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UNCERTAINTY ANALYSIS METHOD BASED ON A COMBINATION OF THE MAXIMUM ENTROPY PRINCIPLE AND THE POINT ESTIMATION METHOD

METODA ANALIZY NIEPEWNOŚCI OPARTA NA POŁĄCZENIU ZASADY MAKSYMALNEJ ENTROPII I METODY OCENY PUNKTOWEJ

Uncertainty is inevitable in product design processes. Therefore, to make reliable decisions, uncertainty analysis incorporating all kinds of uncertainty is needed. In engineering practice, due to the incomplete knowledge, the distribution of some design variables can not be determined. Furthermore, the performance function is highly nonlinear, therefore, the high order moments of the performance function are needed to calculate the probability of failure accurately. In this paper, an uncertainty analysis method combining the maximum entropy principle and the bootstrapping method is proposed. Firstly, the bootstrapping method is used to calculate the confidence intervals of the first four moments for mixed random variables and sample variables. Secondly, the high order moments of limit state functions are estimated using the reduced dimension method. Thirdly, to calculate the probability density function (PDF) and cumulative distribution function (CDF) of the limit state functions, an optimization model based on the maximum entropy principle is formulated. In the proposed method, the assumptions that the distribution of the random variables are known and the calculation of the sensitivity for limit state function with respect to the Most Probable Point (MPP) are avoided. Finally, comparisons of results from the proposed methods and the MCS method are presented and discussed with numerical examples.

Keywords: *uncertainty analysis, bootstrapping , moments, maximum entropy principle.*

Niepewność jest nieodłącznym elementem procesów projektowania produktu. Dlatego też podejmowanie niezawodnych decyzji wymaga analizy niepewności, która uwzględniałaby wszystkie rodzaje niepewności. W praktyce inżynierskiej, z powodu niepełnej wiedzy, wyznaczenie rozkładu niektórych zmiennych projektowych nie jest możliwe. Co więcej, funkcja stanu granicznego jest wysoce nieliniowa, co sprawia, że do poprawnego obliczenia prawdopodobieństwa uszkodzenia potrzebna jest znajomość momentów wyższych rzędów tej funkcji. W niniejszej pracy zaproponowano metodę analizy niepewności łączącą zasadę maksymalnej entropii z metodą bootstrapową. W pierwszej części pracy wykorzystano metodę bootstrapową do obliczenia przedziałów ufności czterech pierwszych momentów dla zmiennych losowych typu mieszanego oraz zmiennych z próby. Następnie, wyznaczono momenty wyższych rzędów funkcji stanu granicznego przy użyciu metody redukcji wymiarów. Po trzecie, w celu obliczenia funkcji gęstości prawdopodobieństwa (PDF) oraz dystrybuanty (CDF) funkcji stanu granicznego, sformułowano model optymalizacji oparty na zasadzie maksymalnej entropii. Proponowana metoda nie wymaga założenia znajomości rozkładów zmiennych losowych ani obliczania wrażliwości dla funkcji stanu granicznego w odniesieniu do najbardziej prawdopodobnego punktu awarii. W końcowej części artykułu porównano na podstawie przykładów numerycznych wyniki otrzymane za pomocą proponowanej metody oraz symulacji Monte Carlo (MCS).

Słowa kluczowe: *analiza niepewności, bootstrapping, momenty, zasada maksymalnej entropii.*

1. Introduction

Uncertainty exists in the whole life-cycle of a product. Therefore, to make reliabile decisions, the representation, quantification, and propagation of uncertainty are needed in design processes, which have been widely studied in many advanced research fields.

Uncertainty analysis is to evaluate the cumulative distribution function (CDF), probability density function (PDF) of a performance function formulated by mutually independent random varaibles. The CDF of the performance function can be evaluated with a multidimensional integral. However, in practice it is very difficult or even impossible to obtain an analytical solution to the probability integration. Many research have been develpoed for approximating the probability integration.

Mainly, there are three approximation approaches for uncertainty analysis including (1) simulation method, (2) agent models method, and (3) analytical method. The most direct reliability analysis method is Monte Carlo simulation (MCS) [5, 18, 19]. However, the efficiency of MCS is very low for high dimension problems or when the requirement of design accuracy is high. The main merit of the agent methods [7, 11, 12] is easy to solve. However, the accuracy of the agent methods usually does not meet engineering requirements. Analytical methods focus on simplifying the multi-dimensional integral calculation. The first order reliability method (FORM) and second order reliability method (SORM) [3, 6, 8, 9, 22] were widely used by first order or second order Taylor expansion of the performance function at the most probable failure point (MPP). In the MPP based analysis methods, the random variables were needed to be transformed into standard normal distribution, and the sensitivity analysis was required in both the FORM and SORM. Further, the MPP search was an iterative optimization process, which might be trapped into local optimum. The accuracy of the two methods was determined by the non-linearity of the performance function. When the performance function is highly non-linear, the results calculated with the two methods may cause huge errors. Another altenative analytical method [10, 15, 20] for uncertainty analysis have been developed with dimension reduced method combined numerical integration methods. Rahman and Xu [20] proposed a univariate dimension reduction method for multi-dimensional integration using moment based quadrature rule. Huang and Du [10] presented an uncertainty analysis method based on the combination of dimension reduction integration and saddlepoint approximation. In their method, all the random variable should be transformed into standard normal distribution, and the Gauss-Hermite integration was used to calculate the moments of the limit state functions. Lee and Choi, et al [15] developed an inverse analysis method using MPP based dimension reduction for reliability based design optimization. In their method, the MPP calculation was needed and all the random variables were transformed into standard normal distribution.

Maximum entropy principle as a measure of uncertainty has beed developed rencently for uncertainty analysis and reliability based design optimization. As the performance function is highly nonliear or the MPP is not unique, the high order moments of the performace function are needed for estimating the CDF of the performance function accurately. Kang and Kwak [14] applied the maximum entropy principle to reliability based design optimization with the improved moment based quadrature rule. Li and Zhang [16] presented the combined reliability analysis approach with dimension reduction method and maximum entropy principle. The moment based quadrature rule was used to calculate the moments of the performance function. Sung and Kwark [21] proposed reliability bound analysis method based on maximum entropy method with respect to the first truncated moment. Ching and Hsieh [4] developed an estimation method to calculate the confidence interval of the probabilty of failure for the performance function with maximum entropy principle. Volpe and Bagan [23] analyzed the Maximum entropy PDFs and the moment problem of random variables under near-Gauss distribution. A constrained optimization problem is needed to solve in the maximum entropy principle based uncertainty analysis methods. Abramov [1-2] proposed BFGS methods to solve this nonliear optimization problem.

In the above related work, the distribution of the random variables were assumed to be known, and were needed to transform into standard normal distribution. In this paper, an uncertainty analysis method combined maximum entropy principle and bootstrapping method is proposed. When the distribution of some random variables can not be exactly determined, the high order moments of limit state functions are estimated by bootstrapping method. Confidence intervals of the probability density function (PDF), and cumulative distribution function (CDF) of performance functions are calculated based on maximum entropy principle.

The structure of this paper is as follows. In the second section of this paper, the bootstrapping method to estimate distribution information of uncertainty variables is introduced. The process to calculate moments of limit state functions are provided in the third section. An optimization model based on maximum entropy principle is formulated in the forth section. Numerical examples are analyzed using the proposed method in the fifth section. Finally the conclusions and disscussion are given in the sixth section.

2. Bootstrapping method to estimate the distribution of the uncertainty variables

A general uncertainty analysis problem as in Eq. (1) is considered in this work. Performance function $y = g(\mathbf{x})$ which is also referred to limit-state function is modeled as the output of mutually independent random variables $\mathbf{x} = [x_1, x_2, \cdots, x_n]$.

$$F(y) = P\left\{ y \le y^a \right\} = \int_{g(\mathbf{x}) \le y^a} \cdots \int f(\mathbf{x}) d\mathbf{x} \tag{1}$$

where $F(y)$ is the CDF of the limit state function, y^a denotes a upper bound of the performance function, $f(\mathbf{x})$ is the joint probability density funtion of $\mathbf{x}$.

Bootstrapping method is a statistical method for estimating the sampling distribution of a random by sampling with replacement from the original samples. The steps of bootstrapping method are analyzed as follows.

Given the m sample points $x_{i,1}, x_{i,2}, \cdots, x_{i,m}$ for a random variable x_i: Step (1) Construct an empirical probability distribution function f_{x_i} from the samples by placing a probability of $1/m$ for each point $x_{i,1}, x_{i,2}, \cdots, x_{i,m}$ of the samples. Step (2) from the empirical distribution function f_{x_i}, draw a random sample of size m with replacement. Step (3) calculate the statistic of the resample points $T_{x_{i,k}}$. Step (4) repeat step 2 and step 3 k times, where k equals to 1000. Step (5) construct the relative frequency histogram from the k number of T_{x_i} by placing a probability of $1/k$ at each point. $T_{x_{i,1}}, T_{x_{i,2}}, \cdots, T_{x_{i,1000}}$. $T_{x_{i,(1)}}, T_{x_{i,(2)}}, \cdots, T_{x_{i,(1000)}}$ denote the bootstrap values by ranking $T_{x_{i,1}}, T_{x_{i,2}}, \cdots, T_{x_{i,1000}}$ from bottom to top. Then the bootstrap percentile confidence interval at 95% level of confidence would be $[T_{x_{i,(25)}}, T_{x_{i,(975)}}]$. x_i lies within the centered bootstrap 95% percentile confidence interval $[2\bar{x}_i - T_{x_{i,(975)}}, 2\bar{x}_i - T_{x_{i,(25)}}]$.

2.1. Calculation of moments for sample variables

Given n samples of a random variable x_i, the first four moments μ, σ, μ_3, μ_4 of a random variable can be calculated by Eq. (2):

$$\begin{aligned} &E(\bar{x}_i) = \mu \\ &E(\bar{x}_i - \mu)^2 = \frac{\sigma^2}{n} \\ &E(\bar{x}_i - \mu)^3 = \frac{1}{n^2}\mu_3 \\ &E(\bar{x}_i - \mu)^4 = \frac{1}{n^3}\mu_4 + \frac{3(n-1)}{n^3}\sigma^4 \end{aligned} \tag{2}$$

where $\bar{x}_i = \frac{\sum_{j=1}^{n} x_{i,j}}{n}$.

The centered bootstrap 95% percentile confidence interval of a random varaible x_i is calculated by $[2\bar{x}_i - T_{x_{i,(975)}},\ 2\bar{x}_i - T_{x_{i,(25)}}]$.

3. Moments estimation for the limit state functions

In engineering practices, the limit state function $g(\mathbf{x})$ is a nonliear function of large input variables $\mathbf{x} = [x_1, x_2, \cdots, x_n]$. The mean of the limit state funtion can be calculated by point estimation method using m^n points. The computational burden is extremely large if n becomes large. In order to reduce the computational burden, a dimension reduced method [24] is introduced to approximate the limit state function which is expressed in Eq. (3):

$$g'(\mathbf{x}) = \sum_{i=1}^{n}(g_i - g_\mu) + g_\mu \tag{3}$$

where $g_\mu = g(\mu_1, \mu_2, \cdots, \mu_n)$ is the performance function value with all input variables taking the mean values. $g_i = g(\mu_1, \mu_2, \cdots, x_i, \cdots, \mu_n)$ denotes the response value with all input variables taking the mean except the i^{th} input variable. From Eq. (3), the computational burden is reduced largely and the number of the function calls is reached $m \times n$. Since x_i is mutually independent, g_i is also mutually independent. The first four moments of the limit state $g(\mathbf{x})$ can be calculated by Eq. (4):

$$\mu_g = \sum_{i=1}^{n}(\mu_i - g_\mu) + g_\mu \tag{4a}$$

$$\sigma_g^2 = \sum_{i=1}^{n}\sigma_i^2 \tag{4b}$$

$$\mu_{3g}\sigma_g^3 = \sum_{i=1}^{n}\mu_{3i}\sigma_i^3 \tag{4c}$$

$$\mu_{4g}\sigma_g^4 = \sum_{i=1}^{n}\mu_{4i}\sigma_i^4 + 6\sum_{i=1}^{n-1}\sum_{j>i}^{n}\sigma_i^2\sigma_j^2 \tag{4d}$$

where μ_i, σ_i, μ_{3i}, μ_{4i} are the first four moments of g_i which can be calculated with the point estimation method of the single variable.

Considering incomplete knowledge of some random variables, the confidence interval of μ_i $[2\mu_i - T_{\mu_i,975},\ 2\mu_i - T_{\mu_i,25}]$ can be calculated by bootstrapping method.

4. Maximum entropy principle for calculation of CDF and PDF

Entropy has been widely studied for uncertainty analysis and reliability design optimization since entropy was analyzed by Jaynes [13] as a measure of uncertainty . Maximum entropy method is developed to estimate the probability distribution of a random variable by maximizing the entropy subject to constraints supplied by the moments of the random variable.

Generally, Eq. (5) and Eq. (6) are used to calculate the entropy for both the discrete and continuous variables respectively:

$$H(x) = -\sum_{i=1}^{n} p_i \ln p_i \tag{5}$$

$$H(x) = H(p(x)) = -\int_x p(x)\ln p(x) \tag{6}$$

where p_i is the probability of the discrete variable x_i, and $P(x)$ is the PDF of the continuous variable x_i.

4.1. Optimization formulation to calculate PDF and CDF

Maximum entropy formulation of a function can be expressed by Eq. (7):

$$\begin{aligned} &\max: H = -\int_R f(g(\mathbf{x}))\ln f(g(\mathbf{x}))dg(\mathbf{x}) \\ &\text{s.t. } \int_R f(g(\mathbf{x}))dg(\mathbf{x}) = 1 \\ &\quad\ \int_R g(\mathbf{x})f(g(\mathbf{x}))dg(\mathbf{x}) = \mu_g \\ &\quad\ \int_R (g(\mathbf{x}) - \mu_g)^r f(g(\mathbf{x}))dg(\mathbf{x}) = \mu_g^r \end{aligned} \tag{7}$$

where R is the integral domain, μ_g is the mean value of $g(\mathbf{x})$, and μ_g^r is the r^{th} central moment for the limit state function $g(\mathbf{x})$.

Lagrange method can be used to solve problem in Eq. (7) and the Lagrange multipliers are denoted as $(\lambda_0, \lambda_1, \cdots, \lambda_n)$, and the maximum entropy formulation for the PDF can be expressed in Eq. (8) , which is the optimal solution to Eq. (7):

$$f(g(\mathbf{x})) = \exp(\lambda_0 + \sum_{i=1}^{n}\lambda_i(g(\mathbf{x}) - \mu_g)^r) \tag{8}$$

4.2. Calculation of the probability of failure for limit state function

The steps to calculate probability of failure for limit state functions based on maximum entropy approach can be summered as follows.

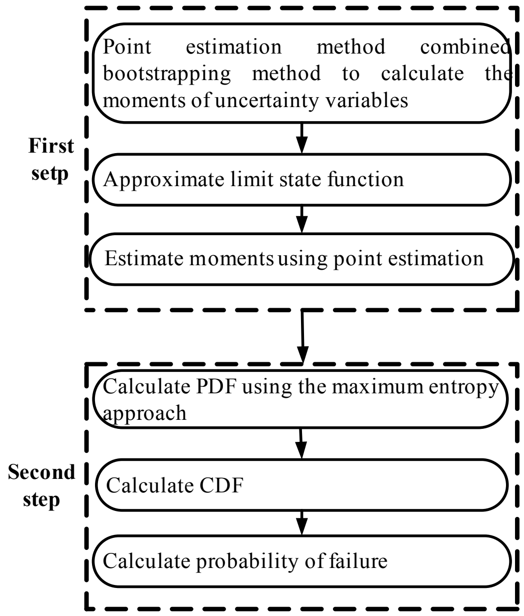

Fig. 1. Flowchart of the proposed method

(1) The first four moments of random variables are calculated by point estimation method combined bootstrapping method.
(2) Estimate moments of the limit state functions where only one random variable is involved, shown in Eq. (2).
(3) Estimate moments of the limit state function where n random variables are involved, shown in Eq. (4).
(4) Estimate PDF of the limit state functions according to Eq. (7) and Eq. (8).
(5) Calculate CDF and probability of failure.

The flowchart of the calculation process is shown in Figure 1.

5. Numerical examples

5.1. Disk edge design

The disk edge design problem used in [20] is expressed as in Eq. (9):

$$y = M_b = g(x) = \sqrt{\frac{fs}{3 \times 385.82\delta (N\frac{2\pi}{60})^2 (R^3 - R_0^3)(R - R_0)}} \tag{9}$$

where $\mathbf{x} = [f, s, \delta, N, R, R_0]^{\mathrm{T}}$; f is the material utilization; s is the tensile strength limit; δ is the density; N is the rotor speed; R is the outer radius; and R_0 is the inner radius.

Table 1. Distributions of random variables

Variable	Distribution type	Parameter 1	Parameter 2
f	Sample	-	-
s	Sample	-lb/in²	-
δ	Normal	0.28 lb/in³	0.30 lb/in³
N	Normal	21,000 rpm	1,000 rpm
R	Normal	24 in	0.5 in
R_0	Normal	8 in	0.3 in

The distributions information of the variables are given in Table 1.

The samples of design variable f and s are displayed as follows:

f=[0.9598 0.8596 0.8850 0.9389 0.9304 0.9346 0.9751 0.9649 0.9474 0.9725 0.8936 0.9767 0.9730 0.9796 0.9728 0.9511 0.9638 0.8676].

s=[215560 215800 224130 215880 218690 219900 220500 226770 226890 212110 211250 219830 217280 214640 214710 222540 218570 216010].

According to the method proposed in Section 2, the confidence interval of moments for the limit state function are shown in Table 2.

Table 2. Confidence interval of moments for the limit state function

First moment	Second moment	Third moment	Fourth moment
$[2.1317, 2.88]\times10^{-5}$	$[5.376, 7.2734]\times10^{-10}$	$[1.3634,1.8446]\times10^{-14}$	$[3.4772,4.7044]\times10^{-19}$

The PDF of the limit state function at the lower bound and upper bound can be expressed as Eq. (10) and Eq. (11) according to the maximum entropy approach:

$$\begin{aligned} f_{lower}(g) = \exp(&0.5153 - 1.26\times10^6 \times g - 3.79\times10^7 \times g^2 \\ &+8.76\times10^{15}\times g^3 \text{-} 2.39\times10^{20}\times g^4) \end{aligned} \tag{10}$$

$$\begin{aligned} f_{upper}(g) = \exp(&8.8779 \text{-} 4.46\times10^5 \times g - 1.0036\times10^9 \times g^2 \\ &+1.032\times10^{16}\times g^3 \text{-} 2.1549\times10^{20}\times g^4) \end{aligned} \tag{11}$$

The comparisons for the PDF and CDF of the limit state function from the proposed method and MCS are displayed in Fig. 2 and Fig. 3, respectively.

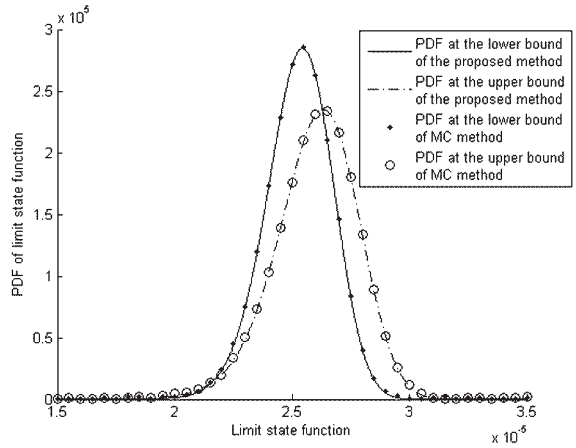

Fig. 2. PDF of the limit state function for disk edge design

5.2. Fortini's clutch problem

The second example is the over running clutch assembly known as Fortini's clutch [17]. The contact angle y in is de-

Table 3. Distribution information for variables

Variable	Distribution type	Mean value[mm]	Deviation[mm]	Parameters
x_1	Beta	55.29	0.0793	q=r=5.0
x_2	Normal	22.86	0.0043	
x_3	Normal	22.86	0.0043	
x_4		Sample		

The samples of design variable are listed as follow.

$\mathbf{x}_4$ = [154.4042,107.4187 115.6844 145.8643 156.3655 156.9087 109.7149 193.6139 158.2305 212.9646 205.6109 383.8824 231.2218 130.8089 110.0401].

termined by the independent random variable, x_1, x_2, x_3, x_4 as shown in Eq. (12). The distribution of design variables is displayed in table 3.

$$y = \arccos\left[2x_1 + (x_2 + x_3)/2x_4 - (x_2 + x_3)\right] \quad (12)$$

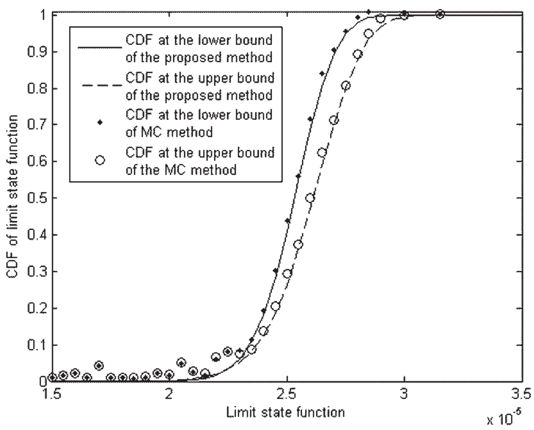

Fig. 3. CDF of the limit state function for disk edge design

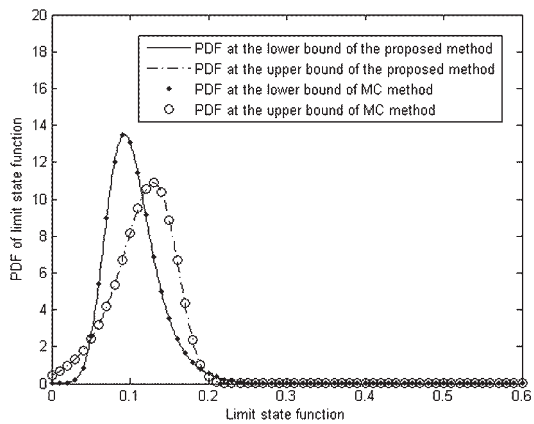

Fig. 4. PDF of the limit state function for clutch

The confidence interval of the first four moments for the limit state function are given in Table 4. And the PDF of the limit state function at the bounds are expressed by Eq. (13) and Eq. (14).

The comparisons for the PDF and CDF of the limit state function from the proposed method and MCS are displayed in Fig. 4 and Fig. 5, respectively.

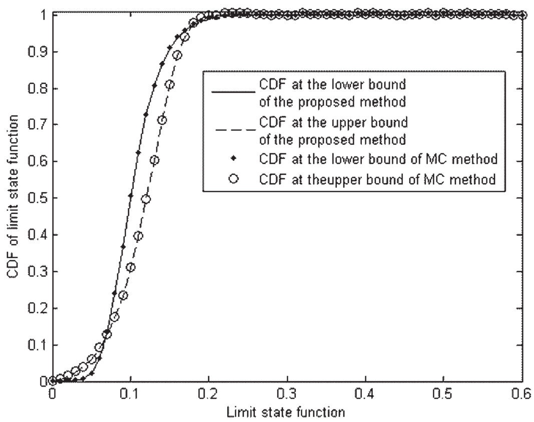

Fig. 5. CDF of the limit state function for clutch

6. Conclusions

In this paper, an uncertainty analysis method with bootsrapping method combined maximum entropy method is proposed. The exact distribution functions of some random variables are not determined using a limited mumber of observations. Therefore, the bootstrapping method is used to estimate the confidence intervals for the stochastic moments of the random variables. Further, the confidence interval of PDF and CDF for the limit state functions are calculated using maximum entropy approach.

In the proposed method, neither derivative nor the MPP search are needed. And the random variables are not needed to be transformed into standard normal distribution. The comparison of results form the proposed method with MC method presents the accuracy of the proposed method.

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