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VERIFICATION OF A MATHEMATICAL MODEL OF THE POLARIMETRIC CURRENT SENSOR WITH SINGLE-MODE FIBER MEASUREMENT COIL BASED ON COMPUTER SIMULATION BY FINITE ELEMENT METHOD

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Summary: This article presents the verification of a mathematical model polarimetric current sensor proposed in [1]. The verification was based on computer simulation, which was based on the finite element method. Simulation model was prepared for one of the options for deployment of high voltage cables 110 kV [2]. The simulation was performed for two operating conditions of high-voltage power line – and rated the state short-circuit conditions. In addition, estimated absolute and relative error of measurement.

Keywords: Verdet constant, Faraday effect, mangeto-optical phenomenon, polarimetric sensor, telecommunication optical fiber, high-voltage line, Finite Element Method (FEM)

1. A MATHEMATICAL MODEL OF THE POLARIMETRIC CURRENT SENSOR [1]

The sensor operation is based on the analysis of the properties of a light wave that propagates through the optical fibre and is transformed when subjected to an external magnetic field generated by a live conductor – energy line.

Optical fibres are not optically active when not exposed to an external magnetic field but become active when a magnetic field is applied – the plane of polarization of a light beam is rotated by a certain angle, this is so-called **Faraday effect** described with the following formula [1]:

$$\alpha = V \cdot L \cdot B \tag{1}$$

where:

α – the angle of the plane of polarization rotation [rad],

V – the Verdet constant (proportionality constant) $\left[\frac{\text{rad}}{\text{T}\cdot\text{m}}\right]$,

L – the length of the path where the light and the magnetic field interact [m],

B – magnetic flux density $[\text{T}]$.

The Verdet constant in the formula (1) is an empirical value that characterizes the material of the medium as a constant of proportionality between the magnetic excitation and the reaction of glass. Standard types of oxide-based glass – diamagnetic materials have positive and low Verdet constants [1]. Also, as far as diamagnetic materials are concerned, the constant depends to a large extent on the length of a light wave, and to a small extent on the temperature [1].

It is possible to describe the relationship between the current in the electric conductor and a change of polarization angle for a single coil of optical fibre with the length of $l = 2\cdot\pi\cdot R$, where: R – the distance between a fibre coil and the centre of the live conductor (fibre curl radius). In order to do so, the Ampère law must be applied in the integral form:

$$I = \oint_{l=2\cdot\pi\cdot R} H dl = H\cdot l = H\cdot 2\cdot\pi\cdot R \tag{2}$$

where:

I – current [A],

H – magnetic field strength $\left[\frac{\text{A}}{\text{m}}\right]$.

For a dielectric medium such as a telecommunication fibre, the relationship between the induction and the magnetic field strength may be defined as follows:

$$B = \mu_0 \cdot H \tag{3}$$

where:

$\mu_0 = 4\cdot\pi\cdot 10^{-7}$ $\left[\frac{\text{V}\cdot\text{s}}{\text{A}\cdot\text{m}}\right]$ – permeability of vacuum.

On the basis of the above formulas (2) and (3) the following relationship may be described:

$$B = \frac{\mu_0 \cdot I}{2\cdot\pi\cdot R} \tag{4}$$

The modification of formula (1) using formula (4) shows that for a sensor fitted with an optical fibre with the length of $L = N\cdot l = N\cdot 2\cdot\pi\cdot R$ a change of light polarization angle may be described with the following relationship:

$$\alpha = V\cdot\mu_0\cdot I\cdot N \tag{5}$$

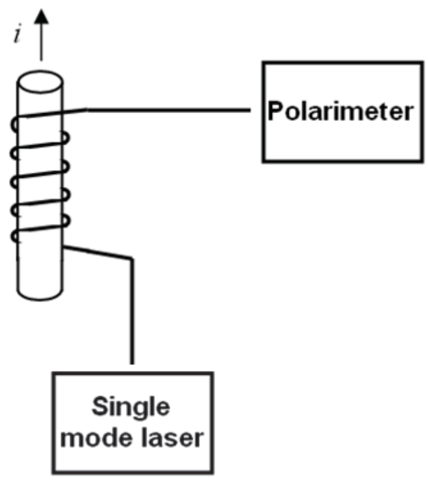


Fig. 1. A block diagram of polarimetric current sensor

After determining a polarization angle α with the polarimeter measurement, the equation (5) may be used to determine the current intensity:

$$I = \frac{\alpha}{\mu_0 \cdot V \cdot N} \tag{6}$$

The value of current I (6) is influenced by the Verdet constant V – a parameter characteristic of the type of optical fibre used for the construction of a sensor. Verdet constant value for single-mode optical fibers are presented in the article [1], have been obtained based on mathematical calculations.

Table 1. Verdet constant value depending on the wavelength and the molar concentration of dopant GeO_2 [1]

λ	V **for the silicon doping** GeO_2 $\left[\frac{\text{rad}}{\text{T}\cdot\text{m}}\right]$			
	3,1 M%	**5,8 M%**	**7,9 M%**	**13,5 M%**
1310 nm second optical window	4,3784	4,3755	4,4090	4,0649
1550 nm third optical window	5,4579	5,4437	5,4642	4,9073

2. CHARACTERISTICS OF THE SIMULATION MODEL AND RESULTS

In order to verify the mathematical model of the polarimetric current sensor [1], was chosen one type of column high-voltage power lines 110 kV [2] (Fig. 2).

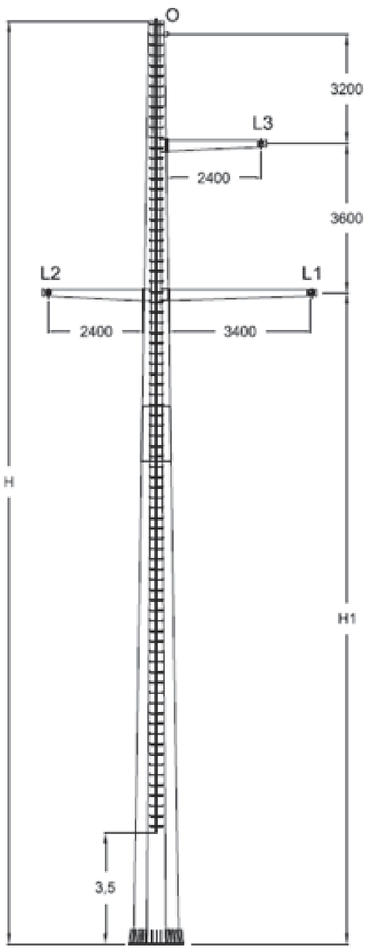
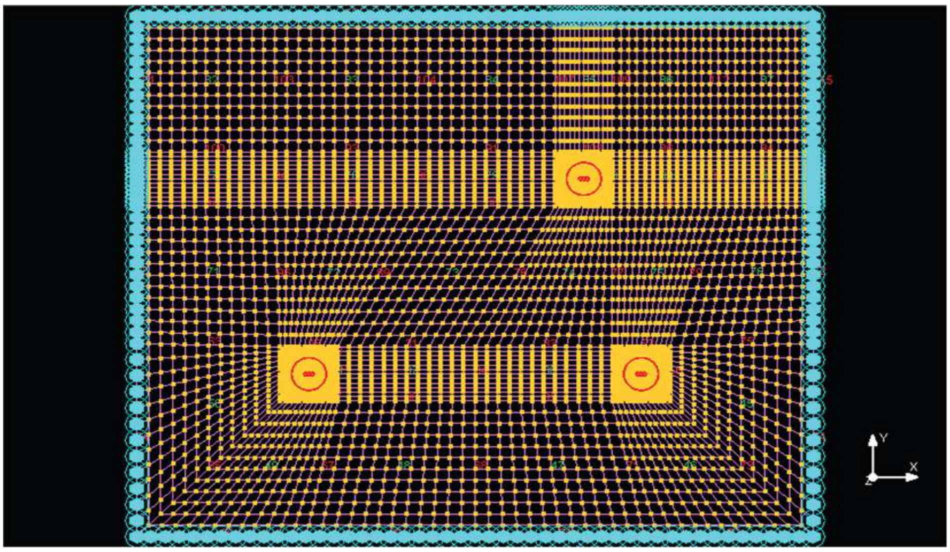


Fig. 2. Continuous pole 110 kV single circuit line ESJ series P [2]

Using the simulation environment EMRC NISA developed numerical model of wire and the surrounding environment (Fig. 3). The grid model contains 6192 elements with 6301 nodes. On the banks of the area erected conditions of zero for the vector potential (dimension of the analyzed area $11{,}8\ \text{m} \times 11{,}8\ \text{m}$). This model leads according to the guidelines for the construction of power systems [3], with a radius $r_0 = 8{,}74\ \text{mm}$ and the cross sectional area $S = \pi \cdot r_0^2 = 239{,}8\ \text{mm}^2 = 239{,}8 \cdot 10^{-6}\ \text{m}^2$.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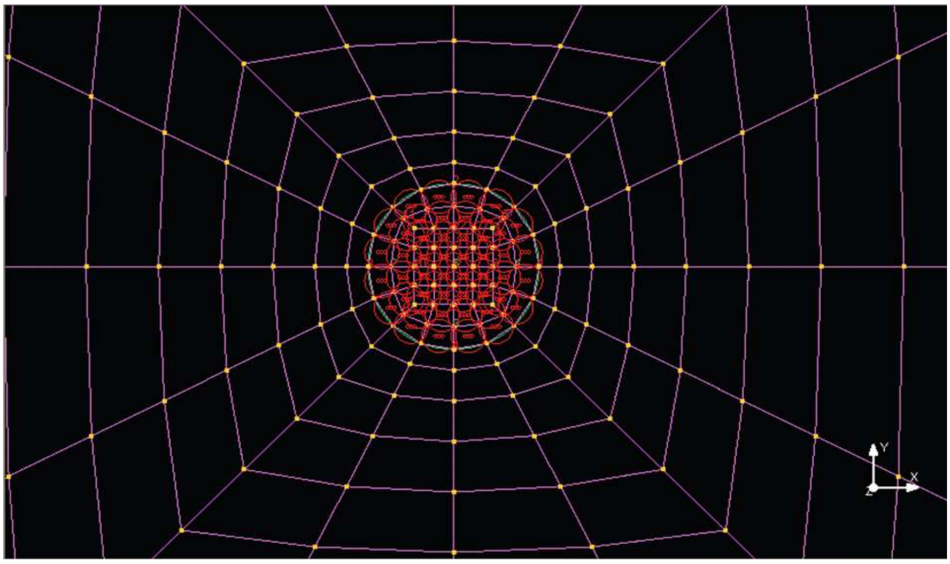
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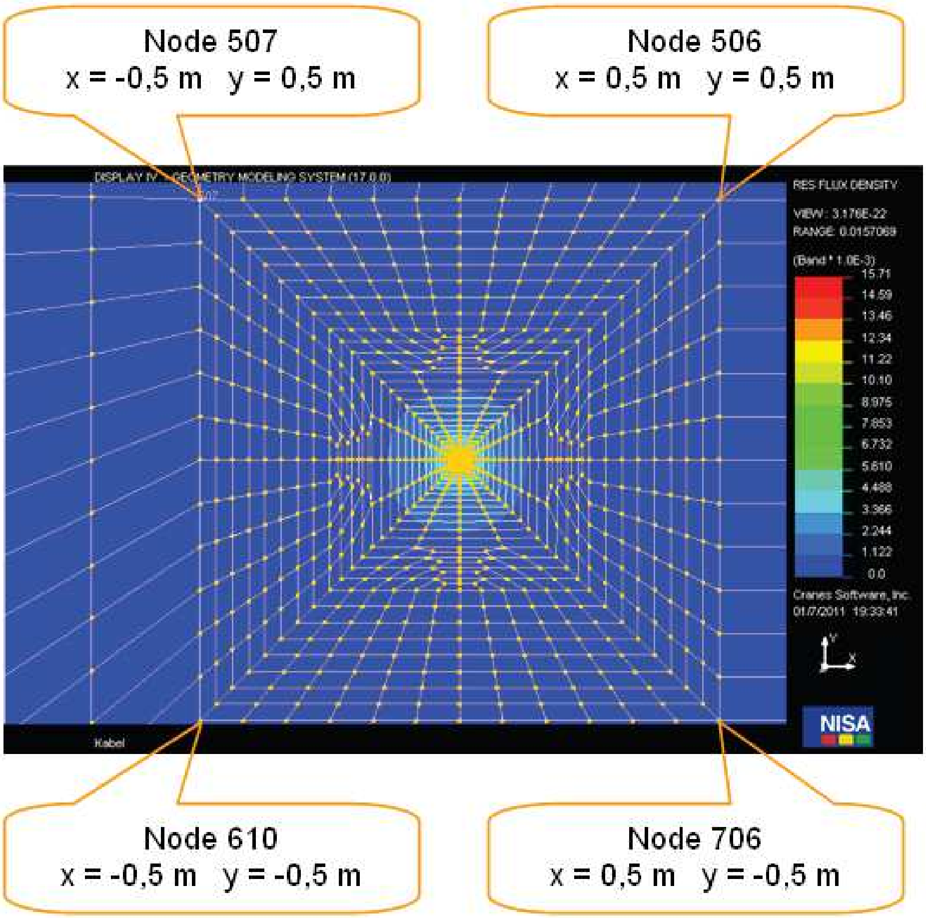
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Node 506
x = 0,5 m   y = 0,5 m
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RANGE: 0.0157069
(Band * 1.0E-3)
15.71
14.59
13.46
12.34
11.22
10.10
8.975
7.853
6.732
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4.488
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Node 610
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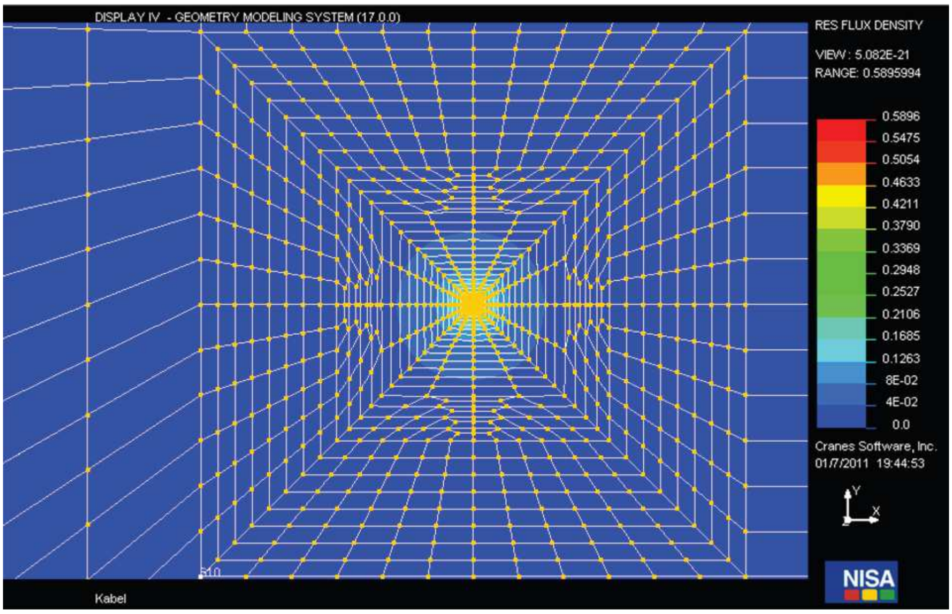
Node 706
x = 0,5 m   y = -0,5 m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wavelength, a varying number of turns (of differen length – $L = N \cdot 2 \cdot \pi \cdot R$, where R = 55 mm) and state of the high-voltage power line. The results are shown in Table 3 and Table 4;

Table 3. The angle of polarization depending on the number of turns of the coil fiber, measurement wavelength, the type of fiber for rated operation condition of power line

The angle of polarization α [rad]									
		1310 nm (second optical window)				**1550 nm (third optical window)**			
	GeO_2	**3,1 M%**	**5,8 M%**	**7,9 M%**	**13,5 M%**	**3,1 M%**	**5,8 M%**	**7,9 M%**	**13,5 M%**
	V	4,3784	4,3755	4,4090	4,0649	5,4579	5,4437	5,4642	4,9073
N	1	0,00421	0,00420	0,00424	0,00391	0,00524	0,00523	0,00525	0,00471
	10	0,04206	0,04204	0,04236	0,03905	0,05243	0,05230	0,05249	0,04714
	100	0,42063	0,42035	0,42357	0,39051	0,52434	0,52298	0,52495	0,47144
	1000	4,20632	4,20354	4,23572	3,90515	5,24340	5,22976	5,24945	4,71444

Table 4. The angle of polarization depending on the number of turns of the coil fiber, measurement wavelength, the type of fiber for short-circuit condition of power line

The angle of polarization α [rad]									
		1310 nm (second optical window)				**1550 nm (third optical window)**			
	GeO_2	**3,1 M%**	**5,8 M%**	**7,9 M%**	**13,5 M%**	**3,1 M%**	**5,8 M%**	**7,9 M%**	**13,5 M%**
	V	4,3784	4,3755	4,4090	4,0649	5,4579	5,4437	5,4642	4,9073
N	1	0,16011	0,16001	0,16123	0,14865	0,19959	0,19907	0,19982	0,17945
	10	1,60113	1,60007	1,61232	1,48648	1,99589	1,99069	1,99819	1,79454
	100	16,01127	16,00066	16,12317	14,86484	19,95887	19,90694	19,98191	17,94539
	1000	160,11269	160,00664	161,23169	148,64838	199,58867	199,06940	199,81906	179,45391

- determine the values of current flowing in a cable of high-voltage power line 110 kV using data presented in Table 3 and Table 4, and the pattern (6). The results are shown in Table 5 and Table 6;

Table 5. Values of current flowing in a cable of high-voltage power line depending on the number of turns of the coil fiber, measurement wavelength, the type of fiber for rated operation condition of power line

Values of current flowing in a cable of high-voltage power line I_m [A]									
		1310 nm (second optical window)				**1550 nm (third optical window)**			
	GeO_2	**3,1 M%**	**5,8 M%**	**7,9 M%**	**13,5 M%**	**3,1 M%**	**5,8 M%**	**7,9 M%**	**13,5 M%**
	V	4,3784	4,3755	4,4090	4,0649	5,4579	5,4437	5,4642	4,9073
N	1	789,3	789,3	789,3	789,3	789,3	789,3	789,3	789,3
	10	789,3	789,3	789,3	789,3	789,3	789,3	789,3	789,3
	100	789,3	789,3	789,3	789,3	789,3	789,3	789,3	789,3
	1000	789,3	789,3	789,3	789,3	789,3	789,3	789,3	789,3

Table 6. Values of current flowing in a cable of high-voltage power line depending on the number of turns of the coil fiber, measurement wavelength, the type of fiber for short-circuit condition of power line

Values of current flowing in a cable of high-voltage power line I_m [A]									
		1310 nm (second optical window)				1550 nm (third optical window)			
	GeO_2	3,1 M%	5,8 M%	7,9 M%	13,5 M%	3,1 M%	5,8 M%	7,9 M%	13,5 M%
	V	4,3784	4,3755	4,4090	4,0649	5,4579	5,4437	5,4642	4,9073
N	1	29100,5	29100,5	29100,5	29100,5	29100,5	29100,5	29100,5	29100,5
	10	29100,5	29100,5	29100,5	29100,5	29100,5	29100,5	29100,5	29100,5
	100	29100,5	29100,5	29100,5	29100,5	29100,5	29100,5	29100,5	29100,5
	1000	29100,5	29100,5	29100,5	29100,5	29100,5	29100,5	29100,5	29100,5

- determine the absolute error (7) and relative error (8) [4] of measurement of current using polarimetric sensor. The results are shown in Table 7.

$$\Delta I = I - I_m \tag{7}$$

$$\delta_I = \frac{\Delta I}{I_m} \cdot 100\% \tag{8}$$

where:

ΔI – absolute error of measurement of current, [A],

I – value of the current in the cable of high-voltage power line, unable to work, considered as the reference value, [A],

I_m – value of the current in the cable of high-voltage power lines measured by the polarimetric sensor, [A],

δ_I – relative error of measurement of current, [%].

Table 7. Values of the absolute error and relative error of measurement of current using polarimetric sensor for two operating states of high-voltage power line 110 kV

States of the high-voltage power line	Absolute error of measurement of current [A]	Relative error of measurement of current [%]
rated operation condition; I = 800 A	10,70	1,36
short-circuit condition; I = 30 kA	899,50	3,09

4. CONCLUSIONS

Environment EMRC NISA has been used to simulate the distribution of magnetic induction around the cables of high-voltage power lines 110 kV.

Based on the results can draw conclusions concerning the correctness of the mathematical model of the polarimetric current sensor described in [1]:

- presented in [1] model of the polarimetric current sensor, in which the measurement coil is made of single-mode optical fiber (telecommunication fiber) is properly designed – it confirms the analysis of measurement errors included in Table 7. It should be noted that the estimation of measurement errors affects the accuracy of determining the module of magnetic induction based on the results of the simulation;

- measured by a polarimeter angle of polarization of light must be in the range of 0 to 2π radians. Currently used polarimeters can measure the angle of polarization with an uncertainty 0,001%, therefore, the results in Table 3 and Table 4 are presented with such accuracy. Analyzing the results included in these Tables it can be concluded, that if we want to measure currents in the cables of high-voltage power line 110 kV, both in the rated operation condition and short-circuit condition, using one polarimetric sensor, it should have fiber-optic coil built of ten turns.

If we replace the conventional current transformers fber-optic current transformers (polarimetric current sensors), we can make more precise measurements of the current in the cables of high-voltage power line 110 kV and adequately protect the power lines against short-circuits (e.g. damage to equipment on the generation side).

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WERYFIKACJA POPRAWNOŚCI MODELU MATEMATYCZNEGO POLARYMETRYCZNEGO CZUJNIKA NATĘŻENIA PRĄDU Z JEDNOMODOWĄ ŚWIATŁOWODOWĄ CEWKĄ POMIAROWĄ W OPARCIU O SYMULACJĘ KOMPUTEROWĄ METODĄ ELEMENTÓW SKOŃCZONYCH

Streszczenie

W artykule przedstawiono weryfikację modelu matematycznego polarymetrycznego czujnika prądu w [1]. W tym celu wykorzystano symulację komputerową, opartą na metodzie elementów skończonych. Opracowany model symulacyjny dotyczył jednej z możliwych opcji rozmieszczenia przewodów na słupie linii elektroenergetycznej wysokiego napięcia 110 kV [2]. Symulacja została przeprowadzona dla dwóch stanów pracy linii wysokiego napięcia – pracy znamionowej i zwarcia. Ponadto został oszacowany błąd bezwzględny i względny pomiaru natężenia prądu za pomocą czujnika polarymetrycznego [1].

Słowa kluczowe: stała Verdeta, zjawisko magnetooptyczne Faradaya, czujnik polarymetryczny, światłowód telekomunikacyjny, linia wysokiego napięcia, Metoda Elementów Skończonych (MES)