

## Chattering attenuation in sliding mode control systems

by

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**Abstract:** In this paper sliding mode control systems are considered and a new technique to attenuate chattering is proposed. The technique employs simple, first-order dynamical devices, i.e. an integrator or a low-pass filter, to compensate in part for unknown and changing disturbance. As a result of this compensation, the magnitude of the discontinuous control term is essentially reduced. The term does not depend on the admissible disturbance or its change rate, but only on the uncertainty of the disturbance at the initial time  $t = t_0$ . The technique proposed in this paper attenuates the undesirable chattering without any deterioration of the system performance.

**Keywords:** sliding mode control, variable structure systems.

### 1. Introduction

In recent years much of the research in the area of control systems theory focused on the design of a discontinuous feedback which switches the structure of the system according to the evolution of its state vector. This technique, usually called sliding mode control, provides an effective and robust means of controlling nonlinear plants, DeCarlo et al. (1988), Hung et al. (1993), Utkin (1992). The main advantage of the technique is that systems in sliding mode are insensitive to disturbances and parameter uncertainties. However, favourable performance of the systems is usually achieved at the price of chattering, i.e. high-speed switching of the control signal. The chattering is undesirable not only because it may excite the non-modelled high-frequency dynamics of the plant, but also because it will result in unnecessary wear and tear on the actuator components.

In order to alleviate the problem of chattering, continuous approximations of the discontinuous control laws, Ryan and Corless (1984), Slotine and Li (1991), Yung and Liu (1994) and dynamical sliding mode controllers, Sin, Panj

proposed. However, if a continuous approximation of the discontinuous sliding mode control law is applied, the total invariance properties associated with the ideal sliding mode are lost and the system performance, at least to some extent, deteriorates. On the other hand, the robustness of dynamical controllers was not analysed in the research reported. This is mainly due to the fact that for the systems of this type, the disturbance typically affects the system input, while the discontinuous action counteracting the disturbance affects the input derivatives rather than the input itself. Consequently, the so called matching conditions are not satisfied in the systems and direct conclusions concerning the robustness cannot be drawn.

In this paper a new technique to attenuate chattering without compromising the system robustness is proposed. The technique is based on the observation that when the system is in sliding mode, information about the disturbance can be extracted from the equivalent value of the discontinuous control. Therefore, sliding mode strategies without reaching phase are considered and an auxiliary continuous control signal which partly compensates for the disturbance is introduced. The signal is generated by a single integrator or a low-pass filter according to the equivalent value of the discontinuous control term. Consequently, the discontinuous term can be essentially reduced without any deterioration of the system performance. In fact, the magnitude of the discontinuous control term is determined only by the uncertainty of the disturbance at the initial time  $t = t_0$ . The most important difference between the technique proposed in this paper and the previous work on dynamical controllers is that in this paper the disturbance is explicitly considered and insensitivity to the disturbance is guaranteed.

## 2. Problem statement

Let us consider a single-input single-output, possibly time-varying and nonlinear, system

$$\dot{x}^{(n)} = f(\mathbf{x}, t) + bu + d(t) \quad (1)$$

where  $x$  is the output of interest,  $u$  is the control input,  $\mathbf{x} = [x, \dot{x}, \dots, x^{(n-1)}]^T$  is the state vector,  $d(t)$  is an unknown disturbance,  $f(\mathbf{x}, t)$  is a known function of time and the state vector, and  $b \neq 0$  is a known constant. The system is supposed to track a demand trajectory  $x_d(t)$ . We define the tracking error  $e = x - x_d$  and the error state vector  $\mathbf{e} = [e, \dot{e}, \dots, e^{(n-1)}]^T = [e_1, e_2, \dots, e_n]^T$ . The purpose of this work is to propose a sliding mode strategy which drives the error of the system to zero, makes the system completely insensitive to the disturbance  $d(t)$  for any  $t \geq t_0$  (i.e. from the very beginning of the proposed control action), and causes as little chattering as possible. In other words, we wish to attenuate chattering as much as possible without compromising the

### 3. Integral compensation

In this section the disturbance  $d(t)$  is not required to be bounded by a known constant at any time  $t \geq t_0$ . The only assumptions made are:

- the initial disturbance  $d(t_0)$  is bounded

$$d_1 < d(t_0) < d_2; \quad (2)$$

- the disturbance does not change infinitely fast, i.e. there exists a possibly very big constant  $\delta$  such that at any time  $t \geq t_0$

$$|\dot{d}(t)| < \delta. \quad (3)$$

The following notation is used throughout this paper

$$d_0 = \frac{d_1 + d_2}{2}, \quad k = \frac{d_2 - d_1}{2}. \quad (4)$$

Note that if the initial disturbance is known precisely, then  $d_0 = d(t_0)$  and  $k$  is an arbitrarily small constant. We choose a (possibly nonlinear and/or time varying) switching surface  $\sigma(e, t) = 0$  in such a way that  $\sigma[e(t_0), t_0] = 0$ , i.e. the error state vector  $e(t_0)$  belongs to the surface at time  $t = t_0$ . This is possible either if a time-varying switching surface adaptable to initial conditions of the system, Bartoszewicz (1995, 1996), Choi et al. (1994), Lu and Chen (1995), is applied or if integral sliding mode control realised in an extended state space, Slotine and Li (1991), is considered. In the latter case the controlled plant is augmented by an extra state  $e_0(t) = C + \int_{t_0}^t e_1(\tau) d\tau$  where  $C$  is a constant which can be appropriately selected to satisfy  $\sigma[e(t_0), t_0] = 0$ . This can be easily realised in practice since no extra differentiation is required.

Provided  $\frac{\partial \sigma}{\partial e_n} \cdot b \neq 0$  we introduce the following switched feedback control law

$$u = u_0 + u_d + u_c \quad (5)$$

where

$$u_0 = b^{-1} \cdot x_d^{(n)} - b^{-1} \cdot f - \left( \frac{\partial \sigma}{\partial e_n} \cdot b \right)^{-1} \left( \frac{\partial \sigma}{\partial e_1} \cdot e_2 + \frac{\partial \sigma}{\partial e_2} \cdot e_3 + \dots + \frac{\partial \sigma}{\partial e_{n-1}} \cdot e_n + \frac{\partial \sigma}{\partial t} \right) \quad (6)$$

is the so-called equivalent control in the absence of disturbance,  $u_d$  is a discontinuous term

$$u_d = -\frac{k}{b} \operatorname{sgn}(\sigma) \operatorname{sgn} \left( \frac{\partial \sigma}{\partial e_n} \right) \quad (7)$$

and  $u_c$  is a continuous compensation term

with an initial condition  $u_c(t_0) = -\frac{d_0}{b}$  and  $T > 0$ . This control law is illustrated in Fig. 1.

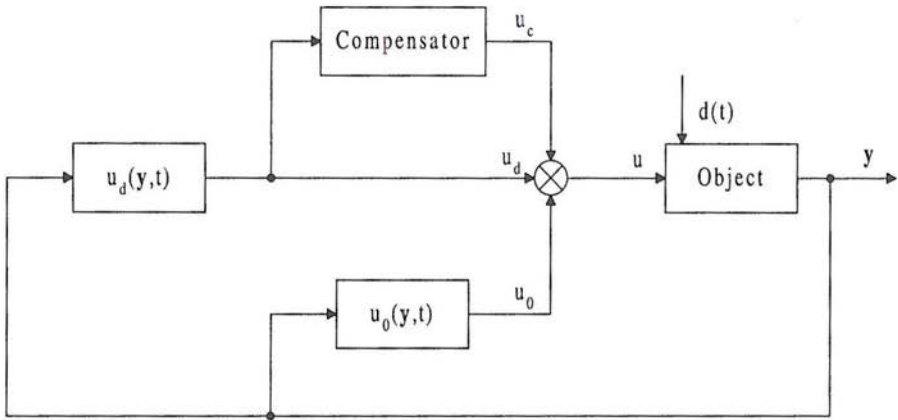


Figure 1. Control system

**DEFINITION 1** Let us define an equivalent value  $u_{deq}(t)$  of the discontinuous term  $u_d(t)$  as follows:

- if the system is not in sliding mode  $u_{deq}(t) = u_d(t) = \pm \frac{k}{b}$
- otherwise, i.e. when the system is in sliding mode,  $u_{deq}(t)$  is a continuous control signal which drives the system along the discontinuity surface in the error state space. In other words  $u_{deq}(t)$  is a continuous average of  $u_d(t)$ .

This definition is different from the conventional definition of the equivalent control, DeCarlo et al. (1988), since it concerns the discontinuous control only, and according to this definition the continuous term  $u_0$  does not contribute to  $u_{deq}$ .

It can be easily verified that the condition  $\sigma \frac{d\sigma}{dt} < 0$  for the existence of the sliding mode is satisfied by the system (1) controlled according to (5)–(8) at the initial time  $t = t_0$ . To this end we take

$$\sigma \frac{d\sigma}{dt} = \sigma \left( \frac{\partial \sigma}{\partial e_1} \cdot e_2 + \frac{\partial \sigma}{\partial e_2} \cdot e_3 + \dots + \frac{\partial \sigma}{\partial e_n} \cdot e^{(n)} + \frac{\partial \sigma}{\partial t} \right). \tag{9}$$

Then, substituting (1) into (9)

$$\sigma \frac{d\sigma}{dt} = \sigma \left\{ \frac{\partial \sigma}{\partial e_1} \cdot e_2 + \frac{\partial \sigma}{\partial e_2} \cdot e_3 + \dots + \frac{\partial \sigma}{\partial e_n} \cdot e^{(n)} + \frac{\partial \sigma}{\partial t} \right\} \tag{10}$$

Taking into account (5), (6) and (7) we get

$$\begin{aligned} \sigma \frac{d\sigma}{dt} &= \sigma \frac{\partial \sigma}{\partial e_n} \cdot [b(u_d + u_c) + d(t)] \\ &= \sigma \frac{\partial \sigma}{\partial e_n} \cdot \left[ -k \cdot \operatorname{sgn} \left( \sigma \frac{\partial \sigma}{\partial e_n} \right) + bu_c + d(t) \right]. \end{aligned} \quad (11)$$

Since at  $t = t_0$ ,  $u_c(t_0) = \frac{d_0}{b}$ , we have

$$\sigma(t_0) \frac{d\sigma(t_0)}{dt} = -k \left| \sigma(t_0) \frac{\partial \sigma(t_0)}{\partial e_n} \right| + \sigma(t_0) \frac{\partial \sigma(t_0)}{\partial e_n} [-d_0 + d(t_0)] < 0 \quad (12)$$

which is by definition (4) negative. Consequently, the system is in sliding mode at  $t = t_0$  and

$$|u_{deq}(t_0)| < \frac{k}{|b|}. \quad (13)$$

On the other hand when the system is in the sliding mode the following relation holds

$$b[u_{deq}(t) + u_c(t)] + d(t) = 0. \quad (14)$$

Differentiating equation (14) we obtain

$$b\dot{u}_{deq}(t) + b\dot{u}_c(t) = -\dot{d}(t), \quad (15)$$

then, substituting relation (8) and replacing  $u_d(t)$  with  $u_{deq}(t)$ , we get

$$T\dot{u}_{deq}(t) + u_{deq}(t) = -\frac{T\dot{d}(t)}{b}. \quad (16)$$

Since this equation describes a typical first order dynamics, taking into account inequality (3), we conclude that  $u_{deq}(t)$  evolves in such a way that for any  $t \geq t_0$

$$|u_{deq}(t)| \leq |u_{deq}(t_0)| < \frac{k}{|b|} \text{ or } |u_{deq}(t)| < \frac{T\delta}{|b|}. \quad (17)$$

Consequently, we have

$$|u_{deq}(t)| < \max \left[ \frac{k}{|b|}, \frac{T\delta}{|b|} \right]. \quad (18)$$

On the other hand, it follows directly from Definition 1 that the system is in sliding mode if

$$|u_c(t)| < \frac{k}{|b|} \quad (19)$$



Therefore, taking into account (18), one may easily conclude that the system will always remain in sliding mode if

$$T \leq \frac{k}{\delta}. \quad (20)$$

By this means a new variable structure control strategy which essentially attenuates chattering and guarantees existence of the sliding mode on the switching surface  $\sigma(\mathbf{e}, t) = 0$  was proposed. In this strategy, the discontinuous control term does not depend on the magnitude of disturbance or its change rate. The only factor which determines the magnitude of the discontinuous control term is the uncertainty of the disturbance at the initial time  $t = t_0$ , and therefore proper evaluation of the bounds  $d_1$  and  $d_2$  in relation (2) is critical for the effectiveness of the proposed strategy.

#### 4. Low-pass filter compensation

In the sequel, again system (1) is considered and similarly to the previous section it is assumed that inequalities (2) and (3) hold. Furthermore, it is assumed that there exists, a possibly very big, constant  $D$  such that at any time  $t \geq t_0$

$$|d(t)| < D \quad (21)$$

Similarly as in the previous section, a switching surface  $\sigma(\mathbf{e}, t) = 0$  is chosen in such a way that  $\sigma[\mathbf{e}(t_0), t_0] = 0$  and control law (5)–(7) is proposed. However, now a modified continuous compensation term is introduced

$$u_c + T\dot{u}_c = Ku_d \quad (22)$$

with an initial condition  $u_c(t_0) = -\frac{d_0}{b}$ ,  $T > 0$  and  $K > 0$ . Now we recall relations (9)–(12) to point out that the considered system with compensation term (22) is in sliding mode at  $t = t_0$ . Therefore, we have  $|u_{deq}(t_0)| < \frac{k}{|b|}$ .

On the other hand, again when the system is in sliding mode, relations (14) and (15) hold. Substituting these relations into (22) and replacing  $u_d(t)$  with  $u_{deq}(t)$  we obtain

$$\frac{T}{K+1}\dot{u}_{deq}(t) + u_{deq}(t) = -\frac{T\dot{d}(t) + d(t)}{(K+1)b}. \quad (23)$$

Since equation (23) represents first order dynamics, we conclude that  $u_{deq}(t)$  evolves in such a way that for any  $t \geq t_0$

$$|u_{deq}(t)| \leq |u_{deq}(t_0)| < \frac{k}{|b|} \quad (24)$$

Now we get

$$|u_{deq}(t)| < \max \left[ \frac{k}{|b|}; \frac{D + T\delta}{(K + 1)|b|} \right]. \quad (25)$$

Therefore, the system will always remain in the sliding mode if big enough  $K$  and small enough  $T$  are chosen. Formally, if the following condition is satisfied

$$K \geq \frac{D + T\delta}{k} - 1 \quad (26)$$

existence of the sliding mode on the surface  $\sigma(e, t) = 0$  is guaranteed. This relation is consistent with the assumption  $K > 0$ , since  $D$  is always equal to or greater than  $k$ . In this strategy, again the discontinuous control term does not depend on the magnitude of disturbance or its change rate, but only on the uncertainty of the disturbance at the initial time  $t = t_0$ . Finally, let us point out that (26) is sufficient but not a necessary condition for the existence of the sliding mode in the system.

## 5. Simulation example

In order to verify the performance of the control techniques proposed in this note we simulated the following system

$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= -y_2 - \frac{1}{2}y_1 + u + d(t) \end{aligned} \quad (27)$$

with the initial condition  $y_1(0) = 100$ ,  $y_2(0) = 0$ . The demand trajectory was defined as  $y_{d1}(t) = y_{d2}(t) = 0$ . We assumed for disturbance  $|d(0)| < 1$  and for disturbance change rate  $|\dot{d}(t)| < 10$ . Consequently, we have  $d_0 = 0$  and  $k = 1$ . Furthermore, for the purpose of comparison with conventional sliding mode control we assumed  $|\dot{d}(t)| < 10$ . The actual disturbance simulated in this example was

$$d(t) = 9.99 \sin(t + 0.1). \quad (28)$$

We choose the switching surface

$$\sigma(e_1, e_2, t) = e_1 + e_2 - g(t) = 0 \quad (29)$$

where

$$g(t) = \begin{cases} 100 - 40t & \text{for } t < 2.5 \\ 0 & \text{for } t \geq 2.5 \end{cases} \quad (30)$$

and the following parameters of the compensators:

- integral compensator  $T = 0.1$ ;

Our control strategies are compared with the conventional sliding mode control scheme without disturbance compensation. The scheme employs a fixed switching line  $\sigma(e_1, e_2) = e_1 + e_2 = 0$ , and the control signal in the scheme is determined as  $u = 0.5y_1 - 10 \operatorname{sgn}(\sigma)$ .

Simulation results for the case of infinitely fast switching are shown in Figs. 2 and 3. Fig. 2 illustrates control signal in the considered system. Fig. 2a shows

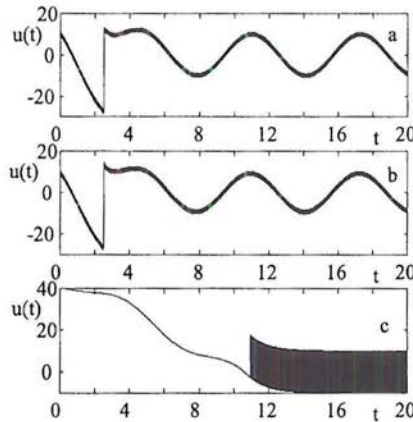


Figure 2. Control signal: (a) — proposed strategy with integral compensation, (b) — proposed strategy with first-order filter compensation, (c) — conventional sliding mode

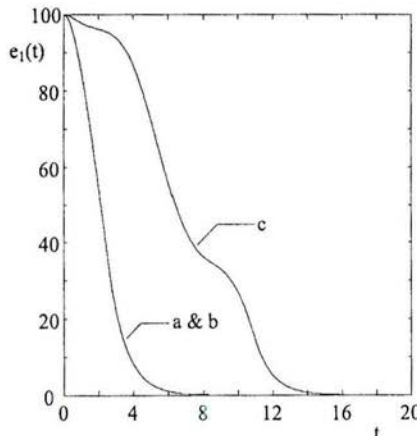


Figure 3. Error convergence: (a) — proposed strategy with integral compensation (b) — proposed strategy with first-order filter compensation, (c) —



the signal for the strategy with integral compensation, Fig. 2b for the strategy with first-order filter compensation, and Fig. 2c for conventional sliding mode control scheme. The chattering for both our strategies is reduced ten times when compared with the conventional scheme. In both our strategies the control signal evolves in a very similar (although not exactly the same) way. This is a natural consequence of the enforced error dynamics and the fact that both strategies have to compensate for the same disturbance. Fig. 3 shows error convergence in the system (1). It can be easily seen from this figure that the undesirable chattering is reduced without any deterioration of the system performance. Our strategies guarantee faster error convergence than the conventional scheme and they indeed assure insensitivity of the system to the external disturbance since the very beginning of the system motion ( $t = 0$ ) while the conventional scheme makes the system insensitive only after its representative point (RP) has reached the switching line  $\sigma(e_1, e_2) = e_1 + e_2 = 0$ , i.e. for  $t \geq 10.945$ . Finally, Fig. 4 shows the evolution of the switching variable  $\sigma$  in the case of finite switching frequency  $f = 500$ . From this figure, it can be seen again that both our strategies effectively attenuate chattering in the controlled system.

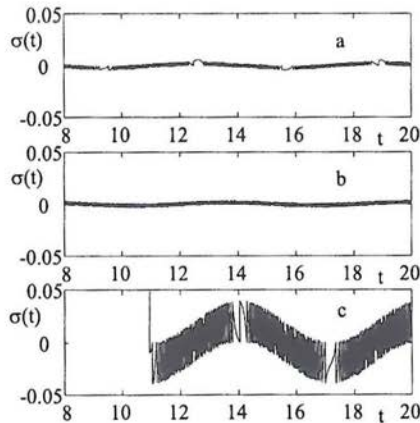


Figure 4. Evolution of the switching variable  $\sigma$  (sampling rate  $f = 500$ ): (a) — proposed strategy with integral compensation, (b) — proposed strategy with first-order filter compensation, (c) — conventional sliding mode

## 6. Conclusions

In this paper two new sliding mode control strategies which effectively attenuate chattering in the controlled systems are proposed. The strategies employ simple, continuous compensation devices — i.e. an integrator or a first-order filter — to partly compensate for the unknown disturbance. Consequently, the

on the magnitude of the disturbance or its change rate. The only factor which determines the discontinuous control term is the uncertainty of the disturbance at the initial time  $t = t_0$ . Both of the strategies presented in the paper can be directly applied to the control of multi-input multi-output (MIMO) systems. Simulation results comprised in Section 5 illustrate feasibility and favourable performance of the proposed strategies.

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