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RELIABILITY ANALYSIS OF MECHANICAL VIBRATION COMPONENT USING FUZZY SETS THEORY

ANALIZA NIEZAWODNOŚCIOWA MECHANICZNEGO ELEMENTU WIBRACYJNEGO Z WYKORZYSTANIEM TEORII ZBIORÓW ROZMYTYCH

The conventional reliability analysis of mechanical vibration component only considers the randomness of vibration but rarely for the fuzziness that may exist. It is therefore difficult to be consistent with the engineering practices. Based on the mechanical vibration theory, a novel fuzzy reliability approach by integrating the fuzzy comprehensive evaluation and fuzzy set theory is proposed in this paper. The fuzzy comprehensive evaluation is used to optimize the fuzzy factors of the reliability analysis of vibration component. With the aim of comparing the performance of the proposed approach with the conventional approach, two engineering examples are presented. The results demonstrate that the proposed approach is better than the conventional approach for its capability of covering fuzzy factors in the engineering problems.

Keywords: *reliability analysis, mechanical vibration, fuzzy reliability, fuzzy comprehensive evaluation.*

Tradycyjna analiza niezawodnościowa wibracyjnego elementu mechanicznego bierze pod uwagę jedynie losowość drgań, rzadko zaś wyjaśnia mogącą występować rozmytość. Taka analiza nie odpowiada zatem praktyce inżynierskiej. Opierając się na teorii drgań mechanicznych, w niniejszym artykule zaproponowano nowatorskie podejście w ramach teorii rozmytej niezawodności, które łączy rozmytą ocenę kompleksową oraz teorię zbiorów rozmytych. Rozmytej oceny kompleksowej użyto do optymalizacji rozmytych czynników analizy niezawodnościowej elementu wibracyjnego. W celu porównania efektywności proponowanego podejścia z efektywnością podejścia tradycyjnego przedstawiono dwa przykłady z dziedziny inżynierii. Wyniki pokazują, że proponowane podejście jest lepsze od tradycyjnego ze względu na możliwość objęcia w problemach inżynierskich czynników rozmytych .

Słowa kluczowe: *analiza niezawodnościowa, drgania mechaniczne, niezawodność rozmyta, rozmyta ocena kompleksowa.*

1. Introduction

Many component failures of engineering systems are related to vibration [12]. The conventional reliability analysis approach is purely based upon the probabilitistic reliability theory and mechanical vibration theory [3-6]. However, it is assumed that components or systems only have two states, either perfect working or completely failed in the conventional reliability theory [1]. The assumption implies that the state of components or systems can be exactly identified and furthermore there are no intermediate states between these two states. Nevertheless, it is widely observed in the engineering practices that the performance of systems may degrade during their lifetime [8, 14]. On the other hand, it is very difficult or even impossible to collect accurate and sufficient failure data in some real systems when quantifying the reliability characteristics, especially for those systems which consist of new components or components with extremely low failure rates [9, 11]. Many uncertainty factors in mechanical vibration could not be covered only with probability theory. To address the issues, a novel fuzzy reliability analysis method by integrating the fuzzy comprehensive evaluation [7] and fuzzy set theory is developed to analyze the reliability of vibration component.

The organization of this paper is as follows. In Section 2, the analysis of mechannical vibration is briefly reviewed. The fuzzy comprehensive evaluation is introduced in Section 3. In Section 4, fuzzy reliability analysis of vibration components is presented. Two engineering examples are followed to illustrate the proposed method in Section 5. Conclusions are provided in Section 6.

2. Analysis of mechanical vibration

The speed when the resonance occurs is called the critical speed. Critical speed analysis is very important and may be quite complex. Due to the randomness of the load and geometrical shape of components, the critical speed of the vibration component could usually not be expressed by a constant, but a special region with a given probability. Hence, the critical speed has discreteness, which reflects in the special performance. For example, the vibration component has no certain accurate frequency. Amplitude, frequency and phase angle of vibration is not deterministic but random at a given time.

In mechanical vibration theory, the machinery has many critical speeds with different orders in nature. When the running speed is close to the first order critical speed, the state is the most dangerous. Hence we often consider the first order critical speed because the effect of the higher orders could be ignorable. It is required that the running speed does not fall into the resonant region which is determined by the experimental data and the natural frequency of component during the process of calculating vibration of mechanical component. Let n_c and n denote the critical speed and the running speed respectively. Thus the range of running speed has the following properties [2].

If $n < n_c$, then $n < n_c(1-\delta_1)$ and $0 < \delta_1 < 0.3$

If $n > n_c$, then $n > n_c(1+\delta_2)$ and $0 < \delta_2 < 0.3$

where δ_1 and δ_2 are fuzzy factors affecting the reliability of vibration component.

Assume that the critical speed n_c is a random variable with normal distribution, and running speed n is a constant. From the above formulas, in the conventional reliability analysis of mechanical vibration component, when the random variable n_c is over $n/(1-\delta_1)$ or below $n/(1+\delta_2)$, the failure will not occur. When the value of n_c falls into the range from $n/(1+\delta_2)$ to $n/(1-\delta_1)$, the failure will occur. This relationship is illustrated in Fig. 1. In Fig. 1, the dash line represents the characteristic function.

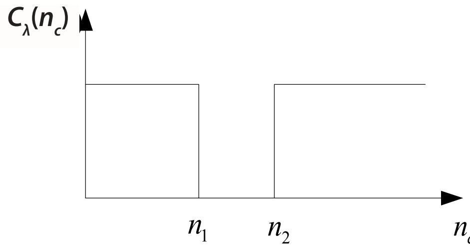

Fig. 1. Characteristic function

The reliability of mechanical vibration component is the probability that no resonance occurs. In other words, the running speed should be not close to the critical speed. The reliability can be computed by:

$$R = \int_{-\infty}^{n_1} f(x)\mathrm{d}x + \int_{n_2}^{+\infty} f(x)\mathrm{d}x \tag{1}$$

where $f(x)$ is the probability density function of the critical speed n_c .

When the critical speed follows a normal distribution with the mean value μ and the standard deviation σ , the reliability is expressed by:

$$R = 1 - [\Phi((n_2-\mu)/\sigma) - \Phi((n_1-\mu)/\sigma)] \tag{2}$$

where $[\Phi((n_2-\mu)/\sigma) - \Phi((n_1-\mu)/\sigma)]$ denotes the failure probability of vibration component.

From the above analysis, it can be concluded that the values of δ_1 and δ_2 are important to reliability analysis of mechanical vibration component because these values directly determine the failure region. Many factors would impact on the selection of the values of δ_1 and δ_2 . Hence, factors associated with the selection of the values of δ_1 and δ_2 must be considered comprehensively. In this paper, the fuzzy comprehensive evaluation is developed to determine the value of δ_1 and δ_2 by considering the fuzziness.

3. Fuzzy comprehensive evaluation

The selection of values of δ_1 and δ_2 involves some different kinds of information of systems. However, when there is no available information, it is usually assumed $\delta_1 = \delta_2$ =0.15 [10]. In other cases, especially involving much information about influence factors, fuzzy comprehensive evaluation can be used to determine the values of δ_1 and δ_2 via fuzzy transformation principle. The procedure of evaluation is summarized as follows.

3.1. Determine factor set and evaluation set

Let $U=\{u_1,u_2,...,u_n\}$ be a set consisting of m influence factors, which represent the attributes of a system. The set is called a factor set. It should be noted that the factors in the factor set often possess fuzziness.

Let $V=\{v_1,v_2,...,v_n\}$ be a set consisting of n remarks, which could be obtained by accounting for the lower and upper boundaries of V, and it is called the evaluation set. V could be determined by the boundaries of V and the number of steps. The aim of fuzzy comprehensive evaluation is to select the optimal result from the evaluation set based on the factor set. Obviously, for each v_i, there are only two options: belonging to this remark or not. Therefore, the evaluation set is a classic set.

3.2. Constructing fuzzy evaluation matrix

First, let $\tilde{R}_i$ be a single-factor fuzzy evaluation set and be expressed as $\tilde{R}_i = (r_{i1}, r_{i2}, \cdots, r_{in})$, where r_{ij} is the membership degree with respect to the remark v_j in terms of the factor u_i. Then, let $\tilde{R}$ be a fuzzy evaluation matrix and be expressed as:

$$\tilde{R} = \begin{bmatrix} \tilde{R}_1 \\ \tilde{R}_2 \\ \vdots \\ \tilde{R}_m \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}$$

where $\tilde{R}_i = (r_{i1}, r_{i2}, \cdots, r_{in})$, the i^{th} row in the matrix $\tilde{R}$, is the single-factor evaluation of the i^{th} factor μ_i , which is a fuzzy subset on V. $(U, V, \tilde{R})$ is called a comprehensive evaluation model.

3.3. Comprehensive evaluation

Let $\tilde{W} = (w_1, w_2, \cdots, w_m)$ be a weight set, which can be determined with the experience of experts or designers. The set denotes the different influence on the evaluation of every factor where w_i represents the weight of the influence factor u_i and $\sum_{i=1}^{n} w_i = 1$ ($w_i \geq 0$).

Based on the single-factor evaluation matrix, the comprehensive effect of every factor with respect to the remark v_j can be denoted by $\tilde{R}_j = \sum_{i=1}^{m} r_{ij}$. However, this approach does not consider the weight of every factor. Then, when $\tilde{W}$ and $\tilde{R}$ are known, fuzzy comprehensive evaluation set $\tilde{B}$ can be obtained by the fuzzy transformation $\tilde{B} = \tilde{W} \circ \tilde{R} = (b_1, b_2, \cdots, b_m)$. The operator '$\circ$' is defined by the equation $b_j = \sum_{i=1}^{n} w_i r_{ij}$ here. However, different models could be obtained depending on other various operation compositions. $v = \sum_{j=1}^{m} b_j v_j \Big/ \sum_{j=1}^{m} b_j$ can be used to denote an evaluation result. Then the values of δ_1 and δ_2 are equal to the evaluation result.

4. Fuzzy reliability analysis of vibration component

The resonant region has a jump in the conventional reliability analysis. However, from the point of practice, there should exist a transition region which can be represented by the fuzzy set. The fuzzy reliability considers the transition region that the conventional approach ignores. Let $[n_1', n_1]$ and $[n_2, n_2']$ denote the transition region of resonance. As shown in Fig. 2, the dash line denotes the membership function of the fuzzy failure.

From Fig. 2, the fuzzy reliability of mechanical vibration component is provided by:

$$R = P(n_c \tilde{\geq} n_2') + P(n_c \tilde{\leq} n_1') = \int_{-\infty}^{+\infty} \mu_1(x) f(x) \mathrm{d}x + \int_{-\infty}^{+\infty} \mu_2(x) f(x) \mathrm{d}x \tag{3}$$

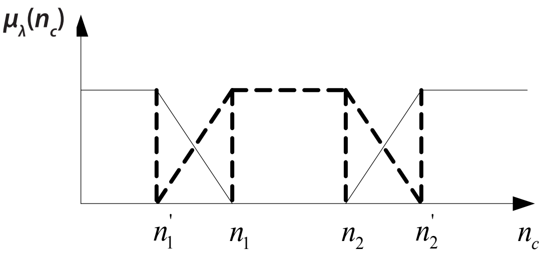

Fig. 2. Membership function

where $\mu_1(x)$ is the membership function when critical speed n_c is greater than n_2' and $\mu_2(x)$ is the membership function when critical speed n_c is less than n_1'. Let $\beta_1 = \frac{n_1' - \mu}{\sigma}$, $\beta_2 = \frac{n_1 - \mu}{\sigma}$, $\beta_1' = \frac{n_2 - \mu}{\sigma}$ and $\beta_2' = \frac{n_2' - \mu}{\sigma}$, the fuzzy reliability is rewritten as:

$$R = 1 - [\Phi(\beta_1', \beta_2') - \Phi(\beta_1, \beta_2)] \tag{4}$$

where

$$\Phi(\beta_1, \beta_2) = \frac{1}{\beta_2 - \beta_1}\{[\beta_2 \Phi(\beta_2) - \beta_1 \Phi(\beta_1)] + \frac{1}{\sqrt{2\pi}}[\exp(-\frac{\beta_2 * \beta_2}{2}) - \exp(-\frac{\beta_1 * \beta_1}{2})]\}.$$

In the fuzzy reliability analysis, other types of membership function could also be used to replace the membership function used here.

5. Case study

5.1. Fuzzy reliability analysis of a shaft

Here a shaft is taken as an example. The manufacturing level and working condition are normal, material quality is good, and importance degree is high. Its running speed is 3000rpm. Then the first order critical speed follows a normal distribution and the mean value and standard deviation are 2640rpm and 145.14rpm respectively. The proposed method will be used to compute the reliability of vibration of the shaft.

Let U ={manufacture level, material quality, work condition, importance degree} and V ={0.1,0.11,0.12,0.13,0.14,0.15}. Then, the evaluation matrix can be constructed as follows:

$$\tilde{R} = \begin{bmatrix} \tilde{R}_1 \\ \tilde{R}_2 \\ \tilde{R}_3 \\ \tilde{R}_4 \end{bmatrix} = \begin{bmatrix} 0.0 & 0.5 & 0.8 & 1.0 & 0.5 & 0.0 \\ 0.0 & 0.2 & 0.4 & 0.8 & 1.0 & 0.5 \\ 1.0 & 0.9 & 0.6 & 0.4 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.5 & 0.6 & 0.8 & 1.0 \end{bmatrix}$$

Moreover, the weight set $\tilde{W} = (0.26, 0.24, 0.30, 0.20)$ and $\tilde{B} = \tilde{W} \circ \tilde{R} = (b_1, b_2, \cdots, b_6) = (0.3,\ 0.488,\ 0.584,\ 0.692,\ 0.56,\ 0.32)$

Therefore, the values of δ_1 and δ_2 equal to 0.1257 ($v = 0.1257$ by using $v = \sum_{j=1}^{m} b_j v_j \Big/ \sum_{j=1}^{m} b_j$). From Eq. (1), the conventional reliability can be computed by:

$$\begin{aligned} R &= 1 - \{\Phi[\frac{3000/(1-0.12) - 2640}{145.14}] - \Phi[\frac{3000/(1+0.12) - 2640}{145.14}]\} \\ &= 1 - [\Phi(5.299) - \Phi(0.266)] \\ &= 0.606 \end{aligned}$$

From Eq. (3), if the values of δ_1 and δ_2 is considered, the fuzzy reliability should be given by:

$$R = 1 - (0.9998 - 0.5755) = 0.5755$$

5.2. Fuzzy reliability analysis of a suspension system

Suspension system is a very important part for cars and trains [13]. One function of suspension system is to transform force and moment while the other one is to reduce the vibration from the rude road. Therefore, fuzzy reliability analysis for the suspension system in a train with the proposed method is conducted. The frenquency from the actuator is 1.50 Hz by accounting for the road condition. The first order inherent frequency is assumed to be normally distributed with the mean value 0.98 Hz and standard deviation 0.12 Hz. The proposed method is employed to compute the reliability of a suspension system under vibration.

Let U ={design level, manufacture level, installation level, work condition} and V ={0.11,0.12,0.13,0.14,0.15,0.16}. The evaluation matrix could be provided as:

$$\tilde{R} = \begin{bmatrix} \tilde{R}_1 \\ \tilde{R}_2 \\ \tilde{R}_3 \\ \tilde{R}_4 \end{bmatrix} = \begin{bmatrix} 0.0 & 0.3 & 0.6 & 0.8 & 1.0 & 0.8 \\ 0.0 & 0.2 & 0.4 & 0.8 & 1.0 & 0.5 \\ 0.0 & 0.0 & 0.4 & 0.7 & 0.9 & 1.0 \\ 1.0 & 0.8 & 0.5 & 0.2 & 0.0 & 0.0 \end{bmatrix}$$

The weight set is given by $\tilde{W} = (0.32, 0.24, 0.22, 0.22)$ and

$$\begin{aligned} \tilde{B} &= \tilde{W} \circ \tilde{R} = (b_1, b_2, \cdots, b_6) \\ &= (0.22,\ 0.32,\ 0.486,\ 0.646,\ 0.758,\ 0.596) \end{aligned}$$

With $v = \sum_{j=1}^{m} b_j v_j \Big/ \sum_{j=1}^{m} b_j$, we can get $\delta_1 = \delta_2 = 0.1405$.

The conventional reliability could be obtained with Eq. (2):

$$\begin{aligned} R &= 1 - \{\Phi[\frac{1.5/(1-0.12) - 0.98}{0.12}] - \Phi[\frac{1.5/(1+0.12) - 0.98}{0.12}]\} \\ &= 1 - [\Phi(6.0375) - \Phi(2.994)] \\ &= 0.9986 \end{aligned}$$

If the fuzziness is considered, the fuzzy reliability of the suspension system by using $\delta_1 = \delta_2 = 0.1405$ could be represented as:

$$R = 0.9974$$

By comparing the results from the conventional method and the proposed method, a conlusion that the reliability becomes lower with consideration of fuzziness is arrived.

6. Conclusion

In this paper, a fuzzy reliability analysis approach of mechanical vibration component is developed. By comparing the proposed approach and the conventional approach, a conclusion that the proposed approach is capable of considering the fuzziness and the reliability of vibration component can be computed in a more comprehensive sense. It should be noted that the proposed approach incorporates the fuzzy comprehensive evaluation into fuzzy reliability theory. From the two engineering examples, it is shown that the proposed approach is more suitable for the complicated reliability analysis because it considers not only aleatory uncertainty but also fuzziness in an integrating framework.

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