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## ADAPTIVE CONTROL OF MACHINING ACCURACY OF AXIAL-SYMMETRICAL LOW-RIGIDITY PARTS IN ELASTIC-DEFORMABLE STATE

### STEROWANIE ADAPTACYJNE DOKŁADNOŚCIĄ OBRÓBKĄ CZĘŚCI OSIOWO-SYMETRYCZNYCH O MAŁEJ SZTYWNOŚCI W STANIE SPRĘŻYŚCIE-ODKSZTAŁCALNYM\*

*The authors developed a method of correction consisting in the introduction, in the control system, of an additional positive feedback relative to the force of milling. Adaptive control was applied for axial feed and for additional force actions causing the elastic-deformable state, which permits the elimination of static errors of control effects and interference in the control of quality parameters.*

**Keywords:** adaptive control, machining accuracy, low-rigidity parts, elastic-deformable state.

*Opracowano sposób korekty ustawienia układu technologicznego obrabiarki polegający na wprowadzeniu do układu sterowania dodatkowego dodatniego sprzężenia zwrotnego względem siły skrawania. Zastosowane sterowanie adaptacyjne posuwem wzdłużnym oraz dodatkowymi oddziaływaniami siłowymi, wywołującymi stan sprężyste-odkształcalny, umożliwia wyeliminowanie błędów statycznych oddziaływania sterującego oraz zakłócającego przy sterowaniu parametrami dokładności obróbki części o małej sztywności w stanie sprężyste-odkształcalnym.*

**Słowa kluczowe:** sterowanie adaptacyjne, dokładność obróbki, części o małej sztywności, stan sprężyste-odkształcalny.

#### 1. Introduction

In the machine-building industry, axisymmetrical parts constitute approximately 34% of the total number of parts, and among those up to 12% can be classified as low-rigidity shafts.

Such parts are characterised by disproportional relations of overall dimensions and by low rigidity in specific sections and directions. Stringent requirements are also posed relative to the parameters of geometric form, mutual positioning of surfaces, linear dimensions, and surface quality.

The specific character of machining of similar parts causes that it is difficult to achieve the required parameters of accuracy of form, dimensions and surface quality. The low inherent stiffness and the relatively low rigidity of the shaft compared to the stiff assemblies of the machine tool cause the appearance of vibrations, under specific conditions. In the course of machining there appear numerous factors that interfere with and destabilise the process of machining (large free deformations of shafts, tools, fixtures, shavings, dust, etc.). This makes it necessary to search for new methods of controlling the machining of low-rigidity shafts.

Positive solution of problems related with improvement of quality of machining of parts of milling machines for metals depends directly on further improvement of methods for designing and testing of automatic control systems (ACS), machine tools and parameters of the dynamic system. The dynamic system (DS) of the process of machining is an MGFT (machine tool-grip-fixture-tool) system – it comprises the mass-dissipative-elastic (MDE) system of the machine tool and the realized process of machining (turning, grinding, drilling, milling) [2, 7, 9, 10].

In principle, the methods of analysis and synthesis of contemporary automatic control systems are based on solving a problem with

notable simplification of the physical and mathematical relations that characterise the processes included in the system. This is largely a result of imperfection of the research apparatus employed, and of complexity of acquired in prior information on the dynamic and static characteristics of the object of control and of external interference factors. The existing methods of synthesis of automatic control systems take into account the possible uncertainty concerning the character and extent of interference only to a limited degree. Even less developed are the issues of scatter of control object parameters, though at present considerable attention is devoted to them.

Designing of ACS capable of operating at uncontrolled variability of control object parameters has led to the appearance of adaptive control (AC) in machine tools [1, 3, 8]. In spite of its flexibility and possibility of assurance of required quality of transition processes for a broad range of objects, in many cases the application of AC on machine tools is difficult due to the necessity of continuous measurement of characteristics of technological systems and interference.

Reserves for increasing the accuracy and quality of machining may be defined in the design of optimum structures of control systems, as the ACS of elastic deformations (by means of relative moves of the cutting tool and the machined part) of the DS is static in terms of both control effects and interference, and their change causes errors of mutual positioning of the part and the cutting edge [4, 8].

The theoretical foundations for improvement of machining quality with adaptive control, considered in this report, are based on mathematical description of the object of control in the DS presented in references [2, 4, 5, 7, 10] and may be applied in DS analyses described mathematically differently than the presented herein and taking into account a specific character of the control object.

(\*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie [www.ein.org.pl](http://www.ein.org.pl)

**2. Some issues of invariability in the control of elastic deformations of the dynamic system**

In principle, systems of automatic control of technological processes comprise the object of control - the dynamic system of the machine tool, made up of the MDE system of the machine tool and the working processes involved in their interaction, and the regulator. Analysis of connections within the dynamic system of the machine tool indicates that it is a multi-contour closed system, and the interaction of the fundamental components is directed by the interactions between the working processes, taking place via the MDE system only MDS [2, 6]. Separation of the zones where the working processes take place, by means of the elements of the elastic system permits relative simple transition to equivalent concepts of the dynamic system of the machine tool as a single-contour system.

The transfer function of an equivalent MDE system has the form:

$$G_{us}(s) = \frac{y(s)}{F_p(s)} = \frac{K_s}{T_1^2 s^2 + T_2 s + 1}$$

where:  $y(s), F_p(s)$  – presentation, acc. to Laplace, of elastic deformations and force of machining;

$K_s = 1/C_s, T_1 = \sqrt{m_y/C_s}, T_2 = n_y/C_s$  – coefficient of proportionality and time constants of MDE system of machine tool;

$m_y, n_y, C_y$  – reduced mass, coefficient of attenuation and rigidity of MDE system.

The transfer functions of elements of adaptive control system (Fig. 1a) have the following form:

- for the MDE system, relative to the control effect

$$G_s^s(s) = \frac{y(s)}{F_p(s)} = \frac{K_s}{T_{1s}s^2 + T_{2s}s + 1}$$

- for the MDE system, relative to interference effect

$$G_s^s(s) = \frac{y(s)}{F_p(s)} = \frac{K_s}{T_{1s}s^2 + T_{2s}s + 1}$$

- for the process of machining

$$G_{sk}(s) = \frac{F_p(s)}{d(s)} = \frac{K_{sk}}{T_{sk}s + 1},$$

- for the executive mechanism

$$G_w(s) = \frac{K_w(s)}{e(s)} = \frac{K_w}{T_w s + 1},$$

where:  $T_{1s} = m_y/C_s, T_{2s} = n_y/C_s, K_s = 1/C_s, K_f = K_{1s}/C_s, T_w, K_w$  – time constant and coefficient of proportionality of transfer function of the executive mechanism.

The accuracy of operation of the system of adaptive control of elastic deformations of DS is characterised by an error  $e_s$  equal to [2, 7]:

$$e_s = e_s^o + e_f, \quad (1)$$

where:  $e_s^o$  – error caused by control effect  $y^0(t)$ ,

$e_f$  – error caused by interference effect  $f(t)$ .

Errors  $e_s^o$  and  $e_f$  are determined from the relation:

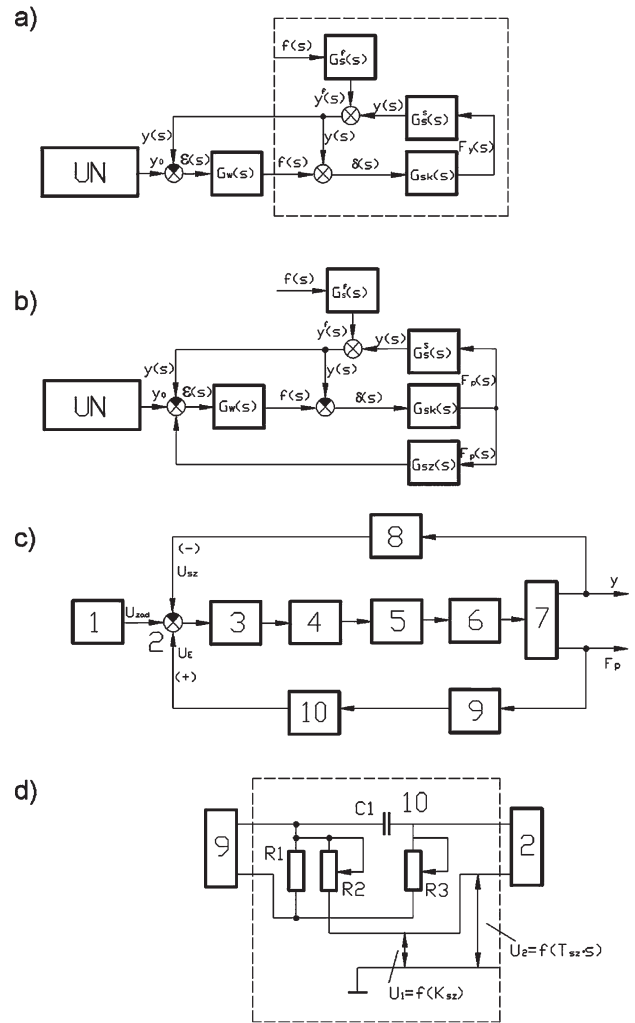


Fig. 1. Block diagram of the system of stabilisation of elastic deformations of technological system – a; schematic of system of adaptive control of elastic deformations of technological system – b; functional schematic of adaptive control – c; node of positive feedback relative to the force of machining – d

$$e_s^o(s) = \frac{1 + G_{sk}(s) \cdot G_s^s(s)}{1 + G_s^s(s) \cdot G_{sk}(s) + G_s^s(s) \cdot G_{sk}(s) \cdot G_w(s)} y^0(s) = \frac{\sum_{i=0}^4 b_i s^i}{\sum_{j=0}^4 a_j s^j} y^0(s), \quad (2)$$

$$e_f(s) = \frac{G_s^s(s)}{1 + G_s^s(s) \cdot G_{sk}(s) + G_s^s(s) \cdot G_{sk}(s) \cdot G_w(s)} f(s) = \frac{\sum_{i=0}^2 d_i s^i}{\sum_{j=0}^4 a_j s^j} f(s), \quad (3)$$

where:  $a_0 = K_s K_{sk} + K_s K_{sk} \cdot K_w; b_0 = 1 + K_{sk} K_s;$   
 $a_1 = b_1 = T_w + K_s K_{sk} K_w + T_{sk} + T_{2s};$   
 $a_2 = b_2 = T_{sk} T_w + T_{2s} T_w + T_{2s} T_{sk} + T_{1s};$   
 $a_3 = b_3 = T_{1s} T_w + T_{2s} T_{sk} T_w + T_{1s} T_{sk};$   
 $a_4 = b_4 = T_{1s} T_{sk} T_w;$

$$\begin{aligned} d_0 &= K_s ; \\ d_1 &= K_s (T_{sk} + T_w) ; \\ d_2 &= K_s T_{sk} T_w . \end{aligned}$$

The determined values of the errors (2) and (3) can be presented as [8]:

$$\begin{aligned} e_{y^0}(t) &= C_0 y^0(t) + C_1 \frac{dy^0(t)}{dt} + \frac{C_2}{2!} \frac{d^2 y^0(t)}{dt^2} + \frac{C_3}{3!} \frac{d^3 y^0(t)}{dt^3} + \frac{C_4}{4!} \frac{d^4 y^0(t)}{dt^4} , \\ e_f(t) &= C'_0 f(t) + C'_1 \frac{df(t)}{dt} + \frac{C'_2}{2!} \frac{d^2 f(t)}{dt^2} + \frac{C'_3}{3!} \frac{d^3 f(t)}{dt^3} + \frac{C'_4}{4!} \frac{d^4 f(t)}{dt^4} , \end{aligned} \quad (4)$$

where  $C_i, C'_i$  ( $i = 0, 1, 2, \dots, 4$ ) – coefficients of error, characterising the accuracy of operation of the control system and dependent on its structure.

In the structure of the control system (Fig. 1), the values of the coefficients of error are defined by the following relations:

$$\begin{aligned} C_0 &= \frac{b_0}{a_0}, \quad C_1 = \frac{b_1 - a_1 C_0}{a_0}, \quad \frac{C_2}{2} = \frac{b_2 - a_1 C_1 - a_2 C_0}{a_0}, \\ C'_0 &= \frac{d_0}{a_0}, \quad C'_1 = \frac{d_1 - a_1 C'_0}{a_0}, \quad \frac{C'_2}{2} = \frac{d_2 - a_1 C'_1 - a_2 C'_0}{a_0}. \end{aligned}$$

It should be noted that the main error is constituted by the error coefficients  $C_0, C'_0$  and  $C_1, C'_1$ , referred to as the static and speed errors. For a 16K20 machine tool, with system parameters of:

$$T_{1s} = 1,6 \cdot 10^{-6} s^2, \quad T_{2s} = 1,2 \cdot 10^{-4} s, \quad K_s = 1,6 \cdot 10^{-6} mm/N, \quad (5)$$

$$T_{sk} = 10^{-3} s, \quad T_w = 10^{-3} s, \quad K_f = 2 \cdot 10^{-4} mm/N, \quad K_{sk} = 1$$

based on data (5), the values of the error coefficients were determined analytically, as follows:

$$\begin{aligned} C_0 &= 0,32 mm^{-1}, \quad C_1 = 1,1 \cdot 10^{-3} s/mm, \quad C_2 = 0,62 \cdot 10^{-6} s^2/mm, \\ C'_0 &= 1,5 \cdot 10^{-3} mm^{-1}, \quad C'_1 = -1,1 \cdot 10^{-6} s/mm, \quad C'_2 = -2,73 \cdot 10^{-9} s^2/mm. \end{aligned}$$

Total or partial elimination of those errors would permit improvement of the accuracy of operation of the control system and, correspondingly, of the machining of the part.

The problem posed can be solved through the introduction of an additional positive feedback relative to the force of machining, with transmittance  $G_{sz}(s)$ , into the system of control of elastic deformations of the DS (Fig. 1b) [2].

In this case, error  $e_s^0(s)$  is determined from the relation:

$$e_s^0(s) = \frac{1 - G_{sk}(s)G_{sz}(s)G_w(s) + G_{sk}(s)G_s^s(s)}{1 - G_{sk}(s)G_{sz}(s)G_w(s) + G_s^s(s)G_{sk}(s)G_w(s)} y^0(s). \quad (6)$$

As follows from expression (6), the error introduced in the system of control of elastic deformations of the TS by the effect  $y^0(t)$  can be totally eliminated if the structure and parameters of the transfer function are presented as follows:

$$G_{sz}(s) = \frac{1 + G_{sk}(s)G_s^s(s)}{G_{sk}(s)G_w(s)}. \quad (7)$$

Expression (7) can be called the condition of total invariability of

the control system relative to the control effect  $y^0(t)$ . If this condition is met, all the error coefficients  $C_i$  ( $i = 1, 2, 3, 4$ ) are equal zero.

Taking into account that meeting the condition (7) basically leads to systems that are physically unrealisable, and that sufficient accuracy can be achieved in practice when only the static error coefficients  $C_0$  or  $C'_0$  and  $C_i$  are equal to zero, it is enough to meet the conditions of  $b_0 = 0$  and  $b_1 = 0$ .

However, with  $b_1 = 0$  also coefficient  $a_1$  equals zero, which causes that the system loses stability. Therefore, in the case under consideration, it is only possible to eliminate the static error  $C_0$ .

Error  $C_0$  equal zero can be achieved with means that are technologically simple enough, through the introduction of an additional positive feedback relative to the force of machining, with transfer function of  $G_{sz}(s) = K_{sz}$ .

The value of the coefficient of proportionality of the feedback is determined from relation (7)

$$b_0 = 1 + K_{sk}K_s - K_{sk}K_{sz} = 0, \quad (8)$$

that is  $K_{sz} = (1 + K_{sk}K_s) / K_{sk}$ ,

however – to maintain stability – it must be lower than 1.

If condition (8) is met, in the control system with parameters (5) considered earlier, the coefficients of error are equal to

$$C_0 = 0; \quad C_1 = 2,33 \cdot 10^{-2} s/mm.$$

Error introduced by an interference effect can be presented as:

$$e_f(s) = \frac{G_s^f(s)[1 - G_{sk}(s)G_w(s)G_{sz}(s)]}{1 - G_{sk}(s)G_w(s)G_{sz}(s) + G_s^s(s)G_{sk}(s)G_w(s) + G_s^s(s)G_{sk}(s)}. \quad (9)$$

Relation (9) can be also used to obtain the conditions of total invariability of the system under the effect of an interference factor

$$G_{sz}(s) = \frac{1}{G_{sk}(s)G_w(s)}. \quad (10)$$

By analogy, condition (10) will be met if the structure of the transfer function of the feedback is presented as:

$$G_{sz}(s) = K_{sz} + T_{sz} \cdot s,$$

and its parameters will be selected from the equations:

$$d_0 = 1 - K_{sz}K_{sk}K_w = 0,$$

$$d_1 = T_{sz} + T_w - T_{sz}K_{sz}K_w = 0,$$

hence  $K_{sz} = \frac{1}{K_{sk}K_w}, \quad T_{sz} = \frac{T_{sk} - T_w}{K_{sk}K_w}.$

In this case, the coefficients of error in the example under consideration will be equal to:

$$C_0 = 0, \quad C_1 = 0, \quad C_2 = 6,25 \cdot 10^{-9} s^2/mm.$$

Fig. 1c presents a block diagram of a system of adaptive control realizing the method of adjustment of settings of the TS [8]. The control system incorporates, in series connection, the elastic deformation

regulator 1, comparing element 2 to one of the inputs, connected to which is the elastic deformation sensor 8, correcting element 3, amplifier 4, power transducer 5, longitudinal feed motor 6, technological system 7, machining force sensor 9, node of positive feedback 10. Node 10, of the positive feedback relative to the machining force  $F_p$  (Fig. 1d) is made as a differential chain of condenser  $C_1$ , resistor a R3 and a voltage divider - resistors R1 and R2 .

In the process of work, preliminary determination is made of the initial value of elastic deformation  $y$ , in the form of signal  $U_z$ , by means of regulator 1. The true value of elastic deformation  $y$  is measured by sensor 8, and the result of the measurement, in the form of voltage  $U_{sz}$ , is summed up algebraically on the comparing element 2 with the regulated voltage  $U_z$ . At the same time, sensor 9 measures the change in the machining force  $F_p$ , caused by changes in machining conditions (material hardness of semi-finished product, allowance amount, width of machined layer, initial error). The change in the value of machining force  $F_p$  in the technological system causes, in turn, deviation of the elastic deformation  $y$  from the adopted value.

Signal from sensor 9, of machining force, is supplied to the input of node 10, of the positive feedback, from which signals  $U_1 = f(K_{sz})$  are taken off from the output of the voltage splitter, on resistors R1 and R2, and  $U_2 = f(T_{sz} \cdot s)$  from the output of the differential chain C1 and R3.

The setting of parameters  $K_{sz}$  and  $T_{sz}$  and the corresponding signals  $U_1$  and  $U_2$  for a specific technological system is realized by means of the adjustable resistors R2 and R3. Signals  $U_1 = f(K_{sz})$  and  $U_2 = f(T_{sz} \cdot s)$  are supplied to the inputs of the comparing element 2, where they are algebraically summed up with  $U_z$  and  $U_{sz}$ . The error signal  $U_e = U_z + U_1 + U_2 - U_{sz}$ , via the correction element 3, is supplied to the input of amplifier 4, and then, through the power transducer, to the longitudinal feed motor 5. Change in the rotation speed of motor 6 causes a change in the value of the longitudinal feed that is a control effect for the technological system 7, and thus a correction is made to the relative positioning of the machined part and the cutting edge, taking into account the change in elastic deformation  $y$  and the machining force  $F_p$ .

### 3. Control of parameters of elastic-deformable state of low-rigidity shafts in turning

The above considerations concerning the creation of a system of adaptive control of accuracy parameters in machining with control effects in the form of longitudinal feed can also be generalised onto the dynamic systems of profiling of elastic-deformable low-rigidity shafts.

References [8, 9] present a mathematical description of systems in turning and grinding of elastic-deformable low-rigidity shafts, the dynamic properties of linearized models being approximated by means of transfer functions of typical dynamic elements. The resultant models of DS and of the parameters of the object of control permit substantiated realization of the search for optimum algorithms of control, selection of control system structure, and synthesis of corrective devices.

Fig. 2 presents a generalised structural schematic of the object of control for the case of controlling the elastic deformable state of low-rigidity parts through the application of tensile force  $F_{x1}$ . The transfer function of the dynamic system, taking into account the assumptions and results of theoretical and experimental studies, can be presented as:

$$G_4(s) = \frac{1 + K_k K_y m_y (1 - e^{-st})}{1 + K_{bz} K_z n_z + K_{xy} n_x + K_{yy} n_y + (1 - e^{-st})^x} \quad (11)$$

$$\frac{1}{[K_k k_y m_y (1 + K_{xy} n_x + K_{bz} K_z n_z + K_{yy} n_y) - (K_k - K_x n_x)(K_{bz} K_z n_z + K_{yy} m_y)]}$$

Fig. 3 presents a structural schematic of a system of adaptive control of elastic deformations of an elastic-deformable part in DS. with an additional feedback  $G_{sz}(s)$  relative to the machining force  $F_p$ , that realizes the condition (7) of invariability in order to eliminate the static error related to the control effect  $y_0(s)$  with positive feedback and to increase stability, speed of operation and insensitivity to a change in material allowance with negative feedback. In Fig. 3 the expression for  $G_5(s)$  can be presented as:

$$G_5(s) = \frac{(1 - e^{-st}) [m_y (K_x n_x - K_k) - K_k K_y m_y n_y] - n_y}{1 + K_k K_y m_y (1 - e^{-st})} \quad (12)$$

Taking into account relations (11) and (12), the transfer function of the corrected control system can be written as:

$$\Phi_{sk}(s) = \frac{1 + K_{bz} K_z n_z + K_{xy} n_x + K_{yy} n_y + n_y K_{F_{x1}} G_w(s) + (1 - e^{-st}) [K_k K_y m_y \times (1 + K_{xy} n_x + K_{bz} K_z n_z + K_{yy} n_y + K_{F_{x1}} G_w(s)) + (1 - e^{-st})^x \times (K_k K_y m_y (1 + K_{xy} n_x + K_{bz} K_z n_z + K_{yy} n_y + K_{F_{x1}} G_w(s)) G_{sz}(s))] + (K_x n_x - K_k) (K_{bz} K_z n_z + m_y K_{yy} - m_y K_{F_{x1}} G_w(s) G_{sz}(s))}{1 + K_{bz} K_z n_z + K_{xy} n_x + K_{yy} n_y + K_{F_N}(s) G_w(s) (1 + n_y G_{sz}(s))} \quad (13)$$

and the static error of the system relative to the control effect is determined from the relation:

$$e_y(s) = y_0(s) \frac{1 + K_{bz} K_z n_z + K_{xy} n_x + K_{yy} n_y - n_y K_{F_{x1}} G_w(s) G_{sz}(s)}{1 + K_{bz} K_z n_z + K_{xy} n_x + K_{yy} n_y + K_{F_N}(s) G_w(s) (1 + n_y G_{sz}(s))} \quad (14)$$

As follows from relation (14), the static error  $e_y(s)$ , introduced into the system of control of parameters of the elastic-deformable state of parts in DS. by the controlling effects, can be eliminated if the structure and the parameters of transfer function of positive feedback  $G_{sz}(s)$  is selected as follows:

$$G_{sz}(s) = \frac{1 + K_{bz} K_z n_z + K_{xy} n_x + K_{yy} n_y}{n_y K_{F_{x1}} G_w(s)} \quad (15)$$

If we leave out the effect of component  $F_f$  of machining force on the increase of elastic deformations along coordinate  $y$ , the structural schematic of the object of control – dynamic system of elastic-deformable low-rigidity shafts can be transformed to the form presented in Fig. 4a, and the transfer function of the object is then defined by the relation:

$$G_6(s) = \frac{1 + m_y K_y K_k (1 - e^{-st})}{1 + K_{bz} K_z n_z + K_{yy} n_y + (1 - e^{-st})^x} \quad (16)$$

$$\frac{1}{[m_y K_y K_k (1 + K_{bz} K_z n_z + K_{yy} n_y) + (K_{bz} K_z n_z + K_{yy} m_y)(K_k - n_x K_x)]}$$

The structural schematic of AC with introduced feedback relative to the force of machining is presented in Fig. 4b, where the transfer function  $G_5(s)$  is defined by relation (12).

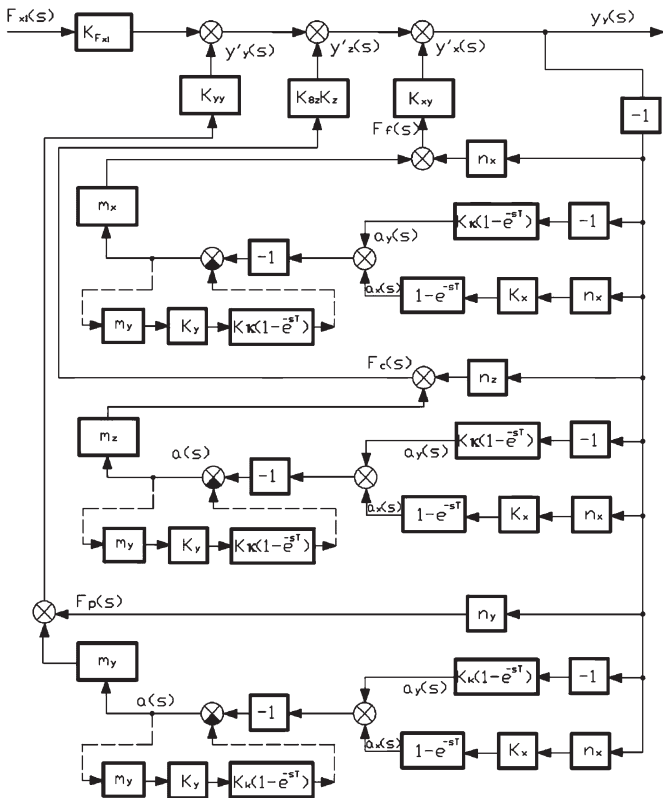


Fig. 2. Structural schematic of dynamic system in turning of elastic-deformable low-rigidity shafts

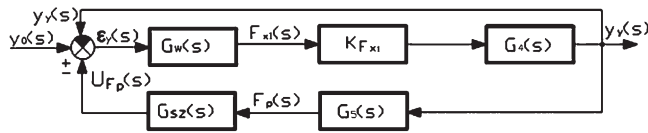


Fig. 3. Structural schematic of adaptive control system with additional positive feedback relative to the machining force

The static error of the control system, introduced by a controlling effect, is defined by the relation:

$$e_{y1}(s) = y_0(s) \frac{1 + K_{bz}K_zn_z + K_{yy}n_y - n_yK_{F_x1}G_w(s)G_{sz1}(s)}{1 + K_{bz}K_zn_z + K_{yy}n_y + K_{F_x1}G_w(s)[1 + n_yG_{sz1}(s)]} \quad (17)$$

To eliminate the static error  $e_{y1}(s)$  introduced into the system by the controlling effects, the structure and parameters of transfer function of positive feedback  $G_{sz1}(s)$  should be defined as follows:

$$G_{sz1}(s) = \frac{1 + K_{bz}K_zn_z + K_{yy}n_y}{n_yK_{F_x1}G_w(s)} \quad (18)$$

In the case when we leave out the effect of elastic deformations along coordinates z and x on the change in machining depth (along coordinate y), the structural schematic for a specific model of the technological system of turning of elastic-deformable shaft can be presented as in Fig. 5a, where the transfer function  $G_5(s)$  is defined by the relation (12). Fig. 5b presents the structural schematic of cor-

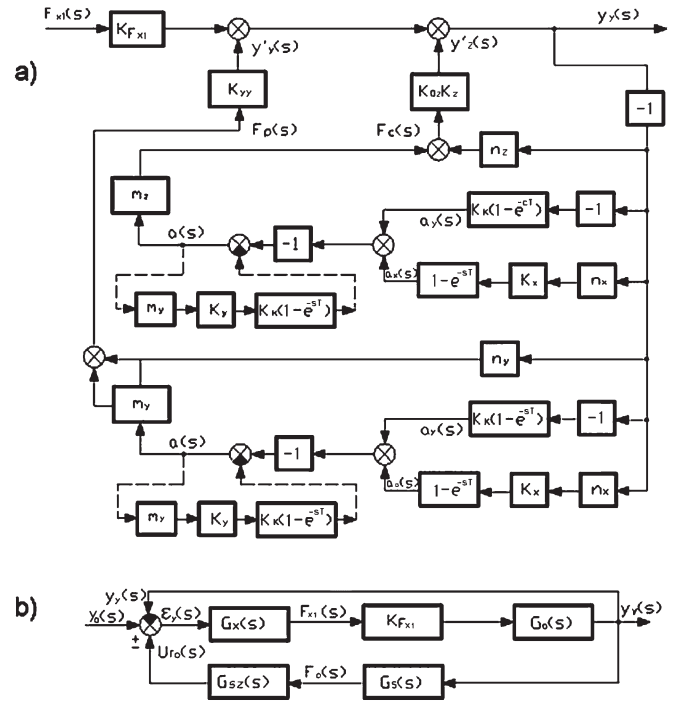


Fig. 4. Structural schematic of control object without the inclusion of the effect of component  $F_p$  of machining force on increase of elastic deformation along coordinate y – a; structural schematic of adaptive control system with feedback relative to machining force  $F_p$  – b

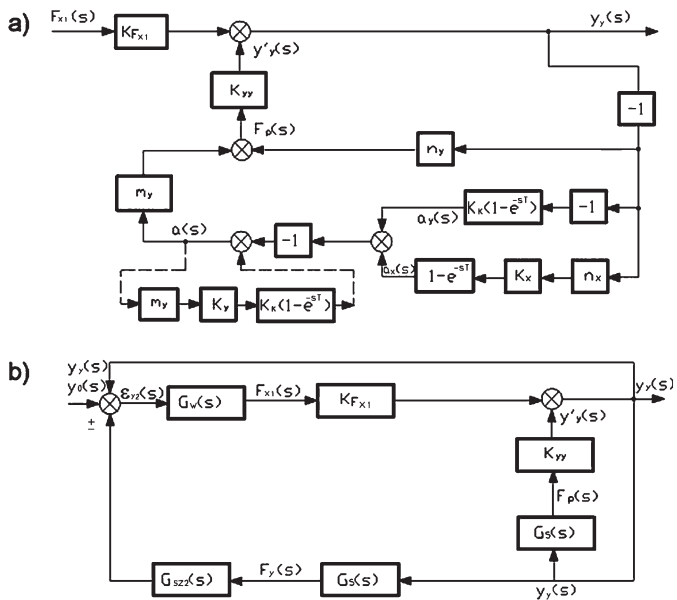


Fig. 5. Structural schematic of specific model of technological system of turning of elastic-deformable low-rigidity shafts – a; structural schematic of adaptive control system – b

rected system of adaptive control of the parameters of the elastic-deformable state of a low-rigidity shaft, that permits the elimination of the static error with the introduction of an additional positive feedback. The transfer function for the corrected system for the specific model is defined by the relation:

$$\Phi_{sk}(s) = \frac{1 + K_{yy}n_y + n_y K_{F_{x1}} G_w(s) G_{sz2}(s) + m_x (1 - e^{-st})}{1 + K_{yy}n_y + K_{F_{x1}} G_w(s) [1 + n_y G_{sz2}(s)] + m_y (1 - e^{-st})} \times \frac{[K_k K_y + (K_{yy} + K_{F_{x1}} G_w(s) G_{sz2}(s)) (n_y K_{yy} K_k - K_x n_x + K_k)]}{[K_k K_y (1 + K_{F_{x1}} G_w(s)) + (K_{yy} + K_{F_{x1}} G_w(s) G_{sz2}(s)) (K_k K_{yy} n_y - K_x n_x + K_k)]} \quad (19)$$

The error of the control system introduced by a controlling effect is defined from the relation:

$$e_{y2}(s) = y_0(s) \frac{1 + K_{yy}n_y - n_y K_{F_{x1}} G_w(s) G_{sz2}(s)}{1 + K_{yy}n_y + K_{F_{x1}} G_w(s) [1 + n_y G_{sz2}(s)]} \quad (20)$$

In the case of determining the structure and parameters of feedback relative to the force of machining in accordance with the relation:

$$G_{sz2}(s) = \frac{1 + K_{yy}n_y}{n_y K_{F_{x1}} G_w(s)}, \quad (21)$$

the static error  $e_{y2}(s)$  relative to the controlling effect assumes the value of zero.

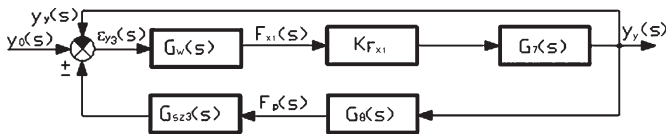


Fig. 6. Structural schematic of system of adaptive control of parameters of elastic-deformable state in longitudinal grinding

#### 4. Control of parameters of elastic-deformable state of low-rigidity shafts in grinding

The generalised model of dynamic system of turning of elastic-deformable parts, taking into account the specific character of profiling the shear section in grinding of elastic-deformable low-rigidity shafts with longitudinal feed, was used to derive the structure of the object of control and the mathematical description of the system [9].

The transfer function of DS in longitudinal grinding is defined by the relation:

$$G_7(s) = \frac{1 + m_x K_x (1 - e^{-st})}{1 + K_{xy}n_x + K_{yy}n_y + (1 - e^{-st}) [m_x K_x - K_{yy} K_x (m_y n_x - m_x n_y)]} \quad (22)$$

#### References

1. Halas W, Taranenko V, Swic A, Taranenko G. Investigation of influence of grinding regimes on surface tension state. Berlin, Heidelberg: Springer – Verlag. Lecture Notes In Artificial Intelligence, 2008; 5027: 749–756.
2. Marchelek K.: Dynamika obrabiarek, WNT: Warszawa, 1991.
3. Kujan K. Badania i analiza powtarzalności rozkładu odchyłek geometrycznych w procesie obróbki skrawaniem. Eksploatacja i Niezawodność – Maintenance and Reliability 2008; 3(39): 45–52.
4. Ratchev S, Liu S, Huang W, Becker A A. A flexible force model for end milling of low-rigidity parts. Journal of Materials Processing Technology. Proceedings of the International Conference in Advances in Materials and Processing Technologies. 2004; 153-154: 134-138.
5. Ratchev S, Liu S, Huang W, Becker A A. Milling error prediction and compensation in machining of low-rigidity parts. International Journal of Machine Tools and Manufacture 2004; 15(44): 1629-1641.

Fig. 6 presents the structural schematic of a system of adaptive control of the parameters of elastic-deformable state of low rigidity shaft in TS in longitudinal grinding, where:

$$G_8(s) = \frac{n_y + (1 - e^{-st}) (m_y n_x K_x - m_x n_y K_x)}{1 + m_x K_x (1 - e^{-st})} \quad (23)$$

The transfer function of the corrected control system, taking into account relations (22) and (23), assumes the form of:

$$\Phi_{sk}(s) = \frac{1 + K_{xy}n_x + K_{yy}n_y + n_y K_{F_{x1}} G_w(s) G_{sz3}(s) + (1 - e^{-st})}{1 + K_{xy}n_x + K_{yy}n_y + K_{F_{x1}} G_w(s) [1 + n_y G_{sz3}(s)] + (1 - e^{-st})} \times \frac{[m_x K_x + (K_{yy} - G_w(s) G_{sz3}(s)) K_{F_{x1}}] (n_y - K_x n_x m_y)}{[m_x K_x (1 + G_w(s) K_{F_{x1}}) + (K_{yy} - G_w(s) G_{sz3}(s)) K_{F_{x1}}] (n_y - K_x n_x m_y)} \quad (24)$$

If the structure and parameters of the transfer function of positive feedback  $G_{sz3}(s)$  are determined from the relation:

$$G_{sz3}(s) = \frac{1 + K_{xy}n_x + K_{yy}n_y}{n_y K_{F_{x1}} G_w(s)}, \quad (25)$$

then, as follows from relation (20), the static error introduced in the adaptive control system in longitudinal grinding by the controlling

effect,  $y_0(s)c$ , can be eliminated.

#### 5. Conclusion

Analysis of functioning of systems of automatic control of elastic deformations in the technological system shows that under dynamic conditions they are static relative to the effect of both controlling and interference effects. This leads to errors of the relative positioning of the machined part and the cutting edge. Optimum structures of control systems, including the developed systems of adaptive control, permit improvement of the quality (accuracy) of machining.

The authors have developed a method for the correction of the setting of the technological system through the application of adaptive control of longitudinal feed and additional force effects, inducing the elastic-deformable state through the introduction into the control system of an additional positive feedback relative to the machining force, permitting the elimination of static errors of controlling and interference effects in the control of parameters of accuracy and quality.

The conditions were developed for the determination of the structure of parameters of the additional positive feedback relative to the machining force, that guarantee the elimination of static errors and impart to the structure adaptive properties.

6. Ratchev S, Govender E, Nikov S. Towards deflection prediction and compensation in machining of low-rigidity parts. Proceedings of the Institution of Mechanical Engineers - Part B 2002; 1(216): 129-134.
7. Świć A, Taranenko W, Szabelski J. Modelling dynamic systems of low-rigid shaft grinding. Eksploatacja i Niezawodność – Maintenance and Reliability 2011, 2 (50): 13 – 24.
8. Taranenko W, Świć A. Urządzenia sterujące dokładnością obróbki maszyn o małej sztywności. Lublin: Politechnika Lubelska, 2006.
9. Taranenko W, Taranenko G, Szabelski J, Świć A. Identyfikacja układu dynamicznego szlifowania wałów o małej sztywności. Modelowanie Inżynierskie 2008, 4(35): 115 – 130.
10. Taranenko G, Taranenko W, Świć A, Szabelski J. Modelling of dynamic system of low-rigidity shaft machining. Eksploatacja i Niezawodność – Maintenance and Reliability 2010, 4 (48): 4–15.

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