$ can be found by the following procedure (Kaliszewski 2001, 2002, 2003)

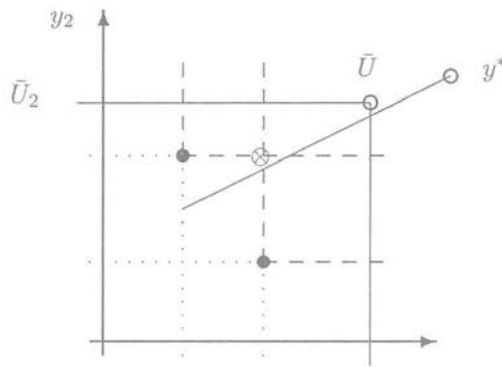
$$t^* = \max_{y \in S} \max t$$

$$\lambda_i (y_i^* - y_i) \geq t, \quad i = 1, \dots, k,$$

(we assume $t = 0$ if $S = \emptyset$) which yields $\check{y} = y^* - \gamma t^*$, where $\gamma = (\gamma_1, \dots, \gamma_k)$, $\gamma_i = 1/\lambda_i$, $i = 1, \dots, k$, and finally

$$U^{GEN}(\lambda)_i = [y(\lambda)]_i = \min_{\{y \in S, y_i \geq \check{y}_i, y_j \leq \check{y}_j, j=1, \dots, k, j \neq i\}} y_i, \quad i = 1, \dots, k. \quad (4)$$

The idea of deriving a lower bound is illustrated in Fig. 2 (\otimes - location of the sought bound $U^{GEN}(\lambda)$, black bullets - shell elements).



4.2. Ensuring assumed approximate representation accuracy

The simplest “ad hoc” method to decide if a shell provides for appropriate bound tightness is as follows. For each element of the shell calculate the lower and upper bounds using all other elements of the shell and compare the calculated bounds with the exact values of components of that element. If bounds are not sufficiently tight (sufficient tightness is problem context dependent) the shell has to be amended with some additional efficient outcomes.

A more systematic procedure is as follows. Let $R_+^k = \{y \mid y_i \geq 0, i = 1, \dots, k\}$. Each efficient element \bar{y} of a shell defines two *dead regions*:

$$\begin{aligned} &(\{\bar{y}\} + R_+^k) \cap \{y \mid \bar{L} \leq y \leq \bar{U}\}, \\ &(\{\bar{y}\} - R_+^k) \cap \{y \mid \bar{L} \leq y \leq \bar{U}\}. \end{aligned}$$

Clearly, no weakly efficient outcome belongs to the interior of any of the dead regions, for otherwise such an outcome would dominate \bar{y} or would be dominated by \bar{y} .

Each efficient element \bar{y} of a shell defines $2^k - 2$ *live regions*:

$$(\{\bar{y}\} + R_{\pm}^k) \cap \{y \mid \bar{L} \leq y \leq \bar{U}\},$$

where R_{\pm}^k is any of the sets $\{y \mid \bar{y}_i \sim y_i, i = 1, \dots, k\}$, \sim is either \leq or \geq and $R_{\pm}^k \neq R_+^k, R_{\pm}^k \neq -R_+^k$.

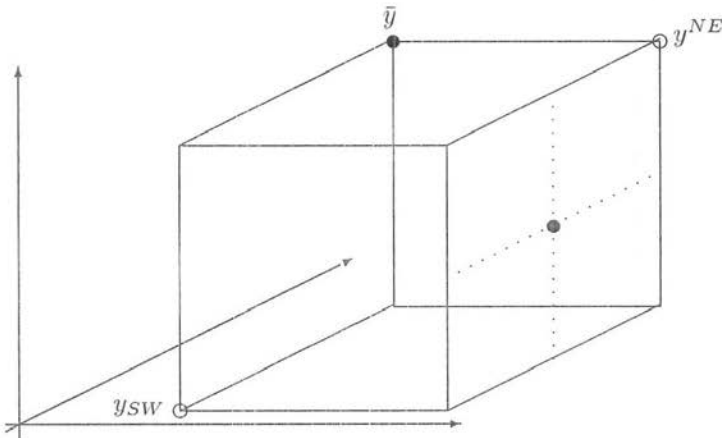


Figure 3.

The element \bar{y} is called *the root* of the live regions it defines. A *live restricted region* is the largest subset of a live region which does not contain in its interior any efficient element. Fig. 3 shows a live restricted region for an element \bar{y} (the root) of a shell (black bullets = shell elements)

Live restricted regions in an obvious way control bound tightness (see Fig. 1 and Fig. 2). Indeed, for any $y(\lambda)$ which is solution of (2) and belongs to a live restricted region,

$$\bar{y}_{SW} \leq L(S, \lambda) \leq y(\lambda) \leq U(S, \lambda) \leq \bar{y}^{NE},$$

where \bar{y}_{SW} is the “South-West” vertex and \bar{y}^{NE} is the “North-East” vertex of the considered live restricted region of \bar{y} . Hence, controlling the size of the maximal live restricted region over all elements of the shell controls bound tightness.

5. Applications of deductive approximate representation of efficient outcomes to MCDM

In this section we would like to demonstrate the general applicability of the deductive approximate representation of efficient outcomes to MCDM problems and a potential impact such a representation may have on MCDM methods. To this aim in Section 5.1 we briefly recall three basic approaches to characterizing the Pareto set. Since any interactive MCDM method employs one of these characterizations (with an exception for methods in which efficiency is verified explicitly, i.e. by definition) we shall use this distinction as a method taxonomy. This taxonomy is meant only to be demonstrative and by no means is exhaustive. In Section 5.2 we present the most representative method(s) for each taxonomy class. In Section 5.3 we discuss how existing methods can be enhanced by our approach. We also discuss related issues: the issue of optimization and of efficiency in the MCDM framework.

5.1. Characterizations

A cornerstone for every interactive MCDM method is the ability to derive efficient outcomes. Some characterizations of the Pareto set are useful for this purpose. Below we present briefly three major characterizations most often exploited in MCDM methods, namely:

- the characterization by weight manipulations,
- the characterization by reference point manipulations,
- the characterization by constraint manipulations.

5.1.1. The characterization by weight manipulations

The idea of characterization of the Pareto set by weight manipulations is to reduce a multiple criteria problem to a scalar one by assigning numerical weights to components of outcomes y and to form a surrogate objective function. Solving the thus obtained scalar problem yields an element (outcome) of the Pareto set. By changing weights one derives different outcomes.

One possible characterization of the Pareto set is possible by the weight parameterized family of optimization problems (2) (for its variants, see Chao

Atkins, 1983, Wierzbicki, 1986, Kaliszewski, 1993, 1994), another by the weight parameterized family of optimization problems

$$\max_{y \in Z} \sum_{i=1}^k \lambda_i y_i \quad (5)$$

(Geoffrion, 1968). By solving (2) for all $\lambda > 0$ one gets all the weakly efficient outcomes of Z whereas by solving (5) for all $\lambda > 0$ one gets all the properly efficient outcomes of Z whenever $Z - R_+^k$ is a convex set.

5.1.2. The characterization by reference point manipulations

A characterization of the Pareto set based on the reference point notion was proposed by Wierzbicki (Wierzbicki, 1980, 1986, 1990, see also Wierzbicki, 1999, Wierzbicki, et al., 2000).

A continuous function of the parameter \bar{y} , where \bar{y} is an element of the real space R^k , (\bar{y} - a *reference point*) is called an *achievement function*. In the context of Pareto set characterizations it is required that achievement functions possess certain properties, namely (for definitions see Wierzbicki, 1980, 1986, 1990) that they be *strictly increasing* or *strongly increasing*, or ϵ -*strongly increasing order representing*, or *order approximating*.

We define the following optimization problem:

$$\min_{y \in Z} s_{\bar{y}}(y). \quad (6)$$

Let outcome \check{y} be a solution of problem (6), i.e.

$$\check{y} = \arg \min_{y \in Z} s_{\bar{y}}(y).$$

The following statements are valid (Wierzbicki, 1980, 1986, 1990).

– Necessary conditions for efficiency –

- if $s_{\bar{y}}$ is order representing and outcome $y \in Z$ is weakly efficient, then y is a solution of problem (6) with $\bar{y} = y$ and $s_{\bar{y}} = 0$,

- if $s_{\bar{y}}$ is order approximating and outcome $y \in Z$ is properly efficient, then y is a solution of problem (6) with $\bar{y} = y$ and $s_{\bar{y}} = 0$.

– Sufficient conditions for efficiency –

- if $s_{\bar{y}}$ is strictly increasing, then outcome \check{y} is weakly efficient,

- if $s_{\bar{y}}$ is strongly increasing, then outcome \check{y} is efficient,

- if $s_{\bar{y}}$ is ϵ -strongly increasing, then outcome \check{y} is properly efficient.

The function (2) for each λ is ϵ -strongly increasing and order representing. This function is an achievement function with $\bar{y} = \lambda^*$.

5.1.3. The characterization by constraint manipulations

It is immediate to show (see, e.g. Benayoun et al., 1971) that a solution of the following optimization problem

$$\max_{y \in Z, y_j \leq \epsilon_j, j=1, \dots, k, j \neq i} y_i$$

is weakly efficient. By manipulating values ϵ_j one can generate different weakly efficient outcomes y . Moreover, every weakly efficient outcome \bar{y} is obviously a solution of the problem

$$\max_{y \in Z, y_j \geq \bar{y}_j, j=1, \dots, k, j \neq i} y_i.$$

5.2. Methods

5.2.1. Weight manipulation methods

In methods of weights manipulations (Zionts, Wallenius, 1976, 1983, Roy, Wallenius, 1991, Dell, Karwan, 1990) the space of weights is systematically searched according to DM preferences formulated in terms of weights. Search can be organized in the form of weight cuts or weight zooming.

Weight cut methods

In the Zionts-Wallenius method (Zionts, Wallenius, 1976, 1983), which applies for so called linear MCDM problems (where set Z is polyhedral), one assumes existence of an implicit pseudoconvex value (or utility) function. For the definition of a value (or utility) function see, e.g., Edwards (1992), Fishburn (1986). This function is locally approximated by a linear function

$$\sum_{i=1}^k \lambda_i y_i, \quad \lambda_i > 0, \quad i = 1, \dots, k.$$

In interaction h , $h = 0, \dots$ an outcome y^h is generated by solving the optimization problem

$$\max_{y \in Z} \sum_{i=1}^k \lambda_i y_i,$$

where $\lambda \in \Lambda^h$, $\Lambda^0 = \{\lambda \in R^k \mid 0 < \lambda_i < 1, i = 1, \dots, k, \sum_{i=1}^k \lambda_i = 1\}$.

Outcomes y^a are identified next. They are the efficient vertices of the polyhedral set Z , adjacent to y^h . Pairs (y^h, y^a) are compared by the DM. Following the DM evaluation, the set of weights Λ^h is reduced by a cut

$$\Lambda^{h+1} = \left\{ \lambda \mid \sum_{i=1}^k \lambda_i y_i^h > \sum_{i=1}^k \lambda_i y_i^a \right\} \cap \Lambda^h,$$

if the DM prefers y^h to y^a , or

$$\Lambda^{h+1} = \left\{ \lambda \mid \sum_{i=1}^k \lambda_i y_i^h < \sum_{i=1}^k \lambda_i y_i^a \right\} \cap \Lambda^h,$$

if the DM prefers y^a to y^h .

In the Dell-Karwan method (Dell, Karwan, 1990) the implicit value function is approximated by a function

$$\max_i \lambda_i (y_i^* - y_i),$$

where $\lambda_i > 0$, $i = 1, \dots, k$. Evaluations of pairs y^h , y^r , where y^r are some reference outcomes, result either in

$$\Lambda^{h+1} = \left\{ \lambda \mid \max_i \lambda_i (y_i^* - y_i^h) < \max_i \lambda_i (y_i^* - y_i^r) \right\} \cap \Lambda^h,$$

or in

$$\Lambda^{h+1} = \left\{ \lambda \mid \max_i \lambda_i (y_i^* - y_i^h) > \max_i \lambda_i (y_i^* - y_i^r) \right\} \cap \Lambda^h.$$

Roy and Wallenius (Roy, Wallenius, 1991) and also Kaliszewski and Zionts (Kaliszewski, Zionts, 2002) proposed a generalization of the Zionts-Wallenius method to nonlinear problems.

The Tchebycheff method

The so called Tchebycheff method (Steuer, 1986, Steuer, Choo, 1983) exploits problem (2) to generate weakly efficient outcomes.

The method consists of the following operations: selecting a number of vectors $\lambda \in \Lambda$, and then, iteratively:

- solving problem (2) for all selected λ to derive a number of efficient outcomes,
- selecting by the DM the most preferred outcome \tilde{y} ,
- selecting a number of vectors λ in the "neighborhood" of $\tilde{\lambda}$, corresponding to the most preferred outcome \tilde{y} .

The above process has an effect of zooming into the set of weights in a quest for weights which yield a sequence of increasingly preferred outcomes.

5.2.2. Reference point method

In the reference point method (Wierzbicki, 1980, 1986, 1990, 1999) the DM articulates his preferences by pointing to a reference point. Then an efficient outcome, which corresponds to the reference point and the achievement function used (see Section 5.1.2.) is generated. The notion of correspondance is intuitively explained in Fig. 4. By manipulating reference points the DM is able to generate a subset of the Pareto set and from this subset select the most preferred outcome.

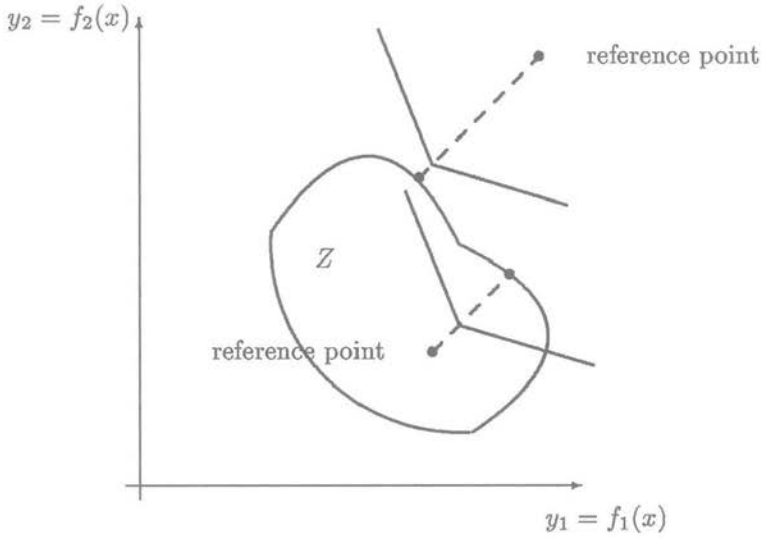


Figure 4.

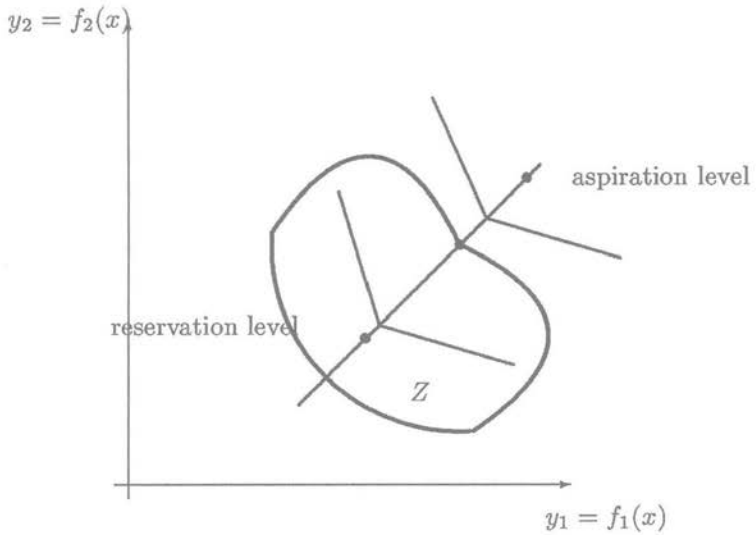


Figure 5.

The reference point method admits also DM pointing to a pair of reference points; a point y_- belonging to Z , called a reservation level and a point $y^* \notin Z$, such that $y^* \in y_- + R_+^k$, called an aspiration level. It is possible then to construct an achievement function such that a solution which maximizes that function over Z is an efficient outcome most distant from the reservation level and at the same time the closest to the aspiration level. This is schematically illustrated in Fig. 5.

5.2.3. Constraint manipulation methods

For this group of methods perhaps the most representative is the STEP method (Benayoun et al., 1971). This method requires that at each interaction the DM indicates outcome components (criteria) i , $i \in I^> \subseteq I = \{1, \dots, k\}$, whose values he wants to be increased at the expense of values of components i , $i \in I^< \subseteq I$.

Given outcome y^h identified in interaction h , in interaction $h+1$ the outcome y^{h+1} is identified by solving the following optimization problem:

$$\begin{aligned} \min_{y \in Z} \max_i \lambda_i (y_i^* - y_i), \\ y_i \geq y_i^h \quad \text{for } i \in I^>, \\ y_i \geq y_i^h - \epsilon_i \quad \text{for } i \in I^<, \end{aligned}$$

where $\lambda_i > 0$ for $i \in I^>$ and $\lambda_i = 0$ for $i \in I^<$, and ϵ_i specifies admissible decrease of the value of the component i , $i \in I^<$.

5.3. Application of deductive approximate representation of efficient outcomes to MCDM

There are three issues to be considered when discussing potential applications of deductive approximate representation of efficient outcomes to MCDM methods. They are the issue of optimization, the issue of methods and the issue of method efficiency. We discuss these issues below.

5.3.1. The issue of optimization

As seen from formulae (3) and (4) the parametric lower and upper bounds are a simple tool for representing efficient outcomes. The cost of bound calculations is negligible as compared to the cost of explicit outcome identification. But the most important feature of the bounds is that they become tighter (but not necessarily in the strict monotone manner) with increasing number of elements in a shell. If a shell consists of a sufficient number of elements (outcomes) to provide satisfactory approximate representation of outcomes, there is no need to perform optimization computations over the underlying mathematical model to explicitly identify efficient outcomes. It is possible to simplify the matter

even further by determining a sufficient number of outcomes “off-line” (e.g. prior to the start of interactive decision making). Then, during the “on-line” decision process, the “machine” phase, i.e. calculation of bounds representing actual outcomes, will be technically simple and take only negligible time (see Fig. 6).

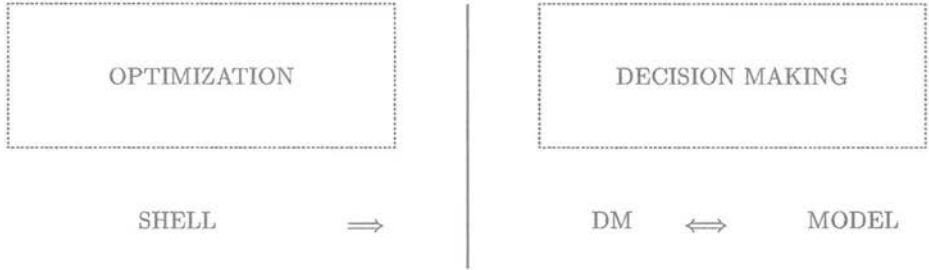


Figure 6.

The approach we propose here can give a new stimulus to applications of MCDM since it radically relocates technicalities related to interactive decision making to the background. Now all the optimization issues can be solved without intimidating the decision maker with optimization methods and software details, and optimization calculations can be left to a computing service provider. Such a provider would cope with the original underlying model, do all necessary optimization computations, and eventually determine a sufficient number of outcomes to guarantee assumed bound tightness. It is left then for the “machine” phase of the interactive decision making to cope solely with simple formulas which can be implemented even in a spreadsheet¹.

5.3.2. The issue of methods

Our taxonomy of interactive MCDM methods (Section 5.1) which follows three distinct principles of Pareto set characterizations forms a framework for discussion of potential applications of deductive outcome approximate representation proposed in Section 4. As seen from bound formulas (3) and (4) and related illustrative drawings (Figs. 1 and 2) a pair of bounds is in fact defined by a line parallel to a vector from R_+^k starting at $\{0\}$ (vectors pointing from “South-West” to “North-East”). As shown below, in interactive MCDM methods such lines are specified as a part of the DM preference revealing process.

Weight manipulation methods

In any weight manipulation method, which uses the optimization problem (2), a point (y^*) and a direction (in terms of a vector of weights λ) and therefore a line are specified at each iteration.

¹A simple spreadsheet implementation of the bounding formulas can be obtained from the author on request.

The characterization of the Pareto set based on optimization problem (5) does not provide for an analogous interpretation, for in methods in which this problem is exploited only a direction (in terms of a vector of weights λ) but no point is specified. Thus, no line can be drawn and therefore bounds (3) and (4) cannot be determined. Moreover, though it is possible to derive bounds on outcomes based on optimization problem (5) rather than (2) such bounds are not, in general, satisfactorily tight (Kaliszewski, 2003).

Reference point method

In the reference point method at each iteration a point (a reference point) and a direction (the direction of steepest ascent of the achievement function) are specified. If the achievement function (2) is used, than a direction is explicitly given in terms of a vector of weights λ .

Constraint manipulation methods

Constraint manipulation methods can be interpreted as restricted weight manipulation methods. Hence, in those methods at each iteration a point and a direction are clearly specified. Lines specified in that way provide bounds on efficient outcomes as outlined above.

5.3.3. The issue of method efficiency

Existence of a tool to assess values of components of efficient outcomes prior to explicitly identifying them has a potential impact on efficiency (we purposely refrain from using the notion "convergence" for convergence considerations usually call for a rigid mathematical framework) of standard interactive decision making methods. Suppose that at a certain point of an interactive decision making process a subset of outcomes containing the most preferred outcome is known. The standard approach is to determine at least one representative outcome from that subset, which, as a rule, involves optimization. An alternative approach we advocate here is to calculate bounds for that representative outcome rather than the outcome itself, which is a much easier task. The bounds are used then to decide if that representative outcome would be of DM's interest (if explicitly determined). If no, this representative outcome needs not to be explicitly determined, which results in saving time and optimization computing.

The extent of the impact this alternative approach may have on interactive decision making methods has to be studied experimentally and should be a subject of further research.

6. Concluding remarks

It is nowadays commonly accepted that decisional tools should follow the soft computing principle. Likewise, one tends to be very modest when estimating the extent to which mathematical methods can be competitive to humans when

it comes to decision making. A complementary rather than competitive role is attributed to such methods and this role consists in handling bulky data and performing laborious data processing.

Our idea to harness deductive approximate representation of efficient outcomes to MCDM framework can be seen as an attempt to soften the computational complexity of the existing MCDM methods without, however, losing any aspect of the MCDM methodologies developed over decades. One of the agents of man – machine interactions, namely the DM, always processes information in a soft manner. It is quite reasonable then to accept, and we advocate this in the work, the same principle for its complementing agent – the “machine”. The game is to lose little with respect to information accuracy and to gain much in terms of reduced computation expenditures.

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