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Zhisheng ZHANG
Yifan ZHOU
Yong SUN
Lin MA

CONDITION-BASED MAINTENANCE OPTIMISATION WITHOUT A PREDETERMINED STRATEGY STRUCTURE FOR A TWO-COMPONENT SERIES SYSTEM

OPTYMALIZACJA ZALEŻNEGO OD STANU TECHNICZNEGO UTRZYMANIA URZĄDZEŃ DLA DWUSKŁADNIKOWEGO SYSTEMU SZEREGOWEGO NIE WYMAGAJĄCA Z GÓRY USTALONEJ STRUKTURY STRATEGII

Most existing research on maintenance optimisation for multi-component systems only considers the lifetime distribution of the components. When the condition-based maintenance (CBM) strategy is adopted for multi-component systems, the strategy structure becomes complex due to the large number of component states and their combinations. Consequently, some predetermined maintenance strategy structures are often assumed before the maintenance optimisation of a multi-component system in a CBM context. Developing these predetermined strategy structure needs expert experience and the optimality of these strategies is often not proofed. This paper proposed a maintenance optimisation method that does not require any predetermined strategy structure for a two-component series system. The proposed method is developed based on the semi-Markov decision process (SMDP). A simulation study shows that the proposed method can identify the optimal maintenance strategy adaptively for different maintenance costs and parameters of degradation processes. The optimal maintenance strategy structure is also investigated in the simulation study, which provides reference for further research in maintenance optimisation of multi-component systems.

Keywords: *semi-Markov decision process, condition-based maintenance, multi-component system.*

Większość badań nad optymalizacją utrzymania systemów wieloskładnikowych bierze pod uwagę jedynie rozkład czasu życia elementów składowych. Kiedy przyjmie się dla systemów wieloskładnikowych strategię utrzymania urządzeń zależną od ich bieżącego stanu technicznego (condition-based maintenance, CBM), struktura strategii staje się złożona w związku z dużą liczbą stanów składowych oraz ich kombinacji. W konsekwencji, często przyjmuje się pewne z góry ustalone struktury strategii utrzymania przed optymalizacją utrzymania systemu wieloskładnikowego w kontekście CBM. Opracowanie takich z góry ustalonych struktur strategii wymaga jednak specjalistycznego doświadczenia, a i tak brak dowodów na optymalność tych strategii. W artykule zaproponowano metodę optymalizacji utrzymania szeregowego systemu dwuskładnikowego, która nie wymaga wcześniej ustalonej struktury strategii. Proponowaną metodę opracowano na podstawie semimarkowskiego procesu decyzyjnego (SMDP). Badanie symulacyjne pokazało, że za pomocą proponowanej metody można ustalać optymalną strategię utrzymania w sposób adaptacyjny dla różnych kosztów utrzymania oraz parametrów procesów degradacyjnych. Za pomocą symulacji badano także optymalną strukturę strategii utrzymania, jako punkt odniesienia dla przyszłych studiów nad optymalizacją systemów wieloskładnikowych.

Słowa kluczowe: *semimarkowski proces decyzyjny, condition-based maintenance, system wieloskładnikowy.*

1. Introduction

Most practical engineering assets are multi-component systems, i.e., they have more than one component. During optimising the maintenance of these multi-component systems, one needs to consider three interactions among components: economic dependence, stochastic dependence, and structural dependence. Economic dependence means that the cost of grouping maintenance can be different from the sum of individual maintenance costs. Stochastic dependence implies that degradation processes of different components influent each other. Structural dependence means that a certain group of components are connected together and should be replaced together. The three interactions make the maintenance strategy optimisation of a multi-component system much more complex than that of a mono-component system.

Various approaches have been developed to optimise the maintenance strategy of multi-component systems [9]. However, most of these approaches were based on the lifetime distribution of system components [5, 10-12]. Only few papers discussed the maintenance optimisation for multi-component systems in the context of CBM. Van Der Duyn Schouten proposed two types of maintenance strategies for multi-component systems [15]. In that paper, an essential condition of a whole system replacement was that the number of components in the doubtful state exceeded a threshold. Gürler further optimised the threshold of the doubtful state based on the research of Van Der Duyn Schouten [3]. Castanier developed a more flexible maintenance strategy; the state dependent inspection interval was adopted in the research [1]. Based on the research by Castanier, Naini considered both preventive replacement and imperfect preventive maintenance to optimise the maintenance strategy of a two-component system [8]. In that paper, the inspection interval was simplified as state independent. These existing approaches to optimising the CBM strategy of multi-component systems largely predetermined maintenance strategy structures. The optimality of these predetermined structures have not been proofed or discussed. Furthermore, identifying an appropriate predetermined maintenance strategy structure also requires expert knowledge and experience that is not always available in reality. Therefore, a maintenance optimisation method that does not require a predetermined strategy structure is more applicable in reality and can be more cost-effective.

This paper proposes a maintenance optimisation approach for multi-component systems without a predetermined strategy structure using the semi-Markov decision process (SMDP). When maintenance strategy optimisation is carried out based on the Markov decision process (MDP) or the SMDP, the optimal maintenance structure can be identified simultaneously with the optimal strategy. Therefore, the MDP and the SMDP are widely used in the maintenance strategy optimisation and the optimal strategy structure investigation of mono-component systems [2, 7, 14, 18]. However, the application of SMDP to multi-component systems is still inadequate. A critical reason is that the health state of a multi-component system is difficult to be expressed, which makes the construction of the relative cost functions for SMDP become challenging. This paper divides the degradation process of a multi-component system into three stages, i.e., normal, partially failed, and completely failed. A SMDP is then developed for the maintenance optimisation of a two-component system. In addition, the optimal maintenance strategy structure of the two-component system under various situations is also investigated.

The body of this paper is organised as follows: Section 2 introduces the formulations of the degradation process of a two-component system and the costs of related maintenance activities. After that, a SMDP for the two-component system is developed in Section 3. The performance of the proposed maintenance optimisation method is investigated by simulation studies in Section 4. Section 4 also investigates the structure property of the optimal maintenance strategy for the two-component system.

2. Description of the System

2.1. The Degradation Model

A two-component system is investigated in this paper. The degradation processes of both components are assumed to follow the stationary Gamma process that are formulated as

$$\lambda^u(t+\Delta t)-\lambda^u(t)\sim Ga(a_u\cdot\Delta t,\xi_u)\quad u=1,2 \tag{1}$$

Here, $\lambda^u(t)$ denotes a degradation indicator of Component u at time t, and $Ga(a_u\cdot\Delta t,\xi_u)$ presents the Gamma distribution with the shape parameter $a_u\cdot\Delta t$ and the scale parameter ξ_u. When the process $\lambda^u(t)$ exceeds a failure threshold L_u, Component u fails. Component u is in a perfect health state when $\lambda^u(t)=0$. The Gamma process is monotonically increasing, which is consisted with the irreversible degradation process of most engineering assets. Therefore, the Gamma process is widely used in degradation modelling [16, 17]. The degradation processes of the two components are assumed to be independent from each other, i.e. the stochastic dependence is not considered in this paper.

The two components are assumed to be connected in series, and the whole system suffers from a failure when one of the two components is failed. The failure of the system cannot be detected immediately. However, operating the system in a failure condition will cause an additional cost, and the normal component still degrades even if the system is operating in a failure condition. A practical example of this scenario is a production line that consists of two machines, and each machine produces a certain part of a product. If one machine fails to produce qualified parts, the final product cannot meet the specifications and the production line is considered as failed. However, the failure of the production line may be not detected until an inspection is conducted on the two machines or final products.

2.2. Maintenance Related Costs and Durations

In this paper, three types of maintenance activities are considered, i.e., inspection, preventive replacement, and corrective replacement. The inspection is assumed to be able to completely reveal the state of the two components. Each inspection entails a cost C_i. Inspections are scheduled according to the health state of the two components to avoid unnecessary inspections. A preventive replacement action for Component u is conducted at a cost C_{pu}, while the cost of corrective replacement for Component u is C_{cu}. The preventive replacement cost is lower than the corrective replacement cost, i.e., $C_{pu}<C_{cu}$. In this paper, both the preventive and corrective replacement can bring a component to an "as good as new" state ($\lambda^u(t)=0$). Any preventive replacement or corrective replacement activity brings about a system set-up cost C_s. The set-up cost is caused by the dismantling and the reassembly of the system, or production losses during the system maintenance. The set-up cost is incurred only once for a group of replacement actions performed simultaneously. For example, correctively replacing the whole system costs $C_s+C_{c1}+C_{c2}$. Subsequently, economic dependence exists between the two components if $C_s>0$. Besides the cost of maintenance activities, running the system in a failure state will cause an additional cost c_d per unit time. The cost rate c_d is assumed to be significant and therefore leaving the system failure after an inspection is not optimal.

In this paper, the expected cost incurred by failure and maintenance activities per unit time is adopted as the criterion of maintenance optimisation. The durations of replacement and inspections can be ignored compared to the life time of components. Resources to carrying out inspections and replacement activities are assumed to be always adequate. The minimum reliability and availability constrains are not considered in this research as well.

3. The Semi-Markov Decision Process Approach

3.1. The Representation of System States and Transitions

Different from a mono-component system, the failure of the two-component system can be caused by the failure of one component or the failures of both the two components. The optimal maintenance action and relative costs in the SMDP under the two situations may be different. Consequently, the states of the two-component system are divided into three types, i.e., normal, partially failed, and completely failed. The normal system state implies that both the two components are running in a normal state. The partially failed system state means that one component is failed, while the other component is still in a normal state. In the completely failed situation, both the two components are in a failure state.

To apply the SMDP, the continuous degradation process of Component u is discretised into M_u different states. The state of Component u at time epoch t is then represented by $x_t^u = 1,2,\ldots,M_u \quad u=1,2$, where the state $x_t^u = 1$ denotes the "as good as new" state and the state $x_t^u = M_u$ stands for the failure state. By combining component states, the system state at time is given by:

$$x_t = \begin{cases} \left(x_t^1 - 1\right)\cdot\left(M_2 - 1\right) + x_t^2 & x_t^1 < M_1, x_t^2 < M_2 \\ \left(M_1 - 1\right)\left(M_2 - 1\right) + x_t^2 & x_t^1 = M_1, x_t^2 < M_2 \\ M_1\left(M_2 - 1\right) + x_t^1 & x_t^1 < M_1, x_t^2 = M_2 \\ M_1 M_2 & x_t^1 = M_1, x_t^2 = M_2 \end{cases} \quad (2)$$

Equation (2) divides the system states into four subsets: when $x_t^1 < M_1, x_t^2 < M_2$, the system is in a normal state; when $x_t^1 = M_1, x_t^2 < M_2$, the system is partially failed, and the failed component is Component one; when $x_t^1 < M_1, x_t^2 = M_2$, the system is failed, and the failed component is Component two; when $x_t^1 = M_1, x_t^2 = M_2$, the system is completely failed. To facilitate the formulation of the SMDP, the state of an individual component given the system state is presented as:

$$x_t^u = g_u\left(x_t\right) \quad u = 1,2 \quad (3)$$

After discretisation, the degradation process x_t^u becomes a continuous time discrete state Markov Chain, and the transition matrix during an interval Δt can be approximated as:

$$\left(\mathbf{P^u}\left(\Delta t\right)\right)_{ij} = p_{i,j}^u\left(\Delta t\right) = \Pr\left(LL_j^u \le \lambda^u\left(t+\Delta t\right) \le UL_j^u \middle| \lambda^u\left(t\right) = \frac{LL_i^u + UL_i^u}{2}\right), \quad (4)$$

where, UL_i^u and LL_i^u denote the upper limit and the lower limit of the i th state of Component u, respectively. The degradation indicator before discretisation, i.e., $\lambda^u\left(t\right)$, follows the Gamma process as in Equation (1). Consequently, Equation (4) can be calculated according to the property of the Gamma process. Because the two components degrade independently, the transition matrix for the system is obtained as:

$$\left(\mathbf{P}\left(\Delta t\right)\right)_{ij} = p_{g_1(i),g_1(j)}^1\left(\Delta t\right)\cdot p_{g_2(i),g_2(j)}^2\left(\Delta t\right) \quad (5)$$

Similarly, the reliability of the system after Δt given that the current system state is i can be calculated as:

$$R\left(\Delta t \middle| i\right) = \prod_{u=1}^{2} \Pr\left(\lambda^u\left(t+\Delta t\right) \le L^u \middle| \lambda^u\left(t\right) = \frac{LL_i^u + UL_i^u}{2}\right), \quad (6)$$

which can be calculated according to the property of the Gamma process [16]. The expected survival time of the system starting at state i during a time interval Δt can be then derived as:

$$\tau\left(\Delta t \middle| i\right) = \int_0^{\Delta t} R\left(s \middle| i\right) ds \quad (7)$$

3.2. The Relative Cost Functions

The relative cost function that formulates the relative cost of a single step in the long-run decision process is a crucial part of constructing and solving the SMDP [6]. In this paper, the relative cost function is a function of the current system state x_t. When the system is in a normal state, i.e., $x_t^1 < M_1$ and $x_t^2 < M_2$, four alternative maintenance activities are available. One is performing an inspection after a certain period of time. The waiting duration till the next inspection depends on the current state of the two system components. The others are preventively replacing Component one, preventively replacing Component two, and conducting a complete system replacement. The relative cost function for a normal system state can be then written as:

$$V\left(x_t\right) = \min\left\{V_{IN}\left(x_t, n_I\Delta_{ID}\right), V_{PR1}\left(x_t\right), V_{PR2}\left(x_t\right), V_{PRAll}; n_I = 1,2,\ldots,N_I\right\} \quad (8)$$

Here, $V_{IN}\left(x_t, n_I\Delta_{ID}\right)$ denotes the relative cost of performing an inspection after a period $n_I\Delta_{ID}$ when the current system sate is x_t, and $N_I\Delta_{ID}$ is the maximum waiting time for the next inspection. The notation Δ_{ID} can be regarded as the minimum time unit of inspection intervals considered in a maintenance strategy. Theoretically, reducing Δ_{ID} can enhance the accuracy of the optimal strategy. However, in reality, the value of Δ_{ID} should be selected based on the application. An unpractical short Δ_{ID} is not beneficial and makes the strategy difficult to implement. For example, when the maintenance strategy of the engine in a locomotive is investigated, Δ_{ID} can be a week instead of an hour. The function $V_{PRu}\left(x_t\right) \quad u=1,2$ is the relative cost when only Component u is preventively replaced. The variable V_{PRAll} is the relative cost of a complete preventive system replacement.

When the system is partially failed ($x_t^u = M_u, x_t^{i\ne u} < M_i$), there are also two optional strategies. One is replacing the failed component only, and the other is a complete system replacement. The corresponding relative cost function is given by:

$$V\left(x_t\right) = \min\left\{V_{PRu}\left(x_t\right), V_{PRAll}\right\} - C_{pu} + C_{cu} \quad u = 1,2 \quad (9)$$

Because a corrective replacement is performed to Component u, the difference between the costs of a corrective replacement activity and a preventive replacement activity should be added to Equation (9).

When the system is failed completely ($x_t^1 = M_1, x_t^2 = M_2$), the only possible maintenance activity is complete corrective system replacement, and the relative cost function is as follows:

$$V(x_t) = C_s + C_{c1} + C_{c2} + V(1) \quad (10)$$

Here, $V(1)$ denotes the relative cost function starting at the "as good as new" system state, i.e., $x_t^1 = 1$, $x_t^2 = 1$.

In Equation (8), the relative cost of conducting an inspection after a given time interval Δt starting at system state $x_t = i$ is calculated as:

$$V_{IN}(x_t = i, \Delta t) = C_i + \sum_{j=i}^{M_1 M_2} V(j) p_{i,j}(\Delta t) - \gamma \cdot \Delta t + c_d \cdot (\Delta t - \tau(\Delta t | i)) \quad (11)$$

, where, γ is the expected cost incurred by failure and maintenance activities per unit time and $p_{i,j}(\Delta t)$ is an element in the system transition matrix during the time interval Δt . The relative cost of preventively replacing Component one and two given that the current system state x_t is i are given by:

$$V_{PR1}(x_t = i) = C_s + C_{p1} + V(g_2(i)) \quad (12)$$

and

$$V_{PR2}(x_t = i) = C_s + C_{p2} + V((g_1(i) - 1) \cdot (M_2 - 1) + 1) \quad (13)$$

respectively. The relative cost for a complete system preventive replacement which is state independent can be calculated as:

$$V_{PRAll} = C_s + C_{p1} + C_{p2} + V(1). \quad (14)$$

3.3. The Policy Iteration

After the relative cost functions are constructed, the policy iteration is used to find the optimal maintenance policy that minimises the expected cost per unit time. A policy is denoted as $\delta(A) = B$, where $A = 1, 2, \ldots, M_1 \cdot M_2$ is a certain discretised system state derived by Equation (2) and B is the corresponding maintenance action. For a normal system state, the maintenance action can be chosen from:

$$B \in \{(IN, n_I \Delta_{ID}), PR_1, PR_2, PR_{all}; n_I = 1, 2, \ldots, N_I\}.$$

The first candidate maintenance activity $(IN, n_I \Delta_{ID})$ implies performing an inspection after a duration $n_I \Delta_{ID}$. The other optional maintenance actions PR_1 , PR_2 , and PR_{all} denote preventively replacing Component one, preventively replacing Component two, and complete system preventive replacement, respectively. When only Component u is failed, the maintenance action space becomes $B \in \{CR_u, CR_u + PR_{i \neq u}\}$. Here, the maintenance action CR_u denotes correctively replacing Component u , while the $CR_u + PR_{i \neq u}$ represents correctively replacing Component u and preventively replacing the other component at the same time. For complete failure, the determinate maintenance action is complete system corrective replacement, i.e., $CR_1 + CR_2$.

The general process of the policy iteration is as shown in Table 1. For a more detailed introduction of the policy iteration, readers can refer to [7, 13].

Table 1: The process of policy iteration

Step 1:

Set an initial policy function

The initial policy function is selected by the rule of thumb, and any policy satisfies the conditions discussed at the beginning of Section 3.3 can be adopted as the initial policy.

Step 2:

Calculate the relative costs $\{V(A); A = 2, 3, \ldots, M_1 M_2 - 1\}$ and the expected cost per unit time γ by solving the following system of linear equations that is constructed according to the current maintenance policy $\delta_k(\cdot)$:

$$V(A)=\begin{cases} I_{(IN,n_I\Delta_{ID})}(\delta_k(A))\cdot V_{IN}(A,n_I\Delta_{ID}) \\ +I_{PR_1}(\delta_k(A))\cdot V_{PR1}(A) & ,2\le A\le (M_1-1)(M_2-1) \\ +I_{PR_2}(\delta_k(A))\cdot V_{PR2}(A)+I_{PR_{all}}(\delta_k(A))\cdot V_{PRAll} \\ \\ I_{CR_1}(\delta_k(A))\cdot V_{PR1}(A) \\ +I_{CR_1+PR_2}(\delta_k(A))\cdot V_{PRAll}-C_{p1}+C_{c1} & ,(M_1-1)(M_2-1)<A\le M_1(M_2-1) \\ \\ I_{CR_2}(\delta_k(A))\cdot V_{PR2}(A) \\ +I_{PR_1+CR_2}(\delta_k(A))\cdot V_{PRAll}-C_{p2}+C_{c2} & ,M_1(M_2-1)<A<M_1M_2 \end{cases},$$

where the formulations of , $V_{PR1}(A)$, $V_{PR2}(A)$ and are given by Equations (11), (12), (13), and (14) respectively, and is the indicator function given by:

$$I_B(x)=\begin{cases}0, & x\ne B\\ 1, & x=B\end{cases} \qquad (15)$$

The relative cost functions when the system is brand new and completely failed are determinate, i.e., and $V(M_1M_2)=C_s+C_{c1}+C_{c2}$.

Step 3:

Calculate the relative costs under different maintenance actions:
$V_{IN}(A,n_I\Delta_{ID})$, $A=2,3,\ldots,(M_1-1)(M_2-1)$, $n_I=1,2,\ldots,N_I$,
$V_{PR1}(A)$, $A=2,3,\ldots,M_1M_2-1$
and
$V_{PR2}(A)$, $A=2,3,\ldots,M_1M_2-1$
given by Equations (11), (12), and (13) using the values of $\{V(A);A=2,3,\ldots,M_1M_2-1\}$ and γ obtained in Step 2.

Step 4:

Obtain the improved policy function $\delta_{k+1}(\cdot)$ using the relative costs calculated in Step 3. The $\delta_{k+1}(\cdot)$ is identified piecewisely as:

When the system is in a normal state, i.e., $1\le A\le (M_1-1)(M_2-1)$, the policy function is:

$$\delta_{k+1}(A)=\begin{cases}(IN,l\cdot\Delta_{ID}), & V_{IN}(A,l\cdot\Delta_{ID})=\min\{V_{IN}(A,n_I\Delta_{ID}),V_{PR1}(A),V_{PR2}(A),V_{PRAll};n_I=1,\ldots,N_I\}\\ PR_1, & V_{PR1}(A)=\min\{V_{IN}(A,n_I\Delta_{ID}),V_{PR1}(A),V_{PR2}(A),V_{PRAll};n_I=1,\ldots,N_I\}\\ PR_2, & V_{PR2}(A)=\min\{V_{IN}(A,n_I\Delta_{ID}),V_{PR1}(A),V_{PR2}(A),V_{PRAll};n_I=1,\ldots,N_I\}\\ PR_{all}, & V_{PRAll}=\min\{V_{IN}(A,n_I\Delta_{ID}),V_{PR1}(A),V_{PR2}(A),V_{PRAll};n_I=1,\ldots,N_I\}\end{cases}.$$

When only Component One is failed, i.e. $(M_1-1)(M_2-1)<A\le M_1(M_2-1)$, the policy function is:

$$\delta_{k+1}(A)=\begin{cases}CR_1, & V_{PR1}(A)<V_{PRAll}\\ CR_1+PR_2, & V_{PR1}(A)>V_{PRAll}\end{cases}.$$

When only Component Two is failed, i.e. $M_1(M_2-1)<A<M_1M_2$, the policy function is:

$$\delta_{k+1}(A)=\begin{cases}CR_2, & V_{PR2}(A)<V_{PRAll}\\ CR_2+PR_1, & V_{PR2}(A)>V_{PRAll}\end{cases}.$$

When both the two components are failed, i.e. $A=M_1M_2$, the whole system should be replaced, and the policy function is therefore predetermined as $\delta_{k+1}(A)=CR_1+CR_2$.

Step 5:

If $\delta_{k+1}(\cdot)=\delta_k(\cdot)$, the optimal maintenance policy $\delta^*(\cdot)$ is obtained as $\delta_k(\cdot)$. Otherwise, go to Step 2 and start a new iteration.

The most time-consuming part of the policy iteration algorithm in Table 1 is Step 2 that entails solving a system of linear equations with $M_1 M_2 - 1$ variables. When the numbers of discretised states (i.e. M_1 and M_2) are large, some iterative methods (e.g. the Jacobi method and the Gauss–Seidel method) are required to solve the system of linear equations. Fortunately, according to the stimulation study in Section 4.2, the policy iteration can obtain a satisfactory approximate optimal maintenance strategy when the resolution of component state discretisation is moderate. Consequently, the system of linear equations in Step 2 is simply solved based on the LU decomposition. Another potential factor relates to the efficiency of the policy iteration is the number of possible inspection intervals N_I . A large N_I can reduce the efficiency of Step 3 and Step 4 in Table1. However, the value of does not change the number of variables in the system of linear equations in Step 2 which is the bottle-neck of the whole algorithm. Consequently, the number of optional inspection intervals does not increase the computing time of the policy iteration significantly.

4. The Simulation Study of the Proposed Approach

4.1. Investigation of the Optimal Maintenance Strategy Structures

Markov decision process (MDP) has been adopted to explore the maintenance strategy structure property of mono-component systems and multi-component systems based on lifetime distribution (i.e. two-state assumption) [4, 6]. However, these strategy structure properties cannot be simply extended to the CBM of multi-component systems. The structure of the CBM strategy of multi-component systems is much more complex due to the large number of component states and their combinations. To address this research gap, this study investigates structure properties of the CBM strategy of a continuous-state two-component system. The results can provide guidelines for approximate maintenance optimisation algorithms of multi-component systems in a CBM context. In addition, investigating the strategy structure can also validate the effectiveness of the proposed SMDP approach.

Maintenance Strategy Structures for Different Set-up Costs

The set-up cost is an important element in the maintenance optimisation of multi-component systems. When the set-up cost covers a considerable proportion of the maintenance cost, significant economic dependence among components exists, and the group maintenance should be adopted. Subsequently, the influence of different set-up costs on maintenance strategy structures were studied first. In this part of simulation study, parameters of system degradation processes and maintenance costs were selected without particular physical meaning, and were for illustrative purpose only. The parameters of the system degradation processes were set as follows: $a_1 = a_2 = 1$, $\xi_1 = \xi_2 = 1/3$, and $L_1 = L_2 = 2$. The inspection cost and the failure cost per unit time were assumed as $C_i = 1$ and $c_d = 10$. The shortest inspection interval was Δ_{ID} =0.2, and the corresponding N_I was selected as 15. As discussed in Section 3.2, the selection of Δ_{ID} is application-dependent in reality, and an unpractical short Δ_{ID} is not preferred. The value of N_I is initially selected by the rule of thumb, and may be modified according to the maintenance optimisation results. When the longest inspection interval in the obtained optimal strategy is equal to $N_I \cdot \Delta_{ID}$, a larger N_I should be used so that the policy iteration can access a potential optimal policy with a longer inspection interval.

First of all, a small set-up cost ($C_s = 1$) was considered, and costs for preventive and corrective replacement were selected as: $C_{p1} = C_{p2} = 39$ and $C_{c1} = C_{c2} = 99$, respectively. After the policy iteration, a minimum average cost per unit time $\gamma = 19.4894$ was derived. The result of the policy iteration is presented as the matrix in Figure 1. Each colour standards for a particular type of maintenance action; the numbers in white rectangles are the waiting durations till the next inspection. Because the degradation processes and the maintenance costs of the two components are the same, the policy matrix is symmetrical about the diagonal line. Figure 1 also shows that the optimal maintenance action for a component is not monotonic in State 9. Preventive replacement for the component in State 9 is required, when the state of the other component is below state 6. On the other hand, a further inspection is optimal when the other component is in State 7 and State 8. A complete system replacement is required when both the components are in or above state 9. This unexpected optimal maintenance structure is caused by the economic dependence: When the other component degrades to a state near the preventive replacement threshold, a more economical way is leaving the component in State 9 along and performing complete system replacement later. To demonstrate the effects of this non-monotonic structure, a monotonic strategy in Figure 2 was also adopted, and the average cost per unit time was $\gamma = 19.5157$. Therefore, the non-monotonic strategy in Figure 1 was more cost-effective.

Then a significant set-up cost ($C_s = 20$) was adopted. To maintain the replacement costs for an individual component (i.e., $C_s + C_{pu}$ and $C_s + C_{cu}$) unchanged, the costs for preventive and corrective replacement were selected as $C_{p1} = C_{p2} = 20$ and $C_{c1} = C_{c2} = 80$, respectively. After the policy iteration, the minimum average cost per unit time was calculated as $\gamma = 17.3396$ and the optimal strategy is presented in Figure 3. Finally a more significant set-up cost ($C_s = 30$) was used, and the costs for replacement were $C_{p1} = C_{p2} = 10$ and $C_{c1} = C_{c2} = 70$. Using the policy iteration, the minimum average cost per unit time was obtained as $\gamma = 15.4537$ and the optimal maintenance strategy is shown in Figure 4.

Some conclusions can be drawn from the maintenance optimisation results for the three different set-up costs. Firstly, the cost reduction by introducing opportunistic maintenance is more significant when the set-up cost covers a larger proportion of the total replacement cost. Secondly, the non-monotonic part of the strategy and the threshold for opportunistic replacement is near the “as good as new” state for a large set-up cost. Finally, besides opportunistic replacement, complete system replacement is required when the two components are both near but still below the preventive replacement thresholds. More cost-effective maintenance strategy structures are expected after the non-monotonic properties that are derived by this simulation study are described appropriately.

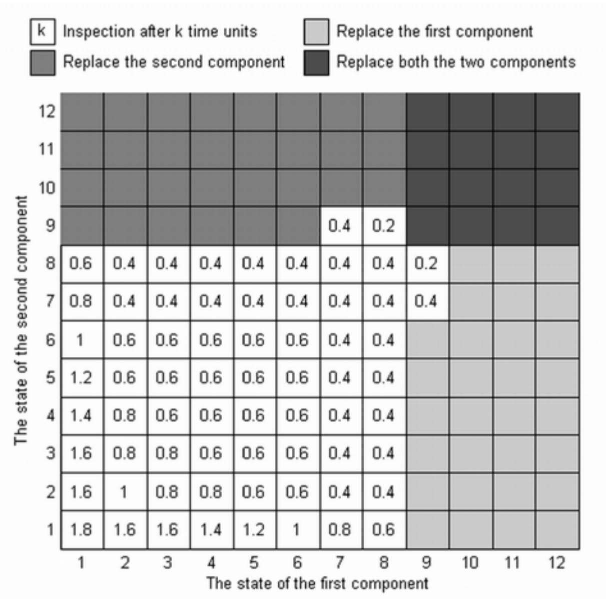

Fig. 1. The optimal maintenance strategy when $a_1 = a_2 = 1$ *,* $\xi_1 = \xi_2 = 1/3$ *,* $C_s = 1$ *,* $C_{p1} = C_{p2} = 39$ *, and* $C_{c1} = C_{c2} = 99$

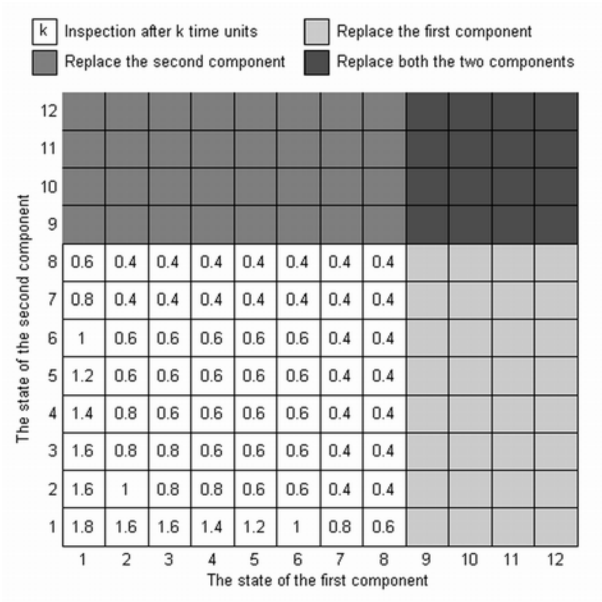

Fig. 2. The monotonic maintenance strategy when $a_1 = a_2 = 1$ *,* $\xi_1 = \xi_2 = 1/3$ *,* $C_s = 1$ *,* $C_{p1} = C_{p2} = 39$ *, and* $C_{c1} = C_{c2} = 99$

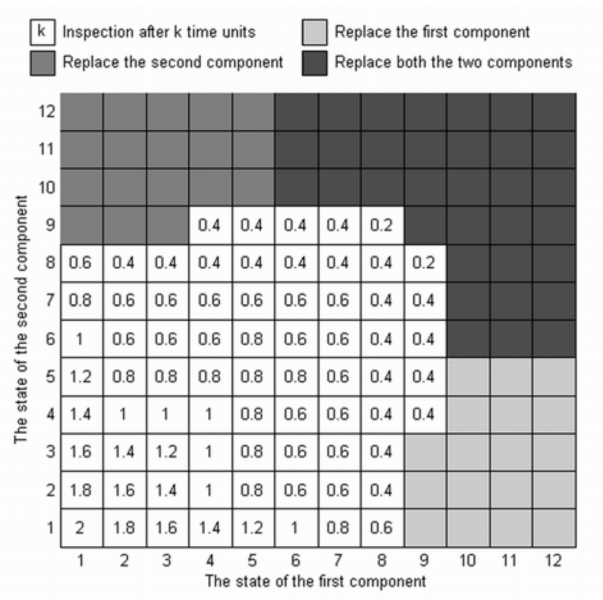

Fig. 3. The optimal maintenance strategy when $a_1 = a_2 = 1$ *,* $\xi_1 = \xi_2 = 1/3$ *,* $C_s = 20$ *,* $C_{p1} = C_{p2} = 20$ *, and* $C_{c1} = C_{c2} = 80$

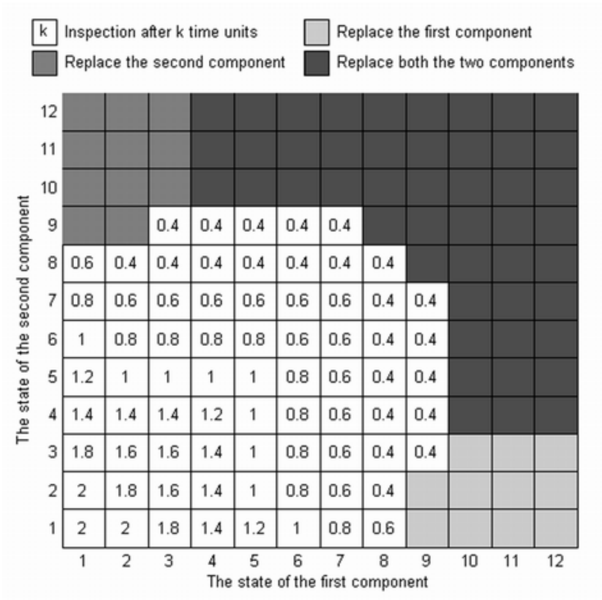

Fig. 4. The optimal maintenance strategy when $a_1 = a_2 = 1$ *,* $\xi_1 = \xi_2 = 1/3$ *,* $C_s = 30$ *,* $C_{p1} = C_{p2} = 10$ *, and* $C_{c1} = C_{c2} = 70$

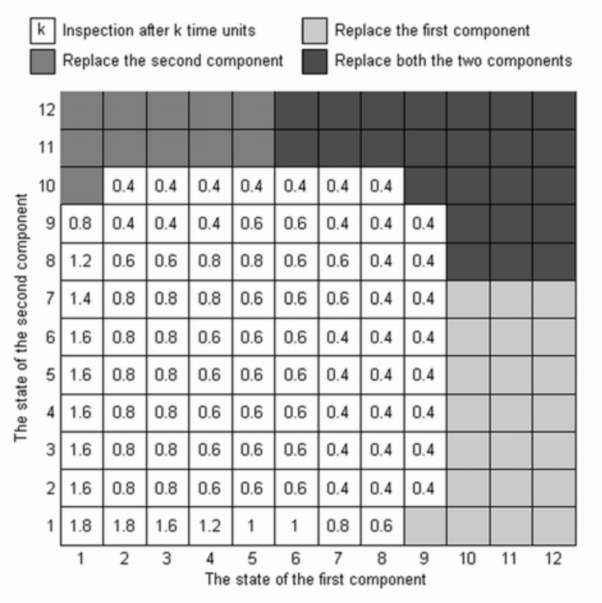

Fig. 5. The optimal maintenance strategy when $a_1 = a_2 = 1$, $\xi_1 = 0.5$, $\xi_2 = 1/6$, $C_s = 20$, $C_{p1} = C_{p2} = 30$, *and* $C_{c1} = C_{c2} = 80$

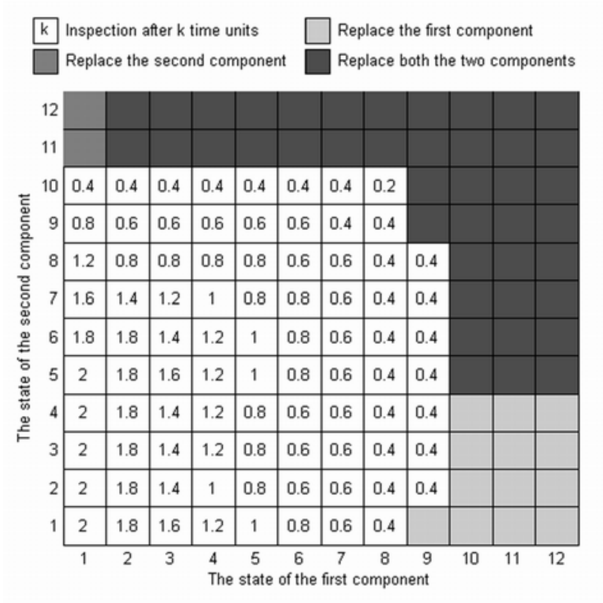

Fig. 6. The optimal maintenance strategy when $a_1 = a_2 = 1$, $\xi_1 = 0.5$, $\xi_2 = 1/6$, $C_s = 47$, $C_{p1} = C_{p2} = 3$, *and* $C_{c1} = C_{c2} = 53$

Maintenance Strategy Structures for Different Degradation Process Parameters

Different from the methods developed in [3, 15], the approach proposed in this paper can process a system whose components follow different degradation processes. This part of simulation study explores the influence of process parameters on maintenance strategy structures. The parameters for the degradation processes of the two components were $a_1 = a_2 = 1$ and $\xi_1 = 0.5$, $\xi_2 = 1/6$. A larger scale parameter ξ_u $u = 1,2$ indicates a faster degradation process. Therefore, Component one degrades more quickly. Two sets of maintenance costs were used: $C_s = 20$, $C_{p1} = C_{p2} = 30$, $C_{c1} = C_{c2} = 80$ and $C_s = 47$, $C_{p1} = C_{p2} = 3$, $C_{c1} = C_{c2} = 53$. The minimum average cost per unit time for the two situations were $\gamma = 21.4651$ and $\gamma = 18.5801$, respectively. The corresponding maintenance strategies are showed in Figure 5 and Figure 6.

Figure 5 and Figure 6 show that lower preventive and opportunistic thresholds are set for Component one due to the faster degradation process of that component. Consequently, the strategy structures become unsymmetrical about the diagonal line. The difference between Figure 5 and Figure 6 shows that the proposed SMDP can adaptively identify the maintenance strategy structure according to different maintenance costs and degradation process parameters.

4.2. Influence of the Number of Discretised Intervals

The system state space is discretised to perform the SMDP, which can introduce errors into the estimate of average cost per unit time. The discretised system state space also leads thresholds for preventive and corrective replacement to be less accurate. Increasing the number of states can reduce the errors that are brought in by discretisation. However, the consumed memory and elapsed time increase quickly with the resolution of the system state space. Therefore, it is necessary to find a balance between the accuracy of a maintenance strategy and the length of computing time.

In this part of simulation study, different numbers of discretised component states were trailed to investigate the relationship between the effectiveness of the maintenance strategy and the elapsed time of the policy iteration. The effectiveness of the maintenance strategy was evaluated through the average cost per unit time of a simulated degradation process. The parameters of the degradation processes were selected as and ; the costs of maintenance actions were $a_1 = a_2 = 1$, $\xi_1 = \xi_2 = 1/3$, and $C_s = 30$. To explore the effects of the resolution of the component states, four different numbers of discretised component states were adopted, i.e., $M_1 = M_2 = 7$, $M_1 = M_2 = 12$, $M_1 = M_2 = 22$, and $M_1 = M_2 = 32$. For the three different resolutions, the policy iteration was carried out and elapsed durations were recorded. The derived maintenance strategies were applied to a simulated degradation process of 106 unit time length. The simulated average costs per unit time were calculated to compare with the approximated results derived by the policy iteration. The results are demonstrated in Figure 7.

Figure 7 shows that the approximated average costs are lower than the simulated average costs, and the difference between the two costs reduces with the growth of the number of discretised states. The increase of the approximated average costs is caused by the reduction of errors in the policy iteration, and the decrease of the simulated average cost is due to more accurate thresholds in maintenance strategies. Figure 7 also shows that when , adopting a finer resolution of component states cannot save the simulated average cost significantly, while the elapsed time is considerably longer. The simulated average cost per unit time when $M_1 = M_2 = 22$ and $M_1 = M_2 = 32$ were 15.6524 and 15.6398, respectively. The corresponding elapsed

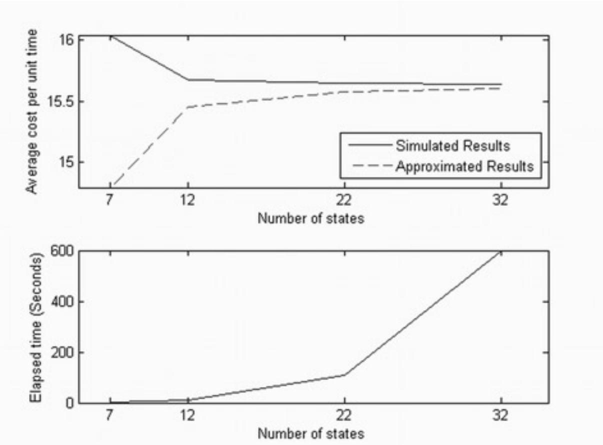

Fig. 7. The simulated and approximated average cost per unit time and the elapsed time

durations were 106.8 seconds and 597.3 seconds. This small variance between the two simulated average costs shows that the proposed approach is able to identify an approximate global optimal strategy for a continuous state two-component system without a predetermined strategy structure.

5. Conclusions

This paper has developed a SMDP approach to optimise the maintenance strategy of a multi-component system without a predetermined strategy structure. The state of the multi-component system has been divided into three different types: normal, partially failed, and completely failed to construct the relative cost function and perform the policy iteration. Compared with other existing approaches, the proposed SMDP do not need to predetermine the maintenance structure and the number of inspection intervals. Therefore, the SMDP developed in this paper is more adaptive and applicable in reality. Furthermore the SMDP divides the long-term degradation process into single time steps. Consequently, the SMDP approach is easier to be carried out in more complex practical situations, e.g., imperfect maintenance, state-dependent maintenance cost, and state-dependent maintenance durations. In addition, the SMDP uses the transition matrix to express the system degradation process. Therefore, the stochastic dependence and the structure dependence can be also processed by the proposed approach when the transition matrix of system states is established.

This research has also explored the structure property of the optimal CBM strategy for a two-component system through a simulation study. The results can provide a guideline to develop an approximate optimal maintenance strategy for multi-component systems. The simulation study also shows that the proposed approach using the SMDP can provide satisfactory optimisation results for a continuous state two-component system. For a more complex multi-component system with intractable number of component state combinations, approximate solving methods for the SMDP can be adopted.

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Prof. Zhisheng ZHANG
Dr. Yifan ZHOU
School of Mechanical Engineering
Southeast University
Nanjing, China, 211189
E-mail: oldbc@seu.edu.cn
E-mail: yfzhou1981@163.com

Dr. Yong SUN
Prof. Lin MA
CRC of Integrated Engineering Asset Management (CIEAM),
School of Engineering Systems, Faculty of Built Environment and Engineering,
Queensland University of Technology,
Brisbane, Australia
E-mail: y3.sun@qut.edu.au
E-mail: l.ma@qut.edu.au