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A note on the origins of this Special Section

The papers contained in this Special Section originate from the 9th International Symposium on Systems, Modelling and Control, held in April-May 1998 in Zakopane, in Polish Tatra Mountains. The selected papers from this meeting were prepared in a journal-proper way, and then subject to the usual reviewing process. The present selection constitutes the talk of those that went through the process. Some other ones will presumably appear in forthcoming issues of Control & Cybernetics.

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Inverse problems for distributed parameter systems solved with combinatorial method

by

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Abstract: An effective new method for solving inverse problems for 2-D systems is introduced using a modern combinatorial analysis. The combinatorial approach consists in the use of new computational tools constructed by authors of this paper during previous investigations in a field of combinatorics and the Fibonacci trigonometry. The formulation of numerical algorithms for solving direct and inverse problems of the 2-D systems is presented with special regard to identification problems. Also possible fields of application in technology and industry are indicated. Advantages of presented method are indicated. These are simplicity of calculations based on recurrence equations defining monic polynomials, high accuracy of calculations, and ease to implement it in the MATLAB language.

Keywords: inverse problems, distributed parameter systems, combinatorial method, field sources identification

1. Introduction

Recently, the studies on problems described by the 2-D and 3-D models have been intensively developed at many world scientific centers in order to explain different phenomena encountered not only in mathematics but also in electrical engineering, mechanics, economics, biology, medicine, and even in social sciences, Kaczorek (1985), Groetsch (1993), Anger (1990), Kurpisz (1995), Tikhonov (1995), Engl (1996), Neittaanmaki (1996). Numerical models of physical phenomena are usually formulated as a set of differential or integral equations. Also, boundary and initial conditions to the problem should be specified.

For such formulated problems, two classes can be distinguished: direct problems and inverse problems. When the coefficients of the governing equations, the geometry of a system, boundary and initial conditions are known, such problems are referred as direct problems. Analysis of direct problems has progressed for almost two centuries. Results of detailed studies on direct problems are widely described in literature. The main objective of direct problems is to find results from the known boundary and initial conditions. Another class of problems arises when the objective is to determine some coefficients and conditions mentioned above. These are the inverse problems. They arise in the course of determining the internal structure of a physical system from system's measured behaviour, or of determining the unknown input that gives rise to a measured output signal. On the contrary, the interest in direct problems is in the system's behaviour given the input and internal structure. The earliest studies on inverse problems date back to the works of Fourier and Kelvin. They considered the problems of the Earth history but these problems were not termed inverse yet. The earliest papers on inverse problems were published at the beginning of the Sixties.

Recently, it can be observed that the inverse problems are of increasing interest both in scientific centers and industry. The studies on inverse problems are carried out in two directions. One is a development of the theory and numerical methods and the second is an improvement of measurement technology. Inverse problems exist in many branches of the natural sciences and engineering such as mathematics (theory and methods), statistics, geophysics, seismology, astrometry, astrophysics, optics, and image restoration, plasma diagnostics, electrodynamics, scattering in elementary particles physics, medicine (medical imaging, impedance tomography, electrocardiogram interpretation). The problem of modelling the physical reality with suitable differential equations systems is relatively uncomplicated in the finite dimensional setting but becomes very difficult for various partial differential equations such as wave, heat, electromagnetic ones. When it is impossible, or difficult, to obtain an exact solution of the partial differential equations governing a continuous system, the system is reduced to discrete form, John (1978), Anger (1990), Tikhonov (1995).

Inverse problems are ill-posed problems according to Hadamard's definition of correctly posed problems, Hadamard (1952). A solution of the correctly posed problem must be unique and stable. The ill-posed nature of inverse problems causes that various methods for direct problems are inapplicable to a wide range of inverse problems. Therefore special numerical procedures must be employed to stabilize the results of calculations, Tikhonov (1995).

In this paper an effective method for computational solutions of direct and inverse problems described by the 2-D models is presented in relation to distributed parameter systems using discrete spatial coordinates. This new approach named combinatorial method consists in the use of new computational tools developed by authors of this paper during previous investigations in a field of combinatorics and the Fibonacci trigonometry, Rydygier (1997), Trza-

ska (1993A, 1993B), Trzaska (1996), Trzaska (1997). Especially, to solve the inverse problems the monic polynomials generated by modified numerical triangles and hyperbolic Fibonacci functions are introduced.

2. Modified numerical triangles and monic polynomials

Presently, a growing interest is observed in development of methods using a combinatorial analysis based on conceptions and objects from modern combinatorics.

The combinatorial analysis is applied in the theory of crystalograghy, cryptology (information encoding) or selected optimization problems of decision making, scheduling and graph theory, Akgul (1992). In this paper, it will be shown that in a field of engineering problems, various structures of the so-called numerical triangles and hyperbolic Fibonacci functions can be used for modelling and numerical analysis of distributed parameter systems. Mathematical bases for solutions of various problems appearing in studies of these systems are taken from the theory of recurrence equations, of the Fibonacci sequence and their generalizations, Bergum (1994), as well as power polynomials, Ross (1996). Monic non-zero polynomials which generate the first modified numerical triangle, *MNT1*, are defined by the following recurrence, Trzaska (1996)

$$T_{n+2}(x) = (2+x)T_{n+1}(x) - T_n(x), \quad n = 0, 1, 2, \ldots \tag{1}$$

with $T_0(x) = 1$ and $T_1(x) = 1+x$ as initial elements. From the above recurrence, the following polynomials can be calculated

$$\begin{aligned}
T_0(x) &= 1 \\
T_1(x) &= 1 + x \\
T_2(x) &= 1 + 3x + x^2 \\
T_3(x) &= 1 + 6x + 5x^2 + x^3 \\
T_4(x) &= 1 + 10x + 15x^2 + 7x^3 + x^4 \\
T_5(x) &= 1 + 15x + 35x^2 + 28x^3 + 9x^4 + x^5 \\
&\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots
\end{aligned}$$

Thus, the polynomial $T_n(x)$ can be written in the form

$$T_n(x) = \sum_{k=0}^{n} a_{n,k} x^k, \quad n = 0, 1, 2, \ldots \tag{2}$$

where the coefficients $a_{n,k}$, $n = 0, 1, 2, \ldots$, $0 \leq k \leq n$, fulfill the relation

$$a_{n,k} = 2a_{n-1,k-1} - a_{n-2,k}, \tag{3}$$

with $a_{0,0} = 1$ and $a_{1,0} = 1$ as initial values.

Based on (3) the *MNT1* can be constructed. It is presented in Table 1.

n/k	0	1	2	3	4	5	6	...	Sum of coefficient values in the same row
0	1								1
1	1	1							2
2	1	3	1						5
3	1	6	5	1					13
4	1	10	15	7	1				34
5	1	15	35	28	9	1			89
6	1	21	70	84	45	11	1		233
...	...	...	...	...	...	...	...	...	...

Table 1. First modified numerical triangle, *MNT1*.

To establish the second numerical triangle *MNT1*, the monic non-zero power polynomials are defined by the recurrence

$$P_{n+2}(x) = (2+x)P_{n+1}(x) - P_n(x), \quad n = 0, 1, 2, \ldots \tag{4}$$

with $P_0(x) = 0$ and $P(x) = 1$ as initial elements.

From (4) the following polynomials can be obtained

$$\begin{aligned} P_0(x) &= 0 \\ P_1(x) &= 1 \\ P_2(x) &= 2 + x \\ P_3(x) &= 3 + 4x + x^2 \\ P_4(x) &= 4 + 10x + 6x^2 + x^3 \\ P_5(x) &= 5 + 20x + 21x^2 + 8x^3 + x^4 \\ &\ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \end{aligned}$$

From the above expressions the polynomial $P_n(x)$ can be written in the form

$$P_n(x) = \sum_{r=0}^{n} b_{n,r} x^r, \quad n = 0, 1, 2, \ldots \tag{5}$$

where the coefficients $b_{n,r}$, $n = 0, 1, 2, \ldots$, $0 \leq r \leq n$ are defined by the recurrence

$$b_{n,r} = 2b_{n-1,r} + b_{n-1,r-1} - b_{n-2,r} \tag{6}$$

with $b_{0,0} = 0$ and $b_{1,0} = 1$ as initial values.

n/k	0	1	2	3	4	5	...	Sum of coefficient values in the same row
0	0							0
1	1							1
2	2	1						3
3	3	4	1					8
4	4	10	6	1				21
5	5	20	21	8	1			55
6	6	35	56	36	10	1		144
...	...	...	...	...	...	...	...	...

Table 2. Second modified numerical triangle, *MNT2*.

Then, based on (6) the *MNT2* can be constructed. It is shown in Table 2.

Formally, both the *MNT1* and the *MNT2* are apparently similar to the classical Pascal triangle, Ross (1996), but their elements cannot be evaluated directly by applying the rule corresponding to the classical Pascal triangle, Trzaska (1995). They must be computed in accordance with recurrence (3) and (6), respectively. The sum of all elements values in a row of *MNT1* or *MNT2* equals to f_{2n}, $n = 0, 1, 2, \ldots$, or f_{2n-1}, $n = 0, 1, 2, \ldots$, respectively, i.e. they are equal to successive elements of the Fibonacci sequence with even or odd indices, respectively, Bergum (1994)

$$f_{n+2} = f_{n+1} + f_n, \quad n = 0, 1, 2, \ldots \tag{7}$$

with $f_0 = 1$ and $f_1 = 1$ as initial values.

Some of the most useful properties of monic power polynomials are

$$xP_{n+1}(x) = T_n(x) - T_{n-1}(x), \quad n = 0, 1, 2, \ldots \tag{8}$$

$$P_n(x)T_{n-1}(x) - P_{n-1}T_n(x) = 1, \quad n = 0, 1, 2, \ldots \tag{9}$$

Properties of monic polynomials were detailly described in Trzaska (1996), Trzaska (1997).

3. Hyperbolic Fibonacci functions and monic polynomials

Hyperbolic Fibonacci functions $sFh(x)$ and $cFh(x)$ are defined as follows

$$sFh(x) = \frac{\phi^{2x} - \phi^{-2x}}{\sqrt{5}}, \quad cFh(x) = \frac{\phi^{(2x+1)} - \phi^{-(2x+1)}}{\sqrt{5}} \tag{10}$$

where $\phi = 1 + \rho \cong 2.618033$, and ρ denotes the golden ratio, Trzaska (1993A). It is easy to demonstrate that when a discrete variable $k \in I$ is used then

the functions $sFh(k)$ and $cFh(k)$ in terms of corresponding elements of the Fibonacci sequence

$$f(p+1) = f(p) + f(p-1), \quad p = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \tag{11}$$

with $f(0) = 0$ and $f(1) = 1$ can be written in the formula

$$sFh(k) = f(2k), \quad cFh(k) = f(2k+1) \tag{12}$$

The generalized Fibonacci hyperbolic functions $cFh_Q(k)$ and $sFh_Q(k)$ can be regarded as generating functions for polynomials $T_k(Q)$ and $P_k(Q)$, $k = 0, 1, 2, \ldots$, respectively

$$cFh_Q(k) = Q^k T_k(Q), \quad sFh_Q(k) = Q^k P_k(Q) \tag{13}$$

where Q means complex matrix or scalar parameter.

For $Q = 1$ there are usual Fibonacci hyperbolic functions

$$cFh_Q(k)_{|Q=1} = cFh(k), \quad sFh_Q(k)_{|Q=1} = sFh(k) \tag{14}$$

Moreover, it is evident that Fibonacci hyperbolic functions and modified numerical triangles above presented can be very useful for practical problem studies.

4. Combinatorial approach to solve inverse problems for 2-D systems

On the basis of the stationary 2-D space-continuous system described by the Poisson equations with specified boundary conditions, the computational algorithms are elaborated for solving direct and inverse problems with special regard to identification problems. This system is described by the second order partial differential equation

$$\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = f(x,y) \tag{15}$$

where $u(x,y)$ is the potential function and $f(x,y)$ is the function of field sources distribution.

At first a direct problem will be solved. This problem consists in finding a solution of equation (15) which is the potential function u for known function f and for given boundary conditions. To solve this problem a discretization using the finite difference method, Dahlquist (1974), and an expanding the function f with a use of the Fourier series, Potter (1973), for parameter $n = 1, 2, \ldots, N-1$ are done. The boundary conditions for function f are in the form

$$f_{m,0} = f_{m,N} = 0. \tag{16}$$

The discrete values of a field sources function can be calculated from following formula

$$f_{m,n} = \sqrt{2} \sum_{k=1}^{M-1} F_m(k) \sin \frac{k\pi n}{M}, \quad m = 1, 2, \ldots, M-1. \tag{17}$$

In the same way a solution for the potential function u can be represented as follows

$$u_{m,n} = \sqrt{2} \sum_{k=1}^{M-1} U_m(k) \sin \frac{k\pi n}{M}, \tag{18}$$

After modification with use some trigonometric identities, the second order difference equation can be established

$$\left[\frac{1}{h^2}\left(U_{m+1}(k) - 2U_m(k) + U_{m-1}(k) - \left(4\sin^2 \frac{k\pi}{2N}\right) U_m(k)\right)\right] = F_m(k) \tag{19}$$

where $m = 1, 2, \ldots, M-1$, values M and N define the limits of the space.

To complete equation (19), the boundary conditions are formulated as follows

$$U_0(k) = 0 \text{ and } U_M(k) = C_k \tag{20}$$

where constants C_k are calculated from $u_{M,n} = 0$, $n = 1, \ldots, N$.

For the new parameter $q_k = 4 \cdot \sin^2 \frac{k\pi}{2N}$ and on the basis of equation (4) generating $P_n(q)$ polynomials, the solution of equation (19) is obtained in the form

$$U_m(k) = P_m(q_k) U_1(k) + \sum_{l=1}^{m-1} P_{m-l}(q_k) h^2 F_l(k), \quad m = 2, 3, \ldots, M-1. \tag{21}$$

From (21) the values $U_m(k)$ can be found in all nodes of discretization. The values $U_1(k)$ in the equation (21) are calculated from the boundary conditions

$$u_{0,n} = 0 \text{ and } u_{M,n} = 0, \quad n = 0, 1, \ldots, N. \tag{22}$$

When $N = M$ and $n = 1$, the second equation shown above is appeared as follows

$$0 = u_{M,1} = \sqrt{2} \sum_{k=1}^{M-1} U_M(k) \sin \frac{k\pi}{M}. \tag{23}$$

Doing similarly for $n = 2, \ldots, M-1$, the system of $M-1$ equations can be obtained to calculate a set of coefficients $U_M(k)$, $k = 1, 2, \ldots, M-1$.

Then after a substitution of these coefficients to the equation (21) for $m = M$, a set of coefficients $U_1(k)$, $k = 1, 2, \ldots, M-1$ can be found. Next, on the basis of equation (18), (21) the solution of equation (15) can be calculated as a set of potential function values at nodes of discretization.

The above algorithm of calculation was implemented in the form of a computer program to analyse the system described by Poisson equation (15). The results of calculations are presented as graphs of potential function for different steps of discretization. Also the analytical solution if exists can be given on the input. It is used to calculate an error's distribution served for the comparison between the calculated and exact solution.

After some modifications of the above algorithm, another algorithm was constructed to solve inverse problem of the system described by equation (15). The task consists in the calculation of unknown field sources function f for known the potential function u and given boundary conditions described by equations (16), (22). The solution is calculated using elaborated algorithm in two stages. Within the first stage, the task consists in calculation of the matrix of Fourier series coefficients for discrete values of potential function $u_{m,n}$ in accordance with equation (18). For example, when $m = 1$ the connection between coefficients $U_1(k)$ in demand and values $u_{1,n}$ can be presented in the matrix equation

$$\vec{U}_1 = \frac{1}{\sqrt{2}} S^{-1} \vec{T}_1, \tag{24}$$

where $\vec{U}_1$ is a vector which consists of coefficients $U_1(k)$ for $k = 1, 2, \ldots, N-1$, the vector $\vec{T}_1$ consists of values of potential function $u_{1,n}$ for $n = 1, 2, \ldots, N-1$, and the matrix S is defined by suitable values of the *sine* function.

During the second stage of calculations, the matrix F of Fourier series coefficients is determined for the field sources function development. Values of elements in rows 1 to $M-2$ can be calculated from following formula

$$F_l(k) = \frac{U_{l+1} - P_{i+1}(q(k))U_1(k) - \sum_{i=1}^{l-1} P_{l+1-i}(q(k))h^2 F_i(k)}{P_1(q(k))h^2} \tag{25}$$

Whereas the calculations $M-1$ row of matrix F are done on the basis of boundary conditions (22).

The last operation is determination of the matrix f_p which corresponds with the matrix f in the algorithm for a direct problem. For calculated elements of matrix F, the elements of matrix f_p can be constructed on the basis of following equation

$$f_p(m,n) = \sqrt{2} \sum_{k=1}^{M-1} F_m(k) \sin \frac{k\pi n}{M}, \tag{26}$$

where m, n means succeeding row and column respectively.

The elaborated algorithms were established in the MATLAB language for PC computer. In order to test an accuracy of calculations, the computer simulations were carried out with special functions called the *benchmark functions* in the form

$$f_1(x,y) = 100(y^2 - y + x^2 - x),$$

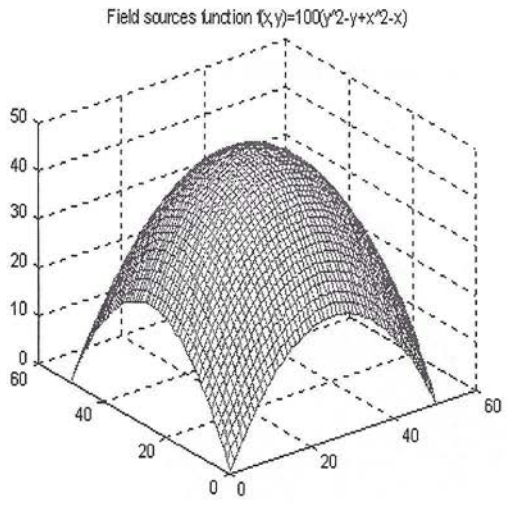

Figure 1. Benchmark source function f_1.

$$f_2(x,y) = 300xy(x^2+y^2-2),$$

$$f_3(x,y) = -2\cos x \sin y,$$

for different steps of discretization.

Some examples of benchmark functions are shown in Fig. 1 and Fig. 2.

The corresponding potential functions illustrated exact solutions of the Poisson equation (15) are presented in Fig. 3 and Fig. 4.

Calculated potential functions were obtained for different steps of discretization. For example, results of computer simulations for benchmark f_1 are shown in Fig. 5 ($M = 10$) and in Fig. 6 ($M = 20$) to illustrate an effect of increase knots number of a discretization grid.

The analytical solutions agree quite well with ones obtained by using the elaborated algorithms of numerical calculations. This proves the efficiency of the established method in the practical use.

5. Identification of field sources

The elaborated algorithms for solving inverse problems were tested using the experimental data to verify numerical calculations. These experimental data were obtained from measurements of potential distribution on the thin conductive plate or thin conductive layer placed on a plate of perfect insulator, Trzaska, Rydygier (1998).

In investigated systems, the point current constraints are sources of potential field. For example several sets of experimental data are shown in Fig. 7 (for

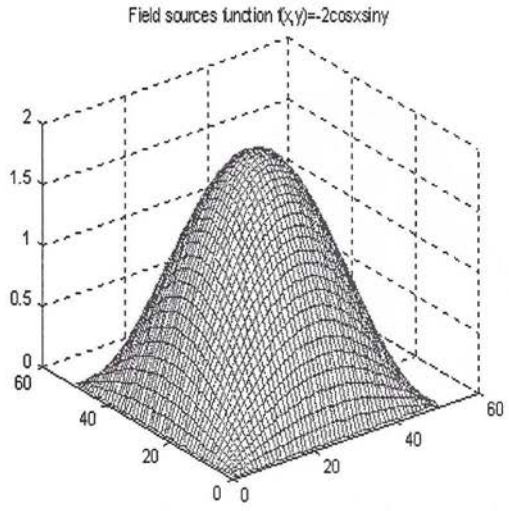

Figure 2. Benchmark source function f_3.

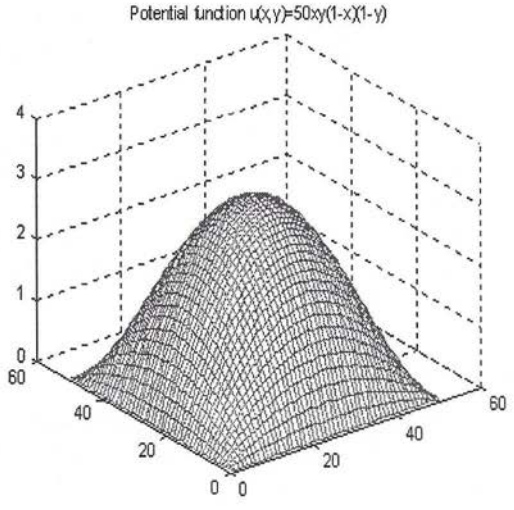

Figure 3. Analytical solution of Poisson equation for f_1.

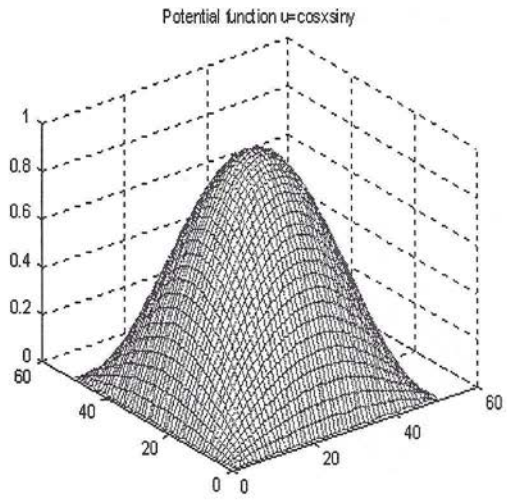

Figure 4. Analytical solution of Poisson equation for f_3.

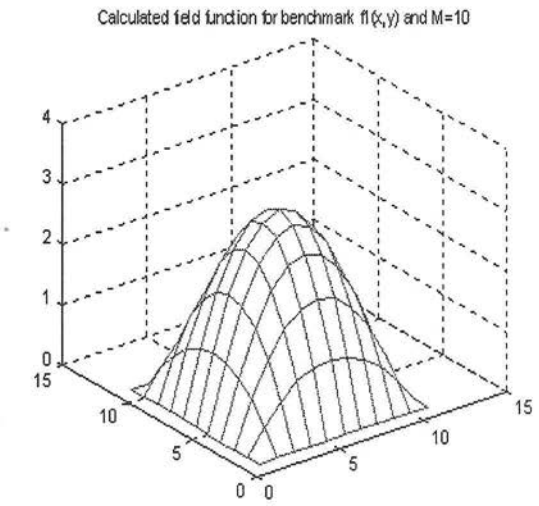

Figure 5. Calculated potential function for $M = 10$.

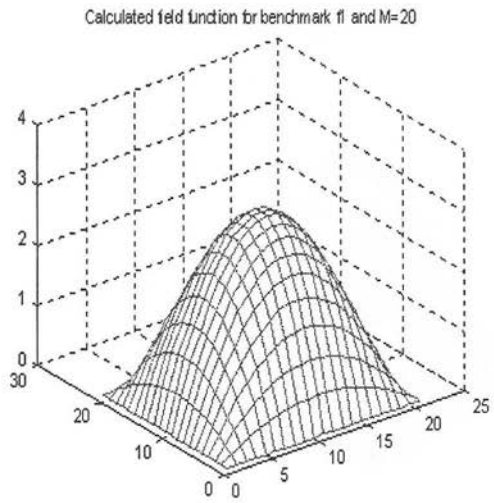

Figure 6. Calculated potential function for $M = 20$.

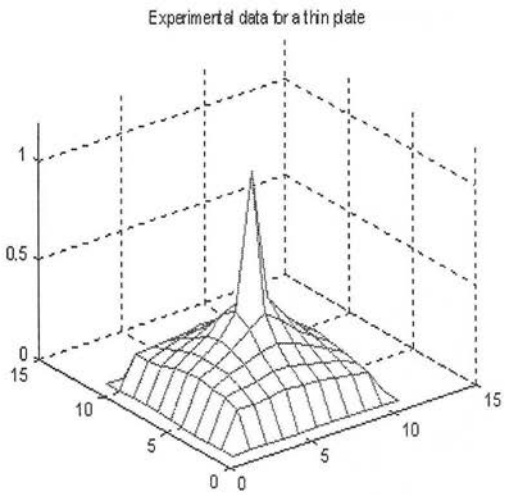

Figure 7. Experimental data for one source.

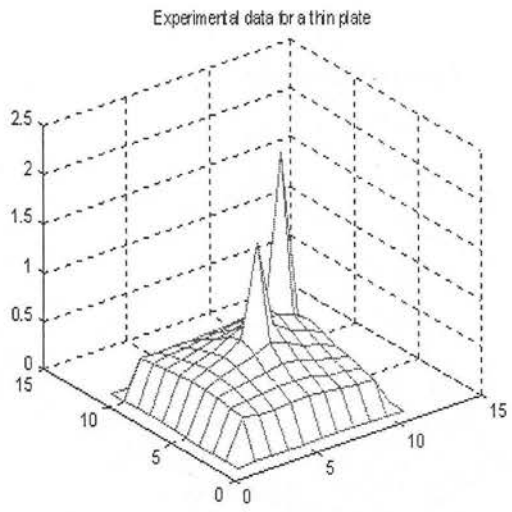

Figure 8. Experimental data for two sources.

one source), in Fig. 8 (for two sources), in Fig. 9 (for another distribution of two sources), and Fig. 10 (for three sources).

Except the data presented in Fig 9 which refer to results of potential measurements on a thin conductive layer, others refer to results of measurements on a thin conductive plate. The data were prepared in a form of two-dimensional tables. These tables were introduced to the computer program as the input data. The input data were treated by the program like given values of potential function at nodes of discretization within the investigated domain. The values of boundary conditions were placed inside the program.

Result of calculations made using the combinatorial method was a set of values determining discrete spatial distribution of field sources function. A location of field sources was defined on the basis of points placed inside the domain which correspond to the maxima of field sources function. Next, the maximum values of calculated functions were used to estimate intensities of sources.

During experimental verification, some detailed problems have been solved. These problems are connected with a data treatment like a two-dimensional interpolation and a smoothing of scattered data as well as an approximation of function circumscribed the field sources distribution. These approximation procedures were used in order to stabilize the results as a form of regularization method, Engl (1996). Together with the estimation limits of a step of discretization, the different approximation procedures were used special methods leading to the self-regularization, Kurpisz (1995). The correct results were obtained for approximation procedure elaborated on the basis of an inverse distance method

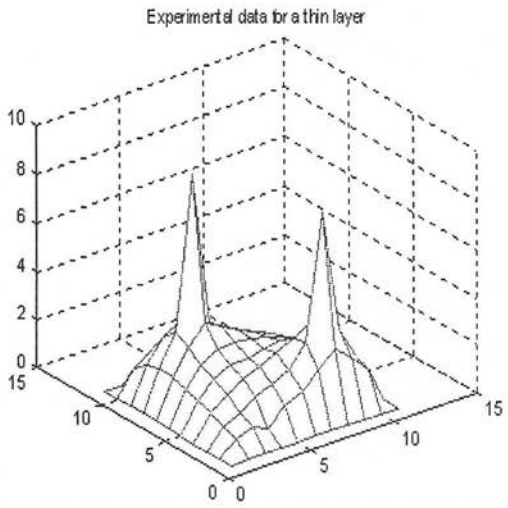

Figure 9. Experimental data for two sources in a thin layer.

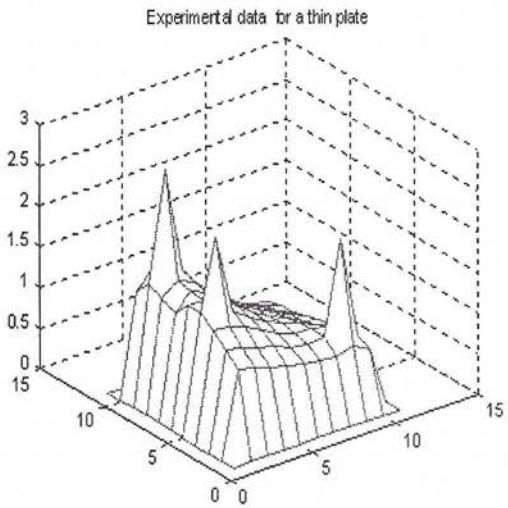

Figure 10. Experimental data for three sources.

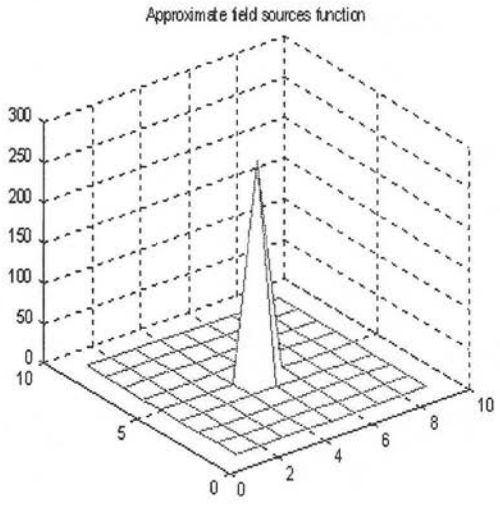

Figure 11. Identification of one source.

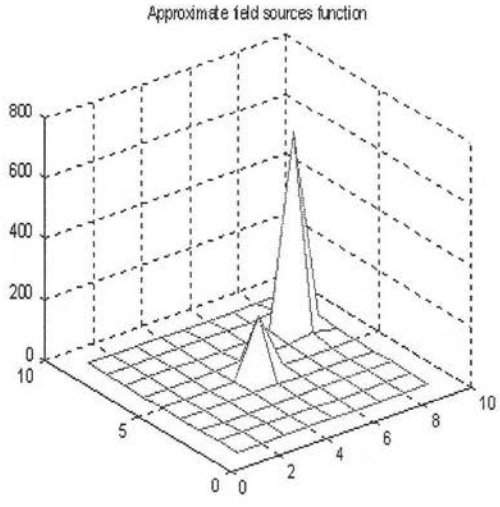

Figure 12. Identification of two sources.

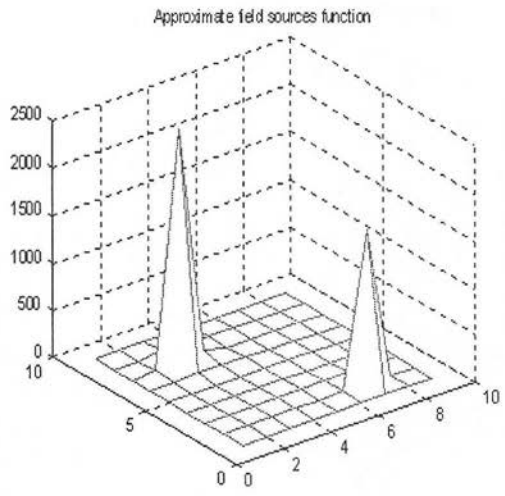

Figure 13. Identification of two sources for thin layer.

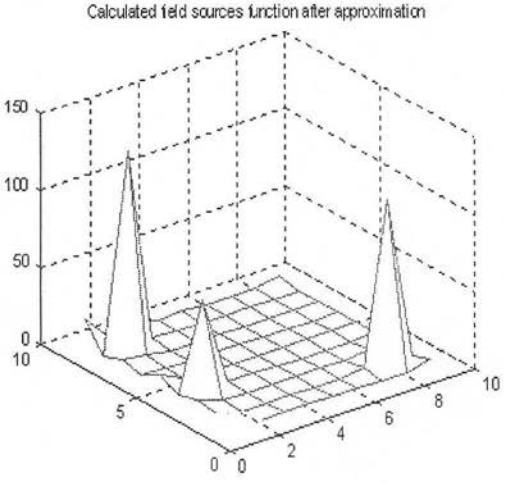

Figure 14. Identification of three sources.

named also the Shepard's method, Allasia (1992), Gordon (1978). Exemplary results of self-regularization method are the graphs of approximated field sources function which are presented in Figs. 11-14. These graphs correspond to the sets of experimental data presented above. After the experimental verification, it should be noted that for solving inverse problems the algorithm using the combinatorial method allows to determine both localization and intensities of field sources with good accuracy.

The elaborated combinatorial method can be applied to determine inverse heat sources which are solutions of different practical heat transfer problems, Malyshev (1989), Matrin (1996) and to locate corrosion domains on iron and carbon steel surfaces, Inglese (1997), Isaacs (1996).

Results of presented research can be utilized to improve usable properties of metal plates in production process in the industry. Also the results of this work can be used to build integrated computer systems for identification of thin layers properties in particular the heterogeneous spots in their structure.

6. Applications of combinatorial method

After the experimental verification, the elaborated combinatorial method was tested for detailed problems. Especially, the heat transfer problem was examined for a resistance sintering of a tungsten rod. It corresponds to a tungsten rod manufacturing that is widely used in the practice. The investigated system is described by the Poisson equation

$$\frac{\partial^2 T(x,y)}{\partial x^2} + \frac{\partial^2 T(x,y)}{\partial y^2} = q \tag{27}$$

where $q = \text{const}$, $T(x,y)$ means temperature distribution on a rectangular plane of cross-section of a rod.

The equation (27) is completed with zero-value boundary conditions. In the resistance sintering problem, heat is generated by an electrical current.

The exemplary temperature data is shown in Fig. 15 for a rod of $0.011 \times 0.011 \times 0.446$ m sizes. This is a distribution of temperature refer to a plane of cross-section at $z = 0.2355$ m (the z axis is putted along the length of a rod).

The inverse problem for the system described by equation (27) consists in a calculation a field sources function q. Calculations were done using square net of 15×15 nodes for discretization. As a result of calculations, the constant field sources function was obtained. The calculated function q is shown in Fig. 16. The conclusion on the investigated above problem is that an estimation of field sources distribution can't be done only on the basis of experimental data of potential function. The reliable identification of field sources can be realized on the basis of field sources function obtained as a suitable inverse problem solution.

Another detailed problem is a use of combinatorial method for 2-D systems that have a complicated shape. The problem from elastostatics is considered for

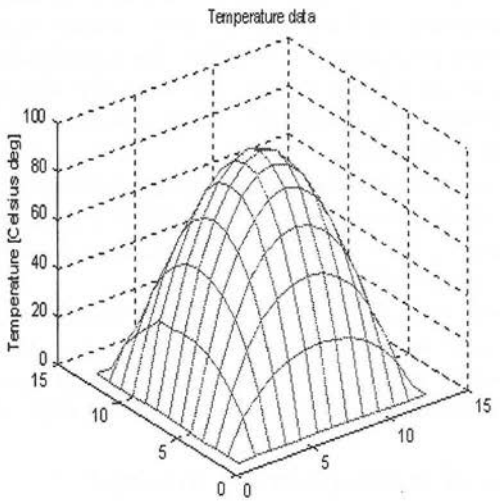

Figure 15. Temperature data for a tungsten rod.

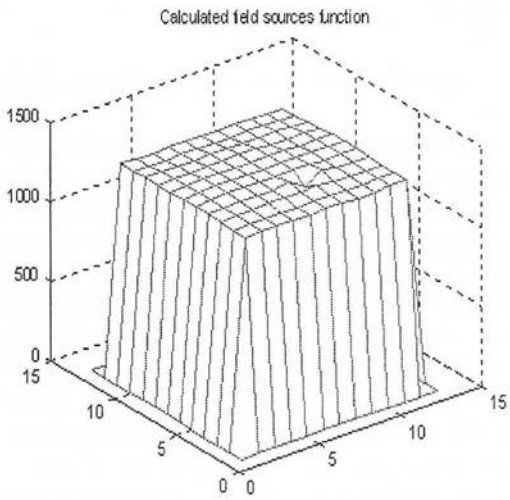

Figure 16. Calculated sources function.

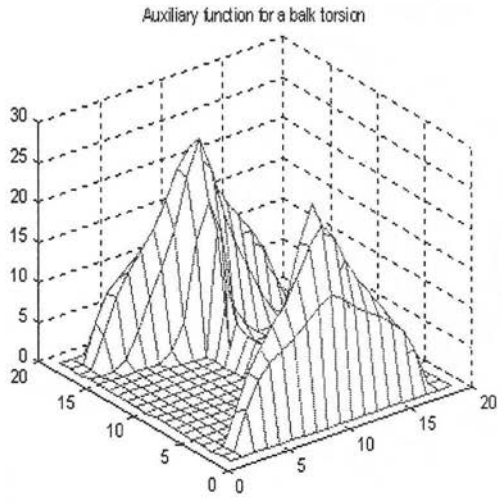

Figure 17. Input data for a torsion of the I-bar.

torsion of a metal I-bar. The 2-D system taken from the practice is described by the Poisson equation

$$\frac{\partial^2 \psi(x,y)}{\partial x^2} + \frac{\partial^2 \psi(x,y)}{\partial y^2} = q \tag{28}$$

where $\psi(x,y)$ means an auxiliary function connected with a torsion angle on a cross-section plane of a bar, q is a constant function.

The equation (28) is completed with zero-value boundary conditions. The input data on a cross-section plane of the I-bar of 16" $\times$ 6" sizes are shown in Fig. 17. The calculated field sources function q is shown in Fig. 18. Calculations are done using a net of 19×19 nodes for discretization.

The application of a combinatorial method to 2-D systems with complicated shapes consists on a substitution zero-values for the nodes placed beyond the limits of a cross-section domain. Then calculations were made like for square domain of discretization.

It should be noted that the proposed approach of calculations for complicated shapes is simple and effective. In the events of disturbances, the additional calculations with a use of smoothing and approximation procedures must be done.

7. Conclusions

In this paper a new approach to solve inverse problems for 2-D systems is presented. This approach is named the combinatorial method. Numerical algo-

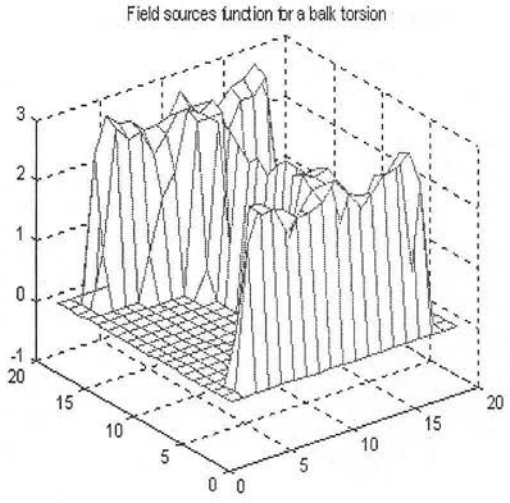

Figure 18. Field sources function for a torsion of the I-bar.

rithms were constructed using monic power polynomials generated by modified numerical triangles. After a comparison of the combinatorial method with another numerical methods used to solve different inverse problems, Anger (1990), Botkin (1995), Flis (1996), Huang (1992), Isaacs (1996), Malyshev (1989), Rydygier (1998), it can be found that this new method is effective and easy to use. The advantages of combinatorial method are simplicity of calculations on account of a use of recurrence equations defining monic polynomials, a high accuracy of calculations, and ease to implement the numerical algorithms in the MATLAB package.

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