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Constructing improvement plan for inputs through dialogue with DM about DEA-Efficiency for DMUs

by

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Abstract: Evaluation of efficiency of the DMUs (Decision Making Units) in a company is a very important problem. Thus, the studies of evaluation of efficiency are being actively carried out on the basis of the production functions, for example Cobb-Douglas production function and fuzzy loglinear production function (fuzzy Cobb-Douglas model). Recently, DEA (Data Envelopement Analysis) was applied to evaluation of efficiency, in the sense of benchmarking evaluation. After having evaluated DMUs by DEA, we obtain the improvement plan for every input in the inefficient DMUs. Though we may not be able to decrease inputs at one time according to the DEA plan, it is natural to construct some phased improvement plans for decreasing the inputs. First, we propose to support the construction of some phased improvement plans by the sensitivity analysis for the evaluated efficiency by application of definite limits to the decrease of inputs. Second, we evaluate this method by applying it to the case of bank data.

Keywords: evaluation of efficiency, sensitivity analysis, partial improvement plan

1. Introduction

The evaluation of efficiency of the DMUs (Decision Making Units) in a company is a very important problem. Because of this, studies of evaluation of efficiency are being carried out very actively on the basis of the formulations involving production functions. Recently, DEA (Data Envelopement Anaysis), Charnes, Cooper, Rhodes (1978), Tone (1993), has been applied to evaluation of efficiency. After DMUs are evaluated with DEA, we obtain the decrease plan for every input in the inefficient DMUs. Though we may not be able to decrease inputs at one time according to the DEA plan, it is natural to construct some phased improvement plans for decreasing the inputs. We propose analysis, Sakawa (1985), of the evaluated efficiency with respect to the limit of the decrease of inputs. Furthermore, we try to apply this method to evaluate a banking problem and explain the consequences for the DMUs with bad DEA efficiency.

2. DEA based on the possibility production set

The first formulation of DEA has been the fractional mathematical programming meant for the direct estimation of efficiency of the DMUs. However, similarly as the direct approach, DEA has been also formulated on the basis of construction of the possibility production set (Tone, 1993). In this section, we explain the approach of DEA modeling based on the possibility production set.

Assume the input set $X_j = (x_{j1}, \ldots, x_{jm})$ in DMU_j $(j = 1, \ldots, n)$ and the output set $Y_j = (y_{j1}, \ldots, y_{js})$. We set the input vector notation to be $\vec{X} = [X_1, \ldots, X_n]$ and the output vector to be $\vec{Y} = [Y_1, \ldots, Y_n]$. The input and output vectors are supposed to take positive values. Using the data set of DMU, $(\vec{X}, \vec{Y})$, we define the possibility production set (x, y) as the set satisfying the following constraints:

$$\begin{aligned} x &\geq \vec{X}\lambda \\ y &\leq \vec{Y}\lambda \\ \lambda &\geq 0 \\ L &\leq e^T\lambda \leq U \end{aligned} \tag{1}$$

where $x \in R^m$, $y \in R^s$, $\lambda \in R^n$, and $e^T = (1, \ldots, 1)$.

By virtue of (1), we constrain the possibility production set in DEA. The different DEA models have been proposed by varying the L and U. Specifically, when $L = 0$, $U = \infty$, we obtain the CCR (Charnes-Cooper-Rhodes) model, and when $L = U = 1$, the BCC (Banker-Charnes-Cooper) model. Let us denote efficiency by Θ. The CCR and BCC models are as follows:

2.1. CCR model 1

$$\begin{aligned} &\min \Theta \\ st.\ \Theta x_0 &\geq \vec{X}\lambda \\ y &\leq \vec{Y}\lambda \\ \lambda &\geq 0 \end{aligned} \tag{2}$$

When $\Theta^* = 1$ in (2), we may have surplus or shortage of inputs or outputs.

2.2. CCR model 2

$$
\begin{aligned}
\min\Theta \quad &- \quad e(e^T s^+ + e^T s^-) \\
st.\ \Theta x_0 - s^+ \quad &= \quad \vec{X}\lambda \\
y + s^- \quad &= \quad \vec{Y}\lambda \\
\lambda, s^+, s^- \quad &\geq 0
\end{aligned}
\tag{3}
$$

2.3. BCC model 1

$$
\begin{aligned}
&\min\Theta \\
st.\ \Theta x_0 \quad &\geq \quad \vec{X}\lambda \\
y \quad &\leq \quad \vec{Y}\lambda \\
e^T\lambda \quad &= \quad 1 \\
\lambda \quad &\geq 0
\end{aligned}
\tag{4}
$$

Again, when $\Theta^* = 1$ in (4), we may have surplus or shortage of inputs or outputs. Hence, we formulate the BCC model using the slack variables as follows:

2.4. BCC model 2

$$
\begin{aligned}
\min\Theta \quad &- \quad e(e^T s^+ + e^T s^-) \\
st.\ \Theta x_0 - s^+ \quad &= \quad \vec{X}\lambda \\
y + s^- \quad &= \quad \vec{Y}\lambda \\
e^T\lambda \quad &= \quad 1 \\
\lambda, s^+, s^- \quad &\geq 0
\end{aligned}
\tag{5}
$$

3. Sensitivity analysis for evaluated DMUs with respect to the limit of input decrease

After having evaluated DMU_j by the DEA method, we obtain the efficiency rating Θ_j^* and the weight coefficient λ_j^*. In order to avoid the calculation difficulties, let us assume that the $s^+ = 0$. DEA suggests such an improvement plan for inputs that we change inputs from $\vec{x}_j$ to $\vec{x}_j\lambda_j^*$. After we carry out this DEA plan, DMU_j becomes efficient. However, if the initial efficiency rating for DMU_j is very bad, it is generally impossible that we change inputs according to the DEA improvement plan at one time. Therefore, we need to construct some partial improvement plan so as to satisfy a certain level of improvement

improvement plan from the viewpoint of the improvement level in this plan in terms of the efficiency rating for DMU_j.

Now, let us assume that the decision maker sets subjectively the lower limit $L_i(i = 1, \dots, n)$ for every input $x_i(i = 1, \dots, n)$. In this case, we can carry out the sensitivity analysis for the efficiency rating of DMU_j with respect to L_i with $\min_i(x_i - L_i)$. When setting up the lower limit L_i, we express the added weight for the weight coefficient as $\Delta\lambda_i$. We obtain L_i as follows:

$$L_i = (\lambda_i^* + \Delta\lambda_i)x_i \tag{6}$$

Along with L_i we obtain the efficiency rating Θ_{L_i} as follows:

$$\Theta_{L_i} = \frac{\lambda_i^*}{\lambda_i^* + \Delta\lambda} \tag{7}$$

Given Θ_{L_i} as in (7), all the L_j except for L_i are calculated automatically as follows:

$$L_j = \Theta_{L_i} x_j \tag{8}$$

where the calculated limit in (8) is bigger than the initial setting.

The DEA model is formulated for the equal significance of every input. Therefore, after the decision maker sets the lower limit L_i for such input that is the most difficult to decrease, he/she obtains the other lower limits L_j automatically. Thus, the setting rule is as follows:

1. We obtain $\lambda_i^*(i = 1, \dots, n)$ by the DEA method.
2. The decision maker selects the input x_i that is most difficult to decrease.
3. He/she sets the satisfying efficiency rating for DMU_i at Θ_{L_i}.
4. By (6) and (7), he/she obtains L_i.
5. By (8), he/she obtains L_j's except for L_i.

By the above described setting rule, he/she obtains the partial improvement plan that changes $x_i(i = 1, \dots, n)$ to $L_i(i = 1, \dots, n)$, with his/her satisfying efficiency rating Θ_{L_i}.

4. Application of the proposed method to the banking problem

We tried to apply this method to a banking problem. Along with the application of the standard DEA to this problem, Sherman, Ladino (1995), di Giokas (1991), we tried to evaluate our proposed method. When the DMU_j's are banks, it is usual that we set three inputs such as: the total assets, the numbers of branches, and the number of employees, and two outputs, namely running earnings and net profits, Sherman, Ladino (1995). Otherwise, DMU_j can be some branch in one bank, and then we usually replace only one input; instead of the number of branches - the floor space of every branch, di Giokas (1991). When trying to

	assets x_1	branch number x_2	employment x_3	revenue y_1	profit y_2	efficiency
BANK1	5223	418	19061	241	22	0.7761
BANK2	1059	211	6128	45	3	1.0000
BANK3	2326	94	8284	148	17	1.0000
BANK4	5246	565	21600	261	16	0.8406
BANK5	4877	368	15701	287	24	1.0000
BANK6	5073	365	16252	287	28	1.0000
BANK7	5225	387	17247	281	24	0.9193
BANK8	1829	243	9604	109	9	1.0000
BANK9	5184	396	14909	274	30	1.0000
BANK10	3086	302	11971	146	14	0.7451
BANK11	2800	437	14436	128	14	0.7479

Table 1. Japanese National Bank data

bank, national bank, etc.). By evaluating the efficiency for DMU_j, we obtain the improvement plan for inputs leading from non-efficiency to efficiency. In this context we may consider that it is difficult, e.g., to discharge workers at one time along this DEA improvement plan for the branch with the bad DEA efficiency, relative to the decrease of the other two inputs. From this point of view, it is natural that DM sets a satisficing efficiency limit and constructs a lower partial improvement plan.

We attempted to apply this method to the Japanese national bank data. There are 11 National Banks in Japan. We assume three following inputs: total assets (in billion yen), branch number and employment. On the output side we assume two outputs: revenue (in billion yen) and profit (in billion yen). The input and output data and DEA-efficiencies are shown in Table 1.

In order to show the construction of the satisficing improvement plan, we focus on Bank7. Because Bank7 has efficiency of 0.9193, the decision maker has to set the inputs to $x_1 = 4803$, $x_2 = 356$, and $x_3 = 15855$, in order for the Bank7 to achieve DEA-efficiency. Therefore, the decision maker needs to reduce x_1, x_2, x_3 by 422, 31, and 1392, respectively. As the decision maker considers that it is most difficult to reduce the number of branches (x_2), we try to design satisficing improvement plan by focusing on x_2.

First, we illustrate the procedure for constructing the satisficing improvement plan through the dialogue with the DM by the example from Table 2. At the beginning, the DM may hope for attainment of the efficiency of 0.98 (Step 1 in Table 2), which would require the reduction of the number of branches by 27. Since this reduction turns out to be too sharp, the efficiency level is decreased in Step 2 to 0.93. The respective requirement of reduction of the number of branches (4) seems to be easily obtainable, and so the DM increases in Step 3

	Step 1	Step 2	Step 3
Θ	0.98	0.93	0.94
x_1	314	56	109
x_2	27	4	10
x_3	1047	184	357

Table 2. Constructing the satisficing improvement plan through a dialogue with the DM about Θ; the numbers denote respective reduction magnitudes

	Step 1	Step 2	Step 3
x_1	109	82	56
x_2	10	6	4
x_3	357	271	184
Θ	0.94	0.935	0.93

Table 3. Constructing the satisficing improvement plan through a dialogue with the DM about x_2; the numbers denote respective reduction magnitudes

of the number of branches by 10, acceptable for the DM.

The basis for the dialogue illustrated in Table 2 was the value of efficiency. Now, in Table 3 a similar dialogue is shown, starting from the end point of the previously illustrated procedure. Here, however, the dialogue refers directly to the value of x_i (here, again, x_2). As can be seen from the table, reduction of efficiency by 0.01 (1%) corresponds to preservation of 6 out of 10 branches to be liquidated, and so the satisficing solution is reached.

5. Conclusions

After having evaluated DMUs by the DEA, we obtain the decrease plan for every input in the inefficient DMUs. Though we cannot decrease respective inputs at one time according to the DEA-produced plan, it is natural to construct some partial improvement plans for decreasing the inputs. Thus we propose to support the construction of some partial improvement plans by the sensitivity analysis for the evaluated efficiency with respect to the limit of the decrease of inputs. Application of the method for a banking problem is cited as illustration.

References

CHARNES, A., COOPER, W.W. and RHODES, E. (1978) Measuring the effi-

search, **2**, 429-444.

DI GIOKAS (1991) Bank branch operating efficiency: A comparative application of DEA and the loglinear model. *OMEGA, Int. J. of Management Science*, **19**, 549-557.

SAKAWA, M. (1985) *Optimization in Linear Systems.* Morikita-bookstore (in Japanese).

SHERMAN, H.D. and LADINO, G. (1995) Managing bank productivity using data envelopement analysis (DEA). *OMEGA, Int. J. of Management Science*, **25**, 60-73.

TONE, K. (1993) *Data Envelopement Analysis.* Nikagiren, 1993 (in Japanese).