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MAINTENANCE SCHEDULING FOR MULTI-UNIT SYSTEM: A STOCHASTIC PETRI-NET AND GENETIC ALGORITHM BASED APPROACH

USTALANIE HARMONOGRAMU OBSŁUGI DLA SYSTEMU WIELOELEMENTOWEGO: PODEJŚCIE OPARTE NA STOCHASTYCZNYCH SIECIACH PETRIEGO ORAZ ALGORYTMIE GENETYCZNYM

Frequent maintenance activities would cause low system availability and require large sums of money. For a multi-unit system, maintenance activities of some units can be combined together to reduce the total maintenance possession time and cost. Therefore, an optimized timetable of the maintenance activities is needed to be planned. Considering the uncertainties in both the deterioration and maintenance process of the units in a system, this paper advances a stochastic Petri-net based simulation optimization model for maintenance scheduling. The genetic algorithm is used to get the solution of the timetable of the maintenance activity schedule such that the overall cost is minimized in a planning horizon taking into account total maintenance possession time, unit condition, life cycle loss and solution feasibility. Some techniques used to reduce the computational effort required to perform the analysis are also described. A case study is given in the end.

Keywords: *Maintenance scheduling; multi-unit system; Petri net; genetic algorithm; deterioration; minimal cut set.*

Częste czynności obsługowe prowadzą do niskiej gotowości systemu oraz wymagają dużych nakładów pieniężnych. W systemie wieloelementowym całkowity czas i koszt obsługi można obniżać łącząc ze sobą czynności obsługowe niektórych elementów. Dlatego też konieczne jest planowanie zoptymalizowanego harmonogramu czynności obsługowych. W artykule zaproponowano model symulacyjny optymalizacji harmonogramu obsługi oparty na stochastycznych sieciach Petriego uwzględniający niepewność zarówno procesu deterioracji jak i procesu obsługi elementów systemu. Algorytm genetyczny wykorzystano do opracowania terminarza czynności obsługowych, który pozwalałby na minimalizację kosztów całkowitych w przyjętym horyzoncie planowania przy uwzględnieniu całkowitego czasu obsługi, stanu elementów, strat wynikających z cyklu życia oraz wykonalności rozwiązania. Ponadto opisano techniki zastosowane w celu zmniejszenia wysiłku obliczeniowego potrzebnego do wykonania analizy. W końcowej części pracy przedstawiono studium przypadku.

Słowa kluczowe*:Ustalanie harmonogramu obsługi; system wieloelementowy; sieć Petriego; algorytm genetyczny; minimalny przekrój niezdatności.*

1. Introduction

Most of the equipments are complex function-integration system, of which the deterioration and failures might incur high costs (e.g due to production losses and delays, service interrupt, unplanned intervention on the system) and safety hazards (e.g if the resistance of a deteriorated structure drops below the applied stress). So there has been a growing interest in the modelling and optimisation of maintenance of multi-unit system which means the units of system depend on each other (i.e., economic/stochastic dependence) [1]. Economic dependence [2-3] implies either cost can be saved when several units are jointly maintained instead of separately, whereas stochastic dependence means that each component's transition probability depends on the other components'. Then optimal maintenance policies for such systems cannot reduce to those for systems with a single unit. A decision must be made to improve the whole system, rather than any subsystem.

The maintenance scheduling problem of multi-unit system has been researched and surveyed by several people. Cho and Palar [1] gives an overview of the multi-unit maintenance literature up to 1991, including machine-interference/repair models, group-replacement models of various types, spare-parts models, and inspection models. Dekker et al. [2] exclusively deals with multi-component maintenance models based on economic dependence. Later [4] presents another paper that surveys this field by different category ways. Furthermore, Wang [3] reviews the maintenance policies of deteriorating systems, of which one section is devoted to opportunistic maintenance policies for multi-unit systems with economic dependence. From these reviews we can find the maintenance model of multi-unit system is too complex to solve, especially considering two dependencies or more, so only the economical dependence is discussed in most references [2,6]. Generally there are two important policies to organize the maintenance of multi-unit system with economical dependence. One is group maintenance [5] under which the system is either entirely replaced with new components or is allowed to remain in operation. The other is opportunistic maintenance [6] under which preventive maintenance is carried out at opportunities, either by choice or based on the physical condition of the system. Most of the existing group/opportunistic maintenance models allow grouping of the maintenance tasks, but few of them are proposed in the context of the condition-based maintenance [7], especially when the number of units increases. The mathematical formulations of these maintenance models become too complicated to get the analytical solution. Therefore, many papers present the simulation method to solve the problem. Ouali [8] proposes a simulation model for opportunistic maintenance strategies. Preventive maintenance activities are combined with corrective one by using Promodel software program. E.Zio [9,10] discussed the simulation method to get the optimal opportunistic policies under several kinds of condition. However, most of literatures [11,12] only focus on the optimisation technique and seldom discuss the influence and allocation of resources.

In the case of a continuous deteriorating process and limited resources in maintenance, we propose, in this paper, three novel developments when compared with the previous work on multi-unit maintenance modeling and optimisation. First, the deterioration model is presented which allow us to investigate the uncertainty of the deterioration and maintenance process. The assumption will not limit the type of probability distribution any more. Second, multi-type resources and the behaviour of competing and sharing can be considered in a stochastic Petri-net (SPN) based model [13]. Finally, a GA-based approach is advanced to find a satisfactory solution. Such approach has been successfully applied to many engineering optimization problems including maintenance scheduling.

The rest of the paper is organized as follows. Section 2 gives the description of this problem, the framework of our approach, the objective function and how to compute the total maintenance possession time. Section 3 presents the SPN-based model for calculating the objective. Section 4 describes the steps of genetic algorithm which is used for searching an optimal solution. Section 5 presents an example of a multi-unit system with 10 units. The final section makes a conclusion.

2. Model formulation

2.1. Problem description

Most units in a system will not fail suddenly but deteriorate from a good condition to an unacceptable condition. This study divided the deterioration process into three phases; good condition, trigger condition and unacceptable condition, as shown in Figure 1. The trigger condition is the deterioration point that can be detected by existing methods and devices. But each unit would be functional in the trigger condition until it deteriorates to the unacceptable condition. Normally, the interval between trigger and unacceptable condition, called the time window, is uncertain. We may assume that it follows a kind of probability distribution based on condition history data analysis.

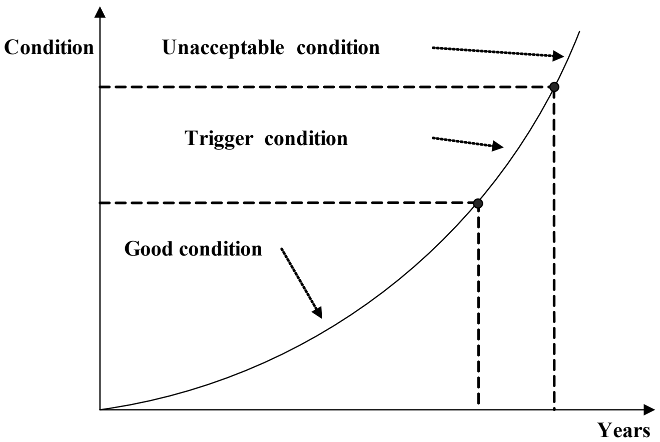


Fig. 1. The deterioration process

It is not possible to wait until the unit deteriorates to an unacceptable condition because it may be very dangerous when it is working in this condition and the cost for interruption of allowing a failure will be huge. Hence, it would be better to repair or renew the units before they deteriorate to an unacceptable condition resulting in system down. So the time window is the best time to start maintenance activity. On the other hand, a long term timetable for maintenance or renewal activities should be planned in order to leave more time for users to arrange the operation schedule.

The problem in this study is that there is a multi-unit system

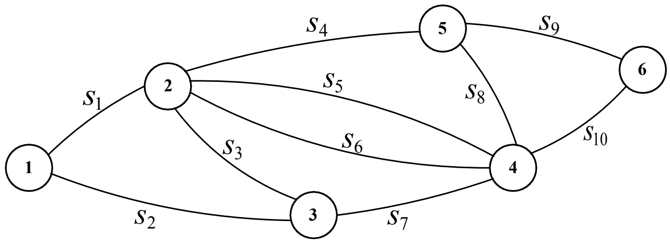


Fig. 2. The transmission network system with 10 units

$S = (S_1, S_2, \ldots, S_N)$. N is the number of units and S_p represents the pth unit, $p = 1,2,...,N$. Let $F = f(S_1, S_2, \ldots, S_N)$ be the system failure logic function. For example, there is a transmission network system with 10 units, where the source and sink node respectively is 1 and 6. Hence, we can get

$$\begin{aligned} F &= S_1S_2 + S_1S_3S_7 + S_2S_3S_4S_5S_6 + S_4S_5S_6S_7 + S_1S_3S_5S_6S_8S_{10} + \\ &+ S_2S_3S_5S_6S_8S_9 + S_5S_6S_7S_8S_9 + S_4S_8S_{10} + S_9S_{10} \end{aligned} \tag{1}$$

The conditions of different units are assumed to be statistically independent. All units are considered to be repaired or renewed in a planning horizon H, the time windows of them are not determinate and is described by a probability distribution, the maintenance resources are limited, the aim is to give an optimal schedule for maintenance activities of N units in a finite horizon taking into account the condition of units, the maintenance possession time, the life loss and the plan feasibility.

2.2. Framework of our approach

There are two key problems that should be solved. One is how to quantify and calculate the benefit of a solution. Another is how to find the best solution.

For the first problem, in this study, the failure time and maintenance time of different units in a system are characterized by different probability distributions. There are also resources shared in the maintenance process. A total cost is defined to quantify the benefit of a solution. And because the SPN is suitable for describing the behaviour of a dynamic system, it is used in this approach for analyzing the system dynamic behaviour and computing the objective of the given solution by simulation.

For the optimization problem, it is known that the solution varies non-linearly with the continuous variables, and that the size of the problem dramatically increases with the number of units considered. Such a scheduling problem is proved to be an NP-hard problem. The standard methods of non-linear programming are not suited particularly when the number of units considered is large. The GA-based approach has been successfully applied to many engineering optimization problems including maintenance scheduling. Hence, GA is also used to solve this problem.

Figure 3 shows the framework of the approach used. The SPN and its simulation are described in Section 3, steps of the GA are described in detail in Section 4.

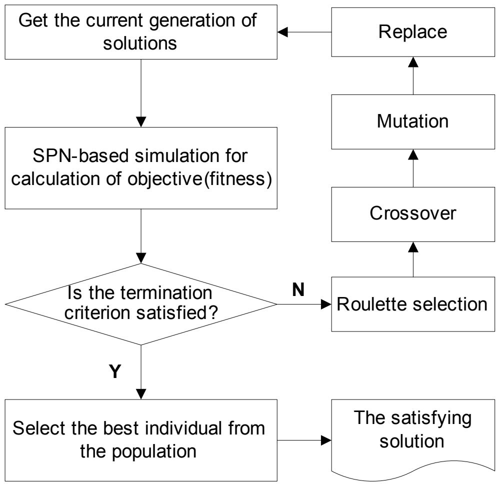


Fig. 3. The framework of SPN and GA-based approach

2.3. The objective function

In this study, the solution is described by the composition of planned maintenance start time of N units in a system given. Let T_p be the planned start time of the maintenance activity for the pth unit, $0 < T_p < H$, $p = 1,2,...,N$. So a schedule solution can be described by the vector of $(T_1,\ ...,\ T_p,\ ...,\ T_N)$. An example is presented in Figure 4, there are five units considered in the planning horizon. Here $(T_1,\ T_2,\ T_3,\ T_4,\ T_5)$ is a solution vector.

To quantify the benefit of a solution, the total cost was defined taking into account four factors in this study.

First, to reduce cost caused by system service interruption, the total maintenance possession time should be reduced. The total maintenance possession time T^{POSS} is a function of all actual maintenance time. If all units are in series, the total maintenance possession time would be sum of all actual maintenance time. However, all units are connected with a reliability logic relationship. The total maintenance possession time could not be given by simple sum. How to calculate possession time is given in section 2.4. Let c^{poss} be the possession cost per time unit. The total possession cost C^{POSS} is given by

$$C^{POSS} = c^{poss} T^{POSS} \tag{2}$$

Second, we should try to cut down the impact of unplanned maintenance activities. In this example, the 5th unit deteriorated to the unacceptable condition before the planned maintenance started. The maintenance activity of the 5th unit should start as soon as possible. The more time the unit is in the unacceptable condition, more dangerous the service is. So the actual start time will be before the planned one. Let T_p^m and T_p^u be the actual maintenance start time and of an unacceptable failure time of the pth unit respectively, $p = 1,2,...,N$.

Let c_p^{unac} be the cost per time unit of the pth unit in an unacceptable condition. Hence, the total cost due to any unit in an unacceptable condition C^{UNAC} is given by sum of that of all units as

$$C^{UNAC} = \begin{cases} \sum\limits_{p=1}^{N} c_p^{unac} \left(T_p^m - T_p^u\right) & T_p^m > T_p^u \\ 0, & other \end{cases} \tag{3}$$

Third, if the time of maintenance for a unit is before the trigger time as a result of it being combined with the maintenance of other units, its life cycle loss may be increased. So it would be better to start the maintenance activity of units in their time windows. In this example, the maintenance activities of the 2nd unit started before the trig-

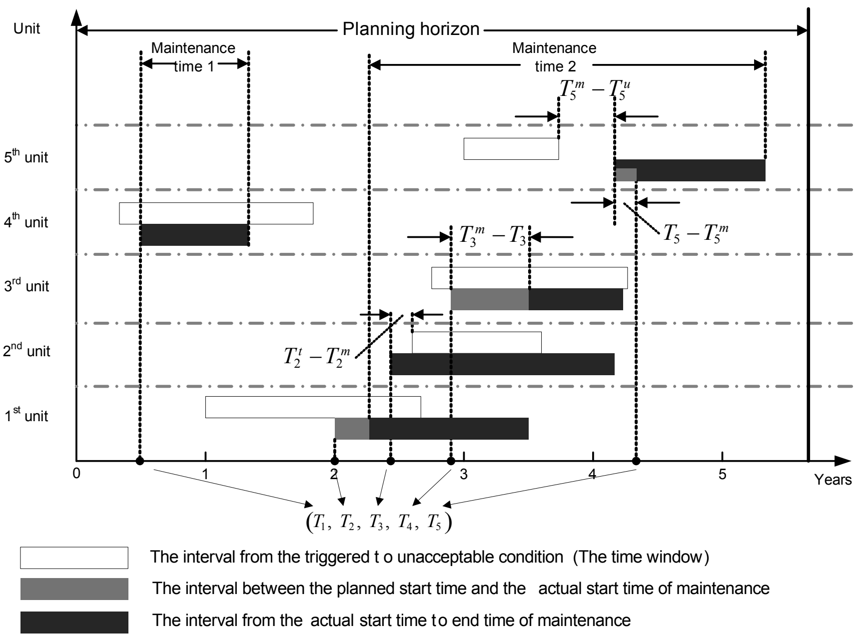


Fig. 4. An example of the maintenance activities of five units

ger time. Let T_p^t be the trigger time of the pth unit and c_p^{window} be the penalty cost per time unit for the advanced time before the trigger time of the pth unit. The penalty cost C^{WIND} caused by the opportunistic maintenance activities starting before the trigger time is given by:

$$C^{WIND} = \begin{cases} \sum_{p=1}^{N} c_p^{window}\left(T_p^t - T_p^m\right) & T_p^t > T_p^m \\ 0, & other \end{cases} \tag{4}$$

Fourth, because of the limit of resources and the uncertainties of the deterioration and maintenance processes, the maintenance activity may not be able to start at the planned time. In the example of Figure 4, the maintenance activities of the 1st, 3rd and 5th units didn't start at the planned time. However, we should try to decrease the waiting time for maintenance resources and guarantee the feasibility of solution. Let c_p^{plan} be the penalty cost per time unit for the activity of the pth unit couldn't start at the planned time. The penalty cost C^{PLAN} caused by the maintenance activities can't start at planned time is given by:

$$C^{PLAN} = \sum_{p=1}^{N} c_p^{plan}\left|\left(T_p - T_p^m\right)\right| \tag{5}$$

where $\left|\left(T_p - T_p^m\right)\right|$ is absolute value of $\left(T_p - T_p^m\right)$, it stands for interval time between the actual maintenance start time and the planned one.

Hence, the total cost is defined as:

$$C^{TOTAL} = C^{POSS} + C^{UNAC} + C^{WIND} + C^{PLAN} \tag{6}$$

The objective of maintenance scheduling, as described above, is to minimize the total cost by searching the best solution.

2.4. Calculation of the total maintenance possession time

Because all units are connected with each other in a system failure logic function described by *F*. The total maintenance possession time cannot be given by simple sum. In this study, the total maintenance possession time is defined as the total time that the system should be interrupted because of maintenance activities. In the system reliability problem, a cut set is a unit set such that if all units in the set are not in working condition, the system cannot work either. And a minimal cut set (MC) is an unit set such that if any unit is removed from the set, then the remaining set is no longer a cut set. For example, $\{S_1, S_3, S_7\}$ is an MC of the system shown in Figure 2. It is easy to know that if all units in any MC are scheduled to do maintenance activities in a same period, the system will be down. How to get all MCs of a system with failure logic function *F* is a classical problem in reliability analysis and there are many methods to solve it [14-15]. Hence, this study assume that all MCs are pre-computed, which are employed to compute the total maintenance possession time.

Let $\left(T_p^{Begin}, T_p^{End}\right)$ be the maintenance time of the pth unit, where T_p^{Begin} is the unacceptable condition begin time or the maintenance start time and T_p^{End} is the maintenance end time. Let $MCs = \{MC_1, MC_2, \ldots, MC_K\}$ be all MCs of the system. K is the number of MC and MC_i is the ith MC. The total maintenance possession time T^{POSS} can be calculated by:

$$T^{POSS} = \int_0^H f(t)dt \tag{7}$$

where

$$f(t) = \begin{cases} 1, & \exists MC_i \in MCs, MC_i \subseteq \left\{S_p \middle| T_p^{Begin} \le t \le T_p^{End}, p = 1,2,\ldots,N\right\} \\ 0, & other \end{cases} \tag{8}$$

For the example shown in Figure 2, all MCs are $\{S_1, S_2\}$, $\{S_1, S_3, S_7\}$, $\{S_2, S_3, S_4, S_5, S_6\}$, $\{S_4, S_5, S_6, S_7\}$, $\{S_1, S_3, S_5, S_6, S_8, S_{10}\}$, $\{S_2, S_3, S_5, S_6, S_8, S_9\}$, $\{S_5, S_6, S_7, S_8, S_9\}$, $\{S_4, S_8, S_{10}\}$ and $\{S_9, S_{10}\}$. If the maintenance time of 10 units are respectively $(10,21)$, $(30,35)$, $(30,41)$, $(153,180)$, $(120,128)$, $(32,54)$, $(180,195)$, $(172,192)$, $(190,220)$, $(175,203)$. Here, the unit of time is a day. The fourth, eighth, and tenth units are not working for the unacceptable condition or maintenance in the 174th day. Hence, $f(174)=1$ because of existing an MC= $\{S_4, S_8, S_{10}\} \subseteq \{S_4, S_8, S_{10}\}$ and it stands for the whole system is not working. But in the 35th day, the second, third, and sixth units are not working and there does not exist an MC which is a subset of $\{S_2, S_3, S_6\}$, the system would be working, so $f(35)=0$. By formula (7), the total maintenance possession time is 20 in this example.

3. SPN-based simulation for computing the objective

Due to the uncertainties and the fact that the maintenance resources are shared in the maintenance process of the system, this problem is complex. The Petri-net method is suitable for describing the dynamic behaviour of the system. First created in 1962 and reported in the thesis of Petri[16], Petri-net are an adaptable and versatile, yet simple, graphical modelling tool used for dynamic system representation. In this study, a stochastic Petri-net is used for analyzing the system dynamic behaviour and computing the objective of a given solution by simulation.

3.1. The deterioration and maintenance process modelling

From section 2.1, it is known that the unit has three states – good condition, trigger condition and unacceptable condition. A SPN representing for the deterioration process including the maintenance process is shown in Figure 5. Three places P_0, P_1 and P_2 stand for three system states respectively, good condition, trigger condition and unacceptable condition. The location of the token indicates the state in which the system resides. The two transitions T_0 and T_1 respectively stand for the deterioration processes from good condition to trigger condition and from trigger condition to unacceptable condition. The transition duration time may follow any probability distribution, such as Weibull, exponential, gamma, normal, lognormal, beta or triangular, as given by the deterioration process of the unit. The place P_3 stands for the maintenance resources, such as maintenance teams, workers, machines and tools. The initial quantity of tokens in place P_3 is decided by the number of available maintenance resources. The transaction T_2 stands for the maintenance process when the unit is in the trigger condition. The transaction T_3 stands for the maintenance process when the unit is in the unacceptable condition. If there are enough maintenance resources (enough tokens in Place P_3) when the unit is in the trigger condition, the transition T_5 will be triggered. But if there is not enough maintenance resource before the unit deteriorates to the unacceptable condition, the transition T_4 will be triggered after enough maintenance resources are ready.

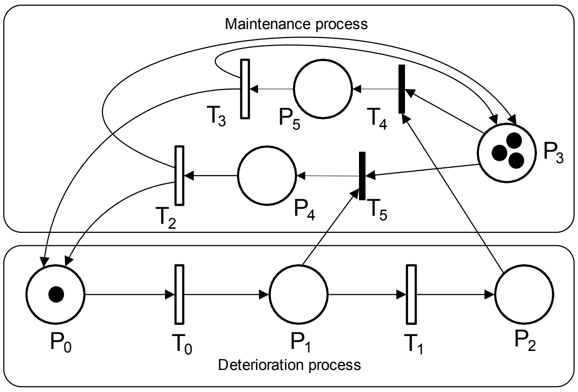


Fig. 5. The SPN of deterioration and maintenance process

3.2. SPN-based model

To calculate the objective function value under a given solution, some parameters need to be given. So we modify the SPN as shown in Figure 6 according to the basic model in Section 3.1. There are four parts in the Petri-net of one unit. The first part is the subnet of the deterioration process as described in Section 3.1. Initially, a token is in the place P_0. The second part is the subnet for maintenance scheduling. Initially, a token is in the place P_3. The transition time of T_4 is the planned start time of this unit and it is determined when the solution is given. When the token enters into the place P_4, it identifies that maintenance is required for this unit. The third part is the maintenance process. It is a little different from Figure 5. The priority of T_6 is higher than the priority of T_5 because the maintenance activity may be different after the unit gets to the unacceptable condition. If there is

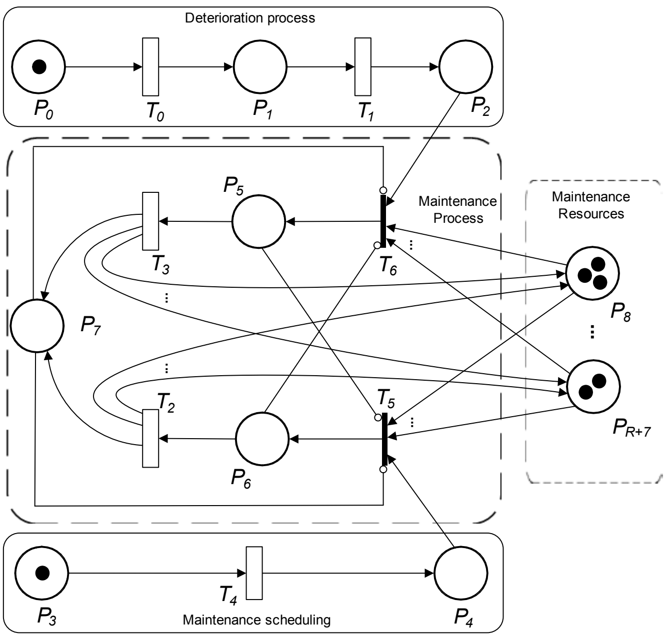


Fig. 6. The basic SPN of one unit for scheduling

a token in the place P_5 or P_7, transition T_5 can't be triggered because an inhibitor arc is activated. If there is a token in the place P_6 or P_7, transition T_6 can't be triggered too. It means only one transition of T_5, T_6 will be triggered. If both transitions T_5 and T_6 are satisfied, transition T_6 will be triggered because of its higher priority. In the fourth part is the subnet of maintenance resources, the places P_8, P_9,..,P_{R+7} stand for free maintenance resources of the 1st, 2nd,…,Rth type. R is the number of maintenance resource types. The number of tokens in these places stands for the number of available resources of the corresponding type.

Because of the assumption that any unit will not fail again after maintenance during the planning horizon, the token in P_7 will not leave again. If there are enough tokens in the places P_8,..,P_{R+7} when place P_2 or P_4 get a token, there are two scenarios of system behaviour. When the token arrival time of place P_4 (planned start time of the unit) is before that of place P_2 (the arrival time of unacceptable condition). Transition T_5, T_2 will be triggered and the scheduled maintenance activity will be done (scenario 1). Otherwise, the token arrival time of place P_4 is behind that of place P_2. Transition T_6, T_3 will be triggered and the maintenance activity caused by the unit in an unacceptable condition should be done (scenario 2). If there are not enough tokens in the resource places when place P_2 or P_4 get a token, there are also another two scenarios. When place P_2 gets a token before the resource places get enough tokens, transition T_6, T_3 will be triggered in any case when the resource places get enough tokens (scenario 3). When the resource places get enough tokens and place P_4 have a token but place P_2 have no token. Transition T_5, T_2 will be triggered when the resource places get enough tokens (scenario 4). These four scenarios suitably stand for four actual cases.

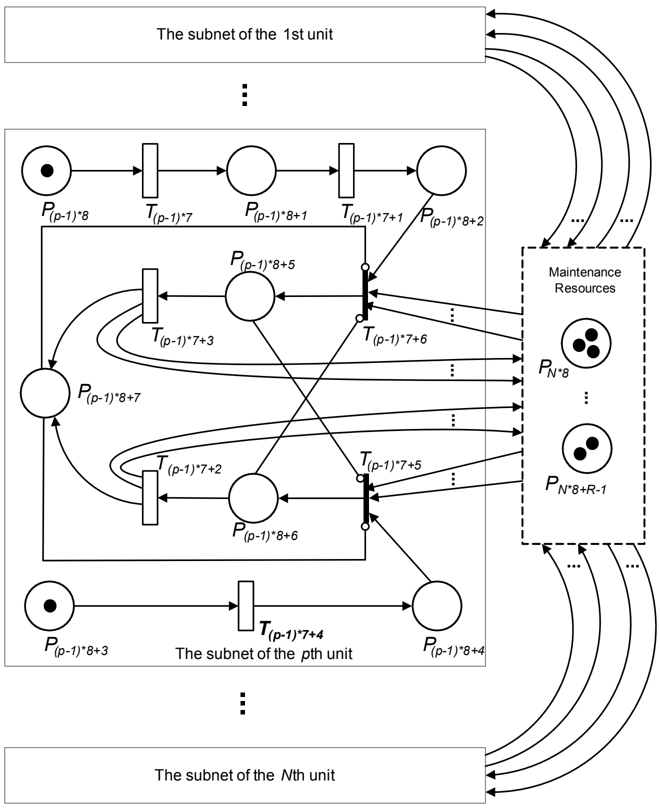


Fig. 7 The SPN of a system with N units

Based on the basic SPN of one unit for scheduling, we can easily build the SPN of the system by combining all SPNs of the considered units, as shown in Figure 7. The maintenance resources in places P_{N*8}, P_{N*8+1}, …, $P_{N*8+R-1}$ are shared by all units. The combination of the transition times of (T_4, …, $T_{(i-1)*7+4}$,…$T_{(N-1)*7+4}$) is the solution of the maintenance schedule. The initial numbers of tokens in P_{N*8}, P_{N*8+1}, …, $P_{N*8+R-1}$ are the numbers of maintenance resources of the corresponding type respectively.

3.3. Calculation of the objective

Let S_N be the number of simulations. All values with subscript i stand for the values calculated in the ith simulation. For example, C_i^{TOTAL} stands for the total cost calculated in the ith simulation. So the average total cost is given by:

$$C^{TOTAL} = \frac{1}{S_N}\sum_{i=1}^{S_N} C_i^{TOTAL} = \frac{1}{S_N}\sum_{i=1}^{S_N}\left(C_i^{POSS} + C_i^{UNAC} + C_i^{WIND} + C_i^{PLAN}\right) \tag{9}$$

Let $T^{Token\_In}(i)$ be the token first arrival time of the ith place. This is obtained during the simulation. For scenarios 1 and 4 in section 3.2, the maintenance possession begin time of the pth unit T_p^{Begin} is the token arrival time of the $((p-1)\times 8+6)$th place. For scenario 2 and 3, it is the token arrival time of the $((p-1)\times 8+2)$th place. So,

$$T_p^{Begin} = \begin{cases} T^{Token\_In}((p-1)\times 8+6), & T^{Token\_In}((p-1)\times 8+6) > 0 \\ T^{Token\_In}((p-1)\times 8+2), & T^{Token\_In}((p-1)\times 8+2) > 0 \ and \ T^{Token\_In}((p-1)\times 8+6) = 0 \\ 0, & other \end{cases} \tag{10}$$

The maintenance possession end time of the pth unit T_p^{End} is the token arrival time of the $(p-1)\times 8+7$ th place. If $T_p^{Begin} \neq 0$ and no token arrives in the $(p-1)\times 8+7$ th place, it means that the maintenance activity can't be finished in the planning horizon. T_p^{End} is considered to be H . Hence,

$$T_p^{End} = \begin{cases} T^{Token\_In}((p-1)\times 8+7), & T^{Token\_In}((p-1)\times 8+7) > 0 \\ H, & T^{Token\_In}((p-1)\times 8+7) \le 0 \ and \ T_p^{Begin} > 0 \end{cases} \tag{11}$$

By formula (6), the total possession time T_i^{POSS} and C_i^{POSS} can be calculated.

As shown in formulae (3), (4) and (5), to calculate C^{UNAC} , C^{WIND} and C^{PLAN} , we only need to get T_p^m, T_p^u and T_p^t . The actual maintenance start time of the pth unit T_p^m is the token arrival time of the $((p-1)\times 8+6)$th or $((p-1)\times 8+5)$th place. If no token arrives in these two places, it means that the maintenance activity can't be started in the planning horizon. We can set it to be H . So,

$$T_p^m = \begin{cases} T^{Token\_In}((p-1)\times 8+6), & T^{Token\_In}((p-1)\times 8+6) > 0 \ and \ T^{Token\_In}((p-1)\times 8+5) = 0 \\ T^{Token\_In}((p-1)\times 8+5), & T^{Token\_In}((p-1)\times 8+5) > 0 \ and \ T^{Token\_In}((p-1)\times 8+6) = 0 \\ H, & other \end{cases} \tag{12}$$

The unacceptable condition begin time of the pth unit T_p^u is when a token arrives in the $((p-1)\times 8+2)$th place. So,

$$T_p^u = \begin{cases} T^{Token\_In}((p-1)\times 8+2), & T^{Token\_In}((p-1)\times 8+2) \ge 0 \\ H, & Other \end{cases} \tag{13}$$

The trigger condition begin time of the pth unit T_p^t is when a token arrives in the $((p-1)\times 8+1)$ th place. So,

$$T_p^t = \begin{cases} T^{Token\_In}((p-1)\times 8+1), & T^{Token\_In}((p-1)\times 8+1) \ge 0 \\ H, & Other \end{cases} \tag{14}$$

Hence, the objective could be calculated by the simulation of the SPN shown in Figure 7.

4. GA-based optimization approach

4.1 Genesis of the population

Planning horizon H is the maximum permitted value of each maintenance planned start time. To be dealt with easily, we use a vector of floating point numbers $(q_1, \ldots, q_p, \ldots, q_N)$ as an individual of the population also called a chromosome in GA terms, $0 < q_p < 1$, $p = 1, 2, \ldots, N$. The chromosome $(q_1, \ldots, q_p, \ldots, q_N)$ stands for the solution as below.

$$(T_1, \ldots, T_p, \ldots, T_N) = (q_1 H, \ldots, q_p H, \ldots, q_N H) \tag{15}$$

The initial population is created randomly. The population size should be large enough to search for an optimal solution. However, the larger the population size, the greater the computing time required.

4.2 Calculation of fitness

Using the SPN described in section 3, the objective of each individual, known as the fitness in the GA, was calculated by simulation. The bigger the number of simulation times, the more accurate the fitness becomes but the more searching time would be consumed. Hence, to improve the efficiency, a buffer and multi-thread programming technology was used. To avoid the repeated simulation of the same solution, the fitness value is saved in the buffer after each simulation. The fitness value would be used directly if the same chromosome appears again in a later generation. On the other hand, the generation would be divided into several groups before calculating their fitness. The simulation of different groups will be executed in different parallel threads. This method is called a multi-thread programming technology. The aim is to improve the execution efficiency.

4.3 Roulette selection

The roulette selection process is to choose the chromosomes to act as parents to perform crossover on. The crossover process is described in Section 4.4. After that, the next generation will be created. In this study, the probability of a chromosome being chosen is inversely proportional to its fitness describing the total cost. So the less the fitness, the greater chance the chromosome will be selected. Hence, the next generation is likely to be better than the previous one.

4.4. Crossover

The parent chromosome pair may crossover with a probability, known as the crossover rate P_c. This study used a one-point crossover process. It consists of selecting the crossover point p randomly, dividing each parent chromosome into two parts, swapping the corresponding part with the other chromosome in the pair to produce two new child chromosomes and replacing the parent chromosomes with child chromosomes. This process is shown as Figure 8.

Parent chromosome 1

$$\left(q_1^1, \ldots, q_{p-1}^1, q_p^1, q_{p+1}^1, \ldots, q_N^1\right)$$

Parent chromosome 2

$$\left(q_1^2, \ldots, q_{p-1}^2, q_p^2, q_{p+1}^2, \ldots, q_N^2\right)$$

$$\xrightarrow[p]{\text{Crossover}}$$

Child chromosome 1

$$\left(q_1^1, \ldots, q_{p-1}^1, q_p^2, q_{p+1}^2, \ldots, q_N^2\right)$$

Child chromosome 2

$$\left(q_1^2, \ldots, q_{p-1}^2, q_p^1, q_{p+1}^1, \ldots, q_N^1\right)$$

Fig. 8. Crossover

4.5. Mutation and replacement

In order to avoid the solution converging to a local best result, a mutation process is necessary. In this study, every value in the chromosomes may mutate with a probability, known as the mutation rate P_m. After mutation, the old population will be replaced by the new one.

5. Case study

This case study bases on the example shown in Figure 2. There is a system with 10 units that should be considered for renewal in the coming five years. Both the deterioration time from now to the trigger condition and the deterioration time from the trigger to unacceptable conditions follow the Weibull distribution. The renewal time follows the triangular distribution. Table 1 gives all distribution parameters of these units. The unit of time is a day.

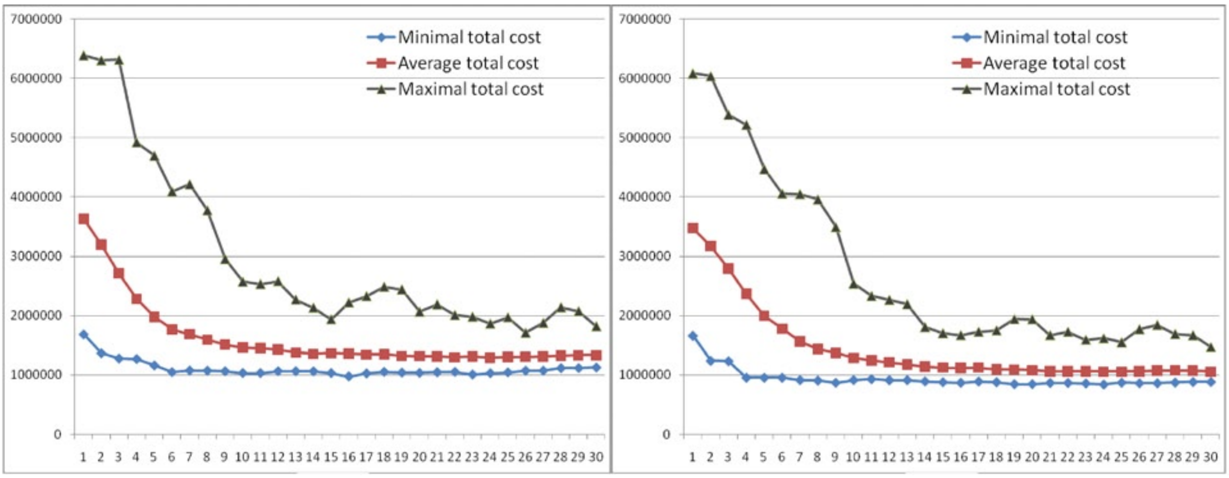
In the SPN of this example, because we take into account the coming 5 years, the planning horizon was set to 1800 days. And for each solution, the simulation time was 1000. The parameters c^{poss}, c_p^{unac}, c_p^{window} and c_p^{plan} of all units were 400, 600, 300 and 1000 respectively.

In the GA-based searching for the optimal solution, it produced 30 generations and the population size of each generation had 200 individuals. The crossover rate P_c was 0.9 and the mutation rate P_m was 0.05. The number of teams is 4. Figure 9(a) and 9(b) respectively show the total cost curve of generation number in two cases; a transmission network system shown in Figure 2 and a series system. We can find that all the minimal, average and maximal total cost becomes lesser during the evolution process of the GA. Figure 10 and 11 respectively show the Gantt graphs of the optimal solution produced by the GA in two cases.

In these solutions, because there are four maintenance teams, the maintenance activities of some units whose deteriorated conditions are similar would be combined taking into account the opportunity to cut down the total maintenance possession time. When all units are not in series, the total maintenance posses-

Table 1. Distribution parameters of the units

Seg. No.	Distribution of deterioration time from now to trigger condition	Distribution of deterioration time from trigger to unacceptable condition	Distribution of renewal time
1	Weibull, Scale=160, Shape=2	Weibull, Scale=200, Shape=2	Triangular. If renewal before the trigger condition, min=80, mean=100 and max=120 If renewal after the unacceptable condition, min=90, mean=120 and max=140
2	Weibull, Scale=210, Shape=2		
3	Weibull, Scale=220, Shape=2		
4	Weibull, Scale=500, Shape=2		
5	Weibull, Scale=650, Shape=2		
6	Weibull, Scale=700, Shape=2		
7	Weibull, Scale=600, Shape=2		
8	Weibull, Scale=950, Shape=2		
9	Weibull, Scale=1200, Shape=2		
10	Weibull, Scale=1230, Shape=2		



(a) (b)

Fig. 9. The total cost curve of the generation number

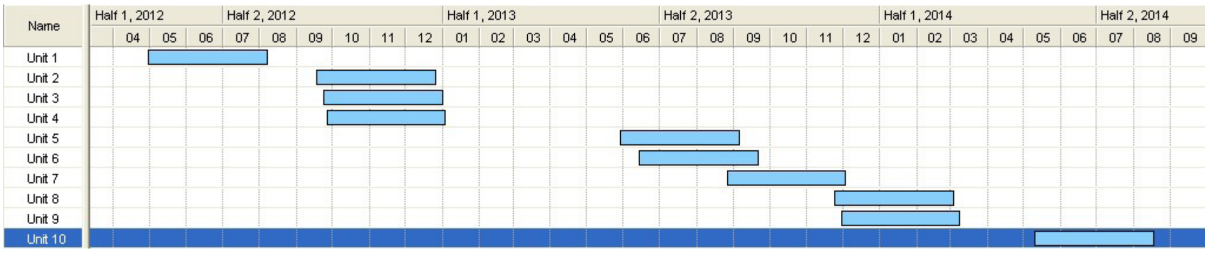


Fig. 10. The Gantt chart of the optimal solution of the system shown in Fig. 2 when the number of teams is 4

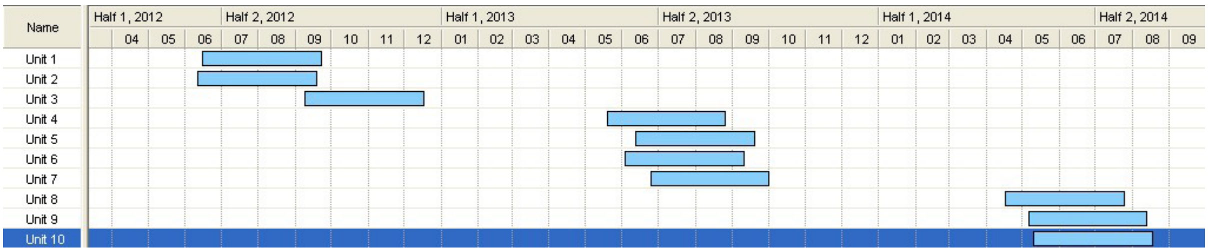


Fig. 11. The Gantt chart of the optimal solution of a series system when the number of teams is 4

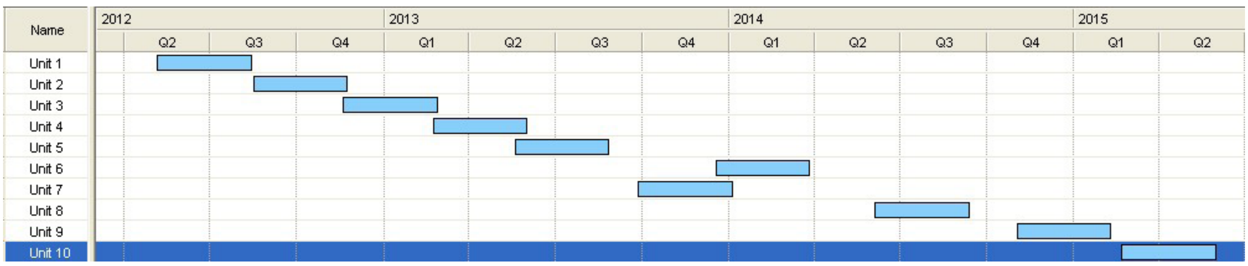


Fig. 12. The Gantt chart of the optimal solution when the number of teams is 1

sion time is not the simple sum of that of all units. It has to be avoided that the units set whose maintenance activities are in a same period belongs to a cut set. Hence, although the conditions of unit 1 and 2 are very close, it is better not to combine the maintenance activities of them. Analogously, it is better not to combine the maintenance activities of the unit 9 and 10. It is different from the case when all units are in series.

Figure 12 shows the Gantt graphs of the optimal solution as produced by the GA when there is only one available maintenance team. All maintenance activities would be scheduled in series because of the limit of the maintenance teams. The worse the condition of the component is, more chance to be scheduled in advance

6. Conclusions

This paper describes an approach for maintenance scheduling of a multi-unit system in a finite planning horizon. In this approach, SPN was used to describe the stochastic and dynamic behaviour of the deterioration and maintenance process of the units. It evaluates the total cost including maintenance possession cost, life cycle loss and penalty cost as the objective which can be calculated by the simulation of the SPN. Because this kind problem was proved to be an NP-hard problem, so the GA was investigated to search for the optimal solution. To improve the efficiency of the GA, a buffer and multi-thread programming technology were used to avoid the repeated simulation for the same solution and increase the execution efficiency. The approach advanced in this study may be used to improve greatly such complex maintenance decision making by taking into account the uncertainties. It can help planners to make decisions regarding solution feasibility, total maintenance possession time and system availability.

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