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ANALYSIS OF THE PARALLEL OPERATION OF THE INDUCTION GENERATORS WITH CAPACITOR EXCITATION

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Summary: In the paper the parallel operation of the induction generators with capacitor excitation is analysed. The most simple methods of paralleling generators are briefly discussed. A mathematical model for the stationary states of the generators operating in parallel has been presented. The conditions of the best use of the power of the generators operating in parallel have been determined.

Keywords: induction generator, parallel operation, stationary states

1. INTRODUCTION

There are various methods of the paralleling of the induction generators with capacitor excitation [3, 4, 5, 6]. The most appropriate seem to be two methods which with ideal fulfilment of the connection conditions guarantee a surge-free connection process [1, 2, 5, 6].

The first method requires the following conditions to be met:
- the voltage of both generators must be equal,
- voltage frequency must be the same,
- the sequence of phases must be identical.

From all these conditions only the third one is obligatory since if is not met then the state is equivalent to shorting which leads to quick voltage decay and complete demagnetizing of the machine.

As far as the two first conditions are concerned, they do not have to be strictly fulfilled. According to the experimental research carried out by the author, with circa 20 percent voltage and 5 percent frequency difference the transient process of the paralleling of generator lasts for 2 up to 4 periods, and the accompanying current surge does not exceed the quintuple value of the rated current.

The second method does not require any earlier excitation of the generator to be paralleled closed. The process of the synchronization of the second generator runs as follows: first, with the first generator connected to the common busbars a capacitor battery of the generator being synchronized has to be connected. This makes some decrease of frequency and increase of voltage. Since induction generators usually run at a significant saturation of the magnetic circuit, the voltage increase normally is not high. Then, the second generator with its shaft revolving with revolutions close to that of synchronic is connected (as quickly as possible) to the common busbars. The connection transition process proceeds quickly and with rather small current surges.

The second method is more convenient, however both of them are relatively simple and no intricate instruments are required. Research on transition processes for induction generators running in parallel, including connection to the common power will be discussed in a separate paper.

2. PARALLEL OPERATION OF INDUCTION GENERATORS IN IDLE MODE

At the analysis of the static states of self-excited induction generators it is convenient to use a equivalent circuit of T type where reactive elements are expressed as shown in Fig. 1.

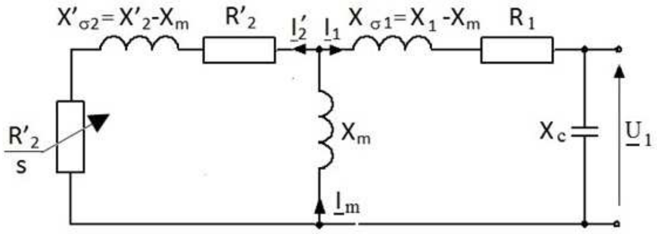

Fig. 1. Equivalent circuit of the induction machine adapted for the considerations (without iron losses)

In the above diagram the reactance X_1 and X_2' represent the total flux linkages of both stator and rotor phases, whereas the reactance X_m – the flux linkage of the common inductance. Following relationships are here relevant:

$$\begin{aligned} X_1 &= X_{\sigma 1} + X_m \\ X'_2 &= X'_{\sigma 2} + X_m \end{aligned} \tag{1}$$

where $X_{\sigma 1}$ and $X'_{\sigma 2}$ are leakage reactances of the both stator and rotor windings. Assuming this notation simplifies to a degree the relationships obtained at the analysis.

The system of equations describing the two A and B induction generators in parallel operation in idle mode can be then presented in the form of:

$$\left.\begin{aligned} &(R_{1A} + jX_{1A})\underline{I}_{1A} + jX_{mA}\underline{I}'_{2A} - jX_C\underline{I} = 0 \\ &js_A X_{mA}\underline{I}_{1A} - \left(R'_{2A} - js_A X'_{2A}\right)\underline{I}'_{2A} = 0 \\ &(R_{1B} + jX_{1B})\underline{I}_{1B} + jX_{mB}\underline{I}'_{2B} - jX_C\underline{I} = 0 \\ &js_B X_{mB}\underline{I}_{1B} - \left(R'_{2B} - js_B X'_{2B}\right)\underline{I}'_{2B} = 0 \\ &\underline{I}_{1A} + \underline{I}_{1B} - \underline{I} = 0 \end{aligned}\right\} \tag{2}$$

where:

$X_{1A,B}; X'_{2A,B}; R_{1A,B}; R'_{2A,B}; X_{mA}; X_{mB}$ – are the parameters of the equivalent circuit of the A and B induction generators,

$X_C = \frac{1}{\omega_1(C_A + C_B)}$ – the reactance of the exciting capacitor phase with the assumption that the machine windings are star connected,

$\underline{I} = \underline{I}_C$ – the current in the equivalent circuit (in idle state it equals to the current of one phase of capacitors.

By solving the first and fourth equation of the system of equations (1) with regard to the secondary side and substituting the obtained expressions in the first and forth equation of this system, we obtain:

$$\left.\begin{aligned} &\left[(R_{1A} + jX_{1A}) - \frac{X_{mA}^2 s_A}{R'_{2A} - jX'_{2A} s_A}\right]\underline{I}_{1A} - jX_C \underline{I} = 0 \\ &\left[(R_{1B} + jX_{1B}) - \frac{X_{mB}^2 s_B}{R'_{2B} - jX'_{2B} s_B}\right]\underline{I}_{1B} - jX_C \underline{I} = 0 \\ &\underline{I}_{1A} + \underline{I}_{1B} - \underline{I} = 0 \end{aligned}\right\} \tag{3}$$

After further transformations the system of equations (2) can be presented in the simplified form:

$$\left.\begin{aligned} \underline{Z}_A \underline{I}_{1A} - jX_C \underline{I} = 0 \\ \underline{Z}_B \underline{I}_{1B} - jX_C \underline{I} = 0 \\ \underline{I}_{1A} + \underline{I}_{1B} - \underline{I} = 0 \end{aligned}\right\} \tag{4}$$

where:

$$\left.\begin{aligned} \underline{Z}_A = R_A + jX_A \\ \underline{Z}_B = R_B + jX_B \end{aligned}\right\} \tag{5}$$

$\underline{Z}_A$ and $\underline{Z}_B$ components of impedance in equations (5) are determined by formulas:

$$\left.\begin{aligned} R_A &= R_{1A} - \frac{X_{mA}^2 R'_{2A} s_A}{(R'_{2A})^2 + (X'_{2A} s_A)^2} \\ X_A &= X_{1A} - \frac{X_{mA}^2 X'_{2A} s_A^2}{(R'_{2A})^2 + (X'_{2A} s_A)^2} \\ R_B &= R_{1B} - \frac{X_{mB}^2 R'_{2B} s_B}{(R'_{2B})^2 + (X'_{2B} s_B)^2} \\ X_B &= X_{1B} - \frac{X_{mB}^2 X'_{2B} s_B^2}{(R'_{2B})^2 + (X'_{2B} s_B)^2} \end{aligned}\right\} \tag{6}$$

The condition for stable voltage generation in the system composed of two generators running in parallel can be determined from the system of equations (4). The mathematical form of this condition is expressed with the formula:

$$\begin{vmatrix} \underline{Z}_A & 0 & -jX_C \\ 0 & \underline{Z}_B & -jX_C \\ 1 & 1 & -1 \end{vmatrix} = 0 \tag{7}$$

from which we obtain, that:

$$\frac{\underline{Z}_A \underline{Z}_B}{\underline{Z}_A + \underline{Z}_B} = jX_C \tag{8}$$

Taking the left hand side of the equation (8) as a certain substitute $\underline{Z}_z$ impedance we can form the relationship

$$\underline{Z}_z = jX_C \tag{9}$$

which informs that in the system of two induction generators operating in parallel in idle mode the voltage can be generated only when the reactive inductive power necessary for magnetizing of both generators is compensated by the wattless capacitive power of the battery of capacitors of the capacity $C = C_A + C_B$.

The frequency of the voltage generated by the system of two generators can be determined from the equation (8), but it would lead to quite intricate relationships. Much more simple and accurate enough relationship for the frequency of the voltage generated in idle state can be obtained by adopting the following simplifying assumptions:

– slips of the both generators in idle state are equal zero,
– active power losses in the generator system is neglected.

With these assumptions, from (3) we directly obtain, that:

$$\begin{vmatrix} jX_{1A} & 0 & -jX_C \\ 0 & jX_{1B} & -jX_C \\ 1 & 1 & -1 \end{vmatrix} = 0 \tag{10}$$

By substituting

$$X_{1A} = \omega_1 L_{1A}$$
$$X_{1B} = \omega_1 L_{1B}$$

and after the calculation of the (10) determinant we obtain the relationship for the pulsation of the generated voltage ω_1

$$\omega_1 = \frac{1}{\sqrt{\dfrac{L_{1A} L_{1B}\left(C_A + C_B\right)}{L_{1A} + L_{1B}}}} \tag{11}$$

and frequency

$$f_1 = \frac{1}{2\pi\sqrt{\frac{L_{1A}L_{1B}(C_A + C_B)}{L_{1A} + L_{1B}}}} \tag{12}$$

To these expressions the equivalent circuit of the system of the two generators running in parallel in idle state corresponds, and it is presented in Fig. 2a. Designating the equivalent generator inductance as

$$L = \frac{L_{1A}L_{1B}}{L_{1A} + L_{2B}} \tag{13}$$

and its capacity as

$$C = C_A + C_B \tag{14}$$

we obtain the equivalent circuit as in Fig. 2b.

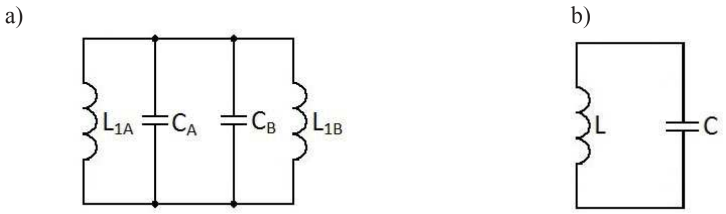

Fig. 2. Equivalent circuit of two induction generators running parallel in idle state – a), and equivalent circuit of generator – b), (L_{1A}, L_{1B} – self-inductances of the phases of the stator of generators A and B, respectively)

Now, the frequency of the voltage generated within the system of two generators operating in parallel in idle mode can be presented with the following, simple relationship

$$f_1 = \frac{1}{2\pi\sqrt{LC}} \tag{15}$$

where:

L – the total inductance of the equivalent generator phase, determined by (13),
C – the capacity of one equivalent generator phase, determined by (14).

In order to obtain a equivalent circuit for two self-excited induction generators running in idle mode with commonly used parameters, in equations (2) the substitutes:

$$\left.\begin{aligned} X_{1A} &= X_{1\sigma A} + X_{mA} \\ X'_{2A} &= X'_{2\sigma A} + X_{mA} \\ X_{1B} &= X_{1\sigma B} + X_{mB} \\ X'_{2B} &= X'_{2\sigma B} + X_{mB} \end{aligned}\right\} \tag{16}$$

should be made with taking into consideration that

$$\left.\begin{aligned}\underline{I}_{1A} + \underline{I}'_{2A} &= \underline{I}_{mA}\\ \underline{I}_{1B} + \underline{I}'_{2B} &= \underline{I}_{mB}\end{aligned}\right\}\tag{17}$$

Then we obtain the system of equations:

$$\left.\begin{aligned}(R_{1A} + jX_{1\sigma A})\underline{I}_{1A} + jX_{mA}\underline{I}_{mA} - jX_C\underline{I} = 0\\ -(R'_{2A}/s_A - jX'_{2\sigma A})\underline{I}'_{2A} + jX_{mA}\underline{I}_{mA} = 0\\ (R_{1B} + jX_{1\sigma B})\underline{I}_{1B} + jX_{mB}\underline{I}_{mB} - jX_C\underline{I} = 0\\ -(R'_{2B}/s_B - jX'_{2\sigma B})\underline{I}'_{2B} + jX_{mB}\underline{I}_{mB} = 0\end{aligned}\right\}\tag{18}$$

The quivalent circuit presented in Fig. 2 corresponds to these equations.

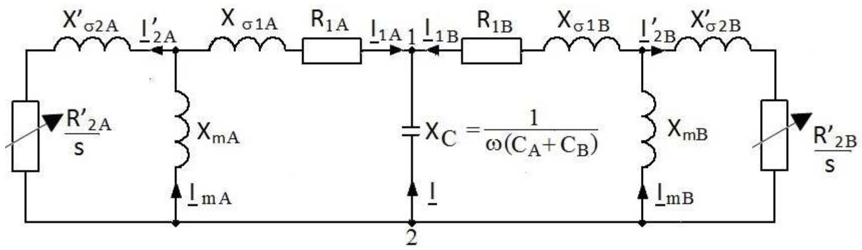

Fig. 3. The quivalent circuit of the two induction generators with capacitor excitation running in idle mode

3. PARALLEL OPERATION OF SELF-EXCITED INDUCTION GENERATORS UNDER LOAD

In general case the relation between the wattless power values of two induction generators operating in parallel under load can be determined by the equation

$$(Q - Q_{load}) = (Q_{1A} + Q_{1B})\tag{19}$$

where:

- Q – the wattless power of the capacitor battery,
- Q_{load} – load wattles power,
- Q_{1A}, Q_1 – the wattless magnetizing power of generators A and B.

From the formula (19) it turns out that the resultant wattless power of the external circuits, regardless of the load nature, is of capacitive character. Thus, the resultant impedance of the external circuits is of resistance-capacitive nature, as shown by

$$\underline{Z} = R - jX\tag{20}$$

Then, for the induction generators operating in parallel under load we obtain an analogous system of equations (2), namely:

$$\left.\begin{aligned}&(R_{1A}+jX_{1A})\underline{I}_{1A}+jX_{mA}\underline{I}'_{2A}+(R-jX)\underline{I}=0\\&js_AX_{mA}\underline{I}_{1A}-(R'_{2A}-js_AX'_{2A})\underline{I}'_{2A}=0\\&(R_{1B}+jX_{1B})\underline{I}_{1B}+jX_{mB}\underline{I}'_{2B}+(R-jX)\underline{I}=0\\&js_BX_{mB}\underline{I}_{1B}-(R'_{2B}-js_BX'_{2B})\underline{I}'_{2B}=0\\&\underline{I}_{1A}+\underline{I}_{1B}-\underline{I}=0\end{aligned}\right\}\tag{21}$$

After the same transformations as for the case of the idle mode we obtain the system of equations in the form:

$$\left.\begin{aligned}&\underline{Z}_A\underline{I}_{1A}+\underline{Z}\underline{I}=0\\&\underline{Z}_B\underline{I}_{1B}+\underline{Z}\underline{I}=0\\&\underline{I}_{1A}+\underline{I}_{1B}-\underline{I}=0\end{aligned}\right\}\tag{22}$$

where:

$$\underline{Z}_A=\left[R_{1A}-\frac{X_{mA}^2R'_{2A}s_A}{\left(R'_{2A}\right)^2+\left(X'_{2A}s_A\right)^2}\right]+j\left[X_{1A}-\frac{X'_{2A}X_{mA}^2s_A^2}{\left(R'_{2A}\right)^2+\left(X'_{2A}s_A\right)^2}\right]$$

$$\underline{Z}_B=\left[R_{1B}-\frac{X_{mB}^2R'_{2B}s_B}{\left(R'_{2B}\right)^2+\left(X'_{2B}s_B\right)^2}\right]+j\left[X_{1B}-\frac{X'_{2B}X_{mB}^2s_B^2}{\left(R'_{2B}\right)^2+\left(X'_{2B}s_B\right)^2}\right]$$

– impedances of the substitute diagrams of generators,

$\underline{Z}$ – impedance of the equivalent circuits, determined by (20).

The condition for the stable operation of induction generators running in parallel under load is, that

$$\begin{vmatrix}\underline{Z}_A & 0 & \underline{Z}\\ 0 & \underline{Z}_B & \underline{Z}\\ 1 & 1 & -1\end{vmatrix}=0\tag{23}$$

or

$$\underline{Z}=-\frac{\underline{Z}_A\underline{Z}_B}{\underline{Z}_A+\underline{Z}_B}\tag{24}$$

i.e.

$$\underline{Z}=-\underline{Z}_{eqv}\tag{25}$$

The above speculation enable us to conclude that:

– the impedances of both the phases of the equivalent circuits and equivalent generator are equal with regard to the value, and opposite to the sign,

– the total resistance of the system is equal zero, which means that the negative resistance of the substitute generator can be considered as a generating element, the entire power of which is developed in the equivalent circuit.
– the total reactance of the system is also equal zero, which means that capacitors only are the source of the system wattless power under the load R and R-L. Their wattless power compensates the wattless powers of the generators and load powers. The values of the frequency of the voltage generated, the saturation of the magnetic circuits, and the slips of the generators become such that the total impedance of the entire system is equal zero.

The system of equations (21) after taking into consideration (16) and (17) can be finally put as:

$$\left.\begin{aligned} \left(R_{1A} + jX_{1A}\right)\underline{I}_{1A} + jX_{mA}\underline{I}_{mA} + \left(R - jX\right)\underline{I} = 0 \\ -\left(\frac{R'_{2A}}{s_A} - jX'_{2A}\right)\underline{I}'_{2A} + jX_{mA}\underline{I}_{mA} = 0 \\ \left(R_{1B} + jX_{1B}\right)\underline{I}_{1B} + jX_{mB}\underline{I}_{mB} + \left(R - jX\right)\underline{I} = 0 \\ -\left(\frac{R'_{2B}}{s_B} - jX'_{2B}\right)\underline{I}'_{2B} + jX_{mB}\underline{I}_{mB} = 0 \\ \underline{I}_{1A} + \underline{I}_{1B} - \underline{I} = 0 \end{aligned}\right\} \tag{26}$$

The equivalent circuit presented in Fig. 3, where between the points 1 and 2 the impedance $\underline{Z}$=R-jX has been connected corresponds to the above system of equations. The obtained system of equations (26), after solving it, can be used for the analysis of the operation of the induction generators running in parallel under load.

4. PARALLEL OPERATION OF THE SELF-EXCITED INDUCTION GENERATORS WITH THE DIFFERENT ROTATIONAL SPEED OF THEIR SHAFTS

The different rotational speed of the shafts of the induction generators running in parallel results in irregular load distribution over individual generators. The consequence of this is the decrease in the use of the power of generators. One of the basic issues of the analysis of parallel operation of generators is to determine the conditions for the maximum use of the power of the system of two generators in various operational circumstances. This issue constitutes the subject of the next part of this paper.

The analysis of the operation of two induction generators running with different rotational speed of their shafts will be carried out with the following assumptions:
– two generators of identical parameters, power, and exciting capacitor battery capacitance are considered,
– the angular velocity of the A generator shaft is higher than that of the B generator shaft, i.e.

$$\omega_{wA} > \omega_{wB}$$

where ω_A = const, and ω_B = const, too.

The induction machine of the shaft lower angular velocity, depending on the generator system load level, can run in various operation modes: as generator, idle, and as motor. If initially this machine was running in the generator mode and the load level was decreased so that $\omega = \omega_w$ (ω – field angular velocity), then this generator goes to idle mode. With further decrease of the load the generator goes to the motor operation mode and it should be disconnected from the common busbars.

In further speculations it is assumed that both generators operate in parallel for the grid in the entire range of load, i.e. from zero to the maximum possible.

Idle mode

In this mode the field angular velocity ω is expressed with the relationship:

$$\omega = \frac{1}{p\sqrt{LC}} \tag{27}$$

or

$$\omega = \frac{\omega_{wA} + \omega_{wB}}{2} \tag{28}$$

The later formula shows, that with the assumption $\omega_{wA} > \omega_{wB}$, $\omega > \omega_{wB}$. Therefore the B machine operates with additional slip and remains in the motor mode. With $|\underline{Z}| = \infty$ the generator system is in idle mode, but individual generators run with the same slip with regard to the value, and opposite to the sign, and are loaded symmetrically. The absolute slip value is proportional to the difference $\Delta\omega_{wA} = \omega_{wA} - \omega_{wB}$.

Summarizing, it can be stated that in the idle mode, when $\omega_{wA} = \omega_{wB}$, the active power generated by one of the generators is consumed by the other. The corresponding to this power electromagnetic moment is acting in the direction opposite to the turning moment of the A generator and accordingly to the direction of the B generator spin. Thus this moment can be named the synchronizing moment.

Symmetrical load state

The relations between the angular velocities of fields and shafts of generators are determined by the relationships applied for all operation modes, namely:

$$\left.\begin{aligned} \omega &= \frac{\omega_{wA}}{(1 - s_A)} \\ \omega &= \frac{\omega_{wB}}{(1 - s_B)} \end{aligned}\right\} \tag{29}$$

With the assumption made earlier that $\omega_{wA} > \omega_{wB}$, the A generator slip remains negative, which means that this generator is always in the generator mode. For working loads it can be put, that:

$$\frac{s_B}{s_A} = \frac{P_B}{P_A} \tag{30}$$

i.e. the slip ratio is then proportional to the ratio of powers developed by the individual generators.

Neglecting the losses it can be stated that

$$P_A + P_B = P \tag{31}$$

where P is the power released in the load.

From (30) and (31) it results that

$$s_B = s_A\left(\frac{P}{P_A} - 1\right) \tag{32}$$

As it has already been mentioned above, the slip of the second generator can reach various levels, depending on the load value. And so with $P < P_A$ the active power balance can be expressed as

$$P_A = P_B + P \tag{33}$$

which means that the B generator is consuming the active power ($s_B > 0$). With $P_A = P$ the B generator operates in idle mode ($s_B = 0$). With $P > P_A$ the slip s_B becomes negative (see formula (32)). Both generators produce active power to the load, and the power generated by them can be determined by (31). However, the loads of both generators are not the same. Since $P_A > P_B$, the power of the B generator is not fully used, and the $|s_B|$ value is smaller then the rated one.

The limit slip value for the B generator corresponds to the rated slip of the A generator. This value can be determined from the formula (32), which in this case can be expressed as

$$s_{Bg} = s_{AN}\left(\frac{P_g}{P_{AN}} - 1\right) \tag{34}$$

where P_g is the limit value of the power of the generator system at the rated load of the A generator and it is equal

$$P_g = P_{AN} + P_{Bg} \tag{35}$$

In the expression (35) P_{Bg} is the limit value of the B generator power corresponding to the negative s_{Bg} limit slip.

The considerations presented above show that the closer the s_{Bg} value is to the s_{BN} rated value the higher is the index of the use of the energy system.

In order to determine the factors affecting the s_{Bg} value we introduce the concept of the generator power use coefficient

$$K = \frac{P}{2P_{AN}} \tag{36}$$

which for s_{AN} , s_{Bg} slips it will be designated as K_N .
Therefore

$$K_N = \frac{P_g}{2P_{AN}} \tag{37}$$

which with taking into consideration (35), where

$$P_{Bg} = \frac{s_{Bg}}{s_{AN}} P_{AN}$$

gives us

$$K_N = \frac{s_{AN} + s_{Bg}}{2s_{AN}} \tag{38}$$

This means that for $|s_A| > |s_{AN}|$ and $|s_B< s_{Bg}|$ slips the use of the power of generators decreases along with the decrease of the use coefficient

$$K = \frac{s_A + s_B}{2s_A} \tag{39}$$

In order to define the relationship between the slip of the generators and the angular velocities of their shafts we introduce the coefficient

$$k = \frac{\omega_{wA}}{\omega_{wB}} \tag{40}$$

and use the relationships (29).
By equating the right hand sides of these formulas we obtain

$$s_B = 1 - \frac{1 - s_A}{k} \tag{41}$$

This relationship above shows that the best use of the generator power is achieved at the rated s_{AN} slip of the A generator and the corresponding negative s_{Bg} slip of the B generator. Then

$$s_{Bg} = 1 - \frac{1 - s_{AN}}{k} \tag{42}$$

As it can be seen the s_{Bg} slip is the function of two independent variables s_{AN} and k, and in general case it can assume positive or/and negative values, as well as it can be equal zero. For the concrete generator s_{AN} = const, whereas $s_{Bg} = f(k)$.
(42) also shows that the s_{Bg} assumes negative values when

$$(1 - s_{AN}) > k \tag{43a}$$

or

$$|s_{AN}| > k - 1 \tag{43b}$$

where

$$k - 1 = \frac{\omega_{wA} - \omega_{wB}}{\omega_{wB}} \tag{44}$$

is equal to the relative value of the difference of angular velocities of generator shafts, whereas the reference value is ω_{wB}. This value can be considered as the slip of the B generator shaft with regard to the A generator rotor, and expressed as a fraction of the ω_{wB} velocity.

Conclusion:
The s_{Bg} slip is negative if the absolute value of the rated s_N slip of generators is bigger then the relative difference of the angular velocity of the generator shafts.

The use of the second generator (B) at operation in parallel makes sense within the range of negative slips only. However, with sudden changing of load, when load peaks exceed the maximum allowable short-term overload of the first generator (A), the simultaneous operation of both generators to supply the load becomes just reasonable.

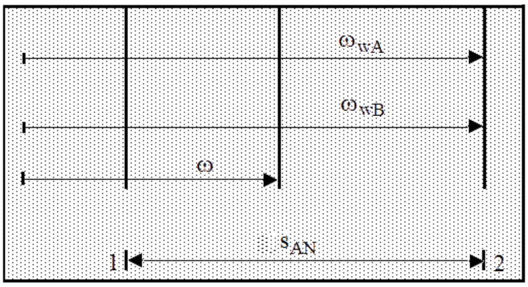

Fig. 4. Operation modes for conditions: $\omega_{wA} = \text{const}$, $\omega_{wB} = \text{const}$, $\omega_{wA} \geq \omega_{wB}$, $s_A = s_{AN}$, $s_{Bg} = s_{AN}$

Below all possible operation modes of generators running in parallel are examined, with the existing difference of the angular velocity of shafts and changing load.

It was mentioned earlier that the operation mode of the generator with the lower angular velocity of its shaft depends on load level and the $(k-1)/|s_N|$ ratio. The possible operation modes for the second (B) generator with s_{AN} = const. can be different. In Fig. 4a case is shown when shaft angular velocities and slips of generators are equal, i.e. $s_{Bg} = s_{AN}$. In these circumstances, with any load level, the slip of the B generator is always negative ($s_B < 0$). Because, as it results from (42)

$$k - 1 = s_{Bg}k - s_{AN} \tag{45}$$

then with k > 1 and the operation of the A generator in the rated mode, the B generator runs in idle mode if

$$k - 1 = |s_{AN}| \tag{46}$$

whereas $\omega_{wB} = \omega$ (see Fig. 5).

With decreasing the load the frequency of the generated voltage rises and the B generator goes to the operation mode with positive slip ($\omega > \omega_{wB}$, $s_B > 0$). The range of negative generator A slips, for which the generator under discussed conditions of operation works in generator mode, presents on Fig. 4 section $\overline{12}$. Similarly sections on Figures 5÷7 represent the same for another discussed conditions of operating.

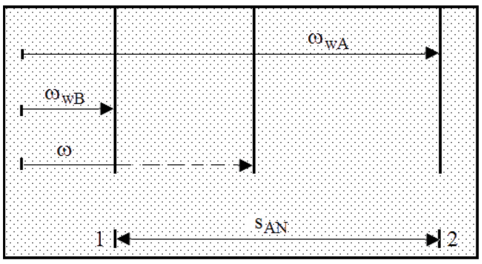

Fig. 5. Operation mode of generators for conditions: $\omega_{wA} = \text{const}$, $\omega_{wB} = \text{const}$, $\omega_{wA} \geq \omega_{wB}$, $s_A = s_{ANg}$, $s_{Bg} = 0$

In idle mode the frequency reaches the maximum value determined by the average angular velocity of generators (dotted line in Fig. 5). If now the load is increased, then with $P = P_{AN}$ we come again to the state $\omega_{wB} = \omega$, $s_{Bg} = 0$. Further increase of the load leads to the situation when $\omega_{wB} > \omega$ and the operation of the B generator with negative slip, but the slip value of the A generator exceeds the rated s_{AN} value of the slip.
The operation of both generators in parallel for the load is possible only when the (43b) condition is met, what is illustrated in Fig. 6.

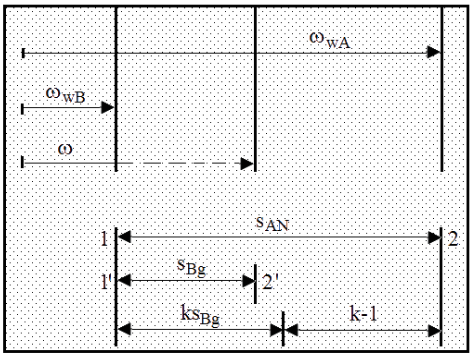

Fig. 6. The operation modes for the conditions: $\omega_{wA} = \text{const}$, $\omega_{wB} = \text{const}$, $\omega_{wA} \geq \omega_{wB}$, $s_A = s_{AN}$, $s_{Bg} < 0$

Since $\omega_{wB} > \omega$, then the s_{Bg} slip is < 0, and from (45) we have that:

$$k - 1 = |s_{AN}| - |s_{Bg}|k \tag{47}$$

The expression (47) determines the relationship between the rated slip of the A generator and the relative difference of the angular velocities of the shafts of both generators at the maximum possible (limit) power of the system of both generators, corresponding to the negative s_{Bg} slip of the B generator. The range of the s_B negative slips of the B generator corresponds with the section $\overline{1'2'}$ in Fig. 6.

In Fig. 7 it is shown that the change of the slip of the A generator from 0 to s_{AN} forces the B generator to run, and this generator, when running, consumes power from the common grid. In this state the

$$k - 1 = |s_{AN}| + |s_{Bg}|k \tag{48}$$

condition is met, which results in the fact that if

$$\frac{k - 1}{s_{AN}} > 1 \tag{49}$$

then the simultaneous operation of the generators for the common load is impossible.

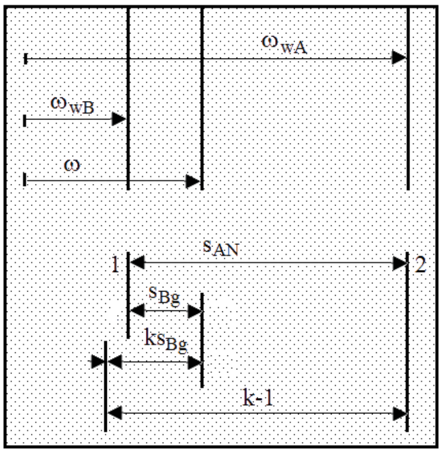

Fig. 7. Operation in parallel for the conditions: $\omega_{wA} = \text{const}$, $\omega_{wB} = \text{const}$, $\omega_{wA} \geq \omega_{wB}$, $s_A = s_{AN}$, $s_{Bg} < 0$

The energy characteristic of the system of two generators operating in parallel can be defined in some different ways [4, 5, 6], but the comparison of them is always based on the comparison of the power use indices. By transforming (38) and (39) with taking into consideration (41) and (42) we have:

$$K_N = \frac{1}{2k}\left(k + 1 + \frac{k-1}{s_{AN}}\right) \tag{50}$$

$$K = \frac{1}{2k}\left(k + 1 + \frac{k-1}{s_A}\right) \tag{51}$$

where s_A is the slip corresponding to any P_1 power lower than the rated one. From the formulas (50) and (51) it results that with the generators running in parallel the inequality $K_N > K$ is always fulfilled, since $|s_{AN}| > |s_A|$.

Conclusion:

The use of the power of induction generators running in parallel depends on the load level and angular velocity of their shafts.

On the basis of the considerations carried out it is possible do determine the value of the coefficients which characterise the maximum possible use of generators depending on operation conditions. This is presented in graphical form in Fig. 8. The conditions of the operation of generators is described by the characteristic family

$$k - 1 = f\left(\frac{s_{Bg}}{s_{AN}}\right) \tag{52}$$

at $s_{AN} = \text{const}$.

The algorithm used at the creation of the Fig. 8 is as follows:

– for the assumed s_{AN} slip value = const. various k ratio values are presumed,
– from (42) the values corresponding to them are found; it is convenient to express the s_{Bg} slip in relative units with reference to s_{AN}.
– The s_{Bg}/s_{AN} ratio is >0 if $s_{Bg}<0$, which demonstrates that to the operation of both generators in the generator mode the first quarter of Cartesian coordinate system corresponds. The s_{Bg}/s_{AN} ratio is <0 if $s_{Bg}>0$, to which the second quarter of coordinate system corresponds;
– after the calculation of the k-1 values corresponding to the k ratios we obtain the data necessary to draw the $k-1 = f(s_{Bg}/s_{AN})$ straight line at s_{AN} = const.
– the characteristic for other slip values is calculated in the same way.

In Fig. 8 the relationship $K_N = f(s_{Bg}/s_{AN})$ at s_{AN} = const. is presented to. This relationship is based on (50) for various k values. If this relationship is to be expressed in relative units then, with the use of this drawing, the limit energy indices can be determined for the two induction generators operating in parallel for various (s_{Bg}/s_{AN}) ratio values, s_{AN} slips, and relative k-1 differences in rotational speeds of the shafts of both generators.

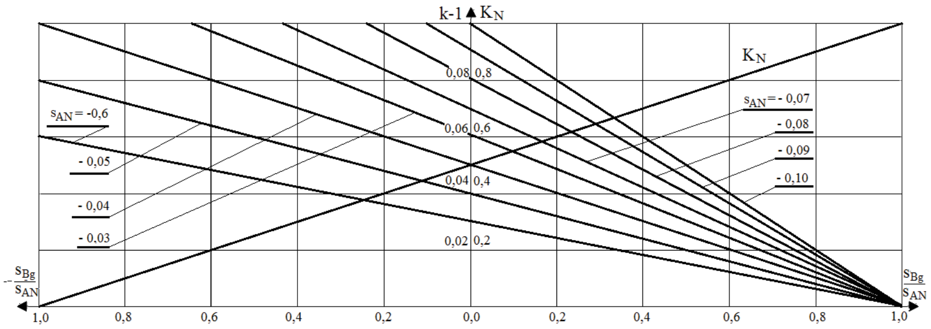

Fig. 8. The relationship between the coefficients of the use of the power of the induction generators operating in parallel and the load conditions

<u>Example:</u>

For the difference in shaft rotational speeds of both generators amounting to 3 percent and the A generator operating with the s_{AN} rated slip = –0.05 the s_{Bg}/s_{AN} ratio = 0.4, and the K_N coefficient of power use = 0.7.

The relationships mentioned above show, that in fact:

$$s_{Bg} = 0.4 s_{AN} = -0.02$$

$$P_{Bg} = \frac{0.02}{0.05} P_{AN} = 0.4 P_{AN}$$

$$K_N = \frac{P_{AN} + 0.4 P_{AN}}{2 P_{AN}} = 0.7$$

Therefore, the second quarter of the coordinate system characterizes the operation of the induction generators in the mode when one of them is loaded with the rated load and supplies its power to the common grid, whereas $\omega_{wA} > \omega > \omega_{wB}$, and the second generator consumes energy from the grid ($s_B > 0$).

With the

$$k = \frac{1 - s_{AN}}{1 + s_{AN}} \tag{53}$$

condition met the power supplied to the load amounts to zero.

Conclusion:
With s_{AN} = const *the use of the power of the induction generators operating in parallel is the higher the lower is the difference in the rotational speed of their shafts. The highest indices of the use of the power generated by generators are achieved when* s_B / s_A *ratios are positive, i.e. when both machines operate in the generator mode.*

With the assumption that k = const the more effective use of the generators operating in parallel can be achieved when machines of increased slip are used. As the s_{AN} slip increases also the s_{Bg}/s_{AN} ratio increases, and in this ratio the increments of the numerator and denominator are the same. In this case, the improvement on the use of the power of generators is achieved by increasing the $|s_{Bg}|$ slip of the B generator with the A generator loaded with the rated load (Fig. 9).

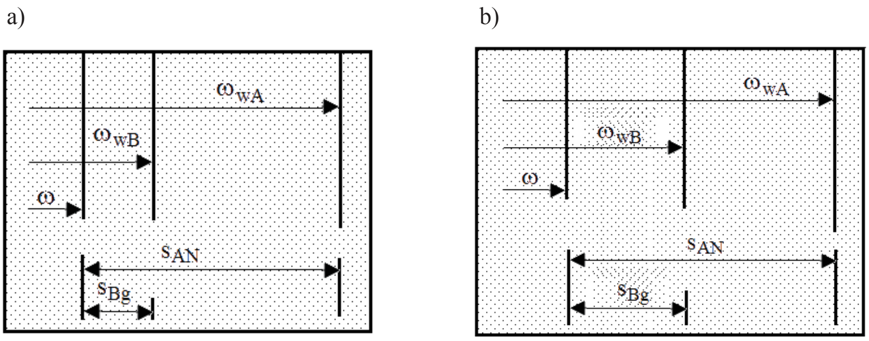

Fig. 9. The impact of the rated slip on the use of the power of the induction generators operating in parallel with different rotational speed of their shafts for slip ratios: a) $s_{Bg}/s_{AN} \approx 0.2$, b) $s_{Bg}/s_{AN} \approx 0.3$

When evaluating the energy indices of the induction generators running in parallel the following relationship can be useful:

$$\left(\frac{s_{Bg}}{s_{AN}} \right) = f(s_{AN}) \text{ at } k = \text{const}$$

which is analogous to that presented in Fig. 10.

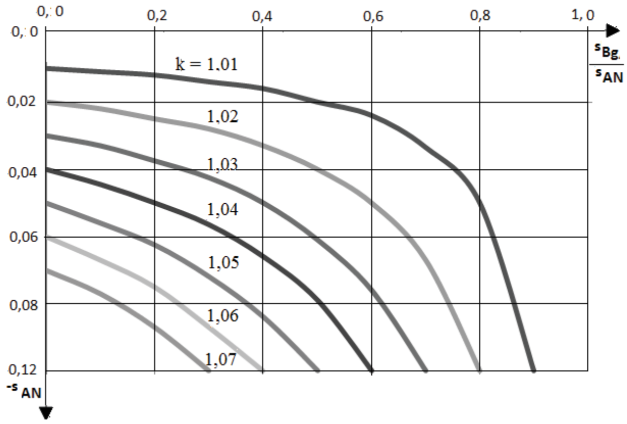

Fig. 10. $K_{Nf} = f(s_{Bg}/s_{AN})$ relationship at s_{AN} = const

Example:

The Fig 7 and 9 show, that for k = 1.02 at $s_{AN} = -0.05$ the coefficient of the use of the power of machines K = 0.8. The B generator power is calculated from

$$P_{Bg} = \frac{s_{Bg}}{s_{AN}} P_{AN} = 0,6P_{AN} \tag{54}$$

If the A generator with the rated s_{AN} slip = –0.1 is applied, then the K_N coefficient of the use of the power of machines = 0.9. Thus, increasing the rated slip of the A generator twice makes the use of the power of the machines increased by circa 10 percent.

The increase of the use the power of the induction generators running in parallel with a given slip is also possible by decreasing the difference of the rotational speed of the shafts of the machines. Decreasing it, e.g. so that at $s_{AN} = -0.05$ the k coefficient drops from k = 1.02 to k = 1.01 also makes the coefficient of the use of the power of machines higher and reaching K = 0.9.

The presented here considerations on the use of the power of machines operating in parallel concerned the most reasonable case when $P_A = P_{AN}$. In the case when $P_A < P_{AN}$, then the coefficient of the use of the power of machines should be determined according to (36) and (51).

CONCLUSIONS

1. Because induction generators with capacitance excitation can run parallel with different rotating speed of shafts, is necessary to define conditions of work ensuring operation of both generators in the range of negative operation slips, keeping as possible, big coefficient of productivity machine power.
2. With different rotational speed of the shafts of generators the load spreads on individual generators unevenly, even when both generators are identical. The generator with higher rotational speed of its shaft supplies higher active power, and consumes higher wattless power. The regulation function of the system is played here by driving motors.
3. The use of the power of the generators running in parallel depends on the difference in the rotational speed of shafts and rated slips. Decreasing the Δn_w difference in the

rotational speed of shafts or increasing the rated slip with unchanged Δn_w value makes the coefficient of the use of the power of machines higher.

4. The best use of the power of the machines running in parallel with different rotational speed of their shafts can be achieved when one of the generators runs with the rated slip, and the second one – with negative limit slip corresponding to the first one. The limit slip of the second slip is negative when the absolute value of the rated slip of the first generator is higher than the relative difference of the rotational speed of both shafts.

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ANALIZA PRACY RÓWNOLEGŁEJ PRĄDNIC INDUKCYJNYCH O WZBUDZENIU KONDENSATOROWYM

Streszczenie

W artykule rozpatrzono pracę równoległą prądnic indukcyjnych o wzbudzeniu kondensatorowym. Omówiono krótko najprostsze sposoby włączenia prądnic do pracy równoległej. Przedstawiono model matematyczny prądnic pracujących równolegle dla stanów statycznych. Określono warunki najlepszego wykorzystania mocy prądnic pracujących równolegle.

Słowa kluczowe: prądnica indukcyjna, praca równoległa, stany statyczne