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THE USE OF COMPLEX SLIDING WINDOW DISCRETE FOURIER TRANSFORMATION IN CURRENT AND VOLTAGE DISTORTION ANALYSIS IN THREE-PHASE CIRCUITS

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Summary: The article concerns the aspects of vector analysis of current and voltage distorted waveforms in three-phase circuits from the point of higher harmonic content. Apart from that, the work contains the results of the analysis of current's vector harmonic based on the complex *SDFT*. These results have been obtained after having conducted a simulation. The article also tackles the proposal of accelerating the *SDFT* algorithm in order to facilitate its implementation in real-time systems.

Keywords: Complex Sliding Window Discrete Fourier Transformation, selective active power filtration, position vector spectrum analysis, voltage and current distorted waveforms

1. INTRODUCTION

The development of microprocessors' market has popularised vector steering methods, which require a relatively big number of calculations. These methods assume transforming a three-phase electric circuit into biaxial Cartesian system and using fictitious current and voltage space vectors that rotate in this spectrum [2, 3,5] (current and voltage are, in fact, scalar values). Basing on these fictitious vectors, including a relatively small amount of mathematical calculations, there is a possibility to determine a real vector of magnetic flux in an electric machine. This makes affecting the torque directly possible [8].

It turns out that not only are vector methods useful in driver automatics, but also in spectrum analysis of distorted waveforms. Information concerning current and voltage spectrum may, therefore, pose the initial data for active filtration algorithm, or even reactive power compensation [2, 5, 6]. However, harmonic analysis with the use of *DFT* requires many mathematical calculations, including complex multiplication and trigonometric functions [1]. These operations are extremely time-consuming for every microprocessor. Taking voltage or current analysis in a three-phase circuit into consideration, an investigated value should be analysed in each phase separately. Yet, after having conducted the distortion mentioned above, analysing only two waveforms suffice. In four-wire installations a zero-sequence component should also be considered. However, this component does not exist in three-wire circuits.

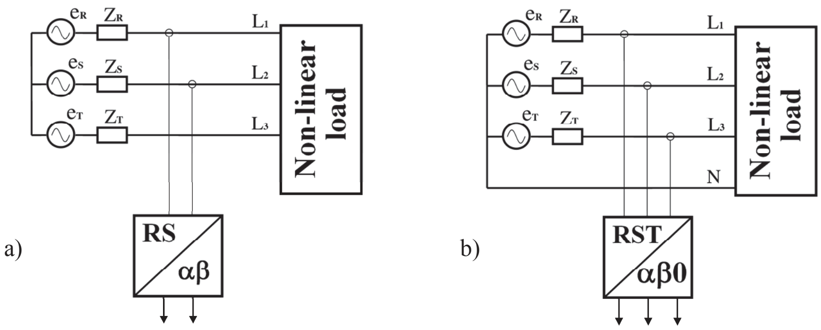


Fig. 1. The current measurement and change of coordinate system: a) in four-wire circuit, b) in three-wire circuit

In the latter, the measurement in one phase can additionally be omitted as according to Kirchhoff's circuit laws, the value in the third phase constitutes the vector total of the two remaining phases.

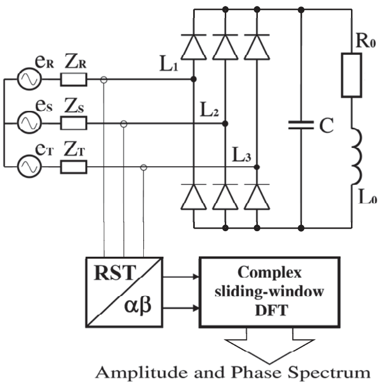


Fig. 2. Six-pulse diode rectifier as a source of higher harmonic of network current.

Since the components in a biaxial system are orthogonal, they can be treated as one complex signal in which one component has a role of a real part and the other of an imaginary part. In the result, two real signals ($\alpha\beta$) are not analysed, but only a complex one is taken into consideration. In fact, the amount of calculations in the process of further analysis is not reduced, as it was in the case of transition from three to two phases mentioned above. However, it still makes it possible to organise obtained information in a way that facilitates further operations (this will be described in details further in the article).

The object used in simulation researches is a six-pulse diode rectifier with RL load and a filtering capacitor in DC circuit. It is supplied from a symmetrical three-phase network (Fig. 2). Such a rectifier constitutes a non-linear load for the network, so it is a source of creating current’s higher harmonics. Due to the voltage drops on network impedance, which is dependent on current, it also creates voltage on clamps [2, 3, 5, 6].

2. DFT OF COMPLEX VARIABLE

DFT can be used for spectrum analysis of a signal. After having brought in the notion of current and voltage space vector in $\alpha\beta$ coordinate system, one can start analysing each compound separately. The trigonometrical form of *DFT* is as follows:

$$X(m) = \sum_{n=0}^{N-1} x(n) \cdot \left(\cos(2\pi nm / N) - j\sin(2\pi nm / N)\right). \tag{1}$$

where:

n – index of sample in time domain;
m – harmonic number;
N – amount of samples in a period of fundamental harmonic.

By substituting *x(n)* respectively by component values α and β of the analysed waveform, the spectra of two real signals are obtained.

Since α and β components are orthogonal, they can be treated as two components of one complex signal. With the assumption that α component stands for a real part, whereas β component for an imaginary part, complex variable *x(n)* can be presented as follows:

$$x(n) = \alpha\left(x(n)\right) + j\beta\left(x(n)\right), \tag{2}$$

or in a more formal way:

$$x(n) = \mathrm{Re}\left(x(n)\right) + j\,\mathrm{Im}\left(x(n)\right). \tag{3}$$

By substituting relation (3) to (1) and ordering, the following formulas are obtained:

$$\begin{aligned} X(m) = &\sum_{n=0}^{N-1}\left[\mathrm{Re}\left(x(n)\right)\cdot\left(\cos(2\pi nm/N) + \mathrm{Im}\left(x(n)\right)\cdot\sin(2\pi nm/N)\right)\right] + \\ &+ j\sum_{n=0}^{N-1}\left[\mathrm{Im}\left(x(n)\right)\cdot\left(\cos(2\pi nm/N) - \mathrm{Re}\left(x(n)\right)\cdot\sin(2\pi nm/N)\right)\right]. \end{aligned} \tag{4}$$

Relation (4) presents a trigonometrical form of *DFT* of complex variable. In this case, in order to determine one harmonic, one needs to conduct $4\times N$ trigonometrical multiplications (twice as many as in the case of real variable). These calculations are extremely time-consuming for a microprocessor. Therefore, taking the algorithm's efficiency into account, there is no difference between the analysis of one complex variable and two real components. In spite of that, a vector approach to current and voltage analysis is still of advance. There are several reasons for this stipulation. Firstly, during analysis of complex signal, onerous symmetry is omitted [1]. The phenomenon of this symmetry results in excessive m ≥ N/2 index harmonics and the necessity of their intentional omission in further analysis (still, onerous aliasing exists, which will be described further in the article). Another reason is obtaining additional information concerning the direction of vector's rotation. The information is obtained while analysing complex variable.

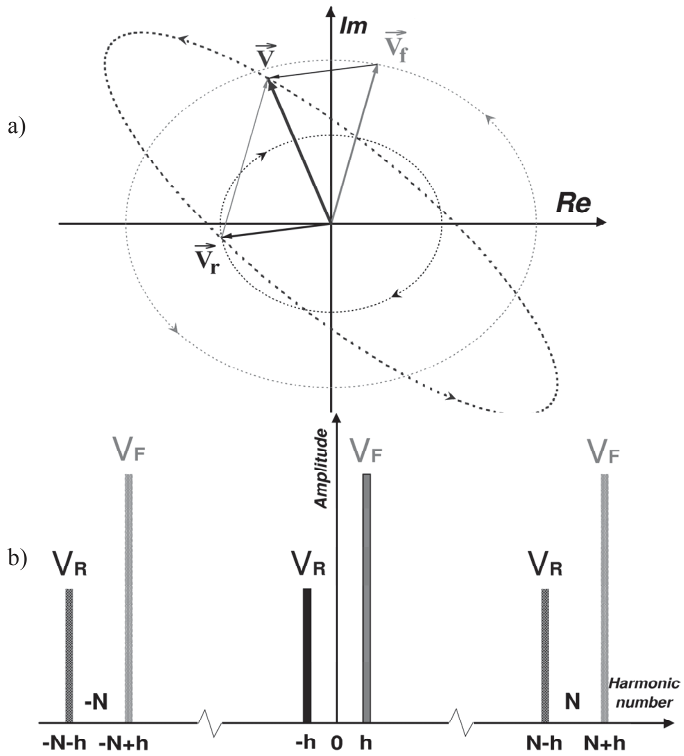


Fig. 3. Complex V signal constituting a total of forward component V_f and reverse component V_r: a) ellipse marked by a position vector totalling two circle-marking vectors, b) amplitude spectrum of a signal totalling two complex harmonics.

Let us take into consideration a case, where the components of α and β signal are sinusoidal waveforms with the same amplitude and frequency. If the phase shift between them equals ±90°, then the end of a position vector marks a circle as the Lissajous figure, while the rotation direction is dependent on the shift's sign. In such situations we talk about one harmonic of a complex signal with a positive number (forward rotation direction) or negative one (reverse direction).

Let us now move to the case where the shift between component α and component β varies 90°, or where the amplitudes of these components vary (or both). Then, the Lissajous figure has a form of an ellipse. It turns out that each complex signal whose position vector makes an ellipse can be split into two complex component signals. Those signals' vectors mark circles while rotating with the same pulsation, but in opposite directions. Each of these component signals ought to be treated as separate complex harmonic. Fig. 3a presents this situation, whereas Fig. 3b shows amplitude spectrum of such a signal. Here, the advantage of spectrum analysis of a complex signal over the spectrum analysis of each real component separately is proven, as the Fourier Transformation of a complex variable provides information on both, forward and reverse harmonics. In the case of analysing signal's real components separately, in order to generate forward and reverse harmonics, each component should undergo an additional treatment. Single harmonics of real α and β components may be, in general, presented as follows:

$$\begin{cases} \alpha(\Theta) = X_\alpha \sin(\Theta + \psi_\alpha) & (5) \\ \beta(\Theta) = X_\beta \sin(\Theta + \psi_\beta). & (6) \end{cases}$$

Since rotating vector that marks an ellipse is a total of two vectors rotating in opposite directions that mark circles, each coordinate α and β components of the vector constitute a total of two sinusoidal waveforms. The former represents Cartesian components of forward harmonic, whereas the latter stands for a reverse one.

Using the theorem that each sinusoidal waveform with X amplitude and ψ phase can be composed of the sum of waveforms not shifted in phase (both sinusoidal and cosinusoidal), with amplitudes A and B respectively, the following formula appears:

$$X \sin(\Theta + \psi) = A \sin\Theta + B \cos\Theta. \tag{7}$$

Relating (7) to (5) and (6), α and β cartesian components of forward and reverse harmonic are determined. Assuming limiting the solutions' set (7) to the case where A > 0 and B >0, the following system of equations is obtained:

$$\begin{cases} X = \sqrt{A^2 + B^2} & (8) \\ \psi = arctg\left(\frac{B}{A}\right). & (9) \end{cases}$$

These are additional, very time-consuming calculations that a computation unit would have to conduct in the case of separate analysis of α and β real components with a view to distinguishing forward and reverse harmonics out of the complex signal.

Fig. 3b illustrates h-numbered band with V_f amplitude (forward component) and $-h$–numbered band with V_r amplitude (reverse component). The remaining bands are created in the result of aliasing, which cannot be avoided while operating on discrete series. The bands are called "aliases", as they represent frequency components which, in reality, do not exist in the investigated signal [1]. Working on N sequence of samples in a period, h-numbered band will additionally appear in a place of each harmonic

$$h_{alias} = h + kN, \tag{10}$$

where:

k – any integer number.

The phenomenon of aliasing cannot be eliminated during operating on quasi-continuous domain. It is very onerous, as in $<-N \div N>$ bandwidth, according to (10), one is not able to distinguish harmonic value h from $-N+h$, or $-h$ from $N-h$. What can be done is to limit the bandwidth of the analysed spectrum or choose an appropriate number of input samples by selecting the sampling frequency.

Taking into account that a harmonic number equalling H is analysed, M amount of bands is expected, where

$$M = 2H + 1. \tag{11}$$

Multiplication by two results from the fact that for a certain pulsation a vector can rotate both in forward and reverse way. Thus, it has to be considered as two separate harmonics. Yet, adding one results from the possibility of constant component's existence, which should also be analysed. The amount of samples needs to be twice as big as the number of analysed bands

$$N = 2M \ . \tag{12}$$

Then, it is certain that no band from the positive bandwidth will appear in a negative part. One should bear in mind, however, that basing on (10), the components with frequencies higher than sampling frequency may come into being. Still, in practice, while analysing several dozen of current and voltage harmonics, the harmonics lying beyond the researched bandwidth are of insignificant value when compared to those within the bandwidth.

Through simulation, the spectrum of current's position vector in the circut presented in Fig. 2 has been researched. Fig. 4 shows the amplitudes and phases of separate current harmonics in the steady state. The results shown on the graphs prove that there are no even harmonics or harmonics with third multiple in the circut being researched [2, 3, 4]. The forward component of the first harmonic is significant (3,7A). At the same time the value of the fifth harmonic's reverse component (1,9A) is relatively high.

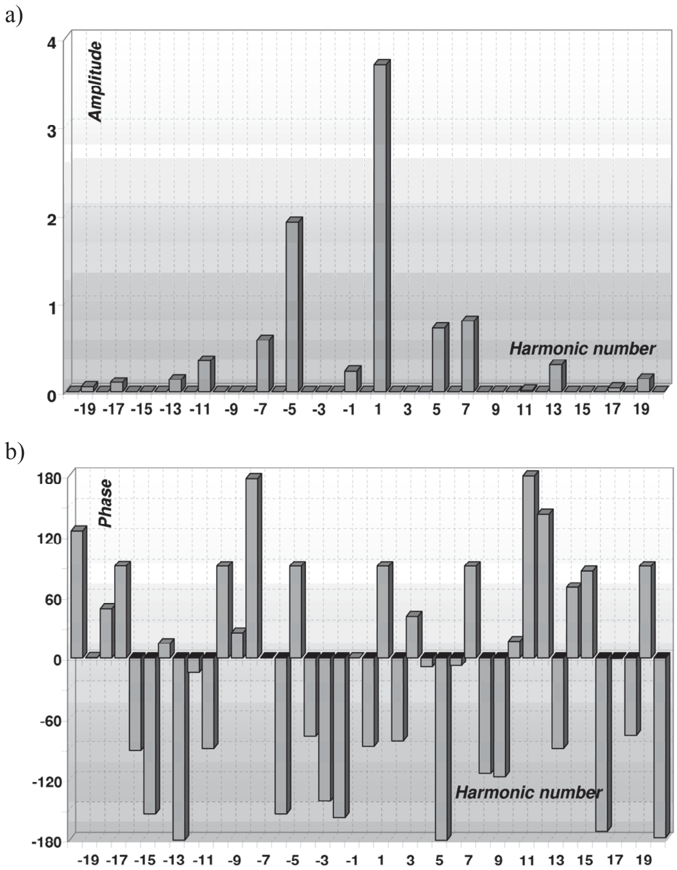


Fig. 4. The spectrum of current intaken by a six-pulse RL-loaded rectifier with a filtering capacitor in DC circuit: a) amplitude spectrum, b) phase spectrum

According to standards, in the case of determining THD factor for current, the analysis ought to include harmonic's measurement up to 40th number. In the results this number has been narrowed down to 20 in order to avoid darkening the picture with such a big amount of bands (40 complex harmonics give, according to (11), 81 bands).

3. COMPLEX SLIDING WINDOW DISCRETE FOURIER TRANSFORMATION

In classic understanding, the implementation of *DFT* is based on collecting *N* number of signal's samples (which constitute the period of fundamental harmonic), then, the sequence of calculations based on (1) is conducted on these samples' set for each *m* harmonic that is of interest. After having conducted all necessary calculations, *N* sequence of samples is collected again and the whole procedure is repeated. This solution, however, has two major disadvantages. First of them is a computation unit's uneven loading, as all the calculations have to be performed in a short time (between collecting the last sample of one calculations' series and the first sample of the next series). The other disadvantage is obtaining calculations' results only after having collected the last sample in the period of fundamental harmonic. In the case where the harmonic analysis is used for implementation of a selective active filtration [7], the information about higher harmonics' appearence might already be out-of-date at the very moment of its obtainment. In such situation, the test on reactions on transition states turns out to be pointless.

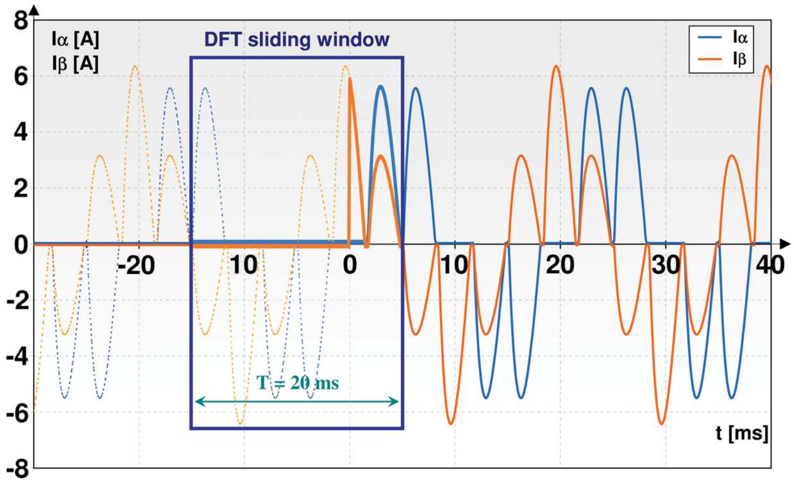
The effects of the first inconvenience can be lessened by using the *FFT* algorithm, which leads to the limitation of computation unit's loading [1]. That, in turn, requires undergoing extra operations, such as additional actions in the number of input samples' selection, taking actions on the windows to limit spectral leakage, etc. Still, the problem of processor's uneven loading can be entirely eliminated by using double buffering of collected samples. In this case, the samples of a present waveform to one buffer are collected, at the same time, the calculations are being done on the other one, where the samples from a previous period are stored. However, such a solution increases disadvantageous lags in obtaining up-to date results.

The useful feature of *DFT* is the fact that it does not matter from which point in the periodic waveform's analysis is begun. What does matter is analysing as many successive samples as constitute one period of fundamental harmonic [1]. The amplitude spectrum for a given periodic waveform will always be the same, no matter the phase of signal's cutaway in the buffer (window) of samples constituting one period. The phase spectrum, however, depending on the analysed waveform's phase, will show, for each harmonic, a shift proportional to the relation of samples's number on which the window has been shifted to the number of samples constituting a period of a given harmonic. This can be written as follows:

$$\varphi_m = 2\pi \frac{m}{N} n, \tag{13}$$

where:

- φ_m – stands for a phase shift of m-numbered harmonic,
- m – harmonic number for which a shift is calculated,
- N – number of samples constituting fundamental harmonic,
- n – number of samples on which the window's content has been shifted.



Iα [A]
Iβ [A]
DFT sliding window
Iα
Iβ
T = 20 ms
t [ms]
8
6
4
2
0
-2
-4
-6
-8
-20
10
0
10
20
30
40

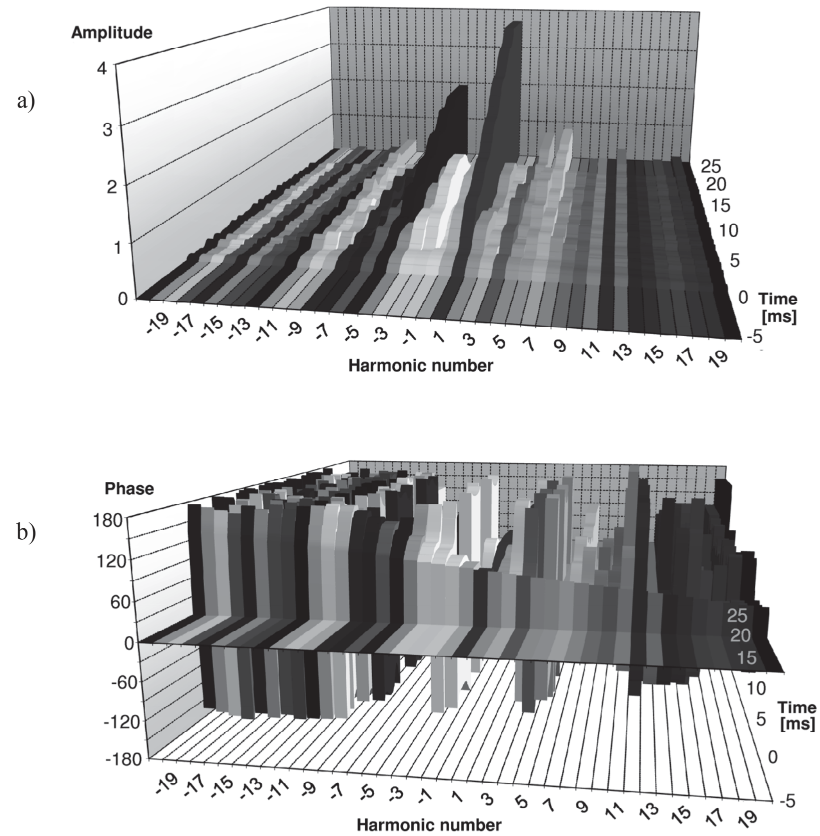


Fig. 6. Creation of complex signal's spectrum in the proceed of filling the sliding window: a) amplitude spectrum, b) phase spectrum

In the result of analysing αβ components of current's position vector in a steady state in a circuit presented in Fig. 2, the spectrum shown in Fig. 4 is obtained. Yet, let us take into account the case where the steady current's waveformis just being slipped in to the sliding window. This has been presented in Fig. 5. The process of slipping the data in to the window starts at the moment t=0. In the graph, the part of current's waveform from before analysis has been marked with the dashed line. The slipped in data overwrites initial zero-values and fills up the sliding window. Assuming that the fundamental harmonic's frequency equals 50 Hz, then the window's width equals 20 miliseconds and this is the period of time of its filling with samples.

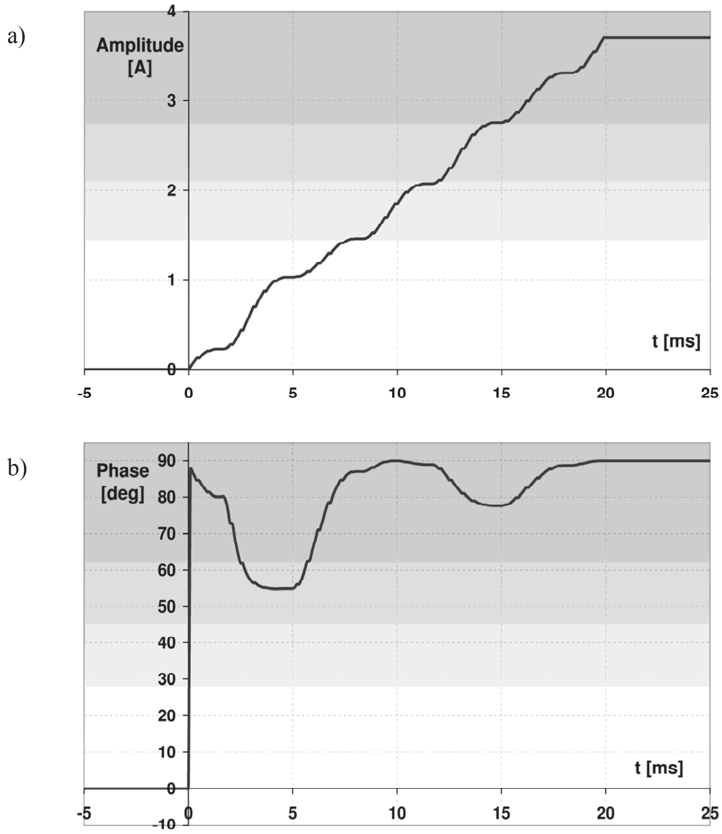


Fig. 7. Creation of the first harmonic of current's position vector in the process of filling the sliding window: a) amplitude's value, b) phase value

The process of spectrum's creation is shown in Fig. 6. The window was filled with initial zero-values at the moment of its creation in digital memory. That is why before filling the window with samples' values, spectrum's bands picture zero-values of both amplitudes and phases of all harmonics. At the agreed moment of t=0, the process of filling the window with the samples' values that project the investigated complex signal began. The amplitude spectrum's bands change their heights at time and they are fixed at correct values after 20 miliseconds. In the result, the spectrum's picture shown in Fig. 4a is obtained. One should pay attention to the fact that the harmonic's bands which, in reality, do not exist in a waveform, present non-zero amplitudes' values before the spectrum's determining. Phase spectrum can be presented in a similar way. Here, the bands are established on the correct angle's values after the period of time equalling the window's width. However, the changes of band's values of phase spectrum are more violent than the amplitude spectrum's ones. Additionally, the higher-numbered harmonic they concern, the more intensive these changes are.

Fig. 7 presents the course of creating the amplitude and phase of only the first harmonic's waveform. As far as the amplitude is established in a mellow way, the phase at the moment of slipping in the first sample to the window discrete reaches the value of 87,8°. During the insertion of next samples the value oscillates reaching the extreme values of 55°, 90° and 77° successively. Only after having entirely filled the window, can one state that the phase is determined at the value of 90° and the amplitude at 3,7A.

4. IMPLEMENTATION OF THE COMPLEX VARIABLE'S SDFT AND THE PROPOSITION OF ITS ACCELERATING

As it has been mentioned before, one of DFT's disadvantages is big and uneven loading of a computation unit with a great amount of calculations. Such loading takes place after each filling the buffer with the samples of an investigated waveform. Two buffers can be used in order to eliminate this phenomenon. Then, the former is filled with up-to-date samples, whereas the latter stores the samples from a previous signal and the calculations are performed on this buffer. At the same time, samples are collected to the first buffer. Such a solution decreases the loading's irregularity of a computation unit and makes the use of a cheaper, less efficient microprocessor possible. However, it delays appearing up-to-date calculations' results.

The *SDFT* algorithm updates the data concerning the whole spectrum after each new sample's collection. However, each conduction of calculations' cycle based on (1) would lead to loading a computation unit *N* times more than in the case of classic *DFT* implementation (which is already very time-consuming). Nevertheless, it turns out that there is no need to conduct such a big amount of mathematical calculations. In accordance with (1), *m*–number harmonic's determining resolves to counting up the sum of products of each *x(n)* sample's value in one signal's period with the following term:

$$\cos(2\pi nm / N) - j\sin(2\pi nm / N) \quad (14)$$

Memorising the up-to-date values of all *M* harmonics and values of all *N* products of

$$x(n)\cdot\left(\cos(2\pi nm / N) - j\sin(2\pi nm / N)\right), \quad (15)$$

one can, after each sample's collection, calculate a new value of m-number harmonic as its up-to-date value reduced by (15), calculated for the oldest sample in the window, and increased by the same (15), calculated for a newly collected sample. After having conducted all those actions, the oldest sample in the window is replaced by a newly calculated one.

According to the example above, calculating each harmonic's value after collecting a new sample resolves to one multiplication (15), one subtraction and one addition. Still, one has to bear in mind the fact that the term (14) appears in each multiplication that requires additional trigonometrical operations, and in the case of complex variable's SDFT, basing on (4), each multiplication requires counting up four trigonometrical products.

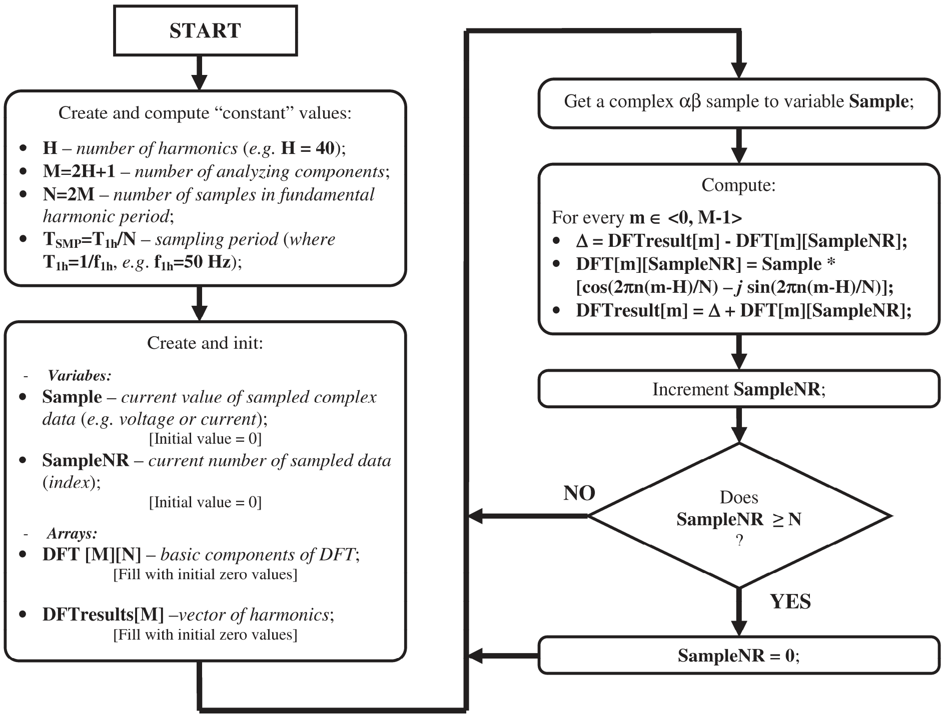


Fig. 8. Algorithm presenting implementation of Complex SDFT

The complete algorithm enabling implementation of Complex SDFT for H harmonics, both forward and reverse, has been presented in Fig. 8.

During the analysis of mathematical operations that are crucial in SDFT algorithm's realisation, it is easy to observe that what is the most time-consuming for a processor is trigonometrical calculations specified in (14). As there are no low-level processor's commands that conduct trigonometrical calculations, in reality, high-level programming languages use algorithms that approximate these functions' results. The approximation of sine and cosine functions that is most often used is their Taylor series expansion.

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots \tag{16}$$

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots \tag{17}$$

In reality, a computation unit does not calculate an infinite sequence, the number of n polynomical depends on the implementation of used programming language and reaches, in practice, several dozens iterations. This is of adequate precision for the majority of algorithms. However, counting up the value of a given angle's sine and cosine remains very time-consuming for every microprocessor, especially for the one not equipped with Floating-Point Unit (FPU).

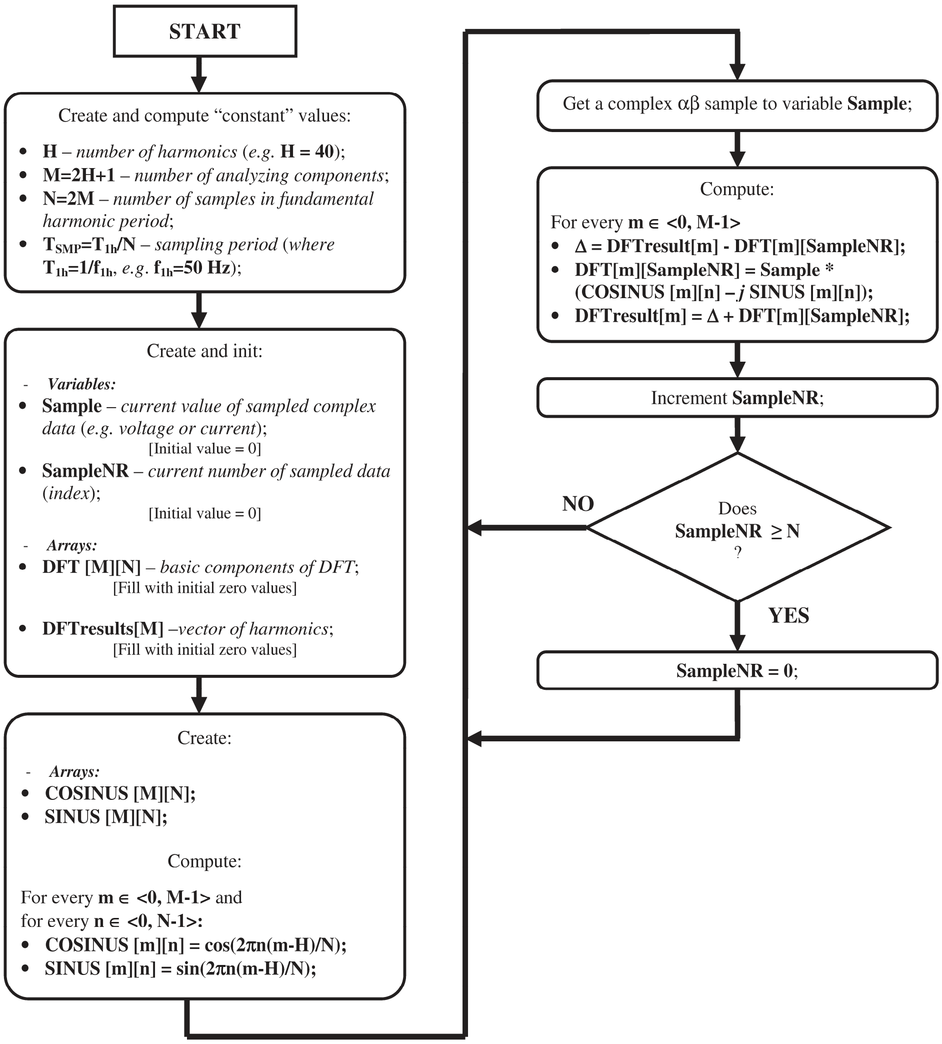


Fig. 9. Algorithm presenting the implementation of Complex SDFT with the use of arrayed values of trigonometric functions

Since one certain value of sine or cosine function corresponds with each angle's value, it can be of help to store some of these values on an array and use them when needed (instead of calculating them for each time). The disadvantage of this solution, however, is increasing the application's demand for memory in which the array is stored. Another inconvenience may be unsatisfactory accuracy of this solution. For example, storing 360 values with a discrete by one degree, there is no possibility to determine an angle's function with the value of a degree's fraction. Certainly, it is possible to increase resolution by arraying the function's values by, for instance, $^{1}/_{10}$ degree. This will make the results more accurate, but increase the demand for memory by ten times.

Looking at the relations (1) and (4), one can notice that the angle's values appearing under trigonometric functions are not accidental. The indexes n and m are integer variables, the values of which comprise in ranges <0, N-1> and <0, M-1> respectively. N value, according to (12) and (11), is constant for the agreed number of analysed H harmonics. Thus, the amount of all possible angles for which sine and cosine values ought to be calculated, is limited to N×M. For instance, for H=40, according to (11) and (12), N×M=13122. Another advantage of this solution is the fact that the values organised in the array in such a way guarantee the same accuracy as their each-time calculating.

The algorithm enabling the above method's implementation has been shown in Fig. 9.

After having implemented the algorithm with the arrayed values of trigonometric functions in a physical circuit based on microcontroller devoid of FPU, its efficiency is over 100 times higher comparing to the version with calculating values for each time. In this case, the demand for memory in respect to the price of a faster processor proves to be of insignificant costs. That is why, if in realised application, SDFT algorithm is the most time-consuming for a computation unit, equipping the application with a cheaper processor and bigger memory turns out to be a more economic solution.

5. SUMMARY

Presented algorithm of current's spectrum analysis with the use of complex variable's Sliding Window Discrete Fourier Transformation (SDFT) can be implemented in the real time systems. The obtained results of spectrum analysis can be used as input data of the active filtration's algorithm. This is the subject of further research carried out in the Department of Power Electronics and Control at University of Technology and Life Sciences in Bydgoszcz.

The algorithm has been developed at University of Technology and Life Sciences in Bydgoszcz with the use of C++ programming language and simulated at Gdynia Maritime University with PSIM v7 simulation environment.

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WYKORZYSTANIE DYSKRETNEJ TRANSFORMATY FOURIERA ZMIENNEJ ZESPOLONEJ ZE ŚLIZGAJĄCYM OKNEM DO ANALIZY ODKSZTAŁCEŃ PRĄDÓW I NAPIĘĆ W UKŁADACH TRÓJFAZOWYCH

Streszczenie

Artykuł przedstawia aspekty wektorowej analizy odkształconych przebiegów prądów i napięć w układach trójfazowych pod kątem zawartości wyższych harmonicznych. Praca zawiera również wyniki analizy harmonicznej wektora prądu opartej o transformatę Fouriera zmiennej zespolonej ze ślizgającym oknem (*complex SDFT*) uzyskane na drodze symulacji oraz propozycję przyspieszenia algorytmu SDFT w celu ułatwienia jego implementacji w układach pracujących w czasie rzeczywistym.

Słowa kluczowe: Dyskretna Transformata Fouriera zmiennej zespolonej ze ślizgającym oknem, selektywna filtracja aktywna, analiza widmowa wektora wodzącego, odkształcone przebiegi prądu i napięcia