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POSSIBILISTIC RELIABILITY ANALYSIS OF REPAIRABLE SYSTEM WITH OMITTED OR DELAYED FAILURE EFFECTS

POSYBILISTYCZNA ANALIZA NIEZAWODNOŚCIOWA SYSTEMU NAPRAWIALNEGO Z POMINIĘTYM LUB OPÓŹNIONYM EFEKTEM USZKODZENIA

Within the practical problems in industrial engineering, the failure effect sometimes can be omitted or delayed if it has less effect on the system. In detail, the prominent features of the system can be described as follows: 1) if a repair time is sufficiently short (less than some threshold value) that does not affect the system operation, i.e. the pessimistic effect of system failure could be ignored. The system can be considered as operating during this repair time. It is called the system with repair time omission (failure effect omitted). 2) if a repair time is longer than the given threshold value and the failure effect is finally suffered. Then the system can be considered to remain operating from the initial stage of the repair till the end of the repair threshold. It is called the system with delayed failure effect. Based on the above two characteristics, model for the related repairable system is introduced in this paper. Two scenarios are discussed where the threshold value is regarded as a constant and non-negative random variable, respectively. Reliability indices such as instantaneous possibilistic availability are formulated for the system with failure effect omitted or delayed.

Keywords: *failure effect omitted or delayed, Markov model, repair time omission, instantaneous possibilistic availability.*

Przy rozwiązywaniu problemów praktycznych w inżynierii przemysłowej można czasami pominąć bądź opóźnić efekt uszkodzenia jeśli ma on niewielki wpływ na system. Ściślej, wiodące cechy systemu można opisać w następujący sposób: 1) jeżeli czas naprawy jest wystarczająco krótki (krótszy niż pewna wartość progowa), tak iż nie ma on wpływu na działanie systemu, to można pominąć negatywny efekt uszkodzenia systemu. Przy takim czasie naprawy można uznać że system nie przerwał działania. Nazywa się go wtedy systemem z pominięciem czasu naprawy (pominięty efekt uszkodzenia). 2) Jeżeli czas naprawy jest dłuższy niż dana wartość progowa i efekt uszkodzenia staje się w końcu odczuwalny, to uznajemy, że system pozostawał aktywny od początkowego etapu naprawy aż do momentu, w którym został przekroczony próg czasu naprawy. Nazywa się go wtedy systemem z opóźnionym efektem uszkodzenia. W oparciu o powyższe dwie charakterystyki, wprowadzono w prezentowanej pracy model systemu naprawialnego. Omówiono dwa scenariusze, w których, odpowiednio, przyjęto, że wartość progowa jest wartością stałą lub nieujemną zmienną losową. Sformułowano wskaźniki niezawodnościowe, takie jak posybilistyczna gotowość chwilowa, dla systemu z pominiętym lub opóźnionym efektem uszkodzenia.

Słowa kluczowe*: pominięty lub opóźniony efekt uszkodzenia, model Markowa, pominięcie czasu naprawy, posybilistyczna gotowość chwilowa.*

Notation

t	time scale
T_S	system lifetime
T_R	system repair time
T_{S_i}	system lifetime within i th period
T_{R_i}	system repair time within i th period
E	state space of the original system
W	working state space of the original system
F	failure state space of the original system
τ	threshold value
$\pi_{T_S}(t_S)$	possibility distribution for variable T_S
$\pi_{T_R}(t_R)$	possibility distribution for variable T_R
Π	possibility measure corresponding to possibility distribution π
$\pi_{(T_S,T_R)}(t_S,t_R)$	joint possibility distribution for variables T_S and T_R
$H(\tau)$	probabilistic distribution for non-negative random variable τ
$A_\Pi(t)$	possibilistic availability of the original system at time t
$\tilde{A}_\Pi(t)$	possibilistic availability of the new system at time t
Z_i	system lifetime plus repair time within ith period $\left(T_{S\_i}+T_{R\_i}\right)$

1. Introduction

Repairable system is defined as a system which, after failing to perform at least one of its required functions, can be returned to performing all of its required functions satisfactorily by any method other than replacement of the entire system [1]. Reliability analysis of repairable system is a momentous branch of system reliability theory, maintaining a high level of reliability is often an essential requisite for repairable systems [11]. Much research has been devoted to analyze system reliability of repairable system and various models have been proposed on repairable systems [12, 17-20].

Traditionally, the models for repairable system in the literature are involved with a basic assumption [12, 17-19]. The system instantaneously falls into the failure state when it is out of work. However, in some practical situations, for instance, if the demands on the system by 'customers' are not too frequent, they more likely miss a small repair time or are at most delayed by a negligibly short time in receiving service. Within such repairable system, the effect of system failure can be neglected if the repair time is sufficiently short (less than some given critical value) or delayed if the repair time exceeds the critical threshold. In other words, if the system is temporarily under repair and has no effect on system operating, the system can be regarded as being operating during such a repair interval (failure effect omitted). If the system is out of work and fails to back operating before the threshold repair time, it can be regarded remain operating during the period of repair until the repair time exceeds the critical value (failure effect delayed).

Recently, several related repairable systems have been studied. Zheng [20] first proposed this omitted failure model with single-unit repairable system in which repair time that is sufficiently short does not result in a system failure. Based on the introduction of the new model for repairable system, researchers [9, 10, 15, 21] analyzed system availability of series-system, parallel system, K-out-of-N: G system and so on. Furthermore, researchers [2, 22] established and analyzed the model for single-unit, and series repairable system in which failure effects could be neglected or delayed.

Among the research for repairable system in which failure effects could be neglected or delayed, probability theory is the most commonly used theory to analyze system reliability indices. In fact, possibility theory has been used increasingly to model epistemic uncertainty in reliability engineering [3, 7, 8, 13, 16]. This type of uncertainty describes subjectivity or lack of information. It is defined as reducible uncertainty and subjective uncertainty, since it can be reduced with increased state-of-knowledge or collection of more data. It exists extensively within the research of reliability, such as poor understanding of initiating events, fault trees, and event trees [14]. The aim of this paper is to theoretically analyze the system possibilistic availability for repairable systems with omitted or delayed failure effects.

In the next section, basic assumptions of the original model and the new model for single-unit repairable system are introduced, and the new model is distinguished from the original model. A comprehensive possibilistic reliability analysis of the new model is presented in Section 3. A numerical example is given to illustrate the results in the subsequent section. Finally, conclusion is given in Section 5.

2. Basic assumptions of the original model and new model for repairable system

In this section, original model and new model for single-unit repairable system are introduced respectively, as well as the difference between them. Firstly, the assumptions of the original model are addressed before presenting the new model [4, 5]:

1) The system is composed of one component and one repair facility. The system is new at the initial time (t=0), and when the component fails, the repair begins immediately.
2) The system has two possible states: up (operating), and down (failed). The repaired component is restored into "as good as new" condition.
3) Assume that the system lifetime T_S follows deflection minor type possibility distribution $\pi_{T_S}(u)$, and the repair time T_R follows deflection minor type possibility distribution $\pi_{T_R}(u)$. All the variables involved are mutually independent.

Suppose that state 0 represents the failed (down) state and state 1 be the operating (up) state, the system state space is denoted as $E=\{0,1\}$. The working state space is $W=\{1\}$ while the failure state space is $F=\{0\}$. Let $X(t)$ denote the stochastic process of the system state at time instant t, thus one has:

$$X(t)=\begin{cases}1, & \text{the system is in the operating state at time } t;\\ 0, & \text{the system is in the failed state at time } t.\end{cases}$$

In fact, $\{X(t), t\geq 0\}$ forms a homogeneous continuous time process in state space E. For such single-unit system, the state of system is just the same with the state of unit. Based on the assumptions of original model, new model that presents a different way of system operation is addressed. The main difference between the original model (single-unit repairable system) and the new model (single-unit repairable system with failure effect omitted or delayed) is that, the new system may still be in the operating state while it turns out to be failed within the original system at the meantime. In details, given a threshold value τ $(\tau \geq 0)$, if the involved repair time is shorter than τ, the new system can be thought of being in the operating state during the repair interval. If the repair time exceeds the threshold value τ, the new system is regarded to be still under the state of operating during the repair interval of $(0,\tau)$ and being in the failed state from the repair time τ till the end of the repair. In other words, for the case that repair finishes before reach the threshold τ, the new system is considered to be under operating state during the repair interval while the original system being in failure state during the same period. Thus, the failure effect has been omitted. For the other case that repair can't finish before the given threshold value τ, the new system is regarded as under the state of operating during the process of repair until the given threshold value τ is exceeded. During the same period, the original system is under the failure state. Thus, the failure effect has been delayed for τ.

As for the same single-unit repairable system that is composed of one component and one repair facility, the new model is described as follows in contrast with the original model:

1) The system is new at the initial time (t=0), and failed unit will receive repair immediately after failure. All the random variables are mutually independent.
2) The system has two possible states: up (operating), and down (failed). The repaired component is restored into "as good as new" condition.
3) The system failure time T_S and the system repair time T_R follow deflection minor type possibility distribution $\pi_{T_S}(t_S)$ and $\pi_{T_R}(t_R)$, respectively as in the original model.
4) The new system is operating if the original system is operating.
5) If the original system fails and the repair time is less than the threshold value τ, then the new system is still operating during the repair time.

6) If the original system fails and it takes longer than τ to finish the repair, then the new system is considered to remain in the operating state during the repair interval of $(0,\tau)$, and being in the failed state after this interval until the repair complete.
7) The threshold value τ can be either a constant or a non-negative random variable. τ follows the distribution of $H(\tau)$ if it is considered as a non-negative random variable.

In particular, when $\tau = 0$, the new system becomes the original system. When $\tau = \infty$, the new system is never down. A possible sequence of system state changes of the original system and the new system is shown in Fig.1.

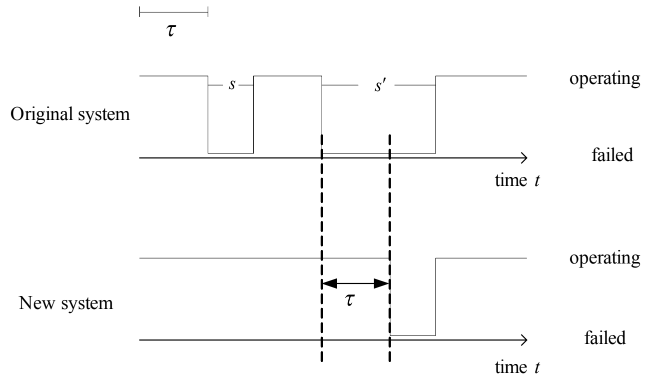

Fig.1. Difference between the original system and the new system

Let the stochastic process $\tilde{X}(t)$ denoting the state of the new system at any time instant t, and one has:

$$\tilde{X}(t) = \begin{cases} 1, & \texttt{the new system is in the operating state at time } t; \\ 0, & \texttt{the new system is in the failed state at time } t. \end{cases}$$

It can be figured out that the Markov property is not held in the new system. In fact, given the present state of the new system, its future is related to its past and is not independent. For instance, take the repair time point of $\tau_1(\tau_1 < \tau)$ as an example, the system state is failed for the original model and is operating for the new model. The future state for the new system is related to the repair time it has been processed. Thus, it is a stochastic process without Markov property or memorylessness.

3. Possibilistic availability analysis

In this section, the instantaneous possibilistic availability for the new system is discussed. Let $A_\Pi(t)$ and $\tilde{A}_\Pi(t)$ denote the instantaneous possibilistic availability of the original system and the new system, respectively.

$$\begin{aligned} A_\Pi(t) &= \Pi(\text{the original system is operating at time } t) \\ &= \Pi\{X(t)=1\} \end{aligned} \tag{1}$$

$$\begin{aligned} \tilde{A}_\Pi(t) &= \Pi(\text{the new system is operating at time } t) \\ &= \Pi\{\tilde{X}(t)=1\} \end{aligned} \tag{2}$$

In the first subsection, instantaneous possibilistic availability of the original system $A_\Pi(t)$ is mathematically derived. Relationship between $A_\Pi(t)$ and $\tilde{A}_\Pi(t)$ are stated in the second subsection, and the expression of $\tilde{A}_\Pi(t)$ as well.

3.1. Mathematical derivation for $A_\Pi(t)$

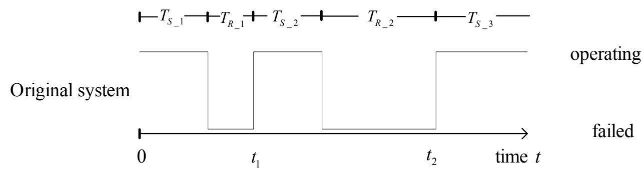

Fig.2. Process progress for the original system

As stated in Eq. (1), it is defined as the possibility that the original system is operating at time *t*. Taking account into the system process progress depicted in Fig.2, $t_1, t_2, \cdots$ are regenerative points, since the failed unit can be regarded "as good as new" after repair. Suppose that $Z_i = T_{S\_i} + T_{R\_i}$, within which T_{S_i} and T_{R_i} denote the system lifetime and repair time within i th period respectively. Thus, $\{Z_i,\ i=1,2,\cdots\}$ is a sequence of variable with independent identical distribution. Towards the event of system operating at time *t* ($X(t)=1$), a restatement of the event can be addressed as follows:

$$\begin{aligned} \{X(t)=1\} = &\{T_{S\_1} > t\} \cup \{Z_1 < t < Z_1 + T_{S\_2}\} \cup \{Z_1 + Z_2 < t < Z_1 + Z_2 + T_{S\_3}\} \cup \cdots \\ &\cup \left\{ \sum_{j=1}^{i} Z_j < t < \sum_{j=1}^{i} Z_j + T_{S\_i+1} \right\} \cup \cdots \end{aligned} \tag{3}$$

Thus, the instantaneous possibilistic availability of the original system can be represented as the possibility of formulation on the right hand side of Eq. (3). As for the right hand side of Eq. (3), it can be rewritten as:

$$\{T_{S\_1} > t\} \cup \bigcup_{i=1}^{\infty} \left\{ \sum_{j=1}^{i} Z_j < t < \sum_{j=1}^{i} Z_j + T_{S\_i+1} \right\}$$

Additionally, based on the memoryless property of each point for $\{Z_i,\ i=1,2,\cdots\}$ and Dubois and Prade's idea [6] in defining the conditional possibility of events ($\Pi(A \cap B) = \Pi(A|B) * \Pi(B)$), it can be inferred that, for $* = \min$,

$$\begin{aligned} \Pi(Z_1 < t < Z_1 + T_{S\_2}) &= \Pi(T_{S\_2} > t - Z_1, 0 < Z_1 < t) \\ &= \min\{\Pi(T_{S\_2} > t - Z_1 | 0 < Z_1 < t), \Pi(0 < Z_1 < t)\} \\ &= \min\left\{ \sup_{u \in (0,t)} \Pi(T_{S\_2} > t - Z_1 | Z_1 = u), \sup_{u \in (0,t)} \Pi(Z_1 = u) \right\} \\ &= \min\left\{ \sup_{u \in (0,t)} \Pi(T_{S\_2} > t - u), \sup_{u \in (0,t)} \Pi(Z_1 = u) \right\} \end{aligned} \tag{4}$$

Similarly,

$$\Pi\left(Z_1+Z_2<t<Z_1+Z_2+T_{S\_3}\right)=\Pi\left(T_{S\_3}>t-Z_1-Z_2,0<Z_1+Z_2<t\right) \tag{5}$$
$$=\min\left\{\Pi\left(T_{S\_3}>t-Z_1-Z_2 \middle| 0<Z_1+Z_2<t\right),\Pi\left(0<Z_1+Z_2<t\right)\right\}$$
$$=\min\left\{\sup_{u\in(Z_1,t)}\Pi\left(T_{S\_3}>t-u\right),\sup_{u\in(Z_1,t)}\Pi\left(Z_1+Z_2=u\right)\right\}$$

$$\Pi\left(\sum_{j=1}^{i}Z_j<t<\sum_{j=1}^{i}Z_j+T_{S\_i+1}\right)=\Pi\left(T_{S\_i+1}>t-\sum_{j=1}^{i}Z_j,\sum_{j=1}^{i-1}Z_j<\sum_{j=1}^{i}Z_j<t\right) \tag{6}$$
$$=\min\left\{\sup_{u\in\left(\sum_{j=1}^{i-1}Z_j,t\right)}\Pi\left(T_{S\_i+1}>t-u\right),\sup_{u\in\left(\sum_{j=1}^{i-1}Z_j,t\right)}\Pi\left(\sum_{j=1}^{i}Z_j=u\right)\right\}$$

It can be deduced from possibility theory and the last formulation of Eqs. (4-6) that the sequence

$$\Pi\left(Z_1<t<Z_1+T_{S\_2}\right),\Pi\left(Z_1+Z_2<t<Z_1+Z_2+T_{S\_3}\right),\cdots,\Pi\left(\sum_{j=1}^{i}Z_j<t<\sum_{j=1}^{i}Z_j+T_{S\_i+1}\right),$$

is decreasing. For instance, as for the items involved within the last formulation of Eq. (5), boundary for variable u is narrower than that in Eq. (4).

Thus, we have,

$$A_\Pi(t)=\Pi\left\{X(t)=1\right\} \tag{7}$$
$$=\Pi\left\{\left\{T_{S\_1}>t\right\}\cup\bigcup_{i=1}^{\infty}\left(\sum_{j=1}^{i}Z_j<t<\sum_{j=1}^{i}Z_j+T_{S\_i+1}\right)\right\}$$
$$=\Pi\left(T_{S\_1}>t\right)\vee\Pi\left\{\bigcup_{i=1}^{\infty}\left(\sum_{j=1}^{i}Z_j<t<\sum_{j=1}^{i}Z_j+T_{S\_i+1}\right)\right\}$$
$$=\Pi\left(T_{S\_1}>t\right)\vee\Pi\left(Z_1<t<Z_1+T_{S\_2}\right)$$
$$=\Pi\left(T_{S\_1}>t\right)\vee\min\left\{\sup_{u\in(0,t)}\left[\Pi\left(T_{S\_2}>t-u\right),\Pi\left(Z_1=u\right)\right]\right\}$$

Therefore, given the possibility distribution of the variables, instantaneous possibilistic availability for the original system can be derived.

Likewise, the event of system under failure state at time t can be represented by specifying that $Z_0=0$.

$$\left\{X(t)=0\right\}=\left\{T_{S\_1}\le t\le Z_1\right\}\cup\left\{Z_1+T_{S\_2}\le t\le Z_1+Z_2\right\}\cup\cdots\cup\left\{\sum_{j=1}^{i}Z_j+T_{S\_i+1}\le t\le\sum_{j=1}^{i+1}Z_j\right\}\cup\cdots$$
$$=\bigcup_{i=0}^{\infty}\left[\sum_{j=0}^{i}\left(Z_j+T_{S\_j+1}\right)\le t\le\sum_{j=0}^{i}Z_{j+1}\right] \tag{8}$$

Respectively, possibility of the first three items within the first formulation of Eq. (8) is restated as follows:

$$\Pi\left(T_{S\_1}\le t\le Z_1\right)=\Pi\left(T_{S\_1}\le t,Z_1\ge t\right) \tag{9}$$

$$\Pi\left(Z_1+T_{S\_2}\le t\le Z_1+Z_2\right)=\Pi\left(T_{S\_2}\le t-Z_1,Z_2\ge t-Z_1\right) \tag{10}$$

$$\prod\left(\sum_{j=1}^{i}Z_j+T_{S\_i+1}\le t\le\sum_{j=1}^{i+1}Z_j\right)=\prod\left(T_{S\_i+1}\le t-\sum_{j=1}^{i}Z_j,Z_{i+1}\ge t-\sum_{j=1}^{i}Z_j\right) \tag{11}$$

Since the sequence of variables $\left\{T_{S\_i},\ i=1,2,\cdots\right\}$ and $\left\{Z_i,\ i=1,2,\cdots\right\}$ are with independent identical distribution respectively, we can deduce the decreasing trend for the sequence of $\Pi\left(T_{S\_1}\le t\le Z_1\right)$, $\Pi\left(Z_1+T_{S\_2}\le t\le Z_1+Z_2\right),\cdots$ from Eqs. (9-11). Thus, similarly to that inferred in Eq. (7), we have:

$$\Pi\left\{X(t)=0\right\}$$
$$=\Pi\left\{\bigcup_{i=0}^{\infty}\left(\sum_{j=0}^{i}\left(Z_j+T_{S\_j+1}\right)\le t\le\sum_{j=0}^{i}Z_{j+1}\right)\right\}$$
$$=\Pi\left\{\bigcup_{i=0}^{\infty}\left(\sum_{j=1}^{i}Z_j\le t\le\sum_{j=1}^{i}Z_j+T_{S\_i+1}\right)\right\} \tag{12}$$
$$=\Pi\left(T_{S\_1}\le t\le Z_1\right)$$
$$=\Pi\left(T_{S\_1}\le t,Z_1\ge t\right)$$

Actually, there is a property for items $\Pi\left\{X(t)=0\right\}$ and $\Pi\left\{X(t)=1\right\}$. Since the union of event $\left\{X(t)=0\right\}$ and event $\left\{X(t)=1\right\}$ comes to the universe, it can be inferred from the possibility theory that $\Pi\left\{X(t)=0\right\}\vee\Pi\left\{X(t)=1\right\}=1$. Furthermore given a critical time t, either $\Pi\left\{X(t)=0\right\}$ or $\Pi\left\{X(t)=1\right\}$ would be the value of 1. Simply speaking, if the system possibilistic availability at time t is not with the value of 1, then the possibility of system being failed at time t would be 1. On the contrary, if the system is not with full possibility to be failed at time t, the system possibilistic availability is 1 at time t.

3.2. Mathematical derivation for $\tilde{A}_\Pi(t)$

Based on the assumption that the new system is operating if the original system is operating, instantaneous possibilistic availability for the new system can be expressed as

$$\tilde{A}_\Pi(t)=\Pi\left(\text{the new system is operating at time } t\right)$$
$$=\Pi\left\{\tilde{X}(t)=1\right\}$$
$$=\Pi\left\{\left[\tilde{X}(t)=1,X(t)=0\right]\vee\left[\tilde{X}(t)=1,X(t)=1\right]\right\}$$
$$=\Pi\left\{\left[\tilde{X}(t)=1,X(t)=0\right]\vee\left[X(t)=1\right]\right\} \tag{13}$$
$$=\Pi\left(\tilde{X}(t)=1,X(t)=0\right)\vee\Pi\left(X(t)=1\right)$$
$$=\Pi\left(\tilde{X}(t)=1,X(t)=0\right)\vee A_\Pi(t)$$

Hence, in order to figure out the instantaneous possibilistic availability for the new system, we only need to analyze $\Pi\left(\tilde{X}(t)=1,X(t)=0\right)$ which representing the possibility that the new system is operating while original system is failed at time t. On the basis of the model assumptions, two cases under the circumstance of new system operating while original system failed are obtained: 1) the original system is under repair at time t, and this repair time is no longer than τ ; and 2) the original system is under repair at time *t* and repair time is longer than τ . At the meantime, the repair have lasted no longer than τ until time *t*.

In the sequel, two scenarios in terms of threshold value are discussed: 1) τ is a constant, and 2) τ is a non-negative random variable.

A. Constant critical repair time

Assuming that the threshold τ is given as a nonnegative constant, it would be distinct to compare the magnitude of *t* and τ . Therefore, the formulation of $\Pi\left(\tilde{X}(t)=1, X(t)=0\right)$ can be represented as follows:

If $t \le \tau$, then

$$\Pi\left(\tilde{X}(t)=1, X(t)=0\right)=\Pi\left(X(t)=0\right)$$

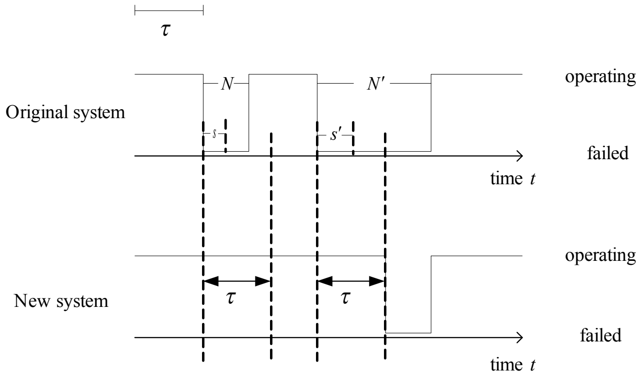

Fig.3. The new system is operating while original system is failed

If $t > \tau$, considering the two cases under the circumstance of new system operating while original system failed which are depicted in Fig.3, then

$$\begin{aligned}&\Pi\left(\tilde{X}(t)=1, X(t)=0\right)\\&=\min\left\{1, \int_0^{\tau} A_{\Pi}(t-s)\cdot w_{10}(t)\cdot\left[\Pi\left(s<T_R<\tau\right)\vee\Pi\left(s<T_R, T_R\ge\tau\right)\right]ds\right\}\\&=\min\left\{1, \int_0^{\tau} A_{\Pi}(t-s)\cdot w_{10}(t)\cdot\Pi\left(s<T_R\right)ds\right\}\end{aligned}$$

in which $w_{10}(t)$ denotes the membership function of transiting from State 1 to State 0 at time *t*. Therefore, system instantaneous possibilistic availability can be expressed as:

$$\begin{aligned}\tilde{A}_{\Pi}(t)&=\Pi\left(\tilde{X}(t)=1, X(t)=0\right)\vee A_{\Pi}(t)\\&=\begin{cases}\Pi\left(X(t)=0\right)\vee A_{\Pi}(t) & t\le\tau\\ A_{\Pi}(t)\vee\int_0^{\tau} A_{\Pi}(t-s)\cdot w_{10}(t)\cdot\Pi\left(s<N\right)ds & t>\tau\end{cases}\end{aligned} \tag{14}$$

B. Random critical repair time

Suppose that the threshold is given as a random variable following distribution function of $H(\tau)$, then

$$\begin{aligned}&\Pi\left(\tilde{X}(t)=1, X(t)=0\right)\\&=\int_0^{\infty}\int_0^{\min(t,\tau)} A(t-s)w_{10}\cdot\left[\Pi\left(s<T_R<\tau\right)\vee\Pi\left(s<T_R, T_R\ge\tau\right)\right]dsdH(\tau)\\&=\int_t^{\infty}\Pi\left(X(t)=0\right)dH(\tau)+\int_0^{t}\int_0^{\tau} A_{\Pi}(t-s)\cdot w_{10}(t)\cdot\Pi\left(s<T_R\right)dsdH(\tau)\\&=\Pi\left(X(t)=0\right)\bar{H}_{\tau}(t)+\int_0^{t}\int_0^{\tau} A_{\Pi}(t-s)\cdot w_{10}(t)\cdot\Pi\left(s<T_R\right)dsdH(\tau)\end{aligned} \tag{15}$$

The last equation holds because of $\int_t^{\infty} dH(\tau)=P(\tau\ge t)=1-H_{\tau}(t)=\bar{H}_{\tau}(t)$. Similarly, system instantaneous possibilistic reliability can be expressed making use of $\tilde{A}_{\Pi}(t)=\Pi\left(\tilde{X}(t)=1, X(t)=0\right)\vee A_{\Pi}(t)$. In which, $A_{\Pi}(t)$ can be figured out taking advantage of Eq. (7).

4. Numerical example

In this section, a numerical example is shown to compare the instantaneous possbilistic reliability between the original system and the new system. Based on the assumption that the repaired component can be restored into "as good as new", each sequence of variables $\{T_{S\_i},\ i=1,2,\cdots\}$ and $\{T_{R\_i},\ i=1,2,\cdots\}$ are with independent identical distribution respectively. Generally, we use variable T_S as a denotation for each variable in the sequence of $\{T_{S\_i},\ i=1,2,\cdots\}$, and simplify each variable within $\{T_{R\_i},\ i=1,2,\cdots\}$ as T_R , respectively. Similarly, the sequence of $\{Z_i,\ i=1,2,\cdots\}$ is simplified as Z in which $Z=T_S+T_R$.

Suppose that the system lifetime T_S follows deflection minor type possibility distribution $\pi_{T_S}(u)$ in which $t_1=100(day)$, and the repair time T_R follows deflection minor type possibility distribution $\pi_{T_R}(u)$ in which $t_2=1(day)$:

$$\pi_{T_S}(t_S)=\begin{cases}1 & t_S\le t_1\\ \exp\left\{-\frac{1}{2}\left(\frac{t_S-t_1}{t_1}\right)\right\} & t_S>t_1\end{cases} \tag{16}$$

$$\pi_{T_R}(t_R)=\begin{cases}1 & t_R\le t_2\\ \exp\left\{-\frac{1}{2}\left(\frac{t_R-t_2}{t_2}\right)\right\} & t_R>t_2\end{cases} \tag{17}$$

Now we come to the possibility distribution for Z . According to the possibility theory and the independence between T_S and T_R ($\pi_{(T_S,T_R)}(t_S,t_R)=\min\left(\pi_{T_S}(t_S),\pi_{T_R}(t_R)\right)$), we have:

$$\begin{aligned}\pi_Z(z)=\pi_{T_S+T_R}(z)&=\bigvee_{t_S+t_R=z}\pi_{(T_S,T_R)}(t_S,t_R)\\&=\bigvee_{t_S+t_R=z}\left[\min\left(\pi_{T_S}(t_S),\pi_{T_R}(t_R)\right)\right]\end{aligned} \tag{18}$$

For the expression of instantaneous possibilistic reliability for original system $A_{\Pi}(t)$, refer to details in appendix. Here in this example, τ is given the value of 2(*day*) and $w_{10}(t)$ is assumed to be a sub-function defined in Eq. (19) . As a matter of fact, according to Eq. (29) the system is with full possibility to be operating when $t \in [0, 2t_1 + t_2]$, which means it is not likely for the occurrence of system state change from operating to failed. Conversely, it is more likely for the system to become failed from operating state when $t \in (2t_1 + t_2, \infty)$. It is because the system possibilistic availability is decreasing.

$$w_{10}(t) = \begin{cases} 0.15 & t \in [0, 2t_1 + t_2] \\ 0.85 & t \in (2t_1 + t_2, \infty) \\ 1 & t = \infty \end{cases} \tag{19}$$

According to the analysis in the previous section, instantaneous possibilistic reliability for new system can be figured out taking advantage of Eq. (14).

$$\tilde{A}_\Pi(t) = \begin{cases} 1 & t \le 2(day) \\ A_\Pi(t) \vee \min\left\{1, \int_0^2 A_\Pi(t-s) \cdot w_{10}(t) \cdot \Pi(s < N) ds\right\} & t > 2(day) \end{cases}$$

$$= \begin{cases} 1 & t \le 201(d) \\ \exp\left\{-\frac{1}{2}\left(\frac{t-201}{200}\right)\right\} \vee \min\left\{1, 0.85 \int_0^2 \exp\left\{-\frac{1}{2}\left(\frac{t-201-s}{200}\right)\right\} \cdot \exp\left\{-\frac{1}{2}(s-1)\right\} ds\right\} & t > 201(d) \end{cases}$$

$$= \begin{cases} 1 & t \le 201(d) \\ \exp\left\{-\frac{1}{2}\left(\frac{t-201}{200}\right)\right\} \vee \min\left\{1, 0.85 \cdot \exp\left\{-\frac{t-401}{400}\right\} \cdot \int_0^2 \exp\left\{-\frac{199}{400} s\right\} ds\right\} & t > 201(d) \end{cases}$$

$$= \begin{cases} 1 & t \le 201(d) \\ \exp\left\{-\frac{1}{2}\left(\frac{t-201}{200}\right)\right\} \vee \min\left\{1, 1.0139 \cdot \exp\left\{-\frac{t-401}{400}\right\}\right\} & t > 201(d) \end{cases}$$

From Fig. 4, it can be figured out that the instantaneous possibilistic availability for the new system is higher than that for the original system. It should be this situation due to the emergence of neglected or omitted failure in the new system. At the meantime, it is shown from Fig.4, $\tilde{A}_\Pi(t) = 1$ holds for the range of $t \le 2t_1 + t_2$. It seems surprisingly for such a consequence. In fact, the result indicates that for the range of $t \le 2t_1 + t_2$, system instantaneous availability is capable to be 1.

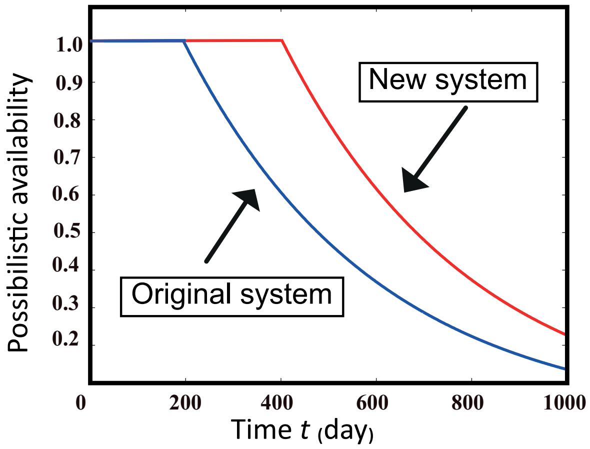

Fig. 4. The curves of the possibilitic availabilities for the new system and the original system

5. Conclusion

On the basis of some practical problems in system maintenances, a new single-unit repairable system is proposed in this paper. In such a new system, a short repair may lead to a system failure. Given a critical value, if the repair time is less than the value, the repair interval can be omitted, i.e., the failure effect is omitted. If the repair time is longer than the value, then the system is considered to remain in the operating state from the initial stage of the repair till the end of the repair threshold, i.e., the failure effect is delayed.

Considering the epistemic uncertainty which widely exists in practical engineering, system possibilistic availability is analyzed based on possibility theory. As for the difference between probability theory and possibility theory, one may be stunned by the result that the possibilistic availability is with a high value. Possibility denotes the capability for the system. Thus, it is with a higher value compared with probability.

We consider the very simple system 'single-unit system' in the paper. More complicated system will be discussed for the application in practical engineering in the future. Furthermore, various indices will be considered to offer more information for the system.

Appendix

Calculation for instantaneous possibilistic reliability of original system $A_\Pi(t)$

In order to figure out the instantaneous possibilistic reliability of the original system which is presented in Eq. (7), four phases for time *t* are distinguished as follows:

I If $t \in [0, t_1]$, it can be easily deduced from the last equation in Eq. (7) that $\Pi(T_S > t) = \Pi(T_S = t_1) = 1$, thus, $A_\Pi(t) = 1$.

II If $t \in (t_1, t_1 + t_2]$, thus it can be obtained from $u \in (0, t)$ that $u \in (0, t_1 + t_2)$. Since that $t_S < t_1$ results in $\pi_{T_S}(t_S) = 1$ and $t_R < t_2$ leads to the consequence of $\pi_{T_R}(t_R) = 1$, we have:

$$\begin{aligned} A_\Pi(t) &= \Pi(T_S > t) \vee \min\left\{\sup_{u \in (0,t)} \left[\Pi(T_S > t-u), \Pi(Z = u)\right]\right\} \\ &= \Pi(T_S > t) \vee \min\left\{\sup_{\substack{u \in (0,t) \\ t \in (t_1, t_1+t_2]}} \left[\Pi(T_S > t-u), \bigvee_{t_S + t_R = u} \left[\min\left(\pi_{T_S}(t_S), \pi_{T_R}(t_R)\right)\right]\right]\right\} \\ &= \Pi(T_S > t) \vee \min\left\{\sup_{\substack{u \in (0,t) \\ t \in (t_1, t_1+t_2]}} \left[\Pi(T_S > t-u), \bigvee_{t_S < t_1, t_R < t_2} \left[\min\left(\pi_{T_S}(t_S), \pi_{T_R}(t_R)\right)\right]\right]\right\} \\ &= \Pi(T_S > t) \vee \sup_{\substack{u \in (0,t) \\ t \in (t_1, t_1+t_2]}} \left[\Pi(T_S > t-u)\right] \\ &= \sup_{\substack{u \in (0,t) \\ t \in (t_1, t_1+t_2]}} \left[\Pi(T_S > t-u)\right] \\ &= 1 \end{aligned} \tag{20}$$

In fact, as long as there is an opportunity for *u* and *t* satisfy that $t - u \le t_1$, the possibilistic availability at time t ($t \in (t_1, t_1 + t_2]$) is 1.

III If $t\in(t_1+t_2,2t_1+t_2]$,

1) When $u\in(0,t_1+t_2)$, it can be similarly inferred as Eq. (19)

$$
\begin{aligned}
A_\Pi(t)&=\Pi(T_s>t)\vee\min\left\{\sup_{u\in(0,t_1+t_2)}\left[\Pi(T_s>t-u),\bigvee_{t_S<t_1,t_R<t_2}\left[\min\left(\pi_{T_s}(t_S),\pi_{T_R}(t_R)\right)\right]\right]\right\}\\
&=\Pi(T_s>t)\vee\sup_{u\in(0,t_1+t_2)}\left[\Pi(T_s>t-u)\right]\\
&=\Pi(T_s>t)\vee\Pi(T_s>t-t_1-t_2)\\
&=\Pi(T_s>t-t_1-t_2)\\
&=1
\end{aligned}
\tag{21}
$$

The last equal mark holds for that the value range of time t is limited by the upper bound of $2t_1+t_2$. Or else, $\Pi(T_S>t-t_1-t_2)=1$ doesn't hold.

2) When $u\in[t_1+t_2,t)$,

a) If $t_S\le t_1,t_R\ge t_2$, then $\pi_{T_S}(t_S)=1$ and $\pi_{T_R}(t_R)=1$. Apparently, it holds for $\min\left(\pi_{T_S}(t_S),\pi_{T_R}(t_R)\right)=\pi_{T_R}(t_R)$.

$$
\begin{aligned}
A_\Pi(t)&=\Pi(T_S>t)\vee\min\left\{\sup_{u\in[t_1+t_2,t)}\left[\Pi(T_S>t-u),\Pi(Z=u)\right]\right\}\\
&=\Pi(T_S>t)\vee\min\left\{\sup_{u\in[t_1+t_2,t)}\left[\Pi(T_S>t-u),\bigvee_{t_S\le t_1,t_R\ge t_2}\left[\min\left(\pi_{T_S}(t_S),\pi_{T_R}(t_R)\right)\right]\right]\right\}\\
&=\Pi(T_S>t)\vee\min\left\{\sup_{u\in[t_1+t_2,t)}\left[\Pi(T_S>t-u),\left(\pi_{T_R}(u-t_1)\right)\right]\right\}\\
&=\Pi(T_S>t)\vee\min_{u=t_1+t_2}\left[\Pi(T_S>t-u),\left(\pi_{T_R}(u-t_1)\right)\right]\\
&=\Pi(T_S>t)\vee 1\\
&=1
\end{aligned}
\tag{22}
$$

b) If $t_R\le t_2,t_S\ge t_1$, it can be similarly derived as in Eq. (21).

$$
\begin{aligned}
A_\Pi(t)&=\Pi(T_S>t)\vee\min\left\{\sup_{u\in[t_1+t_2,t)}\left[\Pi(T_S>t-u),\bigvee_{t_S\ge t_1,t_R\le t_2}\left[\min\left(\pi_{T_S}(t_S),\pi_{T_R}(t_R)\right)\right]\right]\right\}\\
&=\Pi(T_S>t)\vee\min_{u=t_1+t_2}\left[\Pi(T_S>t-u),\left(\pi_{T_S}(u-t_2)\right)\right]\\
&=\Pi(T_S>t)\vee 1\\
&=1
\end{aligned}
\tag{23}
$$

IV If $t\in(2t_1+t_2,\infty)$,

1) When $u\in(0,t_1+t_2)$,

$$
\begin{aligned}
A_\Pi(t)&=\Pi(T_S>t)\vee\min\left\{\sup_{u\in(0,t_1+t_2)}\left[\Pi(T_S>t-u),\bigvee_{t_S<t_1,t_R<t_2}\left[\min\left(\pi_{T_S}(t_S),\pi_{T_R}(t_R)\right)\right]\right]\right\}\\
&=\Pi(T_S>t)\vee\sup_{u\in(0,t_1+t_2)}\left[\Pi(T_S>t-u)\right]\\
&=\Pi(T_S>t)\vee\Pi(T_S>t-t_1-t_2)\\
&=\Pi(T_S>t-t_1-t_2)
\end{aligned}
\tag{24}
$$

2) When $u\in[t_1+t_2,t)$,

a) If $t_S\le t_1,t_R\ge t_2$,

$$
\begin{aligned}
A_\Pi(t)&=\Pi(T_S>t)\vee\min\left\{\sup_{u\in[t_1+t_2,t)}\left[\Pi(T_S>t-u),\left(\pi_{T_R}(u-t_1)\right)\right]\right\}\\
&=\Pi(T_S>t)\vee\min_{u=t-t_1}\left[\Pi(T_S>t-u),\left(\pi_{T_R}(u-t_1)\right)\right]\\
&=\Pi(T_S>t)\vee\Pi(T_R>t-2t_1)
\end{aligned}
\tag{25}
$$

b) If $t_R\le t_2,t_S\ge t_1$,

$$
\begin{aligned}
A_\Pi(t)&=\Pi(T_S>t)\vee\min\left\{\sup_{u\in[t_1+t_2,t)}\left[\Pi(T_S>t-u),\left(\pi_{T_S}(u-t_2)\right)\right]\right\}\\
&=\Pi(T_S>t)\vee\min_{u=\frac{t+t_2}{2}}\left[\Pi(T_S>t-u),\left(\pi_{T_S}(u-t_2)\right)\right]\\
&=\Pi(T_S>t)\vee\Pi\left(T_S>\frac{t-t_2}{2}\right)\\
&=\Pi\left(T_S>\frac{t-t_2}{2}\right)
\end{aligned}
\tag{26}
$$

Thus, if $t\in[0,2t_1+t_2]$, then $A_\Pi(t)=1$. If $t\in(2t_1+t_2,\infty)$, then

$$
\begin{aligned}
A_\Pi(t)&=\Pi(T_S>t-t_1-t_2)\vee\Pi(T_S>t)\vee\Pi(T_R>t-2t_1)\vee\Pi\left(T_S>\frac{t-t_2}{2}\right)\\
&=\Pi(T_R>t-2t_1)\vee\Pi\left(T_S>\frac{t-t_2}{2}\right)
\end{aligned}
\tag{27}
$$

Additionally together with Eqs. (16, 17), it is able to go a step further for the simplification of the expression ahead. Moreover, the last equation within the following formulation is deduced for that $t_1>t_2$.

$$
\begin{aligned}
A_\Pi(t)&=\Pi(T_R>t-2t_1)\vee\Pi\left(T_S>\frac{t-t_2}{2}\right)\\
&=\exp\left\{-\frac{1}{2}\left(\frac{t-2t_1-t_2}{t_2}\right)\right\}\vee\exp\left\{-\frac{1}{2}\left(\frac{t-2t_1-t_2}{2t_1}\right)\right\}\\
&=\exp\left\{-\frac{1}{2}\left(\frac{t-2t_1-t_2}{2t_1}\right)\right\}
\end{aligned}
\tag{28}
$$

Generally, instantaneous possibilistic reliability of original system can be expressed as

$$
A_\Pi(t)=\begin{cases}1 & t\in[0,2t_1+t_2]\\ \exp\left\{-\frac{1}{2}\left(\frac{t-2t_1-t_2}{2t_1}\right)\right\} & t\in(2t_1+t_2,\infty)\end{cases}
\tag{29}
$$

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