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STEREO MATCHING AND DISPARITY CALCULATION BASED ON DISCRETE ORTHOGONAL MOMENTS OF CHEBYSHEV, LEGENDRE AND ZERNIKE

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Summary: In the article we present various theoretical and experimental approaches to the problem of stereo matching and disparity estimation. We propose to calculate stereo disparity in the moments space, but we also present numerical and correlation based methods. In order to calculate disparity vector we decided to use discrete orthogonal moments of Chebyshev, Legendre and Zernike. In our research of stereo disparity estimation all of these moments were tested and compared. Experimental results confirm effectiveness of the presented methods of determining stereo disparity and stereo matching for machine vision applications.

Keywords: orthogonal moments, stereo matching, stereo disparity

1. INTRODUCTION

One of the main research fields in machine and robotics vision is 3D scene perception based on techniques of measuring shapes, positions and relations between 3D objects that are visible in the scene. There are many known methods of retrieving information about the scene basing on 3D perception [2-4,6]. Those methods are based on disparity of stereoscopic scene elements and on information from the common part of stereoimages. After extracting the pair of corresponding points in two stereoimages, which are related to the same point in the scene, we can define the difference between coordinates of those points. Then basing on such differences, it is possible to create depth map for the visible scene by means of simple trigonometric transformations [18].

Recently, discrete orthogonal moments have gained much attention and have been successfully used in many applications of computer vision (e.g. pattern recognition) [1,9,14,16,17]. Therefore, in our research we decided to take advantage of discrete orthogonal moments' properties, in particular those introduced by Chebyshev, Legendre and Zernike, in order to calculate stereo disparity.

In the article, we are concerned with the images acquired from the axe-parallel robotics vision system. In such a system the optical axes of both cameras are parallel to each other and the image planes of stereoscopic pair are situated within the same distance from the centre of the scene coordinates system. Only the common scene area covered by both cameras is further analysed (even though it is only a part of each of the image). In section 2, issues related to stereo-image acquisition in robotics vision system are described in details. In section 3, the discrete orthogonal moments of Chebyshev, Legendre and Zernike are introduced. In section 4, the method of displacement vector calculation in order to determine the margins and common part of stereo images is presented. Then, in Section 5, three approaches to calculate stereo disparity are described. Experimental results, discussion and conclusion are given in the next sections.

2. ACQUISITION OF THE STEREO IMAGES

Hereby we present the model of the used stereo camera system for robotics vision. Moreover, the principles of epipolar geometry and the method of camera system calibration are presented [5,28].

2.1. MODEL OF THE CAMERA SYSTEM

The model of the camera system represents geometrical and physical parameters of the cameras and transfers cameras' coordinates systems onto predefined coordinates system of the visible scene. A description of the model is necessary for correct measuring of the scene objects' shapes, sizes, position, direction and relations within the scene space [20].

Differences in localisation and directions of the cameras and, therefore in cameras' coordinates systems, determine various models of camera systems as presented in Fig. 1. In axe-parallel camera system the coordinates of the point $A^S(X,Y,Z)$ of the left camera $O^{K_L}XYZ$ and the right camera $O^{K_P}XYZ$ related to the scene O^SXYZ are given.

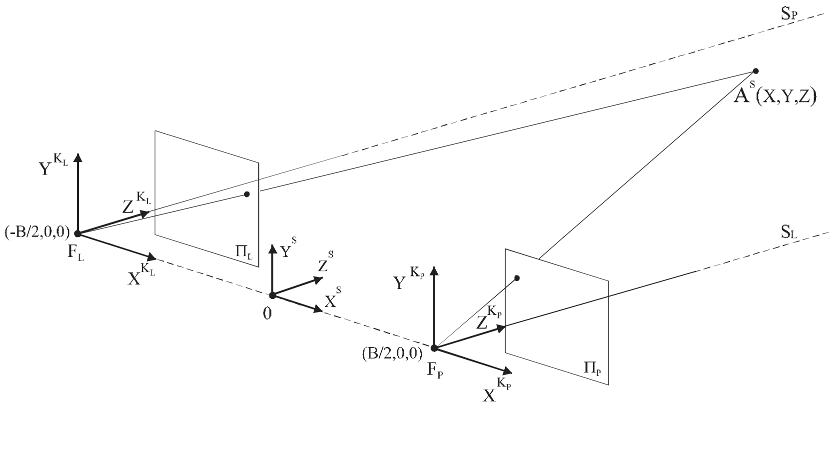

Fig. 1. The axe-parallel model of the stereo cameras vision system

$$\begin{bmatrix} X^{K_L} \\ Y^{K_L} \\ Z^{K_L} \end{bmatrix} = \begin{bmatrix} X^S - \frac{B}{2} \\ Y^S \\ Z^S \end{bmatrix}, \qquad \begin{bmatrix} X^{K_P} \\ Y^{K_P} \\ Z^{K_P} \end{bmatrix} = \begin{bmatrix} X^S + \frac{B}{2} \\ Y^S \\ Z^S \end{bmatrix} \tag{1}$$

The mapping of the point $A^S(X,Y,Z)$ onto the image planes Π_L (left) and Π_P (right) can be written as:

$$\begin{bmatrix} x^{\Pi_L} \\ y^{\Pi_L} \end{bmatrix} = \begin{bmatrix} f\frac{X^S - \frac{B}{2}}{Z^S} \\ f\frac{Y^S}{Z^S} \end{bmatrix}, \qquad \begin{bmatrix} x^{\Pi_P} \\ y^{\Pi_P} \end{bmatrix} = \begin{bmatrix} f\frac{X^S + \frac{B}{2}}{Z^S} \\ f\frac{Y^S}{Z^S} \end{bmatrix}. \tag{2}$$

Then, we can define the coordinates of the point $A^S(X,Y,Z)$ in the scene as:

$$\begin{bmatrix} X^S \\ Y^S \\ Z^S \end{bmatrix} = \begin{bmatrix} \frac{B\left(x^{\Pi_L} + x^{\Pi_P}\right)}{2d} \\ \frac{By^{\Pi_L}}{d} \\ \frac{Bf}{d} \end{bmatrix}, \tag{3}$$

where d is defined as the stereoscopic discrepancy as the difference between projection coordinates of point $A^S(X,Y,Z)$ onto image planes Π_P and Π_L, such as:

$$d = x^{\Pi_P} - x^{\Pi_L}. \tag{4}$$

Stereoscopic images acquired from the axe-parallel camera system can contain errors after the matching process if the chosen image elements do not have the correspondence elements on the second stereo-pair image. The solution for such a problem is to find the translation vector between stereo-images and defining the common part of the left and right image [20]. An example of a stereo-image pair acquired and calibrated in the described system is presented in Figure 2.

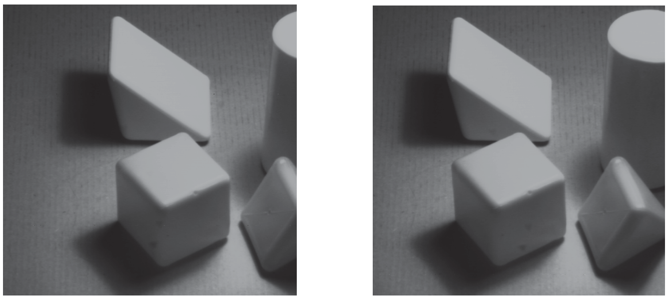

Fig. 2. The stereo-image „blocks" acquired in the axe-parallel camera vision system

2.2. EPIPOLAR GEOMETRY

An important characteristic of stereoscopic systems is the epipolar geometry, meaning that the projections of any point in the scene $A^S(X,Y,Z)$ onto the image planes Π_P and Π_L are localised on the corresponding epipolar lines. Each point placed on the epipolar line of the right image has its corresponding point on the analogical epipolar line of the left image. This relation is true only if we consider the common area of both cameras vision space [4].

The line between the focuses of the left and right cameras in the stereoscopic system is called a base-line, and the length of this line is called a base. The system consisting of the base-line and any point in the scene $A^S(X,Y,Z)$ determines unambiguously the epipolar plane. The intersection of the epipolar plane with the image planes Π_P and Π_L defines the epipolar lines. The number of detected epipolar lines is connected with the resolution of the camera system.

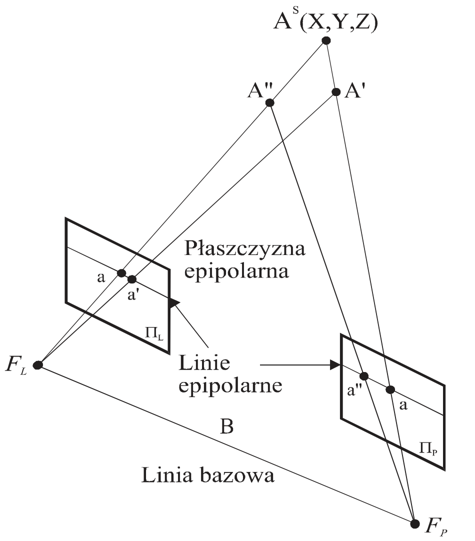

Fig. 3. Epipolar geometry in the stereoscopic camera system

2.3. CALIBRATION OF CAMERA SYSTEM

In order to perform the proper acquisition of stereoscopic images referring to the visible scene it is crucial that the camera system is properly calibrated. The process of calibration of the stereo-system is supposed to define relations between coordinates of the point in the scene $A^S(X,Y,Z)$, known coordinates of that point projection $a^{\Pi L}(x,y)$ and $a^{\Pi P}(x,y)$ onto the image planes Π_L and Π_P, and the geometrical and optical parameters of the camera system. Such parameters are the position and direction of cameras in relation to the defined coordinates system of the scene described by elements of the rotation matrix ΔR and the translation vector ΔT. Those parameters can determine the position and direction of the cameras in relation to the predefined coordinates system of the scene [27]. We have:

$$\begin{bmatrix} X^{K_L} \\ Y^{K_L} \\ Z^{K_L} \end{bmatrix} = \Delta R^{K_L} \begin{bmatrix} X^S \\ Y^S \\ Z^S \end{bmatrix} + \Delta T^{K_L}, \qquad \begin{bmatrix} X^{K_P} \\ Y^{K_P} \\ Z^{K_P} \end{bmatrix} = \Delta R^{K_P} \begin{bmatrix} X^S \\ Y^S \\ Z^S \end{bmatrix} + \Delta T^{K_P}. \quad (5)$$

The translation vector describes the localization of centre points of the camera systems coordinates. The rotation matrix is the orthogonal matrix and its elements are the Euler angles that characterize orientation of camera coordinates systems axis.
The optical parameters are:

- scaling ratios k_x and k_y of the axis Ox^E and Oy^E,
- parameters p_x and p_y of the coordinates systems translations $O^E xy$ and $O^\Pi xy$,
- position of the center point of the coordinates system $O^E xy$,
- length *f* of camera focus,
- geometrical distortions in the camera optical system ϑ_x^Π, ϑ_y^Π.

Then:

$$\begin{bmatrix} x^\Pi \\ y^\Pi \end{bmatrix} = \begin{bmatrix} k_x x^E + p_x + \vartheta_x^\Pi(x,y) \\ k_y y^E + p_y + \vartheta_y^\Pi(x,y) \end{bmatrix}. \quad (6)$$

Description of all the geometrical and optical parameters of the stereoscopic camera system (for left and right camera separately) is the condition for proper measuring of the 3D objects' shapes, position and orientation [10].

The calibration problem is the problem of finding the values of geometrical and optical parameters for a considered camera system in order to define position and orientation of cameras in relation to the scene coordinates system basing on the images acquired from those cameras [26]. The solution of the calibration problem consists of:

- finding optical parameters and the values of distortion introduced by the lenses of both cameras,
- determining the correspondence list for features in the stereo-images,
- calculating the values of geometrical parameters (location and cameras orientation based on the obtained correspondence map).

Moreover, the feedback loop should be taken into account while analysing the camera calibration process. Then, the calculated parameters of position and orientation of the cameras can be tuned recursively until the desired parameters for a specified camera model are achieved. In result of the camera calibration the optical and geometrical parameters characterizing the acquired stereo-images are achieved [11].

3. DISCRETE ORTHOGONAL MOMENTS

In this section, three types of discrete orthogonal moments are presented: Chebyshev and Legendre moments which are realized in the cartesian coordinates system, and Zernike moments which are realized in the polar coordinates system. Due to orthogonalization of their base-functions these moments are characterized by fast algorithms of realization and by simple reconstruction formulas. Hence, these moments are calculated using Geometric moments, Central moments and scaled Geometric moments [7,15,23,24].

3.1. GEOMETRIC MOMENTS

The Geometric moments (GM) m_{ij} of order $i+j$ of a digital image $I(x,y)$ (left or right image of stereopair) of the size $N \times N$ are defined as [13,19]:

$$m_{ij} = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} x^i y^j I(x,y), \tag{7}$$

where $i, j = 0,1,...,N-1$.

The translation invariant Central moments M_{ij} are obtained by placing origin at the centroid of the image.

$$M_{ij} = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} (x-\overline{x})^i (y-\overline{y})^j I(x,y), \tag{8}$$

where

$$\overline{x} = \frac{m_{10}}{m_{00}}, \qquad \overline{y} = \frac{m_{01}}{m_{00}}. \tag{9}$$

Then, the scale invariant Central moments C_{ij} are defined as:

$$C_{ij} = \frac{M_{ij}}{\alpha^{(i+j+2)/2}}, \tag{10}$$

where $\alpha = M_{00}$.

Finally, the scale invariant Radial-Geometric moments R_{ij} are defined as:

$$R_{ij} = \frac{\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\left(\hat{x}^2+\hat{y}^2\right)^{\frac{1}{2}}\left(\hat{x}\right)^i\left(\hat{y}\right)^j I\left(x,y\right)}{\alpha^{\left(i+j+3\right)/2}}, \tag{11}$$

where $\hat{x} = x - \overline{x}$ and $\hat{y} = y - \overline{y}$.

3.2. CHEBYSHEV MOMENTS

The Chebyshev moments (TM) of order $m+n$ of an image $I\left(x,y\right)$ (left or right image of stereopair) of the size $N \times N$ are defined using the scaled orthogonal Chebyshev polynomials $t_n\left(x\right)$, as [14,23]:

$$TM_{mn} = \frac{1}{\rho\left(m,N\right)\rho\left(n,N\right)}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} t_m\left(x\right)t_n\left(y\right)I\left(x,y\right), \tag{12}$$

Where $m,n = 0,1,...,N-1$ and $\left\{t_n\left(x\right)\right\}$ are a set of discrete orthogonal polynomials satisfying the following condition:

$$\sum_{n=0}^{N-1} t_m\left(x\right)t_n\left(x\right) = \rho\left(n,N\right)\delta_{mn}, \tag{13}$$

Where δ_{mn} is the Kronecker function:

$$\delta_{mn} = \begin{cases} 1 & if \quad m-n, \\ 0 & otherwise \end{cases}. \tag{14}$$

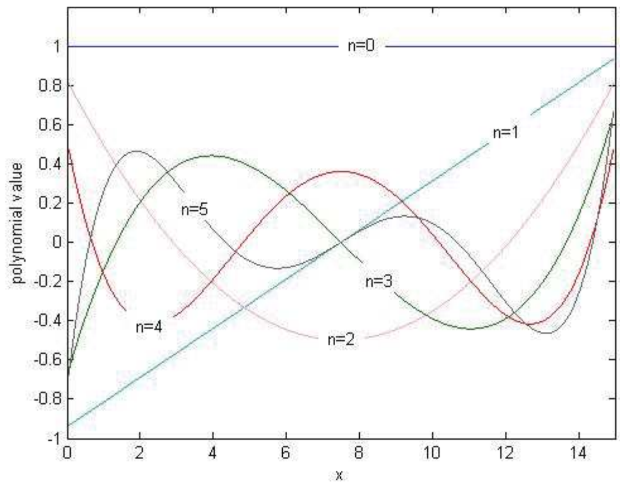

Fig. 4. Chebyshev polynomials $t_n(x)$ of order $n = 0,1,\ldots,5$

The Chebyshev polynomials satisfy the property of orthogonality (13), with:

$$\rho(n,N) = \frac{(N-1)(N-2^2)...(N-n^2)}{(2n+1)N^{2n-1}} \tag{15}$$

and have the following recurrence relation:

$$(n+1)t_n(x) - (2n+1)(2x-N+1)t_n(x) + n(1-n^2N^{-2})t_{n-1}(x) = 0\,, \tag{16}$$

where $n = 2,3,...,N-1$ and $x = 0,1,...,N-1$ with the initial conditions:

$$t_0(x) = 1\,, \quad t_1(x) = (2x-N+1)/N\,. \tag{17}$$

The inverse formula of Chebyshev moments is given by the following equation:

$$I(x,y) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} TM_{mn} t_m(x) t_n(y)\,. \tag{18}$$

The Chebyshev moments of the same image may be expressed in terms of geometric moments as follows [14]:

$$TM_{mn} = A_m A_n \sum_{k=0}^{m} C_k(m,N) \sum_{l=0}^{n} C_l(n,N) \sum_{i=0}^{k}\sum_{j=0}^{l} s_k^{(i)} s_l^{(j)} m_{ij}\,, \tag{19}$$

where

$$A_m = \frac{1}{N^n \rho(m,n)}\,, \tag{20}$$

and

$$C_k(n,N) = (-1)^{n-k} \frac{n!}{k!} \binom{N-1-k}{n-k} \binom{n+k}{n}\,, \tag{21}$$

and $s_k^{(i)}$ are the Stirling number of the first kind, which satisfies:

$$\frac{x!}{(x-k)!} = \sum_{i=0}^{k} s_k^{(i)} x^i\,. \tag{22}$$

The explicit expressions of the Chebyshev moments in terms of geometric moments up to the first order are as follows:

$$TM_{00} = \frac{m_{00}}{N^2}\,,$$

$$TM_{10} = \frac{6m_{10} + 3(1-N)m_{00}}{N(N^2-1)}\,, \tag{23}$$

$$TM_{10} = \frac{6m_{01} + 3(1-N)m_{00}}{N(N^2-1)}\,.$$

3.3. EGENDRE MOMENTS

The Legendre moments (*LM*) of order $r+s$ of discrete image $I(x,y)$ are defined as [12,21]:

$$LM_{rs} = \frac{(2r+1)(2s+1)}{N^2} \sum_{s=0}^{N-1} \sum_{r=0}^{N-1} P_r(\zeta_x) P_s(\zeta_y) I(x,y), \tag{24}$$

where $r,s = 0,1,\ldots,N-1$ and the image coordinate transformation is given by:

$$\xi_x = \frac{2x-N+1}{N-1}, \quad \xi_y = \frac{2y-N+1}{N-1}, \tag{25}$$

where $P_s(\xi_y)$ is the Legendre polynomial of degree s and ζ_x, ζ_y are the normalized coordinates.

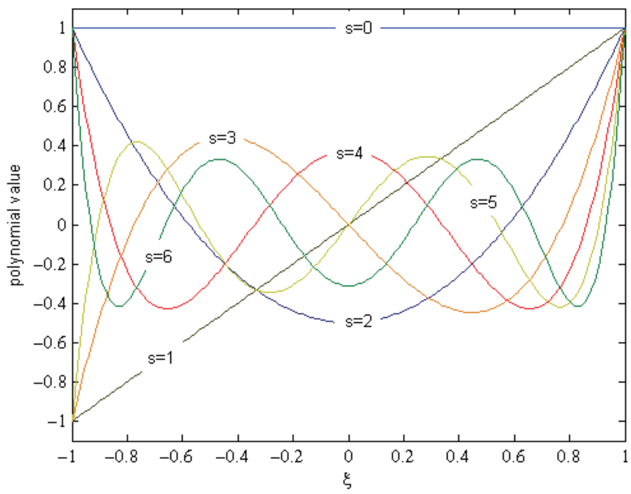

Fig. 5. Legendre polynomials $P_s(\xi_y)$ of order $s = 0,1,\ldots,6$

The Legendre polynomials $P_s(\xi_x)$ are a complete orthogonal basis set within the interval $[-1,1]$, for an order s and satisfying the following condition:

$$\int_{-1}^{1} P_r(\xi_x) P_s(\xi_x) d\xi_x = \frac{2}{2r+1}\delta_{rs}. \tag{26}$$

The Legendre polynomials are defined by:

$$P_s(\xi_x) = \frac{1}{2^s s!} \frac{d^s}{d\xi_x^s} \left(\xi_x^2 - 1\right)^s. \tag{27}$$

The inverse moment transform (only Legendre moments of order $\leq N$) which follows from the orthogonality of Legendre polynomials in the discrete domain, can be approximated by [12]:

$$I(x,y) \approx \sum_{r=0}^{N-1}\sum_{s=0}^{r} LM_{r-s,s} P_{r-s}(\xi_x) P_s(\xi_y). \tag{28}$$

The Legendre moments can be computed by Geometric moments as follows:

$$LM_{rs} = \frac{(2r+1)(2s+1)}{N^2}\sum_{k=0}^{r}\sum_{l=0}^{s} B_{rk} B_{sl} m_{kl}, \tag{29}$$

where

$$B_{rk} = \begin{cases} 0 & \text{if} \quad r-k = odd, \\ \frac{1}{2^k}(-1)^p \binom{r}{p}\binom{2r-2p}{r} & \text{if} \quad r-k = even. \end{cases} \tag{30}$$

and $p = \frac{r-k}{2}$.

3.4. ZERNIKE MOMENTS

Zernike moments (ZM) are the projection of the image $I(x,y)$ on the orthogonal basis $V_{pq}(x,y)$. The Zernike moments of order p with repetition q are defined as follows [8,25]:

$$ZM_{pq} = \frac{p+1}{\pi}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1} I(x,y) V_{pq}^{*}(x,y). \tag{31}$$

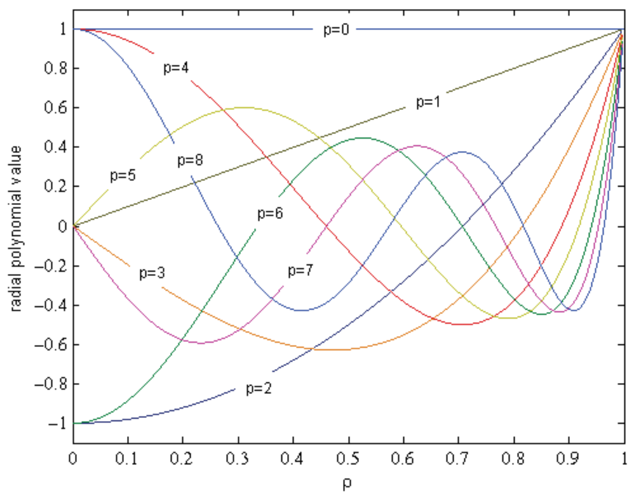

Fig. 6. Zernike polynomials of order $p = 0,1\ldots8$ and $q = 0$ or 1

The Zernike polynomials:

$$V_{pq}\left(x,y\right)=R_{pq}\left(\rho\right)\exp\left(jq\theta\right) \tag{32}$$

are a complete set of complex valued functions orthogonal on the unit disk D: $x^2+y^2\le 1$, where $p\ge 0$, and $p-|q|$ is even positive integer.

The polar coordinates (ρ,θ) in the image domain are related to the Cartesian coordinates (x,y) by:

$$\rho=\sqrt{x^2+y^2}\ , \quad \theta=\arctan(y/x)\ . \tag{33}$$

The Zernike polynomials $V_{pq}(x,y)$ are orthogonal basis set and satisfy the following condition:

$$\iint_D V^*_{pq}\left(x,y\right)V_{p'q'}\left(x,y\right)dxdy=\frac{\pi}{n+1}\delta_{pp'}\delta_{qq'}\ . \tag{34}$$

The radial polynomial $R_{pq}(\rho)$ is given by:

$$R_{pq}(\rho)=\sum_{l=0}^{(p-|q|)/2}(-1)^l\frac{(p-l)!}{l!\left(\frac{p+|q|}{2}-l\right)!\left(\frac{p-|q|}{2}-l\right)!}\rho^{p-2}\ . \tag{35}$$

If Zernike moments of order $\le N$ are given, then the image intensity $I(x,y)$ can be approximated by [22]:

$$I\left(x,y\right)=\sum_{p=0}^{N-1}\sum_{q=0}^{p}ZM_{pq}V_{pq}(x,y)\ . \tag{36}$$

4. STEREO DISPLACEMENT SEARCH

The characteristic feature of the presented robotics vision stereo system is that axe-parallel cameras' visual plane contain different elements of the registered scene. It means that not all the elements on the left and right image have their counterparts. The lack of such a relation eliminates such elements from analysis of stereo images. However, extraction of such image areas, called margins of left and right stereo image, unambiguously characterize common (overlapping) parts of stereo images (the most important parts for stereometry image analysis). The problem of determining the margins and the common part of stereopair images is a problem of appointing the displacement vector between those images acquired from the left and right camera. The value of that vector depends on geometrical and optical parameters of the camera system.

In order to find the displacement vector d_t between stereopair images we perform calculation of the discrete orthogonal moments of the right I_P and left I_L stereo image. We search for the displacement vector d_t by determining the minimum from the set of values:

$$\{U(0),U(1),...,U(d_{t\max})\} \tag{37}$$

calculated accordingly to (38) characterizing adequately subtraction of reconstructed images I_L and I_P regarding to maximal displacement vector $d_{t\max}$

$$U(d_t)=\sum_{x=0}^{N-d_t}\sum_{y=0}^{N-1}\left|I_L(x+d_t,y)-I_P(x,y)\right|+\sum_{x=0}^{d_t-1}\sum_{y=0}^{N-1}I_L(x,y)+\sum_{x=N-d_t}^{N-1}\sum_{y=0}^{N-1}I_P(x,y), \tag{38}$$

where $I_L(x+d_t,y)$ and $I_P(x,y)$ can be calculated on the basis of reconstructed intensity function values, for the Chebyshev (18), Legrande (28) and Zernike (36) moments, respectively.

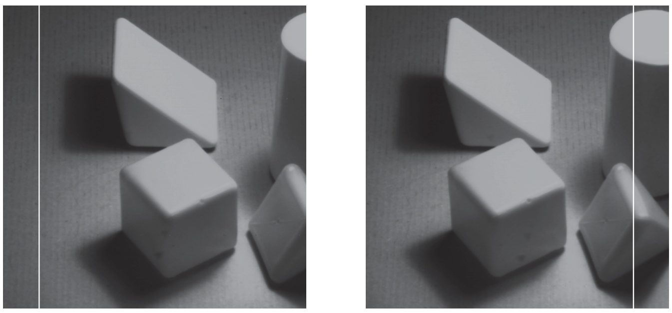

Fig. 7. The stereoscopic image „blocks" with the marked margins

5. MATCHING AND STREO DISPARITY CALCULATION IN MOMENTS SPACE

5.1. DISPARITY ESTIMATION USING NUMERICAL ANALYSIS

Stereo disparity d_x describes the difference in absolute localizations of the corresponding points in the epipolar lines. By analysing corresponding points of the left $I_L(x+d_t+d_x,y_e)$ and righ $I_P(x,y_e)$ images of stereopair we can define the following relation of their intensity function:

$$I_P(x,y_e)\cong I_L(x+d_t+d_x,y_e). \tag{39}$$

By utilizing the reconstruction formulas (18), (28) and (36) of the intensity function on the basis of the calculated moments for the stereopair images and by simplifying the factor dependent on y_e (characterizing the epipolar lines) we can denote:

– for the Chebyshev moments:

$$\sum_{m=0}^{N-1}\sum_{n=0}^{N-1}\left(TM_{mn}^{(P)}t_m(x)-TM_{mn}^{(L)}t_m(x+d_t+d_x)\right)\cong 0, \tag{40}$$

– for the Legendre moments:

$$\sum_{r=0}^{N-1}\sum_{s=0}^{r}\left(LM_{r-s,s}^{(P)}P_{r-s}\left(\frac{2x-N+1}{N-1}\right)-LM_{r-s,s}^{(L)}P_{r-s}\left(\frac{2(x+d_t+d_x)-N+1}{N-1}\right)\right)\cong 0, \tag{41}$$

– for the Zernike moments:

$$\sum_{p=0}^{N-1}\sum_{q=0}^{p}\left(ZM_{pq}^{(P)}V_{pq}(x,y_e)-ZM_{pq}^{(L)}V_{pq}(x+d_t+d_x,y_e)\right)\cong 0\,. \tag{42}$$

Solving equations (40-42) in order to determine the stereo disparity in the analytical way is not possible. Therefore, in practice numerical methods have to be used. Each equation can be written as:

$$F(d_x;x)=0 \tag{43}$$

or for the Zernike moments (42) as:

$$F(d_x;x,y_e)=0\,. \tag{44}$$

We use the known iterative method of Newton-Rahpson to calculate, with the desired accuracy, stereoscopic disparity d_x which is the argument of the function for given parameters x and y_e on the basis of the following formula:

$$d_x^{(i+1)}=d_x^{(i)}-\frac{F\left(d_x^{(i)}\right)}{F'\left(d_x^{(i)}\right)}, \tag{45}$$

where F' is a derivative of F (45) in the i-th iteration.

The starting parameter of the method is in our case $d_x^{(0)}=0$. The achieved values of d_x are verified by the following conditions and properties: d_x is an integer number from the $\langle -d_{\max}, d_{\max}\rangle$, where $d_{\max}$ is a maximal possible value of the stereo disparity for a given pair of stereo images.

5.2. STEREO MATCHING BASED ON CORRELATION OF MOMENTS

In order to determine the stereo disparity d_x of the common part of the stereopair, the corresponding points on the epipolar lines have to be found in the process of stereo matching. In practice, in this approach, we search for the correlation between reconstructed intensity functions (18), (28) and (36) of the images I_L and I_P in the regions bounded by the window function:

$$W_z(x, y_e) = \left\{ u, v \left| \begin{matrix} x - \frac{z}{2} \le u \le x + \frac{z}{2} \\ y_e - \frac{z}{2} \le v \le y_e + \frac{z}{2} \end{matrix} \right. \right\}, \quad (46)$$

where z characterizes the size of the window function, and (u, v) are the coordinates describing the localization of window $W_x(x, y_e)$.

The correlation matching process is realized in the common part of stereopair (determined by the vector d_t) according to the following steps:

– for each point of the right image $I_P(x, y_e)$ choose its neighbourhood by the window function $W_x(x, y_e)$, where (x, y_e) is the centre of the window W_x,
– for all the points from the linear neighbourhood of the left image $I_L(x)$ choose its neighbourhood by the window function $W_x(x + d_x, y_e)$, where d_x characterizes the stereo disparity interval and d_x is within $\langle -d_{\max}, d_{\max} \rangle$,
– for the determined points and their neighbourhoods search for the minimum of the function C_{SSD}:

$$C_{SSD}(d_x) = \sum_{(u,v) \in W_z(x,y)} \left[I_P(u, v_e) - I_L(u + d_x, v_e) \right]^2 . \quad (47)$$

The minimum of the C_{SSD} function determines the value of stereo disparity d_x for the matched points of the left and right stereopair image.

5.3. STEREO MATCHING BASED ON THE SIMILARITY OF VECTORS IN THE MOMENTS SPACE

An alternative approach to correlation matching method based on function (48) minimum search can be a similarity search in the feature vector space according to:

$$\Psi(d_x) = \min_{d_x} \left\| \lambda_x^{(L)} - \lambda_{x+d_x}^{(P)} \right\|, \quad (48)$$

where $\lambda_x^{(L)}$ is a vector consisting of moments values $\phi_i^{(L)}$ of the intensity function in a given window W_x with the centre in point (x, y_e) on the left image of stereopair given by:

$$\lambda_x^{(L)} = \left[\phi_1^{(L)}, \phi_2^{(L)}, \ldots, \phi_i^{(L)}, \ldots \phi_M^{(L)} \right]_{\phi_i \in W_z(x, y_e)} \quad (49)$$

And $\lambda_{x+d}^{(P)}$ is a vector consisting of moments values $\phi_i^{(P)}$ of the intensity function in a given window W_x with the centre in point $(x + d, y_e)$ on the right image of stereopair given by:

$$\lambda_{x+d_x}^{(P)} = \left[\phi_1^{(P)}, \phi_2^{(P)}, ..., \phi_i^{(P)}, ...\phi_M^{(P)}\right]_{\phi_i \in W_z(x+d_x, y_e)}. \tag{50}$$

The similarity measure between the vectors calculated in the moment space according to equations (12), (24) and (31) is calculated on the basis of the Euclidean distance. The obtained minimum of the similarity measure $\Psi(d_x)$ determines the stereo disparity d_x between points (x, y_e) on the left image and $(x + d_x, y_e)$ on the right stereo image.

6. EXPERIMENTAL RESULTS

In the experiments we used stereo images of resolution 512×512 and 256 greyscale levels. All the images were acquired by the well-calibrated axe-parallel camera system. Sample test images are presented in Fig. 2.

Before disparity estimation we calculate the displacement vector d_t between each image stereopair (in other words, we find the common part of stereo images). This stage is based on search of the global minimum of the function $U(d_t)$ in the interval $< 0; d_{t_{\max}} >$, where $d_{t_{\max}}$ is set as ¼ of the image resolution. The ideal displacement vector for the presented sample image blocks is 52 and was equal to the one obtained by the Chebyshev reconstruction formula (18). The results achieved for other moments were characterized by small errors in the case of the Zernike moments (36), and large errors for the Legendre moments (28). Such a situation was caused by imprecise approximations in the reconstruction formulas and by the larger sensitivity of Legendre moments on intensity function deformations in the process of stereoscopic projections. Moreover, the errors were caused by the high orders of the used moments.

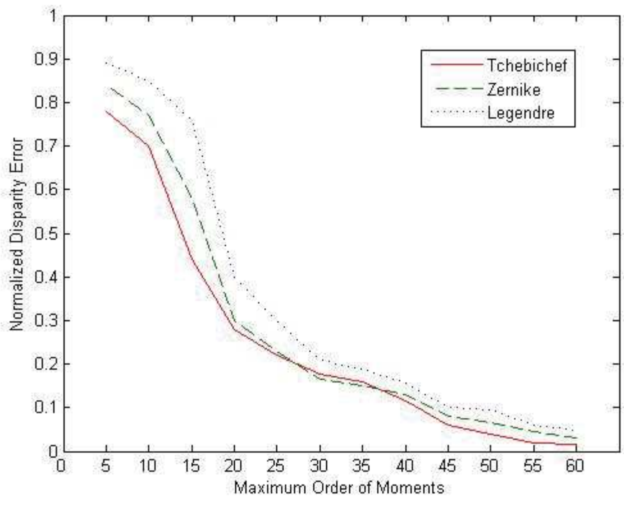

Fig. 8. Stereo disparity calculation error on the basis of the correlation of moments.

Stereo disparity d_x search by numerically solving the numerical equations requires the estimation of derivative of function F (e.g. by differential quotient). Moreover, it requires the verification of the obtained results on the basis of demanded intervals of their changes. In the experiments we found out that we can often achieve the proper values of stereo disparity d_x after a small number of iterations if the starting value is set to $d_x^{(0)} = 0$. Even though, in general, this method is not computationally effective, it could be used as a verification tool for results obtained by other methods of stereo disparity calculation.

In order to verify the proposed methods of stereo disparity calculation (sections 5.2-5.3), we use the normalized disparity error, given by:

$$NDE = \frac{1}{N \times N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \frac{\left| d_x\left(x, y_e\right) - \hat{d}\left(x, y_e\right) \right|}{d_{x\max}}, \tag{51}$$

where $d_x\left(x, y_e\right)$ is the calculated stereo disparity in the specified point $\left(x, y_e\right)$ and $\hat{d}\left(x, y_e\right)$ is the ideal stereo disparity calculated by other methods (e.g. correlation matching) and $d_{x_{\max}}$ is the maximal stereo disparity for the given image.

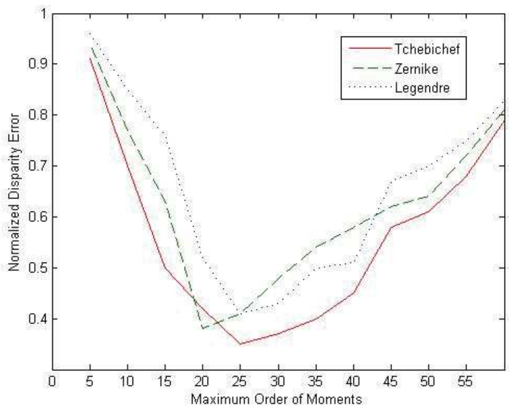

Fig. 9. Stereo disparity calculation error on the basis of the similarity of vectors in the feature space

Due to such formula (51), the normalized stereo disparity error values are varying within the interval $<0;1>$. In the Fig. 8. we presented the influence of the number of the used moments on the NDE. It decreases with the larger number of the used moments. Such situation reflects the impact of moment order on the reconstruction precision of the image intensity function (eq. 18, 28, 36 for different moments).

The results of stereo disparity estimation method implementation on the basis of the vectors similarity in the moments space and an interesting phenomenon of moments order adjustment are shown in the Fig. 9. The obtained value of NDE is optimal for the moments of order 20-25. Then, with the higher order of moments the results become worse, which is caused by the increased distance between vectors (48) in the Euclidean space.

7. SUMMARY

In the article, we presented the idea and implementation of using discrete orthogonal moments of Chebyshev, Legendre and Zernike in the process of matching and stereo disparity estimation. In order to optimalize those procedures, in the first stage we extracted the common parts of stereo images (which is important for matching) and the margins of stereo images.

In the article, three approaches to the problem of stereo disparity estimation were presented and tested. In the first method, we performed the estimation of stereo disparity d_x by numerically solving equations (40-42). The second approach was based on the correlation analysis of the reconstruction of image intensity function on the basis of discrete orthogonal moments. In the third approach, the problem of stereo disparity estimation was solved by similarity search in the vector space for the calculated moments characterizing the corresponding points of stereo images.

In the described methods we used the discrete orthogonal Chebyshev, Legendre and Zernike moments. After experiments we concluded that Chebyshev and Zernike were the most appropriate for stereo estimation moments, respectively. Much worse results were achieved by Legendre moments.

In the further work, we experiment with other discrete orthogonal moments applied to stereo disparity estimation task. We also try to analytically estimate the influence of moments values on stereo disparity d_x .Such a value of d_x could be used for optimalization and enhancement of the stereopair matching process.

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PASOWANIE I OBLICZANIE NIEZGODNOŚCI STEREOSKOPOWEJ NA PODSTAWIE DYSKRETNYCH MOMENTÓW ORTOGONALNYCH CHEBYSHEVA, LEGENDRE'A I ZERNIKE'A

Streszczenie

W artykule przedstawiono teoretyczne i eksperymentalne podejścia do problemu pasowania i oceny niezgodności stereoskopowej. Zaproponowano realizacje obliczeń niezgodności stereoskopowej w przestrzeni momentów otrogonalnych, jak również przedstawiono podstawy do obliczeń numerycznych i metod opartych na korelacji. W celu obliczania wektora niezgodności zdecydowano się na użycie dyskretnych momentów ortogonalnych Chebysheva, Legendre'a i Zernike'a. W procesie badawczym oceny niezgodności stereoskopowej wszystkie proponowane momenty były testowane i porównywane. Wyniki badań potwierdzają skuteczność prezentowanych metod określania niezgodności i pasowania stereoskopowego dla zastosowań widzenia maszynowego.

Słowa kluczowe: momenty ortogonalne, pasowanie stereo, niezgodność stereoskopowa